EoS constraints from Nuclear Collective Motions
『Neutron Star Matter Symposium』
(October 25, 2013 at Kyoto, Japan)

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1. Incompressibility and Giant Monopole Resonances
2. Isospin dependence of GMR
3. Mass model and symmetry energy
4. Summary

Energy Density Functionals

$$E[\rho] = \int d_3 r \; \mathcal{E}(\rho_n(\vec{r}), \rho_p(\vec{r})),$$

where the dependence on $\nabla \rho_q$, on the kinetic energy densities τ_q, and on the spin-orbit densities J_q is not explicitly indicated.

In infinite matter,

$$\mathcal{E}(\rho, \delta \equiv \frac{\rho_n - \rho_p}{\rho}) = \mathcal{E}_0(\rho, \delta = 0) + \mathcal{E}_{\text{sym}}(\rho)\delta^2.$$

$$E_{sym} = \rho S(\rho) = E(\rho, \delta = 1) - E(\rho, \delta = 0)$$

Well-known basics on EDF's

$$E = \langle \Psi | \hat{H} | \Psi \rangle = \langle \Phi | \hat{H}_{eff} | \Phi \rangle = E[\hat{\rho}\,]$$

$|\Phi\rangle$ **Slater determinant** $\Leftrightarrow$ $\hat{\rho}$ **1-body density matrix**

Calculating the parameters from a more fundamental theory (Relativistic Bruckner HF or Chiral field theory and -----------)

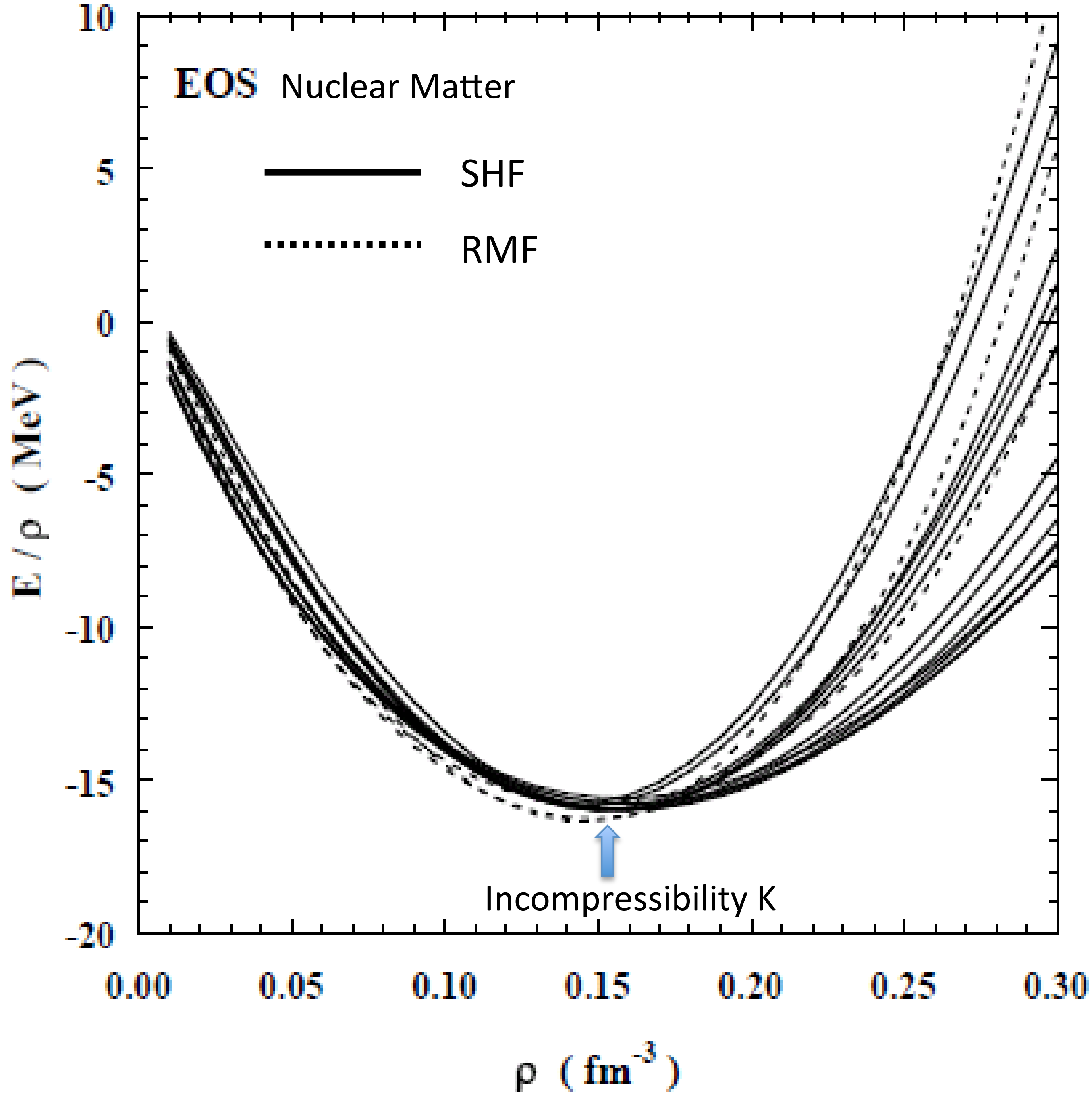
Setting the structure by means of symmetries (spin, isospin --) and fitting the parameters

