The Radius of Neutron Stars

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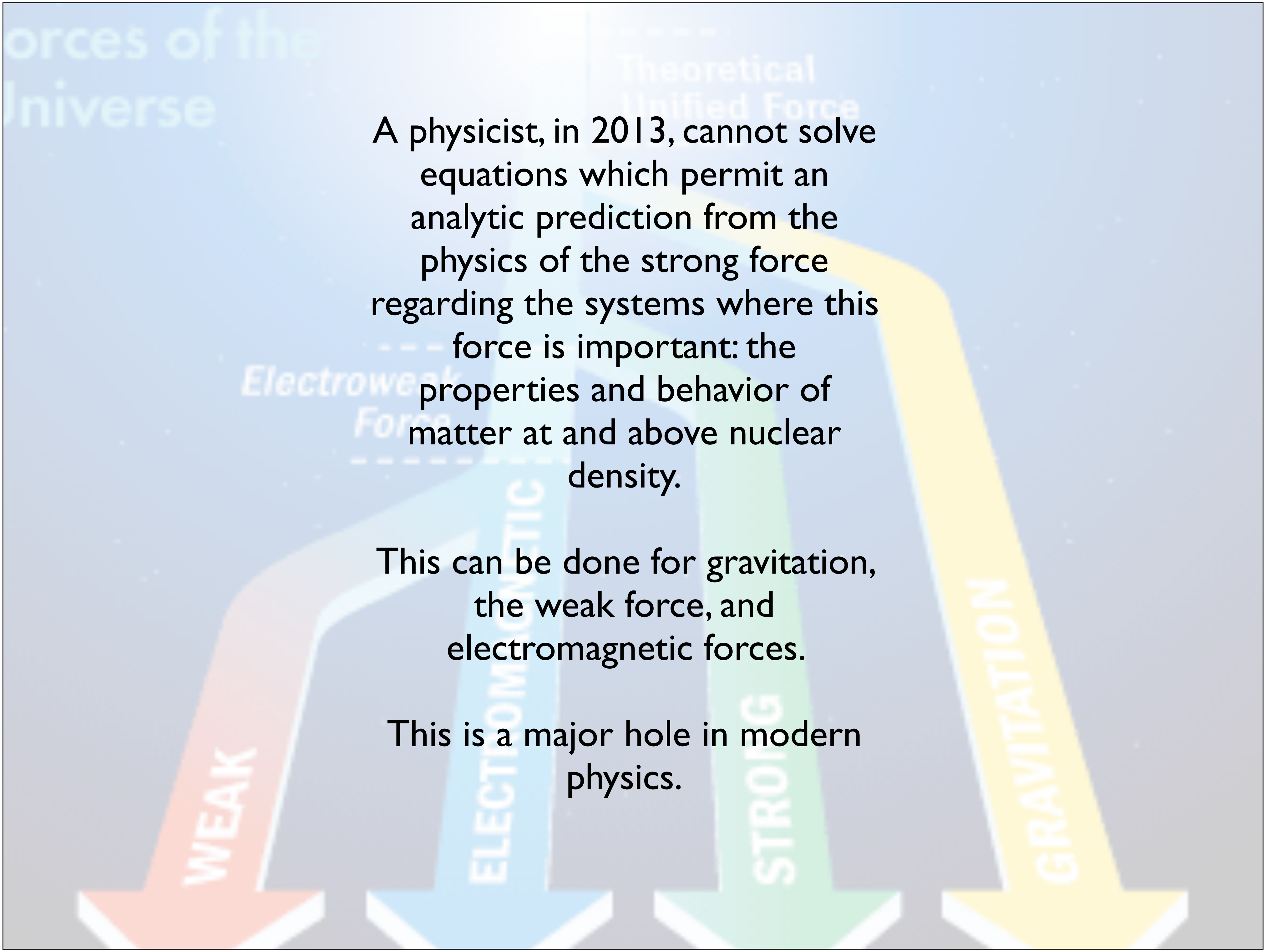


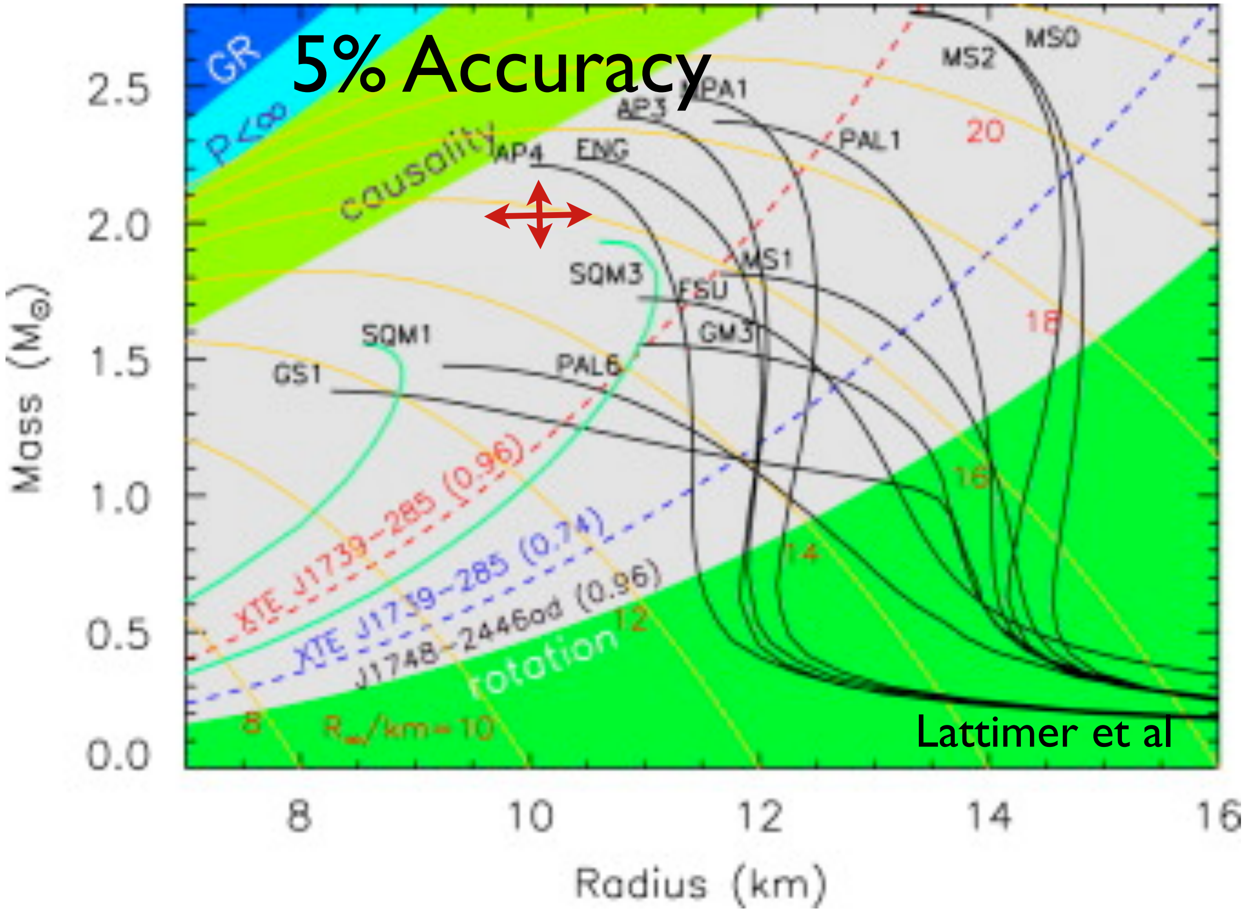
A physicist, in 2013, cannot solve equations which permit an analytic prediction from the physics of the strong force regarding the systems where this force is important: the properties and behavior of matter at and above nuclear density.

This can be done for gravitation, the weak force, and electromagnetic forces.

This is a major hole in modern physics.

The Dense Matter Equation of State is an important Strong Force Regime

- Each different proposed dEOS produces a different mass-radius relationship for neutron stars.
- Thus, measure the mass-radius relationship of neutron stars, and you have a measurement of the dEOS.
- Precision requirement -- 5% in mass and radius, separately.
- A larger uncertainty is useless to nuclear physics.



