General Purification Partners and Quantum Entanglement in Field Theory

B7SM2061

M2 Tomitsuka, T.

QE1

Contents

Review

- Hawking Radiation and Black Hole Information Loss Problem

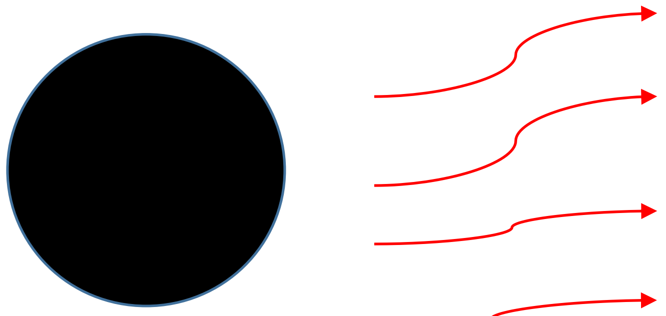
  └Pure State & Mixed State, Quantum Entangled State

- Moving Mirror Model
- Purification Partners Formula

My Research

- Purification Partners in Moving Mirror Model

Black Hole Information Paradox



$$T \propto \frac{1}{M}$$

Black Hole

(Pure State)

Hawking Radiation

(Mixed State)

- Hawking's semi-classical calculation (1975) implies Black Hole radiates and eventually evaporates
- This process violates "Unitarity" since if a pure state collapses into Black Hole, it will eventually evolve mixed state.

$\Rightarrow$ Where is information pure state has ?

Information Paradox

How to solve this problem ? ? ?

No one knows exact quantum gravity…so, there are a lot of conjectures

e.g)

- Firewall Conjecture (Polchinski)
- Black Hole Complementarity (Susskind, ‘t Hooft)
- Remnant scenario (Aharanov, Banks, Giddings)
- Soft Hair (Hawking-Perry-Strominger, Hotta-Trevison-Yamaguchi)
- Vacuum fluctuation (Wilczek, Hotta-Schützhold-Unruh)

Unitarity is preserved

- Unitarity is not preserved

I assume “unitarity is preserved”

What is needed in preserving unitarity ?

⇒Purification partners

Moving mirror model

Purification partners formula

Purification partners formula in moving mirror model

• Pure State &Mixed State

Quantum state is described by "Density Operator''

$$\hat{\rho}^\dagger = \hat{\rho}$$

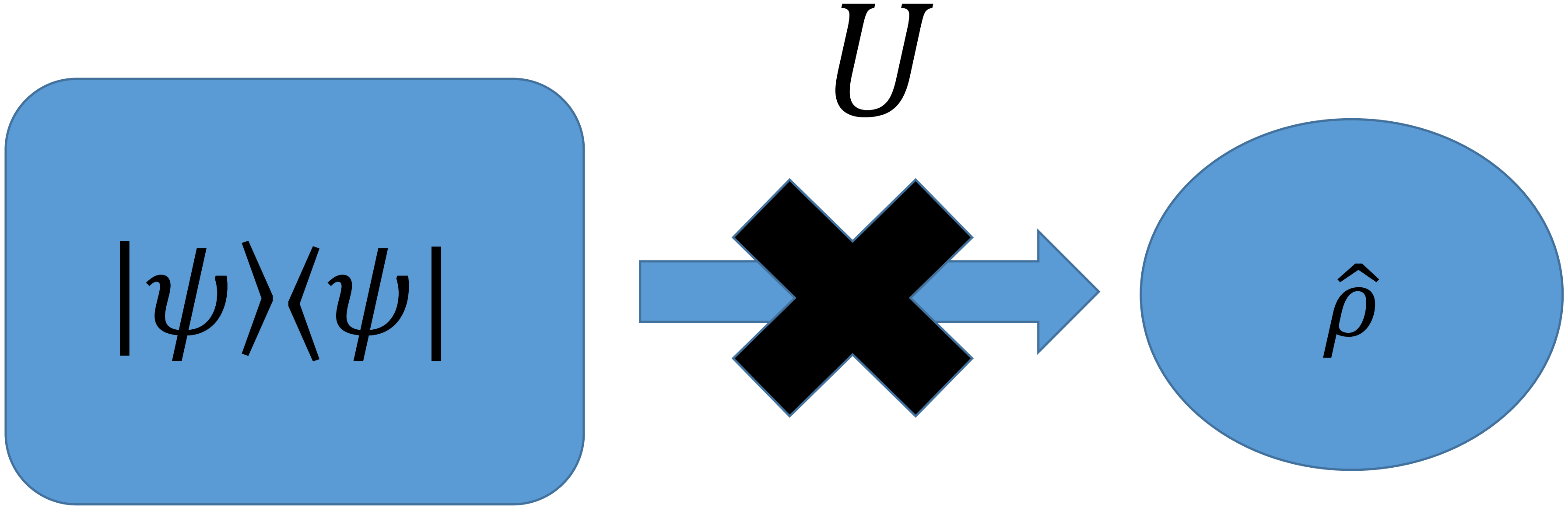
$$\hat{\rho} \geq 0$$

$$\mathrm{Tr}[\hat{\rho}] = 1$$

$$\Rightarrow \qquad \hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

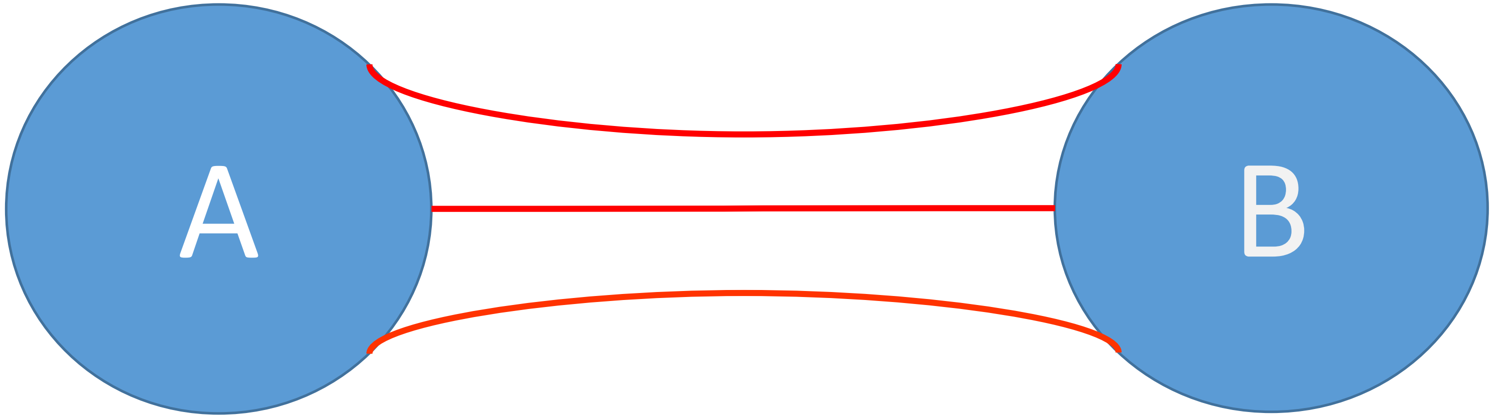
(probability p_i ; state $|\psi_i\rangle$ $\quad (\sum_i p_i = 1)$)