$$E[\rho]$$

Allows calculating nuclear matter and finite nuclei (even complex states), by disentangling physical parameters.

HF/HFB for g.s., RPA/QRPA for excited states.

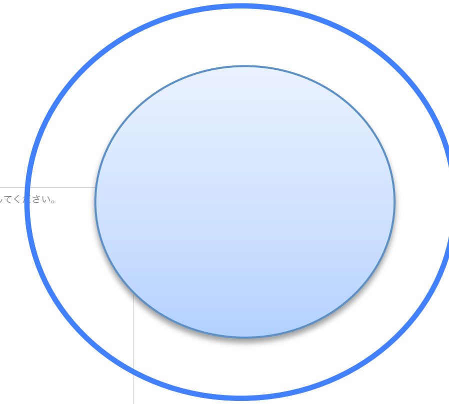
Possible both in non-relativistic and in relativistic covariant form.



EOS Nuclear Matter
SHF
RMF
Incompressibility K
E / ρ (MeV)
ρ (fm-3)
10
5
0
-5
-10
-15
-20
0.00
0.05
0.10
0.15
0.20
0.25
0.30

イメージを表示できません。メモリ不足のためにイメージを開くことができないか、イメージが破損している可能性があります。コンピューターを再起動して再度ファイルを開いてください。それでも赤い x が表示される場合は、イメージを削除して挿入してください。

Nuclear Matter EOS → Supernova Explosion



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Isoscalar Giant Monopole Resonances

Isoscalar Compressional Dipole Resonances

Incompressibility K

$$E_{ISGMR} = \sqrt{\frac{\hbar^2 K_A}{m < r^2 >_m}},$$

$$K_A = K_\infty + K_{surf} A^{-1/3} + K_\tau \delta^2 + K_{Coul} \frac{Z^2}{A^{4/3}},$$

Self consistent HF+RPA calculations

Self consistent RMF+RPA calculations

(α, α') experimtent

The nuclear incompressibility from ISGMR

We can give credit to the idea that the link should be provided microscopically through the Energy Functional E[ρ].

IT PROVIDES AT THE SAME TIME

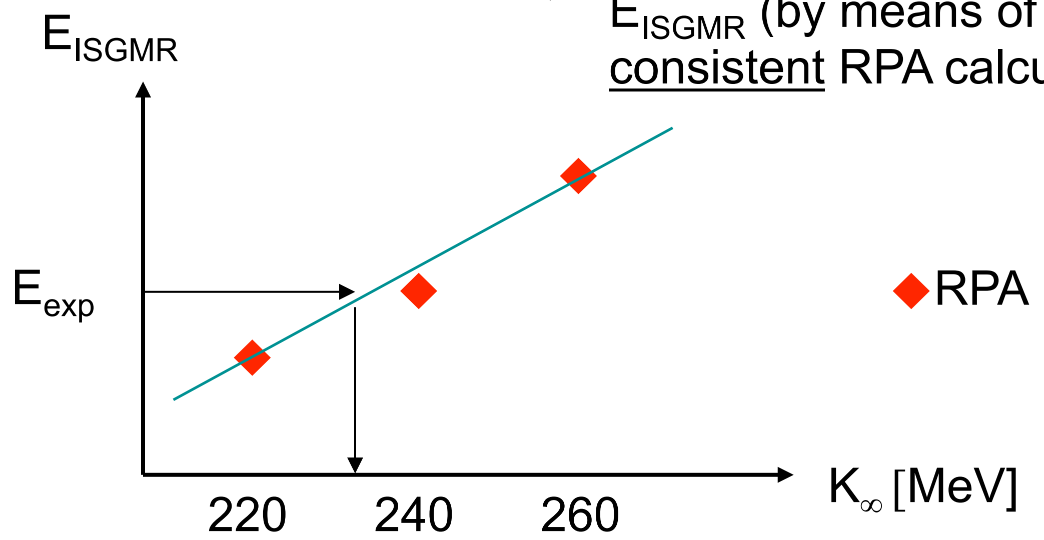
K_∞ in nuclear matter (analytic)