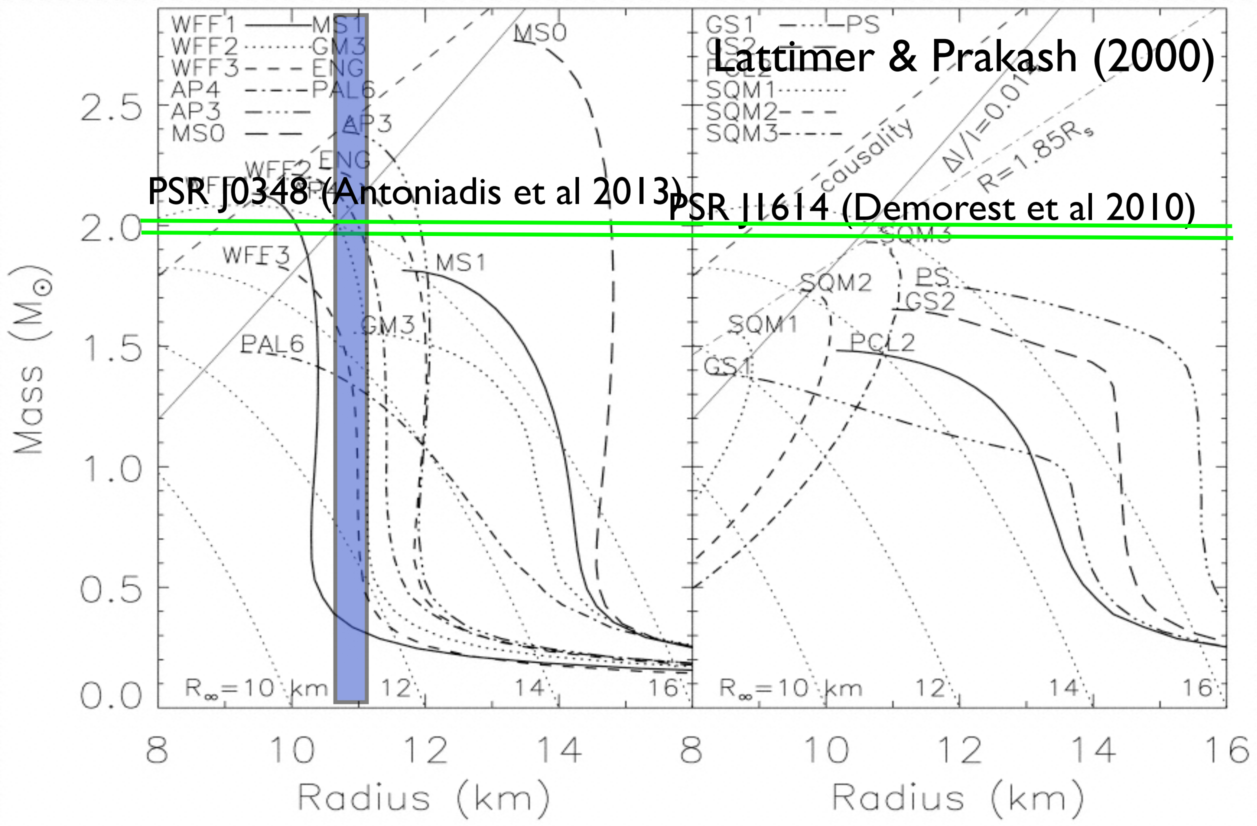
5% Accuracy

Lattimer et al

Mass-Radius Relation from the Equation of State

Measuring the Mass and Radius simultaneously is difficult.

High-mass measurements prefer EOSs which produce a nearly constant radius at astrophysically interesting masses.

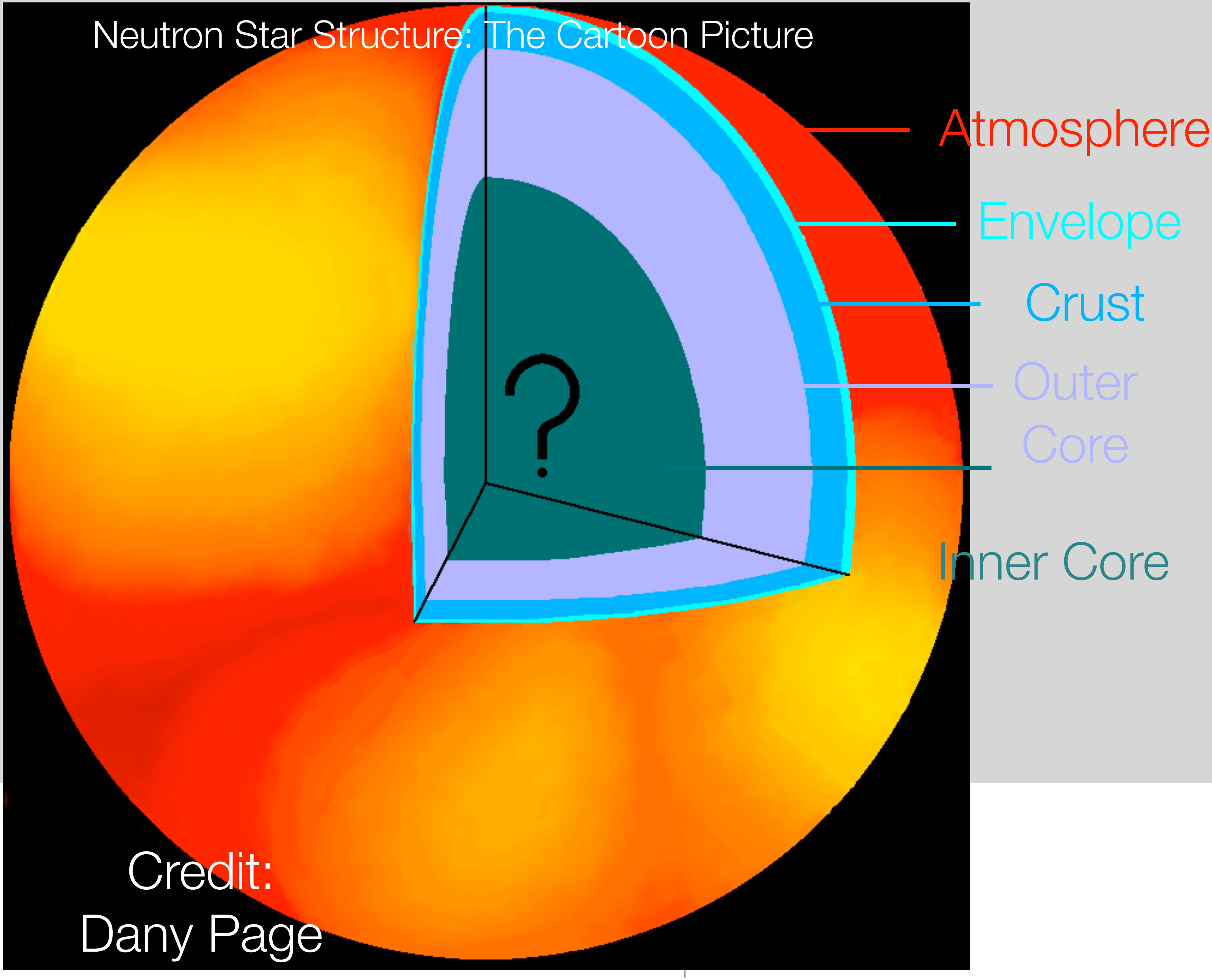

Lattimer & Prakash (2000)

PSR J0348 (Antoniadis et al 2013)

PSR J1614 (Demorest et al 2010)

Precision Radius Measurements (<5%) may be they key to measuring the dEOS.

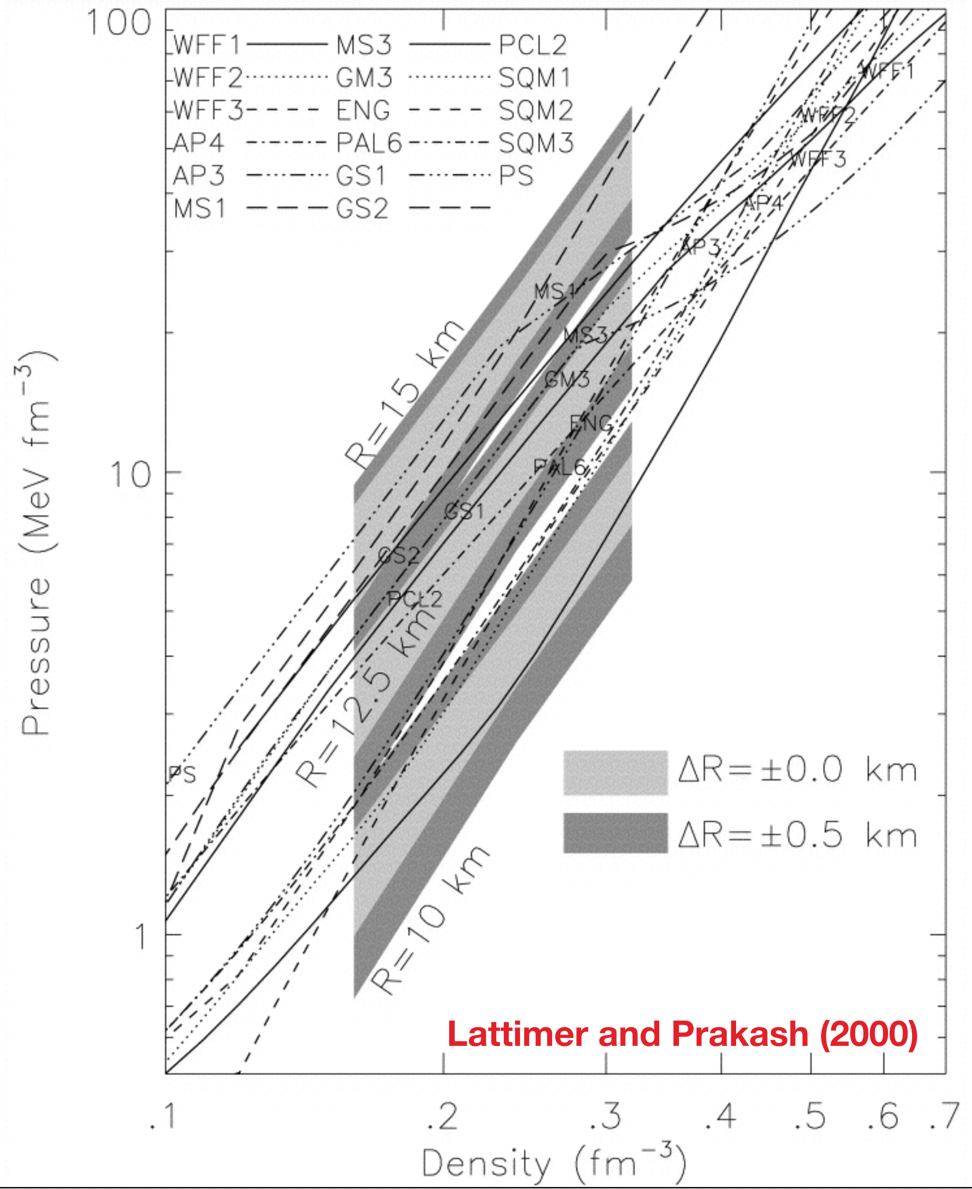
Neutron Star Structure: The Cartoon Picture



Credit:
Dany Page

Estimated Equations of State

- Different calculational (approximation) methods
- Different input physics
- Different nuclear parameters (example: nuclear compressibility as a function of fractional neutron excess).