Pure state is special case $\hat{\rho} = |\psi\rangle\langle\psi|$ (only one state)
Others are Mixed state (some states are mixed)



Pure state cannot evolve into mixed state by unitary process

- <u>Entangled state</u>



A & B composite system is "Entangled state"

$$|\Psi\rangle_{AB} \neq |\phi\rangle_A \otimes |\psi\rangle_B \qquad |\phi\rangle_A \in \mathcal{H}_A, \quad |\psi\rangle_B \in \mathcal{H}_B$$

(example)

Bell state（2-spin system）

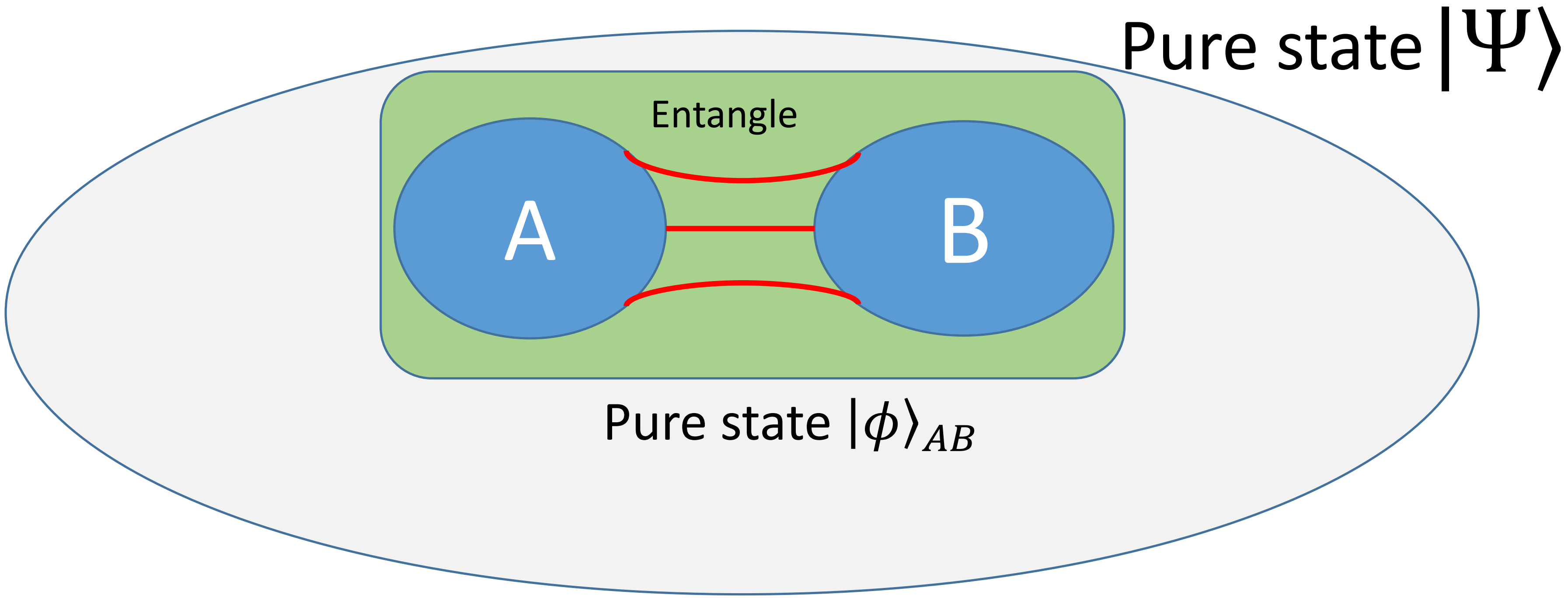
$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B)$$

subsystem A has correlation with subsystem B

- <u>Purification</u>

Subsystem A in mixed sate can be entangled by other subsystem B in mixed state such that composite system AB is pure.

$\Rightarrow$ "Purification"



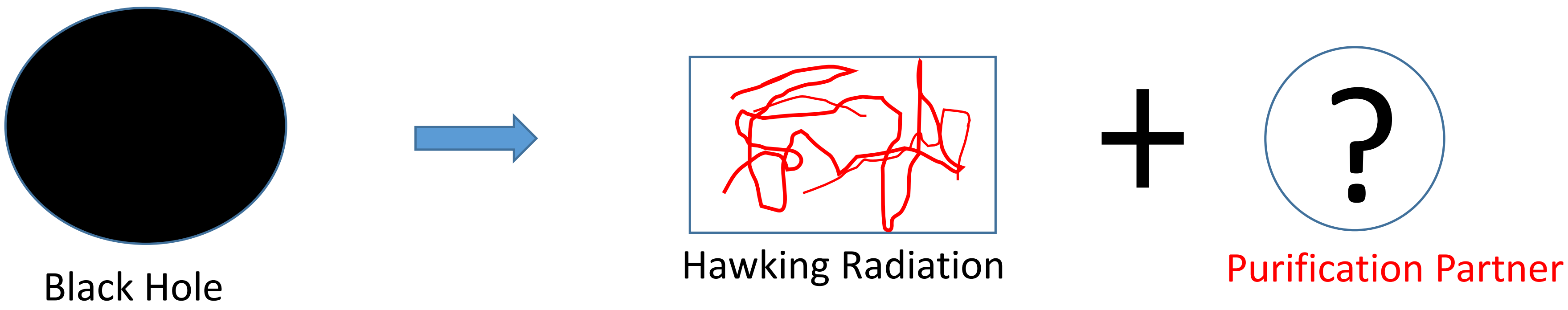
• What is purification partner of Hawking Radiation ?