E_{ISGMR} (by means of self-consistent RPA calculations)

Skyrme

Gogny

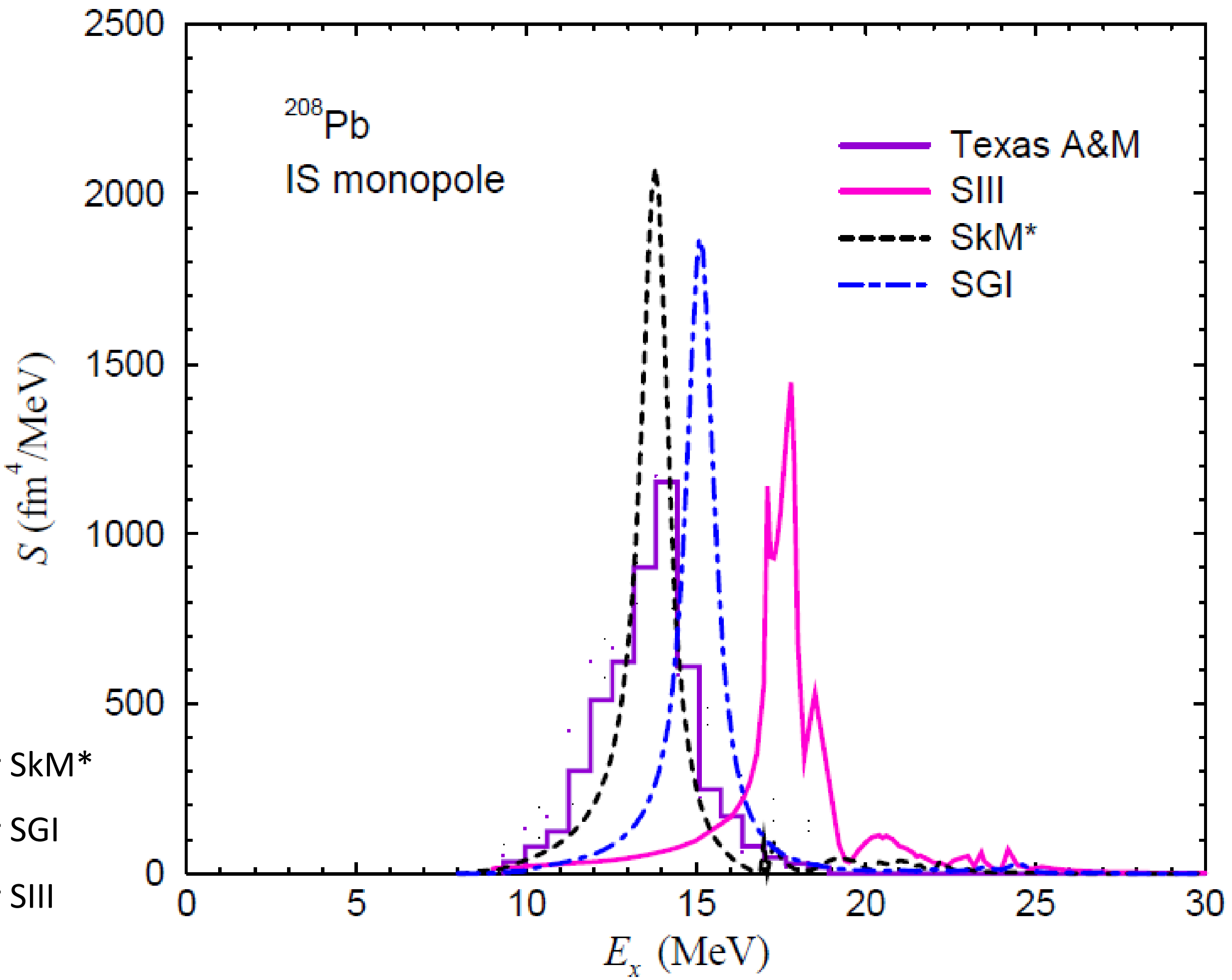
RMF



Extracted value of K_∞

The incompressibility of nuclear matter K_∞

The incompressibility of nuclear matter can not be measured directly, it can be deduced from the response of ISGMR in heavy nuclei, such as ^{208}Pb.



K=217MeV for SkM*

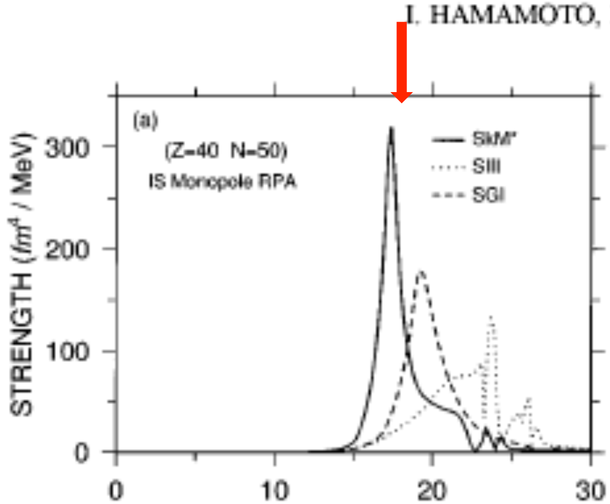
K=256MeV for SGI

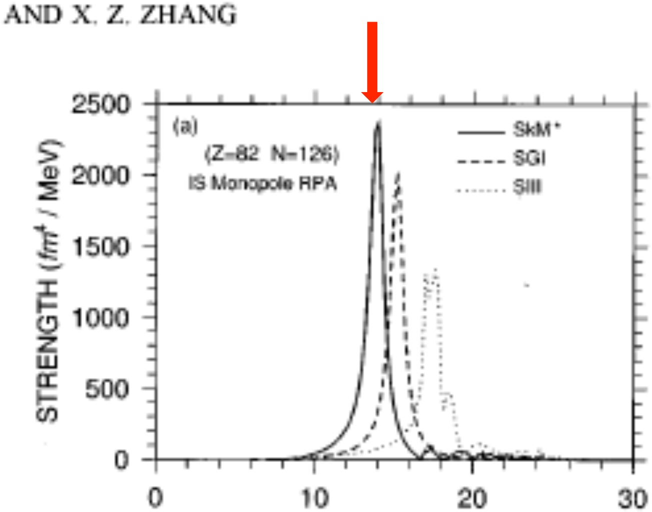
K=355MeV for SIII

TABLE II. Peak energies, widths, and EWSR fractions for the ISGMAR and ISGDR. The errors in fitting the $L=0$ and $L=1$ strengths with the Breit-Wigner functions are included.

	ISGMR			LE ISGDR			HE ISGDR		
	E_{ISGMR} (MeV)	Γ (MeV)	EWSR (%)	E_{ISGDR} (MeV)	Γ (MeV)	EWSR (%)	E_{ISGDR} (MeV)	Γ (MeV)	EWSR (%)
^{90}Zr	16.6±0.1	4.9±0.2	101±3	17.8±0.5	3.7±1.2	7.9±2.9	26.9±0.7	12.0±1.5	67±8
^{116}Sn	15.4±0.1	5.5±0.3	95±4	15.6±0.5	2.3±1.0	4.9±2.2	25.4±0.5	15.7±2.3	68±9
^{144}Sm [21]	$15.3^{+0.11}_{-0.12}$	$3.70^{+0.12}_{-0.63}$	84^{+4}_{-25}	14.2±0.2	4.8±0.8	23^{+4}_{-10}	$25.0^{+1.7}_{-0.3}$	19.9±1.4	91^{+25}_{-17}
^{208}Pb	13.4±0.2	4.0±0.4	104±9	13.0±0.1	1.1±0.4	7.0±0.4	22.7±0.2	11.9±0.4	111±6