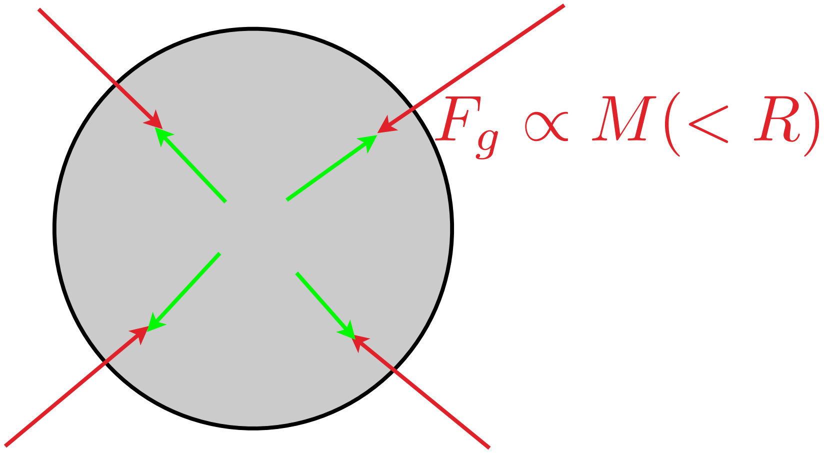
Lattimer and Prakash (2000)

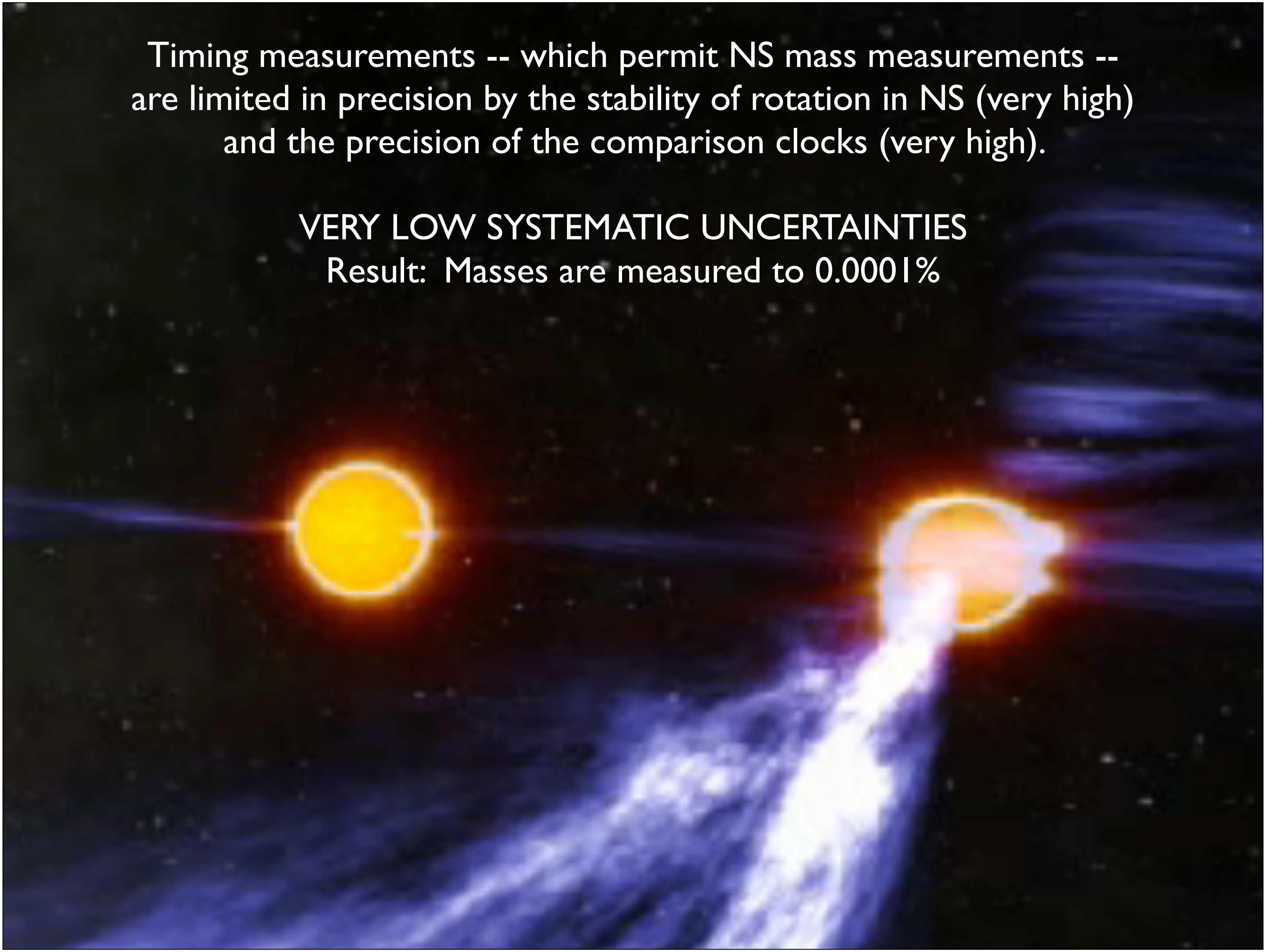
From Neutron Star Mass-Radius Relation to the Equation of State

$$P = f(\rho)$$

- Lindblom (1992) showed that each Dense Matter Equation of State maps to a unique Mass-Radius relationship for neutron stars.
- Ozel and Psaltis (2009) demonstrate how to perform the inverse problem: take the mass-radius relationship, and produce an equation of state. Only ~5-7 such objects are needed, but "with different masses", to derive a new dense matter equation of state.
- Thus, measurement of the neutron star mass-radius relationship would implicate a unique dEOS.

Short Course:
Gravity pulls inward
Pressure Pushes Outward
Result: R=f(M)



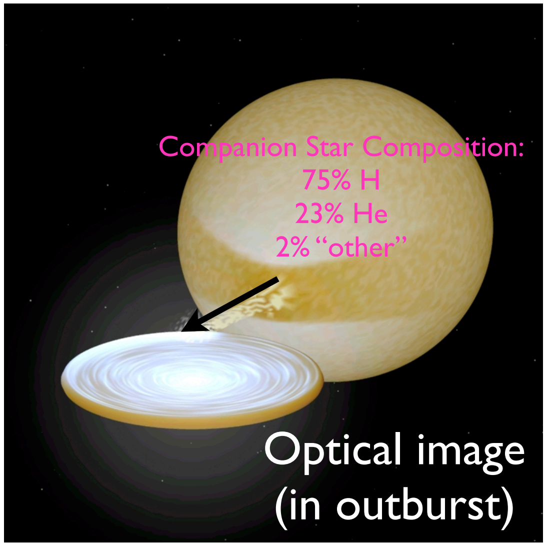


Timing measurements -- which permit NS mass measurements -- are limited in precision by the stability of rotation in NS (very high) and the precision of the comparison clocks (very high).

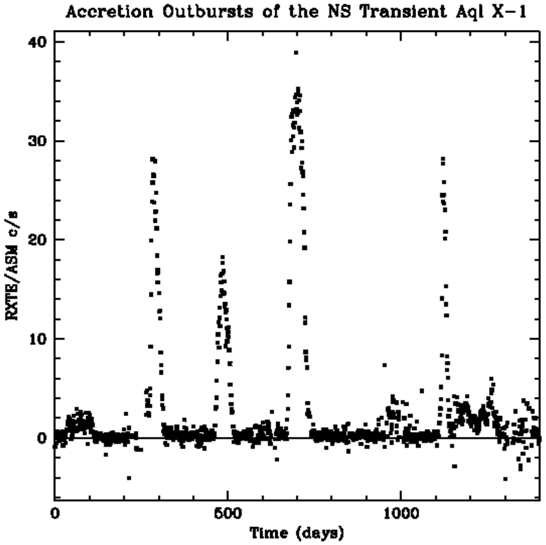
VERY LOW SYSTEMATIC UNCERTAINTIES

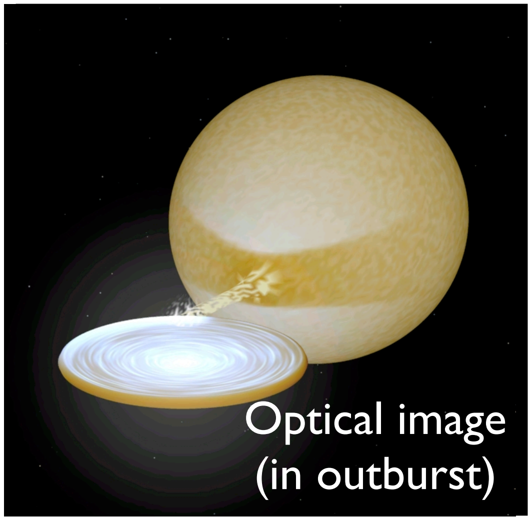
Result: Masses are measured to 0.0001%

Quiescent Low Mass X-ray Binaries (qLMXB)