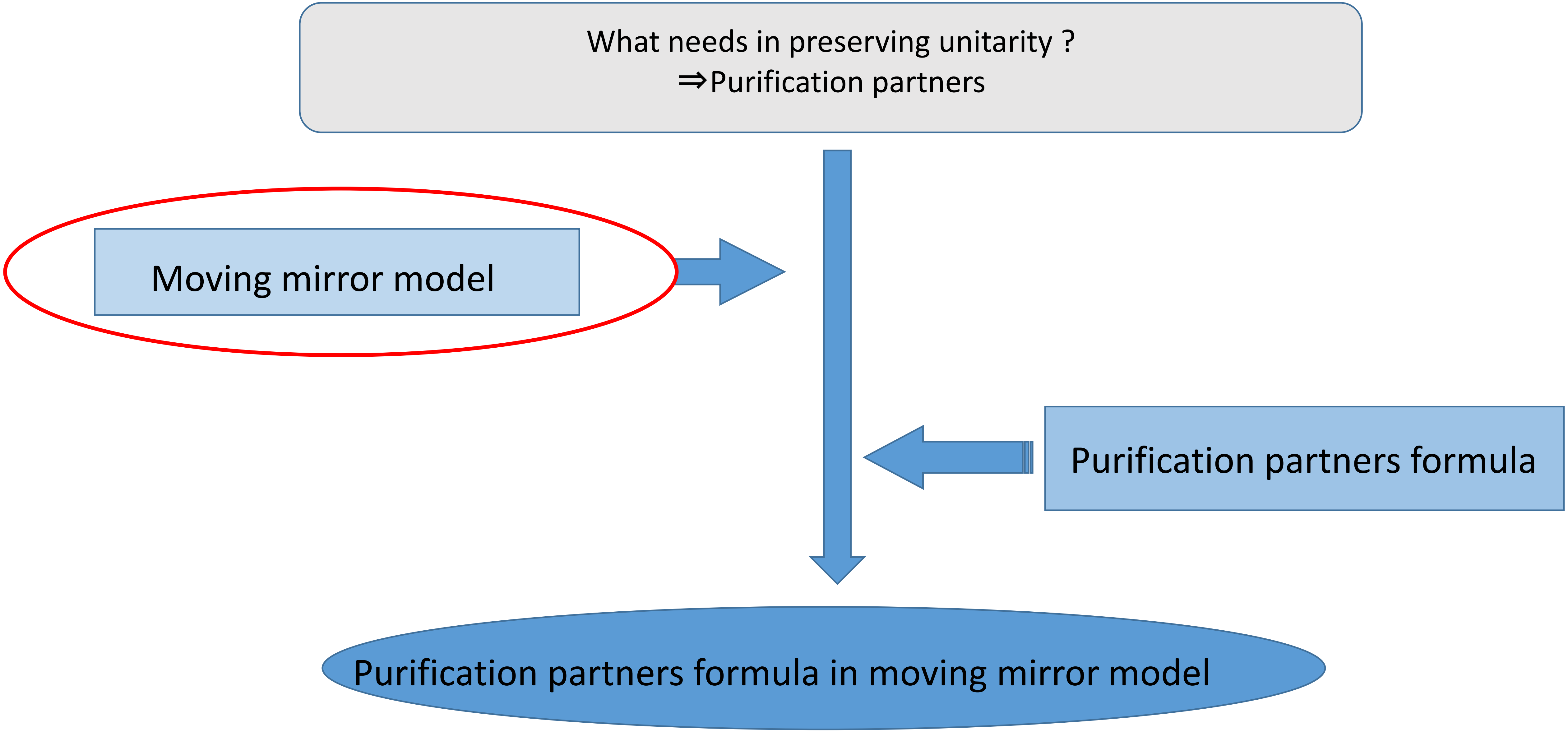
Any mixed state can be purified by appropriate auxiliary system,

that is , there is a purification partner and combined system is pure.

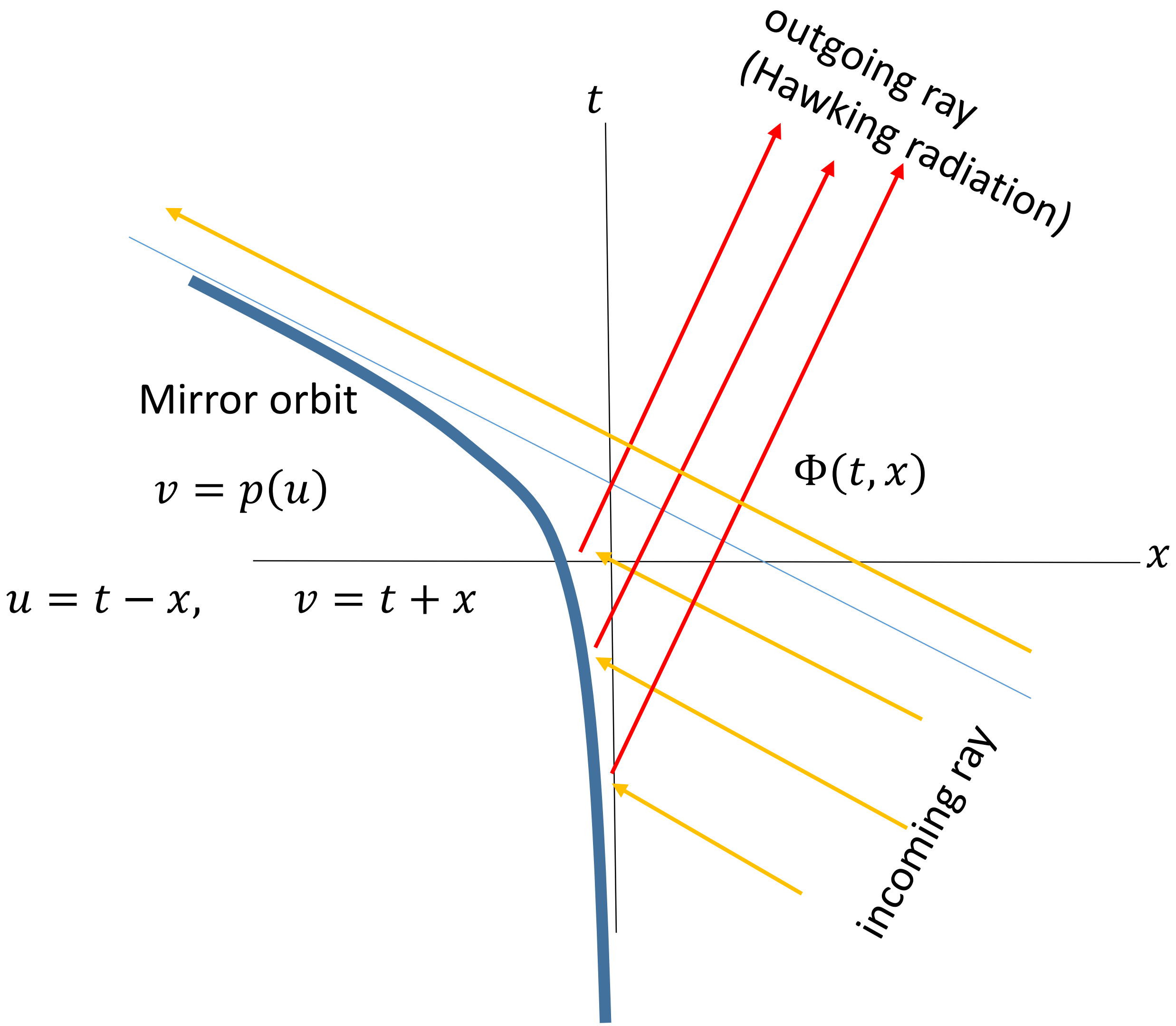
So, if we can find this purification partners, unitarity is preserved.
(pure state→pure state)

→ What purifies Hawking radiation ??