3126 I. HAMAMOTO, H. SAGAWA, AND X. Z. ZHANG 56





K=217MeV for SkM*

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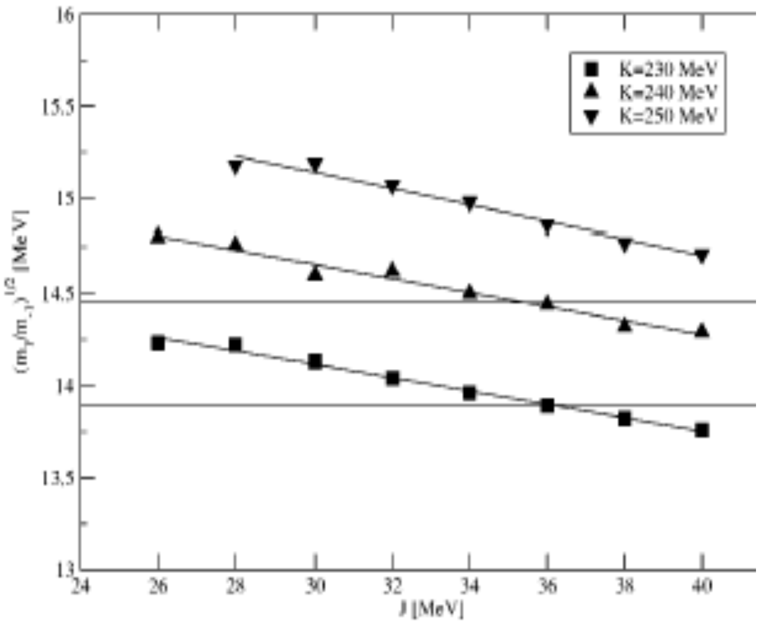
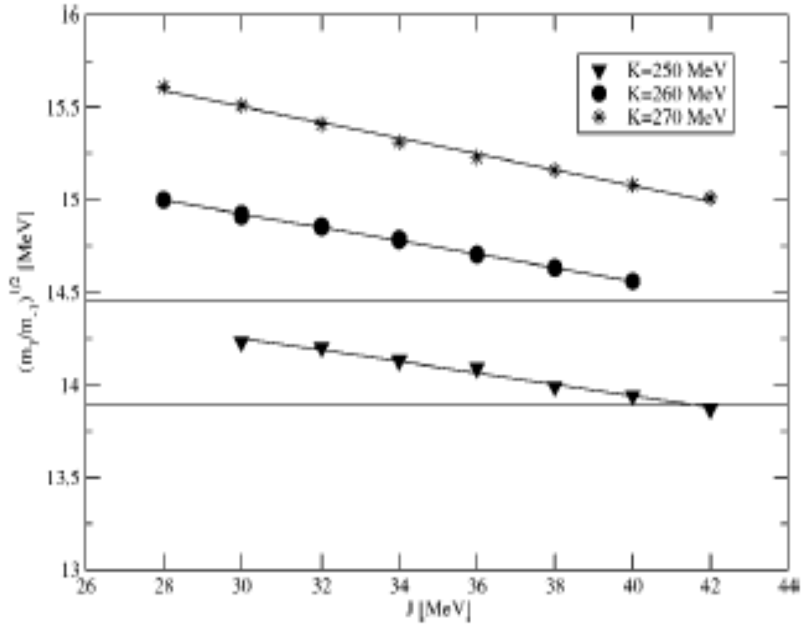




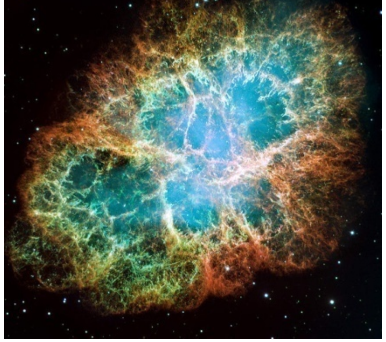
α = 1/6 implies K around 230-240 MeV **α = 1/3 implies K around 250 MeV**

G.Colo, N. Van Giai, J. Meyer, K. Bennaceur, P. Bonche, *Phys. Rev.* C70, 024307 (2004)

<u>Constraint from the ISGMR in ^{208}Pb</u> :

E_{GMR} constrains K_∞ = 240 ± 10 MeV. The error comes from the choice of the density dependence