Outburst





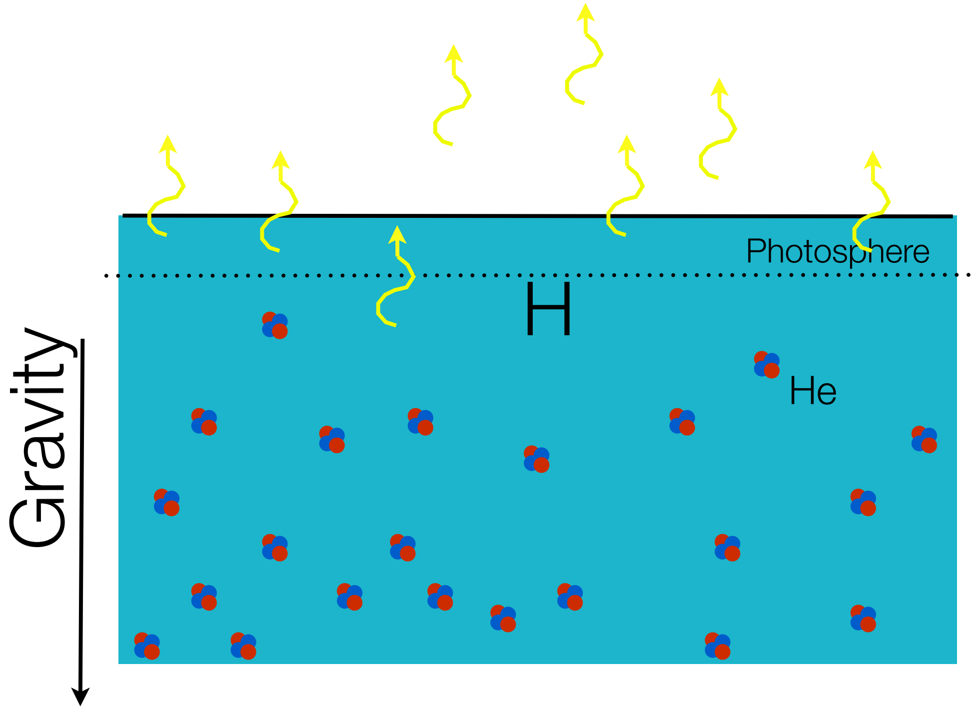
Quiescence

- Transient LMXBs in quiescence are H atmosphere neutron stars, powered by a core heated through equilibrium nuclear reactions in the crust.

Brown, Bildsten & RR (1998)

qLMXBs, in this scenario, have pure Hydrogen atmospheres

- When accretion stops, the He (and heavier elements, gravitationally settle on a timescale of ~10s of seconds (like rocks in water), leaving the photosphere to be pure Hydrogen (Alcock & Illarionov 1980, Bildsten et al 1992).



Brown, Bildsten & RR (1998)

Emergent Spectrum of a Neutron Star Hydrogen Atmosphere

- **H atmosphere calculated Spectra are ab initio radiative transfer calculations using the Eddington equations.**

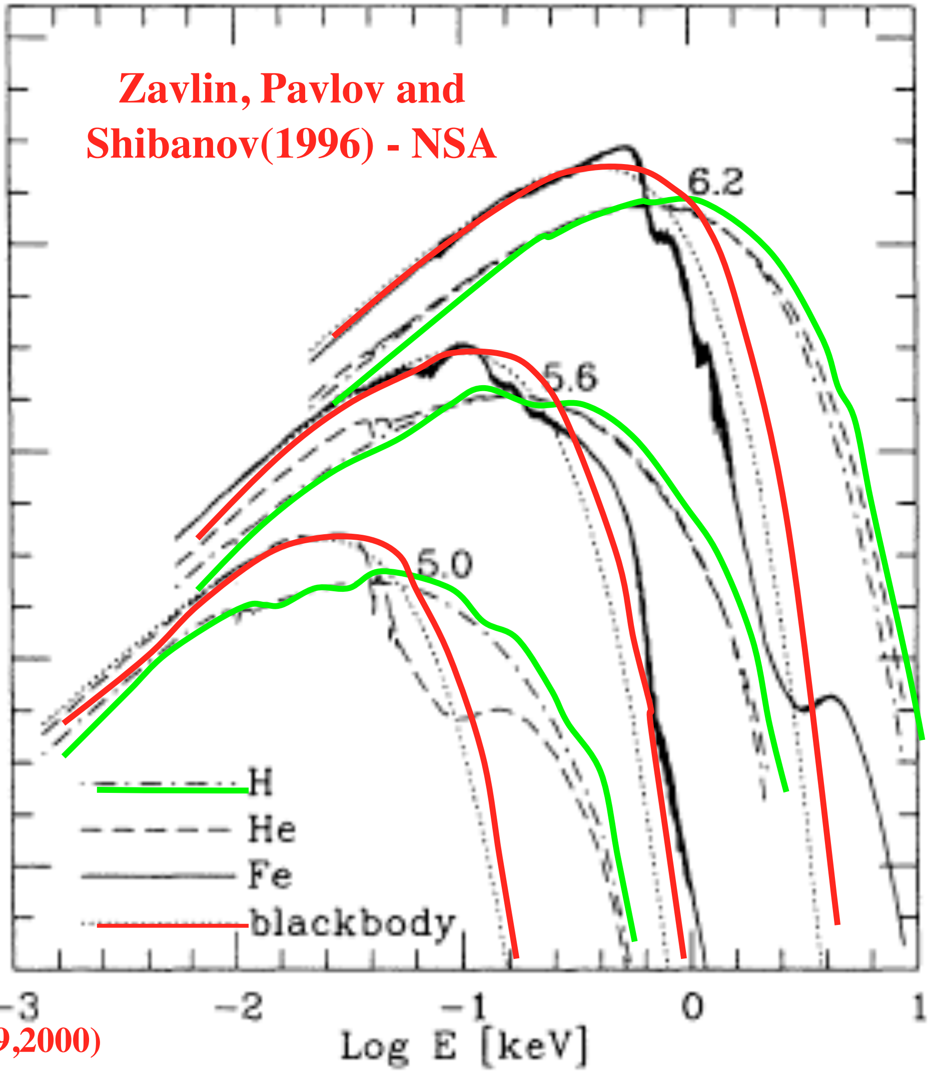
- **Rajagopal and Romani (1996); Zavlin et al (1996); Pons et al (2002; Heinke et al (2006) -- NSATMOS; Gaensicke, Braje & Romani (2001); Haakonsen et al (2012)**

All comparisons show consistency within ~few % (e.g. Webb et al 2007, Haakonsen 2012).

“Vetted”: X-ray spectra of Zavlin, Heinke together have been used in several dozen works.

$$F = 4\pi T_{eff,\infty}^{4}\left(\frac{R_\infty}{D}\right)^{2}$$

$$R_\infty = \frac{R}{\sqrt{1-\frac{2GM}{c^{2}R}}}$$



Zavlin, Pavlov and Shibanov(1996) - NSA

RR et al (1999,2000)

Deep Crustal Heating

Non-Equilibrium Processes in the Outer Crust
Beginning with ^{56}Fe (Haensel &Zdunik 1990, 2003)

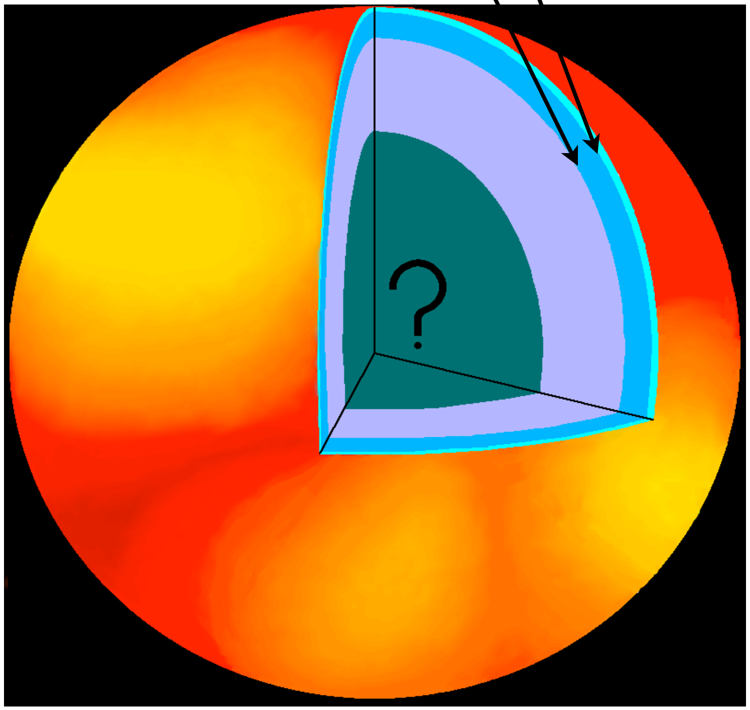
ρ (g cm^{-3})	Reaction	Δρ/ρ	Q (Mev/np)
$1.5 \cdot 10^{9}$	$^{56}\mathrm{Fe} \Rightarrow {}^{56}\mathrm{Cr} - 2e- + 2\nu_e$	0.08	0.01
$1.1 \cdot 10^{10}$	$^{56}\mathrm{Cr} \Rightarrow {}^{56}\mathrm{Ti} - 2e- + 2\nu_e$	0.09	0.01
$7.8 \cdot 10^{10}$	$^{56}\mathrm{Ti} \Rightarrow {}^{56}\mathrm{Ca} - 2e- + 2\nu_e$	0.10	0.01
$2.5 \cdot 10^{10}$	$^{56}\mathrm{Ca} \Rightarrow {}^{56}\mathrm{Ar} - 2e- + 2\nu_e$	0.11	0.01
$6.1 \cdot 10^{10}$	$^{56}\mathrm{Ar} \Rightarrow {}^{52}\mathrm{S} + 4n - 2e- + 2\nu_e$	0.12	0.01