What needs in preserving unitarity ?
⇒Purification partners
Moving mirror model
Purification partners formula
Purification partners formula in moving mirror model

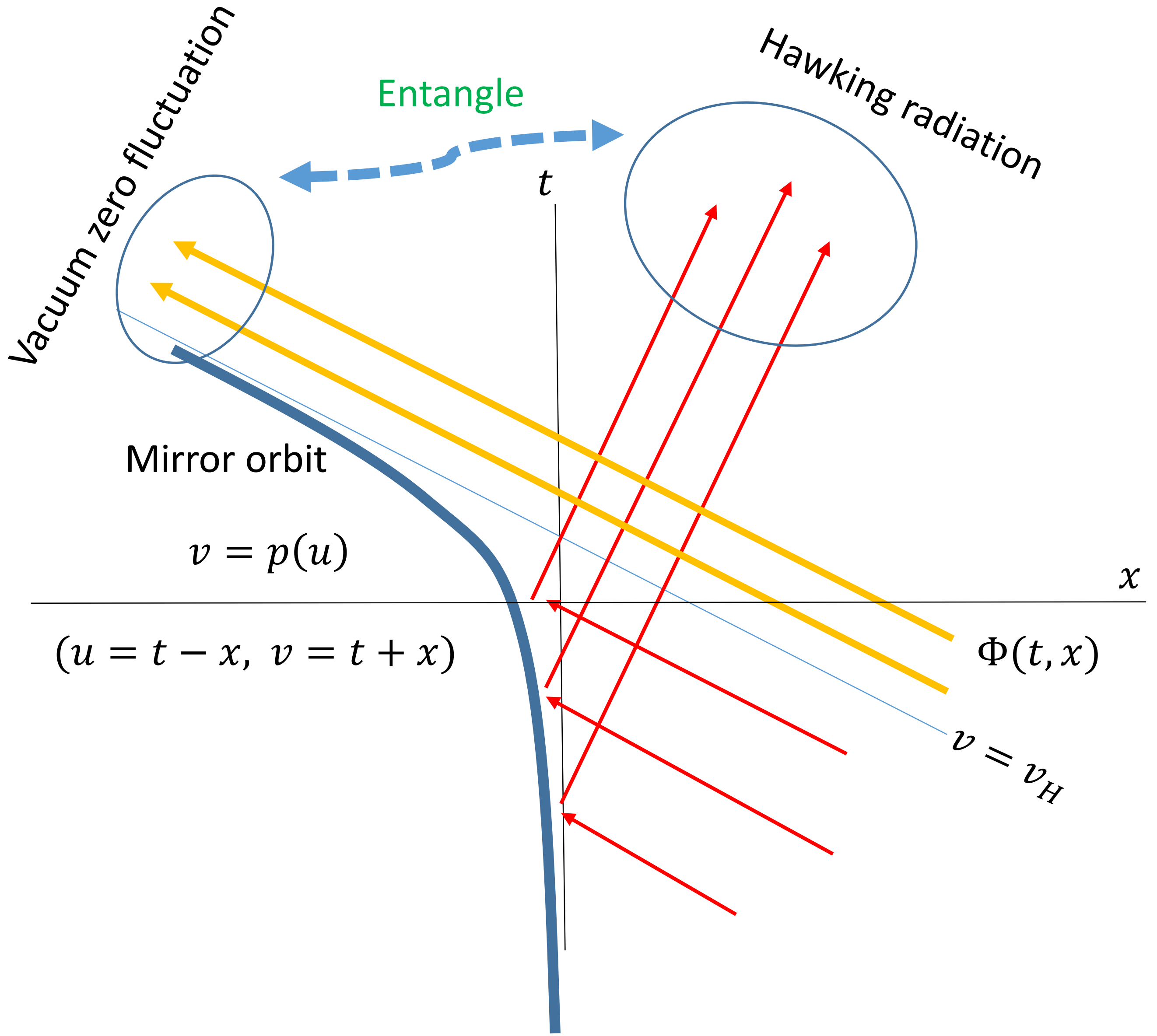
- ## Moving Mirror Model



(1+1)dim accelerated moving mirror on Minkowski spacetime which reflects field $\Phi\big(t, z(t)\big) = 0$ can mimic fields dynamics in collapsing star spacetime

Wilczek used this model and pointed out "vacuum zero point fluctuations" purify Hawking radiation.

(but this model neglects high curvature, so problem does not be perfectly solved)

