Quantum surface friction model for fusion reactions around the Coulomb barrier

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To achieve a consistent description of fusion reactions around the Coulomb barrier, we develop a quantum surface friction model. The method combines the concept of friction with quantum mechanics based on the equation of motion. We apply the method to the ^{16}O + ^{208}Pb system. Our results are in good agreement with the experimental data in a wide energy range, suggesting an importance of quantum effects on fusion reactions at above barrier energies.

KEYWORDS: heavy-ion fusion reactions, coupled-channels method, quantum friction

1. Introduction

Heavy-ion fusion reactions are interesting phenomena, in which both quantum tunneling and macroscopic effects such as friction play a prominent role. On one hand, fusion reactions at energies below the Coulomb barrier take place only through quantum tunneling, to which the coupled-channels method with a few low-lying collective states has successfully been applied [1]. On the other hand, guided by an observation of a large amount of energy loss in deep inelastic collisions, the classical trajectory calculation including a frictional force has been developed, and has succeeded in describing above barrier fusion reactions [2].

Although each method can describe well fusion reactions in each energy range, one method cannot explain fusion reactions in the other energy range. In order to achieve a unified description, a combination of quantum mechanics and friction is a key. A careful discussion regarding a unification is important not only to enlarge our understanding of nuclear fusion, but also from the viewpoint of quantum mechanics in general. For instance, if the classical treatment is really justified at above barrier energies, it implies the existence of the quantum-classical transition between subbarrier and above barrier fusion reactions. Such transition has continuously been discussed in other fields of science, including condensed matter physics, chemistry, and quantum biology.

In this contribution, we report our recent work in this aspect. To combine quantum mechanics with friction, our approach is based on the equation of motion. Extending an existing quantum friction model to a multi-channel system, we investigate fusion reactions for the ^{16}O + ^{208}Pb system.

2. Quantum friction model of Kostin

To incorporate friction into quantum mechanics, we follow a similar approach to classical mechanics. Suppose a particle of mass m moving in a one dimensional space x under influence of potential $V(x)$. In the presence of friction, one usually adds a frictional force to the equation of motion,

$$\frac{dp}{dt} = -\gamma(x)p - \frac{dV}{dx}, \tag{1}$$

with the momentum p and a friction coefficient $\gamma(x)$.

To extend the above approach to quantum mechanics, one needs to find the equation of motion in quantum mechanics. Notice here that the Schrödinger equation can be transformed into a form of the hydrodynamical equation. By adding a frictional force to it, one obtains,

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) + \int^x dy\,\gamma(y)\frac{\partial}{\partial y}S(y,t)\right]\psi(x,t), \tag{2}$$

where $S(x,t)$ is phase of the wave function, $\psi(x,t) = |\psi(x,t)|\exp(iS(x,t)/\hbar)$. A similar equation was first introduced by Kostin in Ref. [3] with a classical fluctuation force. A derivation based on the hydrodynamical equation can be found in Ref. [4].

For an application to fusion reactions, one needs to extend Eq. (2) to a three dimensional space $\vec{x} = \vec{x}(r,\theta,\phi)$. Assuming a spherically symmetric potential, let us first expand the wave function with the Laguerre polynomials $P_l(x)$ as $\psi(\vec{x},t) = \sum_{l=0}^{\infty} u_l(r,t)P_l(\cos(\theta))/r$. One can modify the Schrödinger equation for $u_l(r,t)$ in the same way as Eq. (2) to incorporate a frictional force,

$$i\hbar\frac{\partial}{\partial t}u_l(r,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2} + V(r) + \int^r d\rho\,\gamma(\rho)\frac{\partial}{\partial\rho}S_l(\rho,t)\right]u_l(r,t), \tag{3}$$

where $S_l(r,t)$ is phase of the wave function, $u_l(r,t) = |u_l(r,t)|\exp(iS_l(r,t)/\hbar)$. In this treatment, for simplicity, we take only friction for the radial motion into consideration, and neglect angular momentum dissipation and a fluctuation force. Now, an extension to the coupled-channels formalism is obvious.