Nuclear Matter EOS



Isoscalar Monopole Giant Resonances in ^{208}Pb

Isoscalar Compressional Dipole Resonances

Incompressibility K

K=(240 +/-10 +/- 10)MeV

$$K \approx (240 \pm 10)\ \text{MeV for Skyrme}$$ (G. Colo ,2004)

$$\approx (230 \pm 10)\ \text{MeV for Gogny}$$

$$\approx (250\ \pm 10)\ \text{MeV for RMF}$$ (Lalazissis,2005)

$$\approx (230 \pm 10)\ \text{MeV for Point Coupling}$$ (P. Ring,2007)

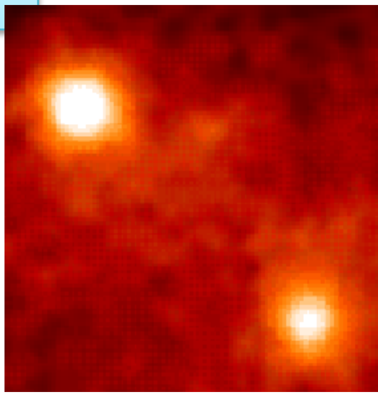
What can we learn about neutron EOS from nuclear physics?

Symmetry energy S ⟷ Neutron EOS

Size ~10fm

size difference ~ 10^{18}

Neutron star ~10km



EDF's

$$E[\rho] = \int d_3 r \; \mathcal{E}(\rho_n(\vec{r}), \rho_p(\vec{r})),$$

where the dependence on $\nabla\rho_q$, on the kinetic energy densities τ_q, and on the spin-orbit densities J_q is not explicitly indicated.

In infinite matter,

$$\mathcal{E}(\rho, \delta \equiv \frac{\rho_n - \rho_p}{\rho}) = \mathcal{E}_0(\rho, \delta = 0) + \mathcal{E}_{\text{sym}}(\rho)\delta^2.$$

The isoscalar GMR constraints the curvature of E/A in symmetric matter, that is,

$$K_\infty = 9\rho_0^2 \frac{d^2}{d\rho^2} \frac{\mathcal{E}_0}{\rho}\bigg|_{\rho_0}.$$

$$\rho S(\rho)\delta^2$$

symmetry energy = S

What can we learn about neutron EOS from Giant resonances and mass formulas?

Isospin dependence of GMR

Dipole polarizability in ^{208}Pb (Tamii)

In infinite matter,

$$\mathcal{E}(\rho,\delta \equiv \frac{\rho_n - \rho_p}{\rho}) = \mathcal{E}_0(\rho,\delta = 0) + \mathcal{E}_{\rm sym}(\rho)\delta^2.$$