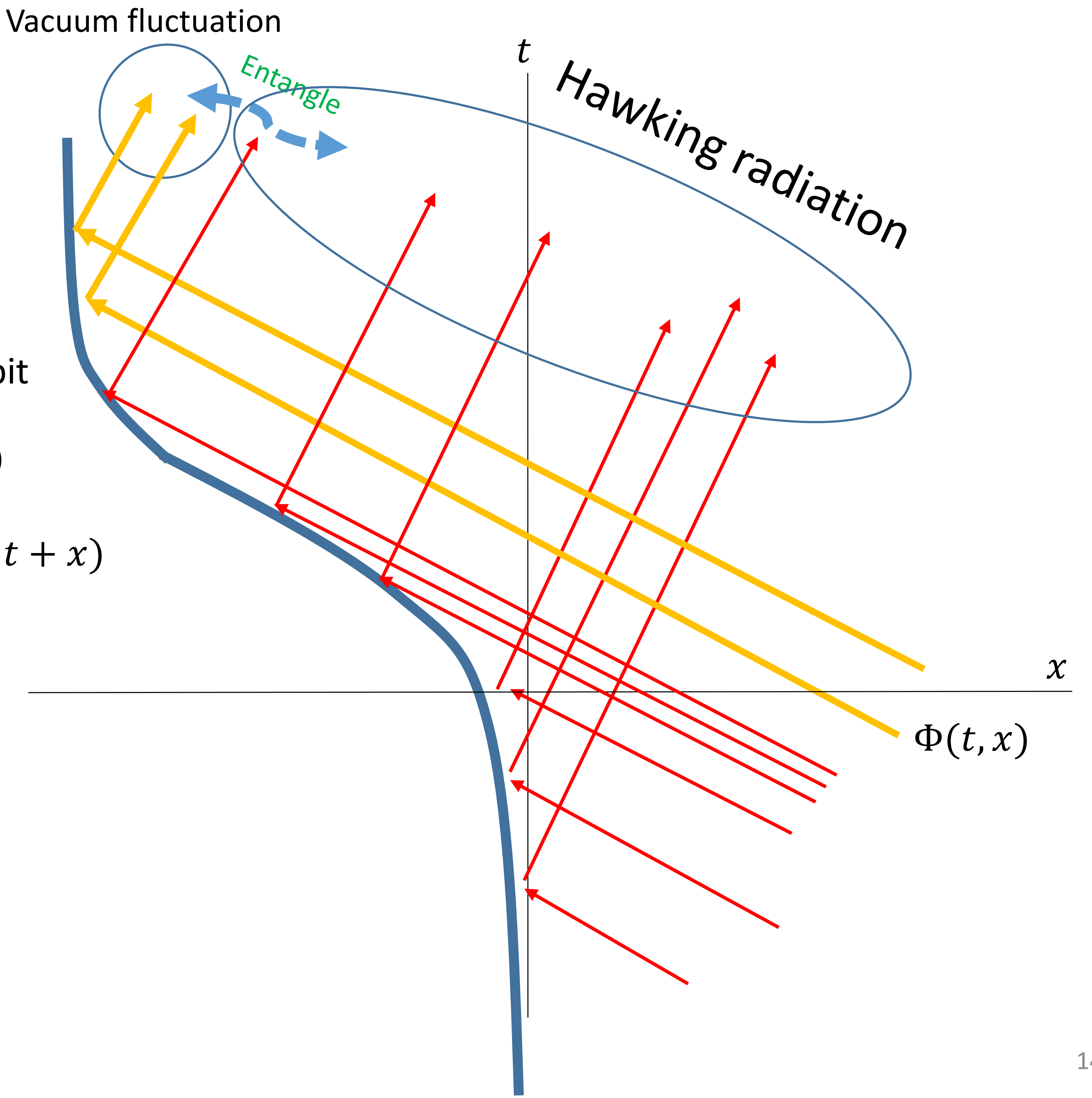

Vacuum zero fluctuation
Entangle
Hawking radiation
t
Mirror orbit
$v = p(u)$
x
$(u = t - x,\ v = t + x)$
$\Phi(t, x)$
$v = v_H$

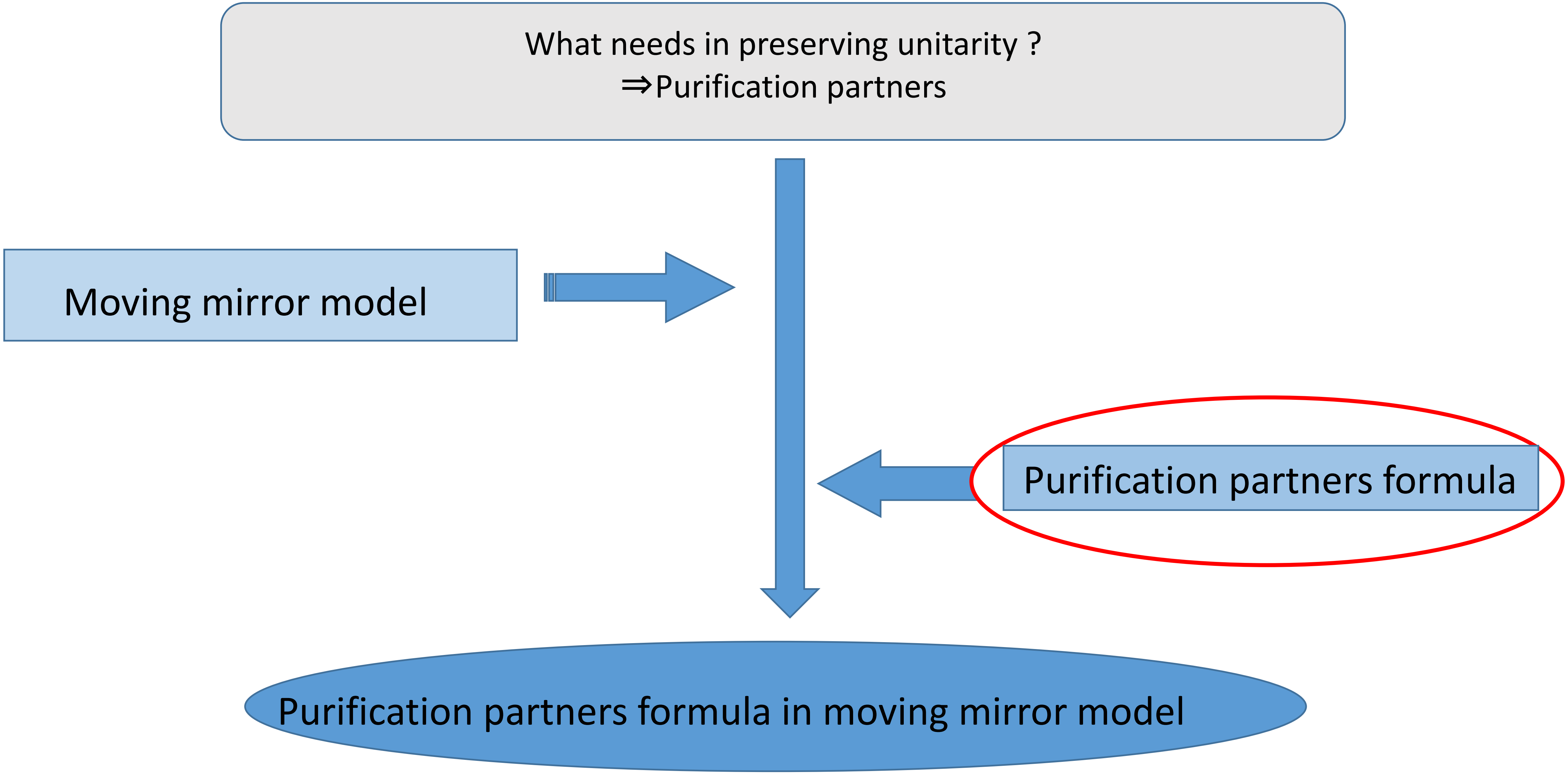
Evaporation case

Mirror orbit

$$v = p(u)$$

$$(u = t - x,\ v = t + x)$$




What needs in preserving unitarity ?
⇒Purification partners
Moving mirror model
Purification partners formula
Purification partners formula in moving mirror model

How to find purification partner ?