3. Application to the ^{16}O + ^{208}Pb system

For a practical application to nuclear fusion reactions, we consider the ^{16}O + ^{208}Pb system for which the experimental data exist in a wide range of energy [5]. We simulate fusion reactions using an optical potential. The Aküze-Winther potential is employed for the real part of nuclear potential V_N [6]. The imaginary potential is in the Woods-Saxon form $W_0/(1+\exp((r-R_w)/a_w))$ with $W_0 = -30$ MeV, $R_w = 8.4$ fm, and $a_w = 0.4$ fm. The channel-coupling effect is treated in the same way as the CCFULL code, and the iso-centrifugal approximation is adopted [7]. For intrinsic states, we take into account only the first excited state of both ^{16}O and ^{208}Pb. The coupling parameters are taken from Ref. [8].

For a friction coefficient $\gamma(r)$, we employ the surface friction model,

$$\gamma(r) = \frac{\gamma_0}{m}\left(\frac{dV_N}{dr}\right)^2, \tag{4}$$

with the reduced mass m and the strength of friction γ_0 which is an adjustable parameter in this work. This is a general form of the friction coefficient obtained perturbatively [9], and has successfully been applied to above barrier fusion reactions and to deep inelastic scatterings [2].

The friction term in Eq. (3) depends on time. Thus, one needs to use the time-dependent approach to calculate the penetrability. In the time-dependent approach, there is an efficient method to calculate the penetrability known as the energy projection method [10]. However, it is not applicable in the present approach since the energy is not conserved. To deal with it, we determine the initial condition from the Gaussian distribution of the center of mass energy $E_{\text{c.m.}}$. By setting the width of the energy distribution σ_E narrow enough, it is expected that the result converges to a certain value. Actually, we have confirmed that $\sigma_E = 0.5$ MeV is sufficient for the present setup.

In solving the time-dependent Schrödinger equation of the form Eq. (3), we employ the fourth order Runge-Kutta method with the time grid $c\Delta t = 0.1$ fm, and the space grid $\Delta r = 0.1$ fm in

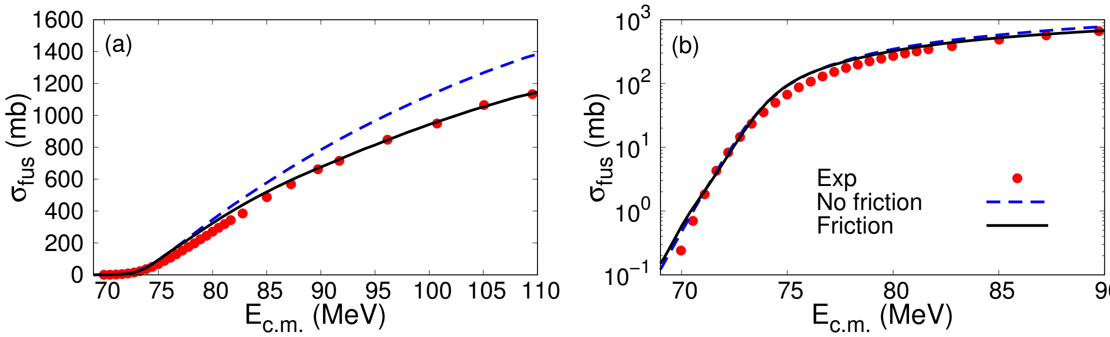

Fig. 1. Comparison among the experimental data (the circles), the no-friction result (the dashed lines), and the friction result with $\gamma_0 = 0.6 \times 10^{-23}$ s/MeV (the solid lines) for the fusion cross sections for the ^{16}O + ^{208}Pb system. The left panel is in the linear scale, while the right panel is in the logarithmic scale. The experimental data are taken from Ref. [5].

$3 < r\,(\text{fm}) < 225$. We compute the phase of the wave function following Ref. [11]. By calculating the weight of the reflected wave, one can estimate the penetrability, and thus fusion cross sections σ_{fus}.