Non-Equilibrium Processes in the Inner Crust

ρ (g cm^{-3})	Reaction	X_n	Q (Mev/np)
$9.1 \cdot 10^{11}$	$^{52}\mathrm{S} \Rightarrow {}^{46}\mathrm{Si} + 6n - 2e- + 2\nu_e$	0.07	0.09
$1.1 \cdot 10^{12}$	$^{46}\mathrm{Si} \Rightarrow {}^{40}\mathrm{Mg} + 6n - 2e- + 2\nu_e$	0.07	0.09
$1.5 \cdot 10^{12}$	$^{40}\mathrm{Mg} \Rightarrow {}^{34}\mathrm{Ne} + 6n - 2e- + 2\nu_e$		
	$^{34}\mathrm{Ne} + {}^{34}\mathrm{Ne} \Rightarrow {}^{68}\mathrm{Ca}$	0.29	0.47
$1.8 \cdot 10^{12}$	$^{68}\mathrm{Ca} \Rightarrow {}^{62}\mathrm{Ar} + 6n - 2e- + 2\nu_e$	0.39	0.05
$2.1 \cdot 10^{12}$	$^{62}\mathrm{Ar} \Rightarrow {}^{56}\mathrm{S} + 6n - 2e- + 2\nu_e$	0.45	0.05
$2.6 \cdot 10^{12}$	$^{56}\mathrm{S} \Rightarrow {}^{50}\mathrm{Si} + 6n - 2e- + 2\nu_e$	0.50	0.06
$3.3 \cdot 10^{12}$	$^{50}\mathrm{Si} \Rightarrow {}^{44}\mathrm{Mg} + 6n - 2e- + 2\nu_e$	0.55	0.07
$4.4 \cdot 10^{12}$	$^{44}\mathrm{Mg} \Rightarrow {}^{36}\mathrm{Ne} + 6n - 2e- + 2\nu_e$		
	$^{36}\mathrm{Ne} + {}^{36}\mathrm{Ne} \Rightarrow {}^{72}\mathrm{Ca}$		
	$^{68}\mathrm{Ca} \Rightarrow {}^{62}\mathrm{Ar} + 6n - 2e- + 2\nu_e$	0.61	0.28
$5.8 \cdot 10^{12}$	$^{62}\mathrm{Ar} \Rightarrow {}^{60}\mathrm{S} + 6n - 2e- + 2\nu_e$	0.70	0.02
$7.0 \cdot 10^{12}$	$^{60}\mathrm{S} \Rightarrow {}^{54}\mathrm{Si} + 6n - 2e- + 2\nu_e$	0.73	0.02
$9.0 \cdot 10^{12}$	$^{54}\mathrm{Si} \Rightarrow {}^{48}\mathrm{Mg} + 6n - 2e- + 2\nu_e$	0.76	0.03
$1.1 \cdot 10^{13}$	$^{48}\mathrm{Mg} + {}^{48}\mathrm{Mg} \Rightarrow {}^{96}\mathrm{Cr}$	0.79	0.11
$1.1 \cdot 10^{13}$	$^{96}\mathrm{Cr} \Rightarrow {}^{88}\mathrm{Ti} + 8n - 2e- + 2\nu_e$	0.80	0.01

Begins Here

Ends Here



1.47 Mev per np

Brown, Bildsten & RR (1998)

Assumptions -- the systematic uncertainties.

- **H atmosphere neutron stars.** Expected from a Hydrogen companion LMXB; can be supported through optical observations of a H companion.
- **Low B-field ($<10^{10}$ G) neutron stars.** This is true for ‘standard’ LMXBs as a class, but difficult to prove on a case-by-case basis.
- **Emitting isotropically.** Occurs naturally when powered by a hot core.
- **Non-Rotating neutron stars.** qLMXBs are observed to rotate at 100-600 Hz. This can be a significant fraction of the speed of light. Doppler boosting and deviation from NS spheroidal geometry are not included in emission models. *These effects should be calculated, but have not yet been.*

If you don’t like these assumptions: “We find the assumptions not strongly supported and therefore ignore this result.”

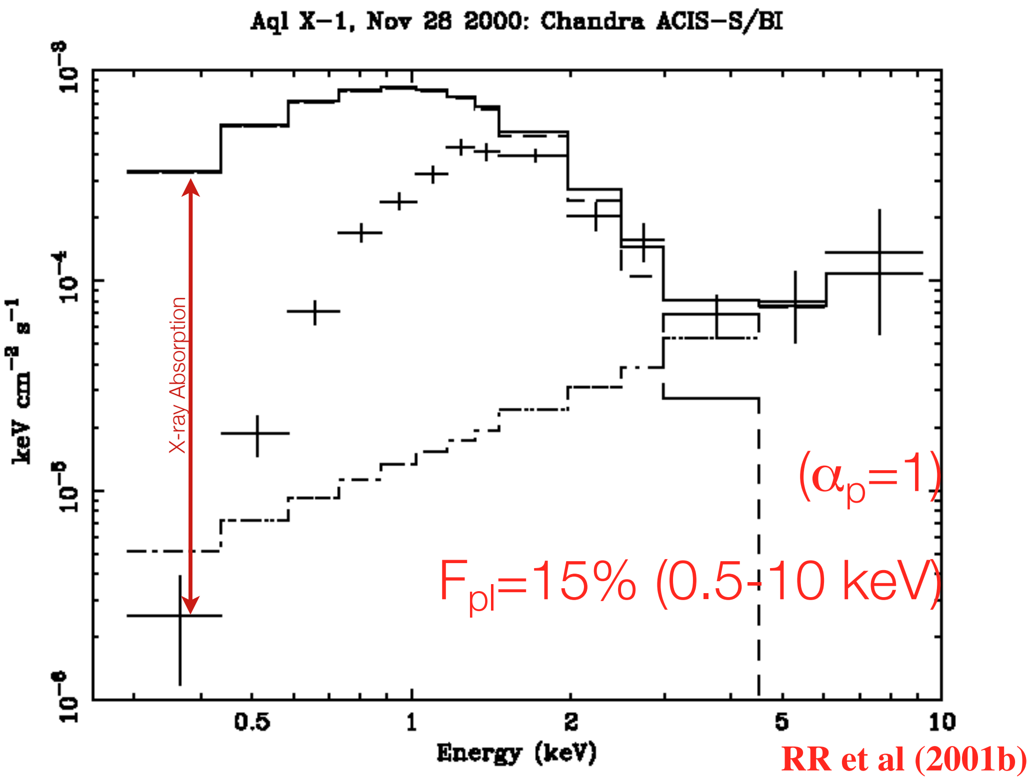
Instruments for measurements of qLMXBs

Chandra X-ray Observatory
- Launched 1999 (NASA)
  - 1” resolution

XMM/Newton
- Launched 1999 (ESA)
- 6” resolution
- ~4x area of Chandra.

Every photon is time tagged (~1 sec), with its energy measured (E/deltaE = 10) with full resolution imaging.

Aql X-1 with Chandra -- Field Source



RR et al (2001b)

R_∞ (d/5 kpc)	$kT_{eff,\infty}$	N_H (1e20 cm^{-2})
13^{+5}_{-4} km	135^{+18}_{-12} eV	35^{+8}_{-7}

The LMXB Factories: Globular Clusters

- GCs : overproduce LMXBs by 1000x vs. field stars
- Many have accurate distances measured.

NGC	D (kpc)	+/-(%)
104	5.13	4
288	9.77	3
362	10.0	3
4590	11.22	3
5904	8.28	3
7099	9.46	2
6025	7.73	2
6341	8.79	3
6752	4.61	2

Carretta et al (2000)

qLMXBs can be identified by their soft X-ray spectra, and confirmed with optical counterparts.