- So far, some people could find the purification partner under particular condition [Hotta-Schützhold-Unruh ('15)]

How to find the partner in general ??

We consider general A and want to find partner B…

Quantum system can be characterized by Characteristic Function via CCR operator

$$\chi(x_A, v_A, x_B, v_B) = \langle\Psi|\, e^{i(v_A\hat{q}_A - x_A\hat{p}_A)} e^{i(v_B\hat{q}_B - x_B\hat{p}_B)}\, |\Psi\rangle$$

$$\langle\bar{x}_A, \bar{x}_B|\,\hat{\rho}_{AB}\,|x_A, x_B\rangle$$
$$= \frac{1}{(2\pi)^2}\int dv_A dv_B \chi(x_A - \bar{x}_A, v_A, x_B - \bar{x}_B, v_B) e^{-\frac{i}{2}(v_A(\bar{x}_A + x_A) + v_B(\bar{x}_B + x_B))}$$

Consider subsystem A (defined by the following CCR operator)

$$\hat{q}_A = \int d^d\boldsymbol{x} \left(X_A(\boldsymbol{x})\hat{\varphi}(\boldsymbol{x}) + Y_A(\boldsymbol{x})\hat{\Pi}(\boldsymbol{x}) \right)$$

$$\hat{p}_A = \int d^d\boldsymbol{x} \left(Z_A(\boldsymbol{x})\hat{\varphi}(\boldsymbol{x}) + W_A(\boldsymbol{x})\hat{\Pi}(\boldsymbol{x}) \right)$$

$$[\hat{q}_A, \hat{p}_A] = i$$

$$\Downarrow$$

want to get subsystem B (defined by the following CCR operator) s.t. $\hat{\rho}_{AB}$ is pure

$$\hat{q}_B = \int d^d\boldsymbol{x} \left(X_B(\boldsymbol{x})\hat{\varphi}(\boldsymbol{x}) + Y_B(\boldsymbol{x})\hat{\Pi}(\boldsymbol{x}) \right)$$

$$\hat{p}_B = \int d^d\boldsymbol{x} \left(Z_B(\boldsymbol{x})\hat{\varphi}(\boldsymbol{x}) + W_B(\boldsymbol{x})\hat{\Pi}(\boldsymbol{x}) \right)$$

$$[\hat{q}_B, \hat{p}_B] = i$$

Conditions

1. Canonical Commutation Relation (CCR)

   $$[q_B, p_B] = i$$

2. Locality

   $$[\hat{q}_A, \hat{q}_B] = 0,\ [\hat{q}_A, \hat{p}_B] = 0,\ [\hat{p}_A, \hat{q}_B] = 0,\ \ [\hat{p}_A, \hat{p}_B] = 0$$

3. Composite System AB is pure

   $\Rightarrow \hat{\rho}_{AB}$ is pure state

Gaussian Vacuum state solution

$$X_B(\boldsymbol{x}) = \frac{\sqrt{1+g^2}}{g} X_A(\boldsymbol{x}) - \frac{2}{g}\int d^d\,\boldsymbol{x}'\Delta_p(\boldsymbol{x}-\boldsymbol{x}')W_A(\boldsymbol{x}')$$

$$Y_B(\boldsymbol{x}) = \frac{\sqrt{1+g^2}}{g} Y_A(\boldsymbol{x}) + \frac{2}{g}\int d^d\,\boldsymbol{x}'\Delta_q(\boldsymbol{x}-\boldsymbol{x}')Z_A(\boldsymbol{x}')$$

$$Z_B(\boldsymbol{x}) = -\frac{\sqrt{1+g^2}}{g} Z_A(\boldsymbol{x}) - \frac{2}{g}\int d^d\,\boldsymbol{x}'\Delta_p(\boldsymbol{x}-\boldsymbol{x}')Y_A(\boldsymbol{x}')$$

$$W_B(\boldsymbol{x}) = -\frac{\sqrt{1+g^2}}{g} W_A(\boldsymbol{x}) + \frac{2}{g}\int d^d\,\boldsymbol{x}'\Delta_q(\boldsymbol{x}-\boldsymbol{x}')X_A(\boldsymbol{x}')$$

$$where \ <\hat{\phi}(\boldsymbol{x})\hat{\phi}(\boldsymbol{x}')>= \Delta_q\,(\boldsymbol{x}-\boldsymbol{x}') = \int \frac{d^d\boldsymbol{k}}{(2\pi)^d}\frac{1}{2E_{\boldsymbol{k}}}e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}, <\hat{\Pi}(\boldsymbol{x})\hat{\Pi}(\boldsymbol{x}')>= \Delta_p(\boldsymbol{x}-\boldsymbol{x}') = \int \frac{d^d\boldsymbol{k}}{(2\pi)^d}\frac{E_{\boldsymbol{k}}}{2}e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}$$

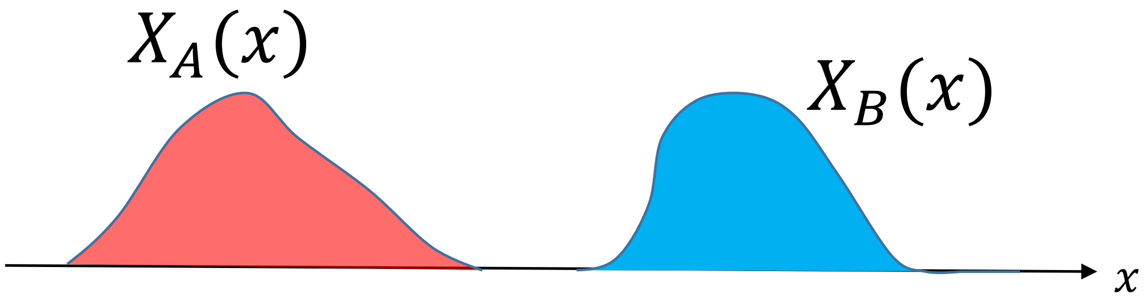
$\Rightarrow$ (proportion to A's weighting function term)＋(2-point function integral term)