In Fig.1, we compare the energy dependence of fusion cross sections. Note that the no-friction model is nothing but the conventional coupled-channels calculation. By fitting the experimental data, we find the strength of friction to be $\gamma_0 = 0.6 \times 10^{-23}$ s/MeV. One sees in Fig.1(a) that the above barrier fusion cross sections obtained from the friction model (the solid line) are suppressed compared to the no-friction result (the dashed line), and it is in agreement with the experimental data. Furthermore, as can be seen in Fig.1(b), the subbarrier fusion cross sections in the friction model are almost the same as those in the no-friction model, hence the predictive power is comparable. Considering overall energies, the present friction model provides a more consistent description of fusion reactions around the Coulomb barrier than the conventional coupled-channels approach.

Notice that the improvement at above barrier energies is not surprising, because the friction model includes one more parameter than the no friction model which inherently suppresses the penetrability. We had naively anticipated that the subbarrier fusion cross sections are suppressed in the same way in the presence of friction and, as a result, the friction model underestimates the experimental data. Contrary to our expectation, we have seen no significant change at subbarrier energies, and this is a key to achieve a consistent description. We believe that this behavior is nontrivial. We mention that the energy loss is larger for the friction model than the no friction model and the friction is apparently active.

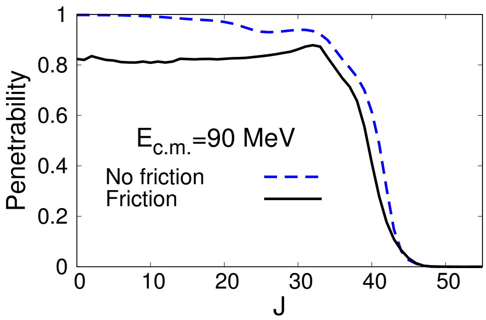

Fig. 2. Dependence of the penetrability on the total angular momentum at $E_{\text{c.m.}} = 90$ MeV. The dashed line is the no friction result, while the solid line is the result with friction.

To gain a deeper insight into this behavior, Fig.2 shows a dependence of the penetrability on the total angular momentum J at $E_{\mathrm{c.m.}} = 90$ MeV. For later discussions, let us first discuss the reason why friction can suppress fusion cross sections in the classical sense. Since friction causes the energy loss, one needs higher energy to overcome the Coulomb plus the centrifugal barrier at a critical point in the presence of friction. Thus, the critical angular momentum is lowered, leading to suppression of fusion cross sections. This is actually observed in Fig.2 at around $J = 40$.

However, one finds in Fig.2 that this is not a major origin of suppression. Rather, damping of the penetrability at small J has a larger impact on fusion cross sections. The penetrability is almost saturated at around 0.8. Since there is no thermal fluctuation in this work, this means that the exit channel is in a quantum superposition state of absorption (fusion) and reflection (scattering) even at small impact parameters at above barrier energies. Such division of probability, which is a purely quantum effect, mainly causes suppression of above barrier fusion cross sections in our calculation. This conclusion implies that quantum effects play an important role, not only in subbarrier fusion reactions, but also in above barrier fusion reactions.

4. Concluding remarks

In summary, we have applied a quantum surface friction model to fusion reactions around the Coulomb barrier, in order to achieve a unified description from subbarrier energies to above barrier energies. We have shown that the experimental data for the ^{16}O + ^{208}Pb fusion reaction can be well reproduced in a wider energy range than the conventional approach. To be more specific, we have found that fusion cross sections are suppressed in the presence of friction at above barrier energies, while not much affected at subbarrier energies. We have discussed that this is attributed to quantum effects at above barrier energies, which has not been paid attention so far.

One might argue that such nontrivial behavior is due to a special form of the Hamiltonian Eq. (3), rather than quantum effects of friction. To address this issue, one can employ the so called system-bath approach, in which a huge number of degrees of freedom is explicitly taken into account in a simple form. In that approach, a fluctuation force is automatically incorporated into the formalism, while the present approach neglects it. A work towards this direction is now in progress [12].

Acknowledgments

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