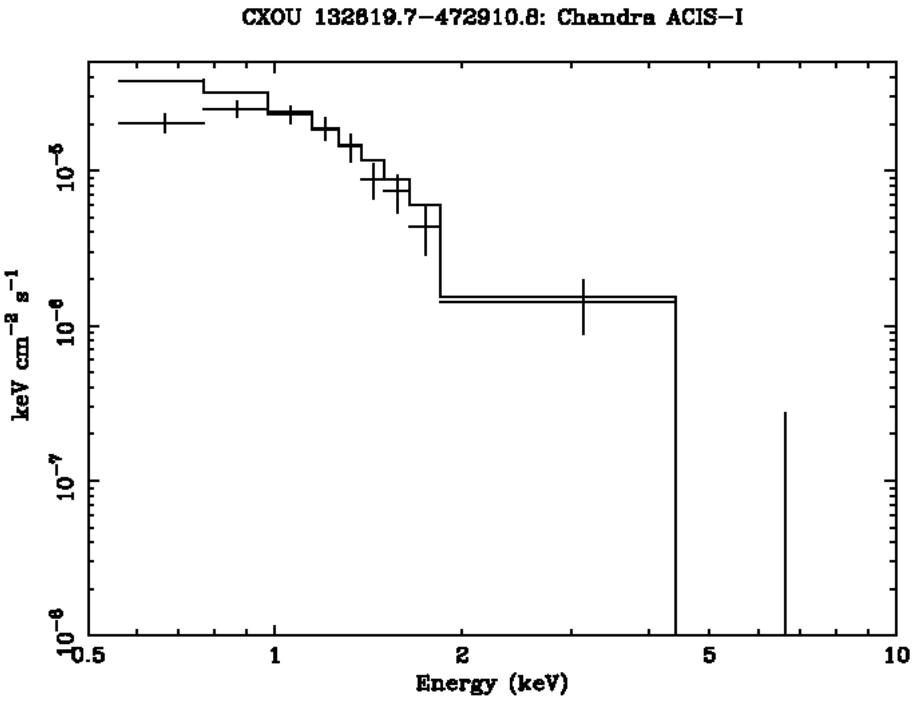
NGC 5139 (Omega Cen)

qLMXBs can be identified by their soft X-ray spectra, and confirmed with optical counterparts.

The identified optical counterpart demonstrates unequivocally the X-ray source is a qLMXB.

An X-ray source well outside the cluster core

NGC 5139 (Omega Cen)



X-ray Spectrum is inconsistent with any other type of known GC source (pulsars, CVs, coronal sources).

Full confirmation as LMXB requires Hubble photometry

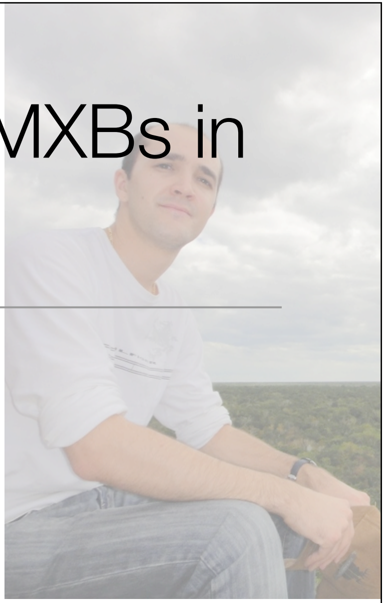
R_∞ (d/5 kpc)	$kT_{eff,\infty}$	N_H (1e20 cm^{-2})
14.3 ± 2.1 km	66^{+4}_{-5} eV	(9)

RR et al (2002)

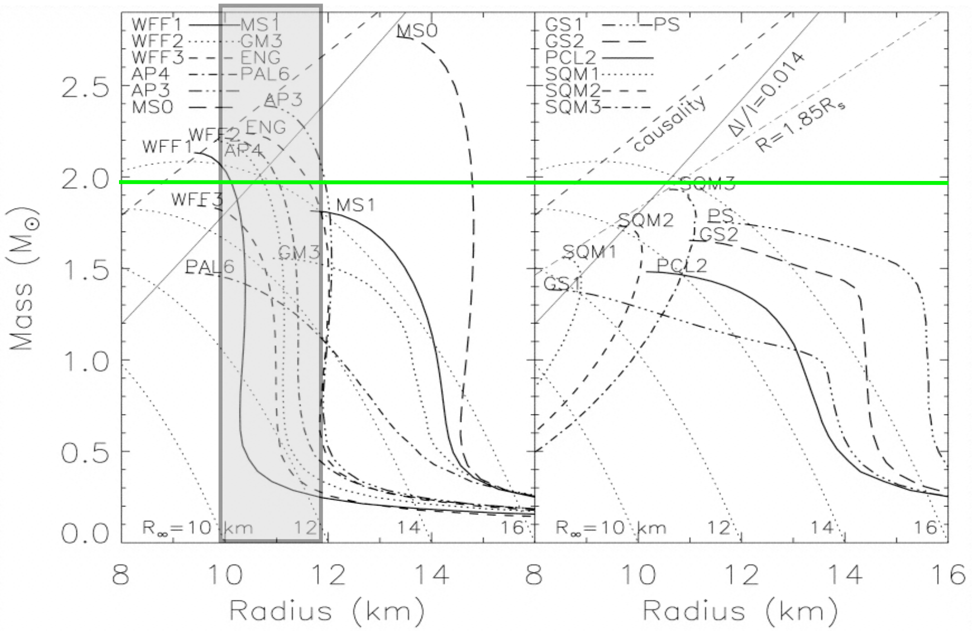
All qLMXBs used in this work were previously identified and analysed, with resulting R published.

Name	kT_eff(infty) (ev)	NH	Fx (10^{-13} cgsflux)	Band (keV)	Ref.	R(infty)
47 Tuc X7	105(5)	0.04(2)	5.3	0.5-10	Heinke et al (2006)	<: 5”
47 Tuc X5	100(20)	0.09(7)	4.3	0.5-10	Heinke et al (2003)	< 5”
M28	90(+30-10)	0.26(4)	3.4	0.5-8	Becker et al (2003)	<5”
NGC 6304 X9	115(20)	[0.266]	1.5	0.5-10	Guillot et al (2009)	+++
NGC 6304 X4	120(20)	[0.266]	1.1	0.5-10	Guillot et al (2009)	<15”
NGC 6397 (U24)	74(18)	0.1-0.26	1.06	0.5-2.5	Grindlay et al (2001	< 15”
M13	76(3)	[0.011]	1.03	0.1-5	Gendre et al (2003)	DONE
NGC 3201 X16	170 (50)	[0.14]	1.0	0.6-6	Webb et al (2006)	Data taken
NGC 6553 X9	100 (20)	[0.35]	1.0	0.5-10	Guillot et al (in prep)	+++
Omega Cen	67(2)	0.09(3)	0.95	0.1-5	Rutledge et al (2002), Gendre et al (2003)	DONE
NGC 6637 X3	100 (40)	[0.11]	0.84	0.5-10	Guillot et al (in prep)	+++
M30 A-1	94(15)	0.03(1)	0.73	0.5-10	Lugger (2007)	<10”
NGC 6553 X3	127(+7-45)	[0.35]	0.65	0.5-10	Guillot et al (in prep)	<: 5”
NGC 6304 X5	70(25)	[0.266]	0.32	0.5-10	Guillot et al (2008)	+++
NGC 6553 x35	88 (60)	[0.35]	0.3	0.5-10	Guillot et al (in prep)	+++
NGC 2808 C2	--	0.86	0.24	--	Servillat et al (2008)	<15”
M80 CX2	82(2)	0.09(2)	0.23	0.5-6	Heinke et al (2003)	<5”
M80 CX6	76(6)	0.22(7)	0.07	0.5-6	Heinke et al (2003)	<15”

Measuring the Radius of Neutron Stars from qLMXBs in Globular Clusters

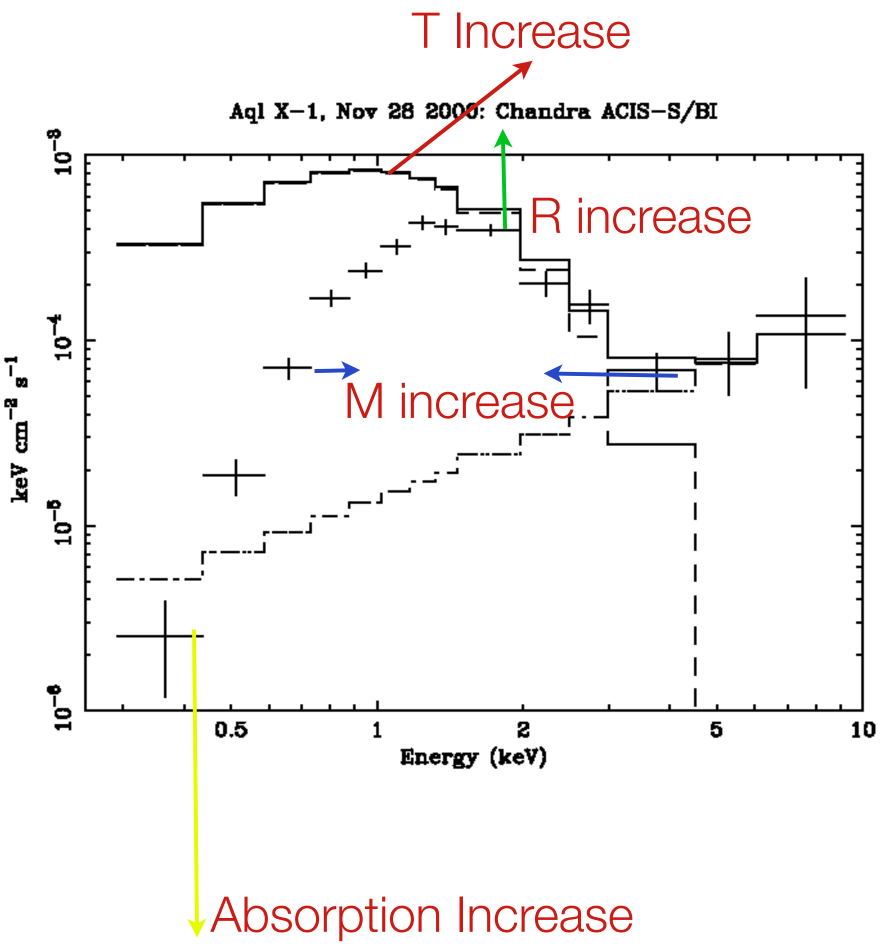


- The 2.0 solar mass neutron stars favor hadronic dEOSs over quark and phase-transition dEOSs. These have the property of a quasi-constant neutron star radius.
- Analysis goal: Using all suitable qLMXB X-ray data sets of targets (there are five) provide the most reliable neutron star radius measurement possible.
- Assume the radius of neutron stars is **quasi-constant** (a constant, at astrophysically important masses, within measurement error).
- Perform a Markoff-Chain-Monte-Carlo (MCMC) and include all known uncertainties and use conservative assumptions.