Similar result in the cases of general Gaussian state in curved spacetime

• Nature of Purification Partners: SSP & SOP

B's weighting functions have no overlapped support with A's

⇒ Spatially Separated Partner

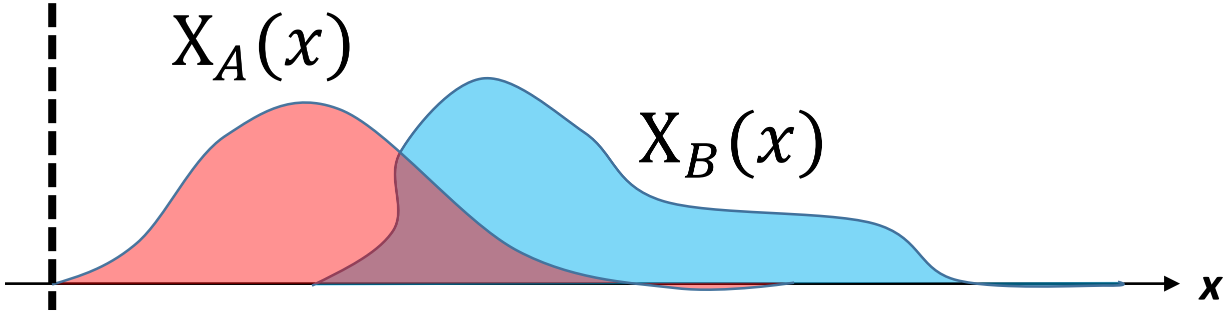


SSP

Reznik (`03)
Hotta-Schützhold-Unruh (`15)

B's weighting functions have overlapped support with A's

⇒ Spatially Overlapped Partner



SOP

This is new class

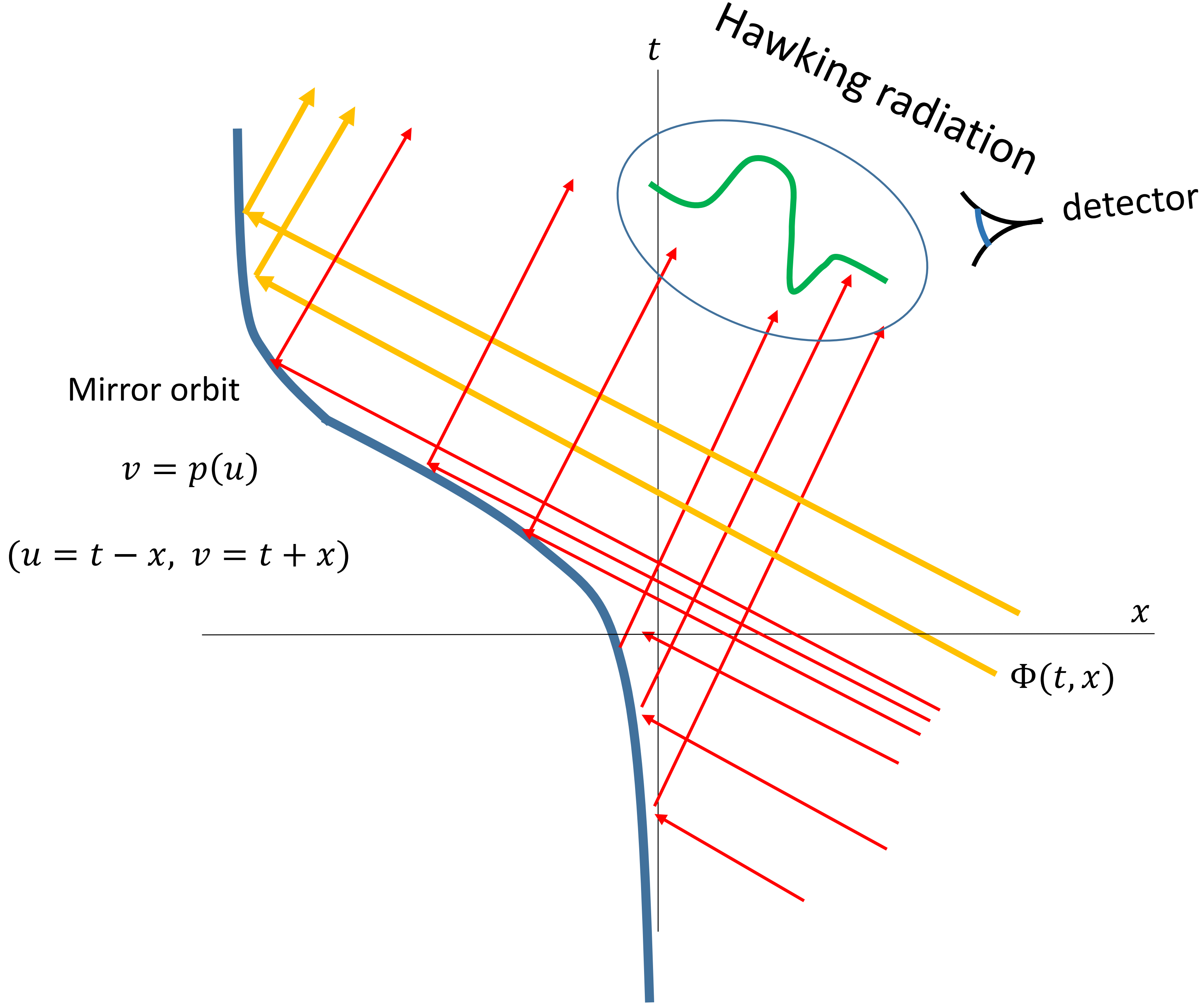
Trevison-Yamaguchi-Hotta (`18)

What needs in preserving unitarity ?
⇒Purification partners

Moving mirror model

Purification partners formula

Purification partners formula in moving mirror model



Hawking radiation
detector
t
x
Mirror orbit
$v = p(u)$
$(u = t - x,\ v = t + x)$
$\Phi(t, x)$

Purification partners in moving mirror model

$$Q_B^{out}(u) = \frac{\sqrt{1+g^2}}{g} Q_A^{out}(u) + \frac{1}{g}\int_{-\infty}^{\infty} \Delta(p(u)-p(u'))P_A^{out}(u')\partial_{u'}p(u')du'$$

$$P_B^{out}(u) = -\frac{\sqrt{1+g^2}}{g} P_A^{out}(u) + \frac{1}{g}\int_{-\infty}^{\infty} \Delta(p(u)-p(u'))Q_A^{out}(u')\partial_{u'}p(u')du'$$

$$\Delta\big(p(u)-p(u')\big) \coloneqq \frac{1}{\pi}\mathrm{Pv}\frac{1}{p(u)-p(u')}$$

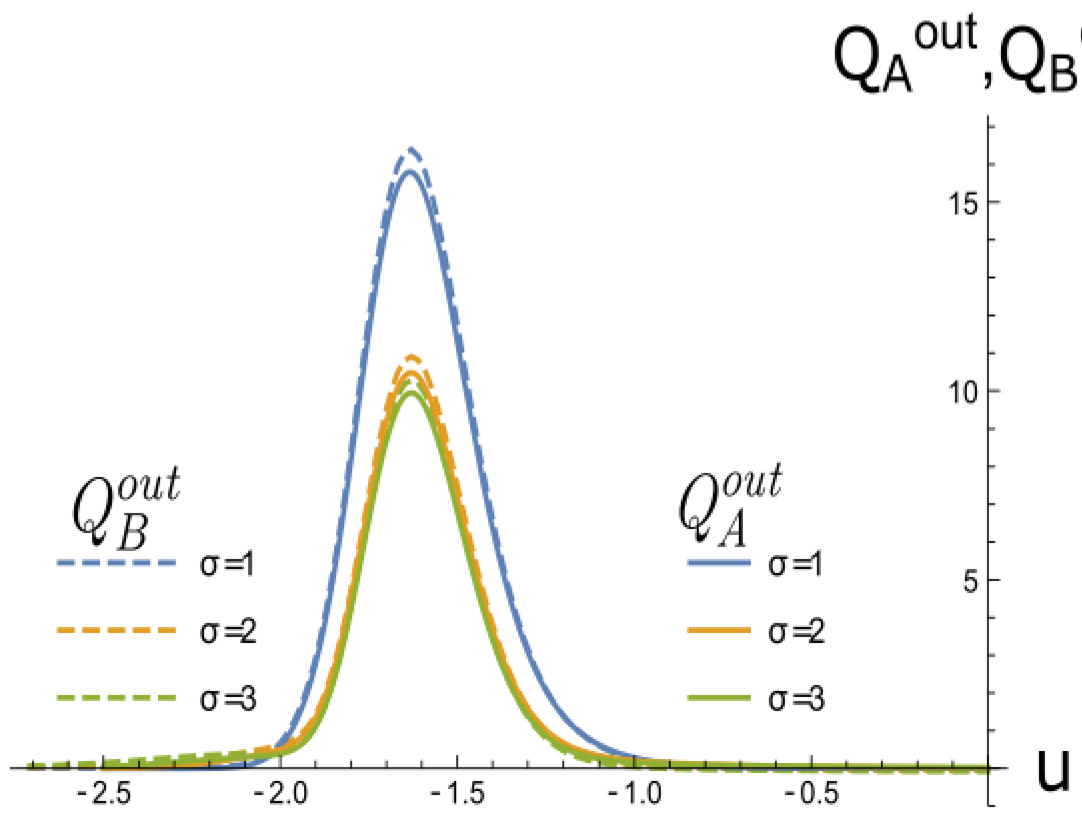
$p(u)$:mirror orbit

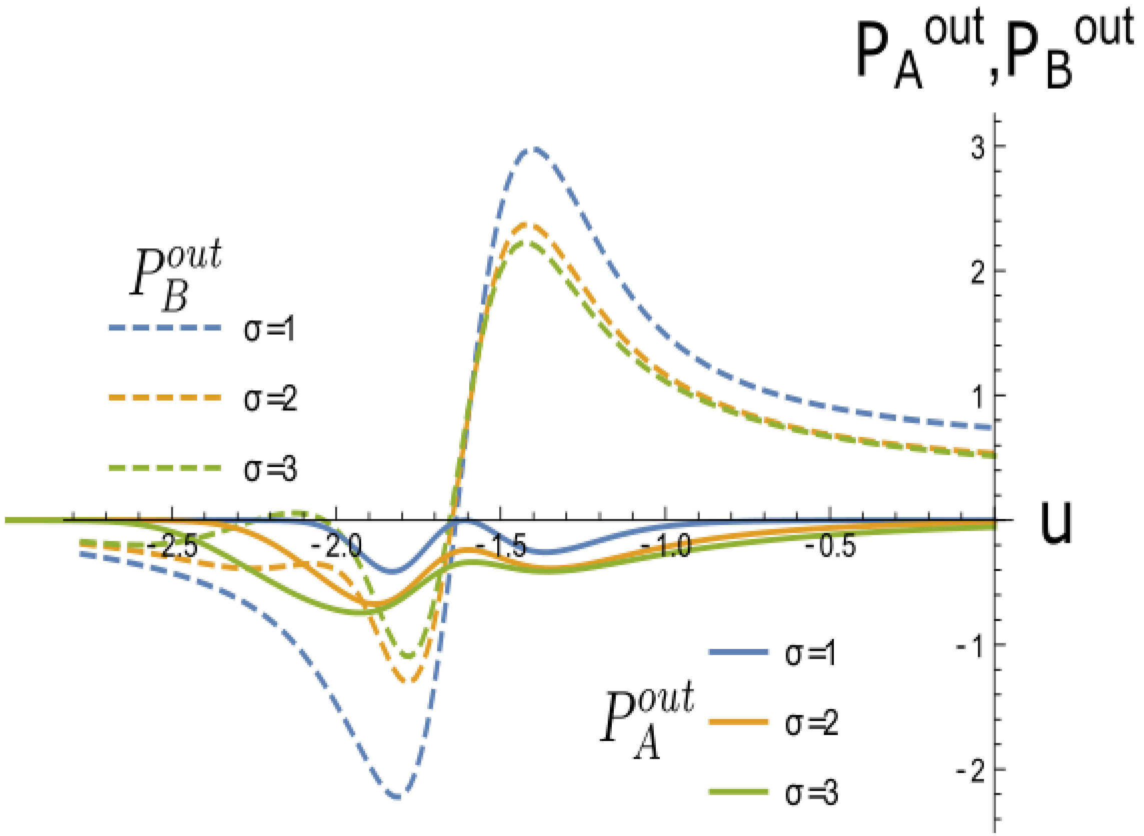
(proportion to A's weighting function term)

+

(mirror orbit integral term)

[Hotta-Tomitsuka-Yamaguchi, arXiv: 19xx.xxxxx]


Q_A^out, Q_B^out
15
10
5
u
-2.5
-2.0
-1.5
-1.0
-0.5
Q_B^out
σ=1
σ=2
σ=3
Q_A^out
σ=1
σ=2
σ=3


P_A^out, P_B^out
3
2
1
-1
-2
u
-2.5
-2.0
-1.5
-1.0
-0.5
P_B^out
σ=1
σ=2
σ=3
P_A^out
σ=1
σ=2
σ=3

Summary

- We need purification partners of Hawking Radiation in blackhole information loss problem, but this has been done in particular case.

- I can get purification partners formula in moving mirror model.

  ⇒ this can be applied to general case.
- SOP class also emerges typically in moving mirror model

Next steps

- What is relation between mirror orbit (=blackhole nature) and purification partners ?

  For example, examine entanglement entropy between Hawking radiation and partners w.r.t orbits