All previous EoS work treated measurements of each NS independently, and combined their statistics inefficiently.

$$\frac{\partial\chi^2}{\partial(p1)\partial(p2)} = \begin{array}{cc}
\begin{array}{cccccccc}
\multicolumn{4}{c}{\text{Source 1}} & \multicolumn{4}{c}{\text{Source 2}} \\
R_1 & M_1 & T_1 & N_{H,1} & R_2 & M_2 & T_2 & N_{H,2}
\end{array} & \\
\left(\begin{array}{cccccccc}
\frac{1}{\sigma^2_{R_1}} & a_{1,2} & a_{1,3} & a_{1,4} & 0 & 0 & 0 & 0 \\
a_{2,1} & \frac{1}{\sigma^2_{M_1}} & a_{2,3} & a_{2,4} & 0 & 0 & 0 & 0 \\
a_{3,1} & a_{3,2} & \frac{1}{\sigma^2_{T_1}} & a_{3,4} & 0 & 0 & 0 & 0 \\
a_{4,1} & a_{4,2} & a_{4,3} & \frac{1}{\sigma^2_{N_{H1}}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\sigma^2_{R_2}} & a_{5,6} & a_{5,7} & a_{5,8} \\
0 & 0 & 0 & 0 & a_{6,5} & \frac{1}{\sigma^2_{M_2}} & a_{6,7} & a_{6,8} \\
0 & 0 & 0 & 0 & a_{7,5} & a_{7,6} & \frac{1}{\sigma^2_{T_2}} & a_{7,8} \\
0 & 0 & 0 & 0 & a_{8,5} & a_{8,6} & a_{8,7} & \frac{1}{\sigma^2_{N_{H2}}}
\end{array}\right) &
\begin{array}{c}
R_1 \\ M_1 \\ T_1 \\ N_{H,1} \\ R_2 \\ M_2 \\ T_2 \\ N_{H,2}
\end{array}
\end{array}$$



T Increase

R increase

M increase

Absorption Increase

“Joint Fits” - the major difference from previous work

Source 1

Source 2

$$\frac{\partial\chi^2}{\partial(p1)\partial(p2)} = \begin{array}{cc} \begin{array}{cccccccc} \mathbf{R} & M_1 & T_1 & N_{H,1} & \mathbf{R} & M_2 & T_2 & N_{H,2} \end{array} & \\ \begin{pmatrix} \boxed{\frac{1}{\sigma_R^2}} & a_{1,2} & a_{1,3} & a_{1,4} & \boxed{\frac{1}{\sigma_R^2}} & a_{1,6} & a_{1,7} & a_{1,8} \\ a_{2,1} & \frac{1}{\sigma_{M_1}^2} & a_{2,3} & a_{2,4} & a_{2,5} & \frac{1}{\sigma_{M_1,M_2}^2} & a_{2,7} & a_{2,8} \\ a_{3,1} & a_{3,2} & \frac{1}{\sigma_{T_1}^2} & a_{3,4} & a_{3,5} & a_{3,6} & \frac{1}{\sigma_{T_1,T_2}^2} & a_{3,8} \\ a_{4,1} & a_{4,2} & a_{4,3} & \frac{1}{\sigma_{NH1}^2} & a_{4,5} & a_{4,6} & a_{4,7} & \frac{1}{\sigma_{NH1,NH2}^2} \\ \boxed{\frac{1}{\sigma_R^2}} & a_{5,2} & a_{5,3} & a_{5,4} & \boxed{\frac{1}{\sigma_R^2}} & a_{5,6} & a_{5,7} & a_{5,8} \\ a_{6,1} & \frac{1}{\sigma_{M_2,M_1}^2} & a_{6,3} & a_{6,4} & a_{6,5} & \frac{1}{\sigma_{M_2}^2} & a_{6,7} & a_{6,8} \\ a_{7,1} & a_{7,2} & \frac{1}{\sigma_{T_2,T_1}^2} & a_{7,4} & a_{7,5} & a_{7,6} & \frac{1}{\sigma_{T_2}^2} & a_{7,8} \\ a_{8,1} & a_{8,2} & a_{8,3} & \frac{1}{\sigma_{NH2,NH1}^2} & a_{8,5} & a_{8,6} & a_{8,7} & \frac{1}{\sigma_{NH2}^2} \end{pmatrix} & \begin{array}{c} R_1 \\ M_1 \\ T_1 \\ N_{H,1} \\ R_2 \\ M_2 \\ T_2 \\ N_{H,2} \end{array} \end{array}$$

Every parameter (M, R, T, N_H) of all five sources affects every other parameter of every source

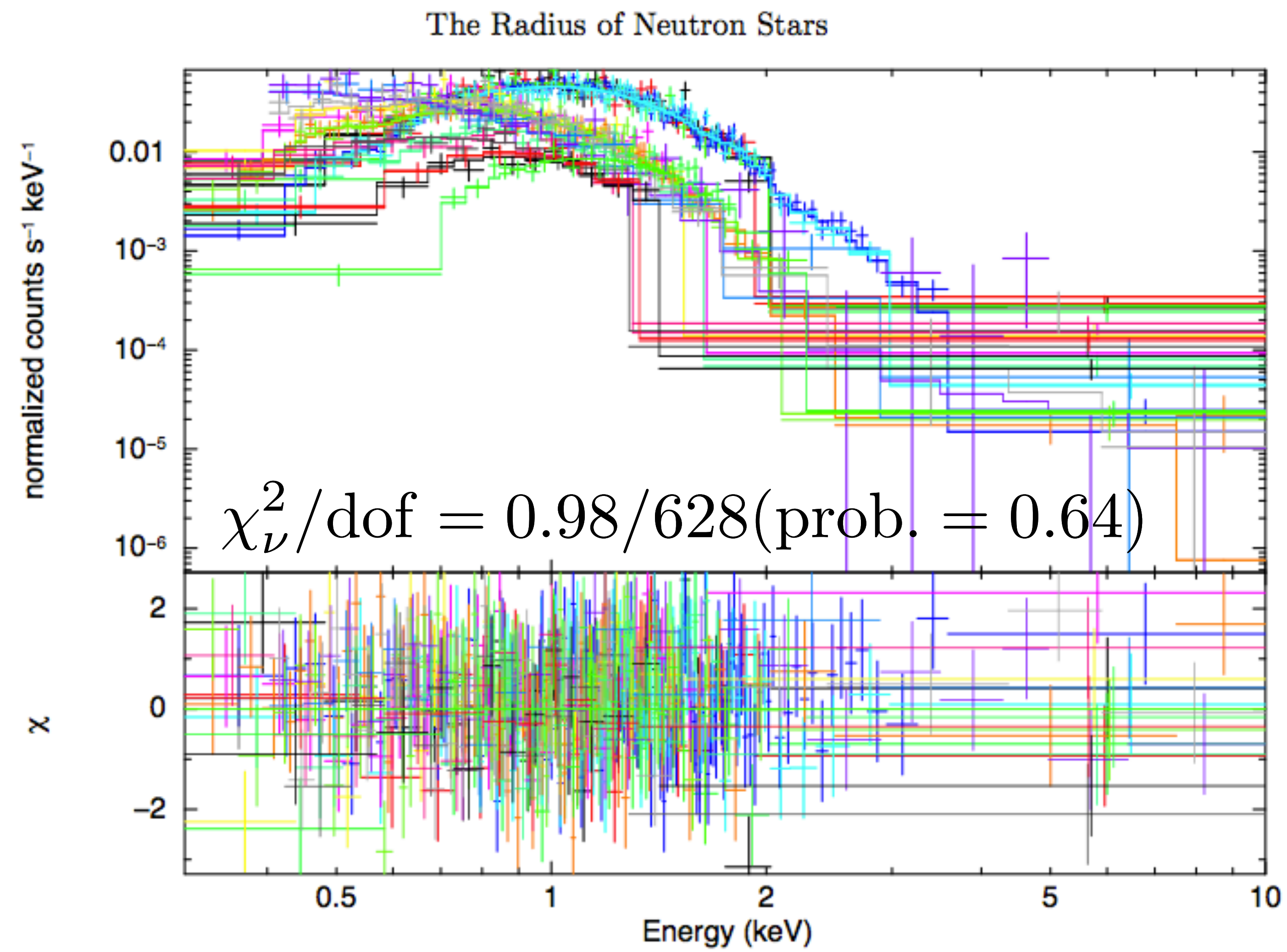
$$\frac{S}{N} \propto \frac{N^2(\text{\# of Source Matrix Elements})}{N(\text{\# of sources})} = N$$

In comparison to using the sources “independently”, its as if we have 25 sources, instead of 5 sources.

Best H atmosphere (+ PL) spectral fit of all 5 qLMXBs

- This model is a statistically acceptable fit to the X-ray spectral data. This is an a posteriori confirmation that the data are consistent with our assumptions.

- After finding the best fit a MCMC method was used to find the uncertainty regions for all parameters - -- the Radius, Mass, Temperature, absorption, distance, and power-law normalization.



$$\chi^2_\nu/\mathrm{dof} = 0.98/628(\mathrm{prob.} = 0.64)$$

Guillot et al (2013)

NH free, D gaussian bayesian priors, PL included.

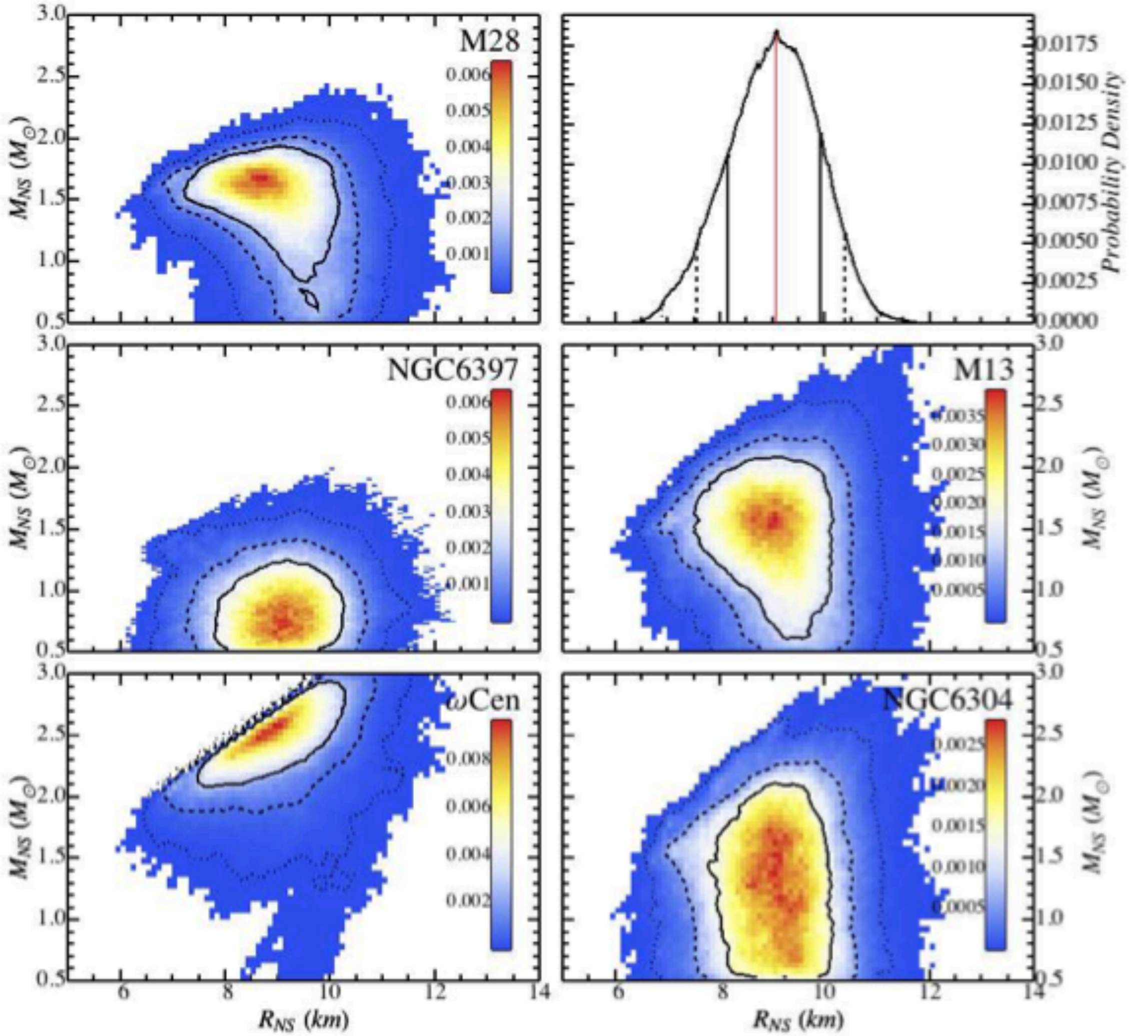
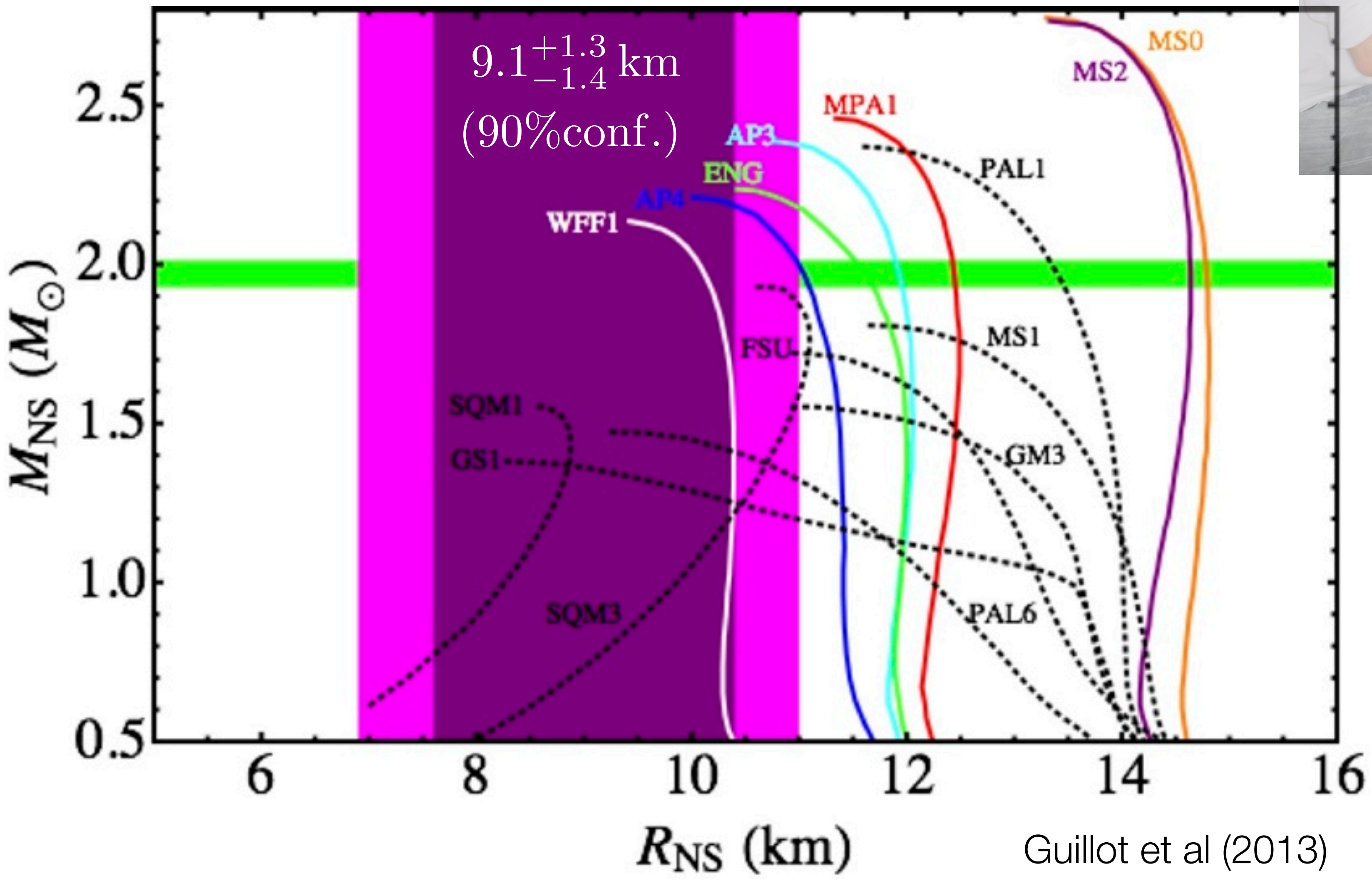


FIG. 15.— Figure similar to Figure 9 corresponding to Run #7. Here, all the possible assumptions have been relaxed to obtain a $R_{\rm NS}$ measurement the least affected by systematic uncertainties. The N_H parameters are left free; and Gaussian Bayesian priors and PL components are included. This results in an $R_{\rm NS}$ measurement: $R_{\rm NS} = 9.1^{+1.3}_{-1.5}$ km

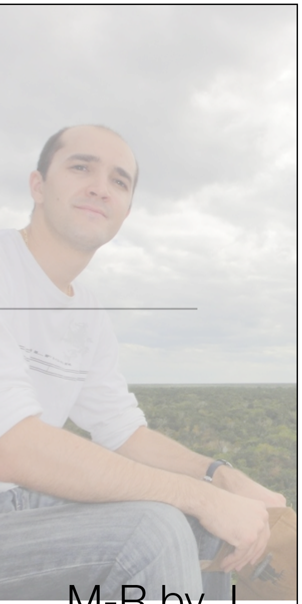
Guillot et al (2013)

The Neutron Star Radius

<11 km (99% conf).



Guillot et al (2013)



M-R by J. Lattimer

WFF1= Wiring, Fiks and Fabrocini (1988)

Contains uncertainties from:
Distance
All spectral parameters
Calibration