Symmetry Energy

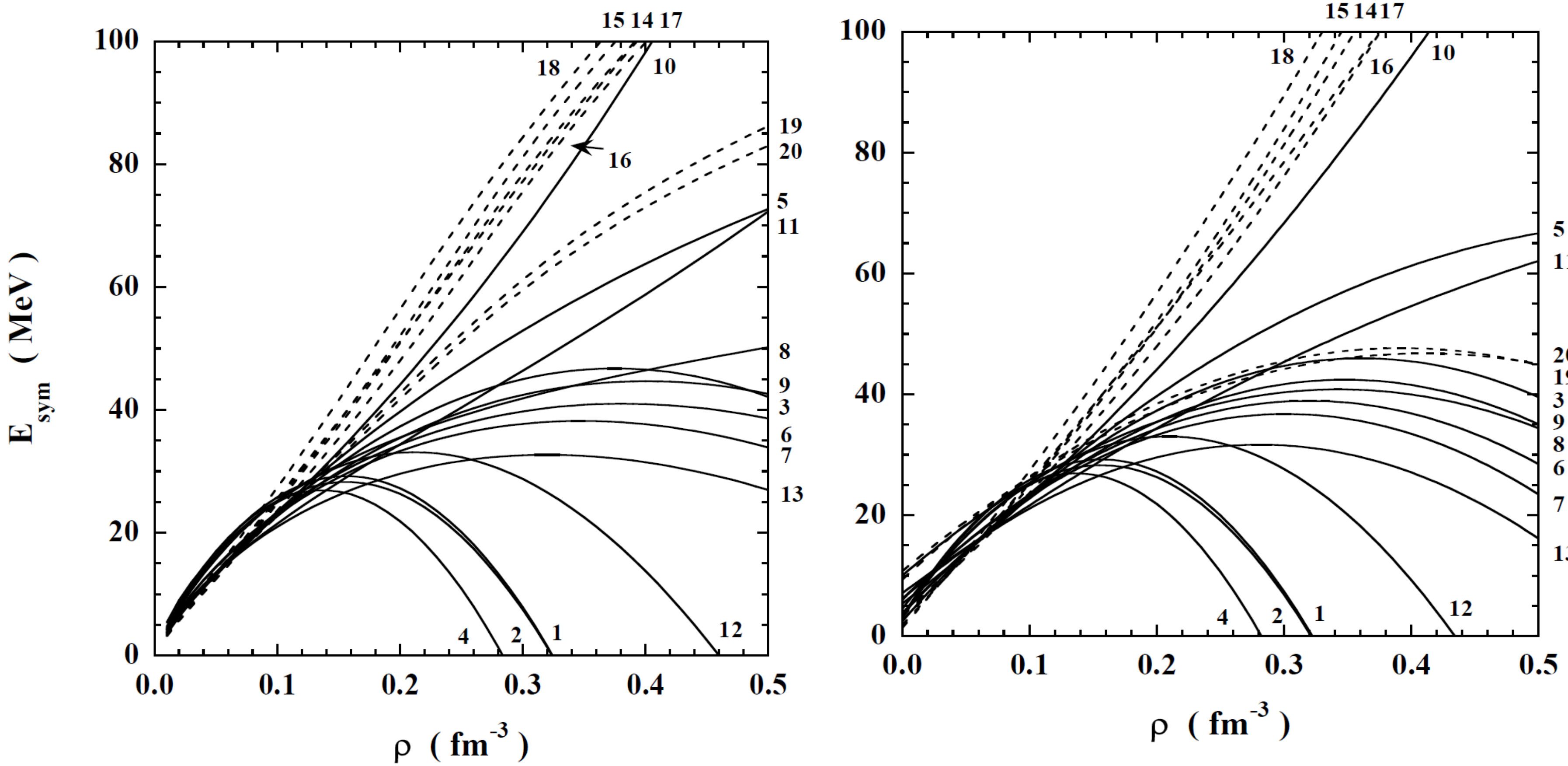
$$S(\rho)\ (=E_{sym}(\rho)/\rho) = \frac{1}{2}\frac{\partial^2(\varepsilon/\rho)}{\partial\delta^2} \quad \text{where } \delta=(\rho_n-\rho_p)/\rho$$

$$S(\rho) = J + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{1}{2}K_{sym}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2$$

$$\text{where } J=S(\rho_0),\ L=3\rho_0\left.\frac{\partial S}{\partial\rho}\right|_{\rho_0},\ K_{sym} = 9\rho_0^2\left.\frac{\partial^2 S}{\partial\rho^2}\right|_{\rho_0}$$

$$\varepsilon_\delta(\rho) = \frac{1}{2} \lim_{I \to 0} \frac{\partial^2}{\partial I^2} \left(\frac{H_{nm}}{\rho} \right)$$

$$\varepsilon_\delta(\rho) = J + \frac{L}{3}\frac{\rho - \rho_{nm}}{\rho_{nm}} + \frac{K_{sym}}{18}\left(\frac{\rho - \rho_{nm}}{\rho_{nm}}\right)^2.$$



Isospin dependence of GMR in Sn and Cd isotopes

$$E_{ISGMR} = \sqrt{\frac{\hbar^2 K_A}{m < r^2 >_m}},$$

$$K_A = K_\infty + K_{surf} A^{-1/3} + K_\tau \delta^2 + K_{Coul} \frac{Z^2}{A^{4/3}},$$

$$K_{surf} = 4\pi \, r_0{}^2 \left[4\sigma(\rho_{nm}) + 9\rho_{nm} \left. \frac{d^2\sigma}{d\rho^2} \right|_{\rho=\rho_{nm}} + \frac{54\sigma(\rho_{nm})\rho_{nm}^2}{K_\infty} \left. \frac{d^3h}{d\rho^3} \right|_{\rho=\rho_{nm}} \right]$$

$$K_{Coul} = \frac{3}{5}\frac{e^2}{r_0} \left(1 - \frac{27\rho_{nm}^2}{K_\infty} \left. \frac{d^3h}{d\rho^3} \right|_{\rho=\rho_{nm}} \right),$$

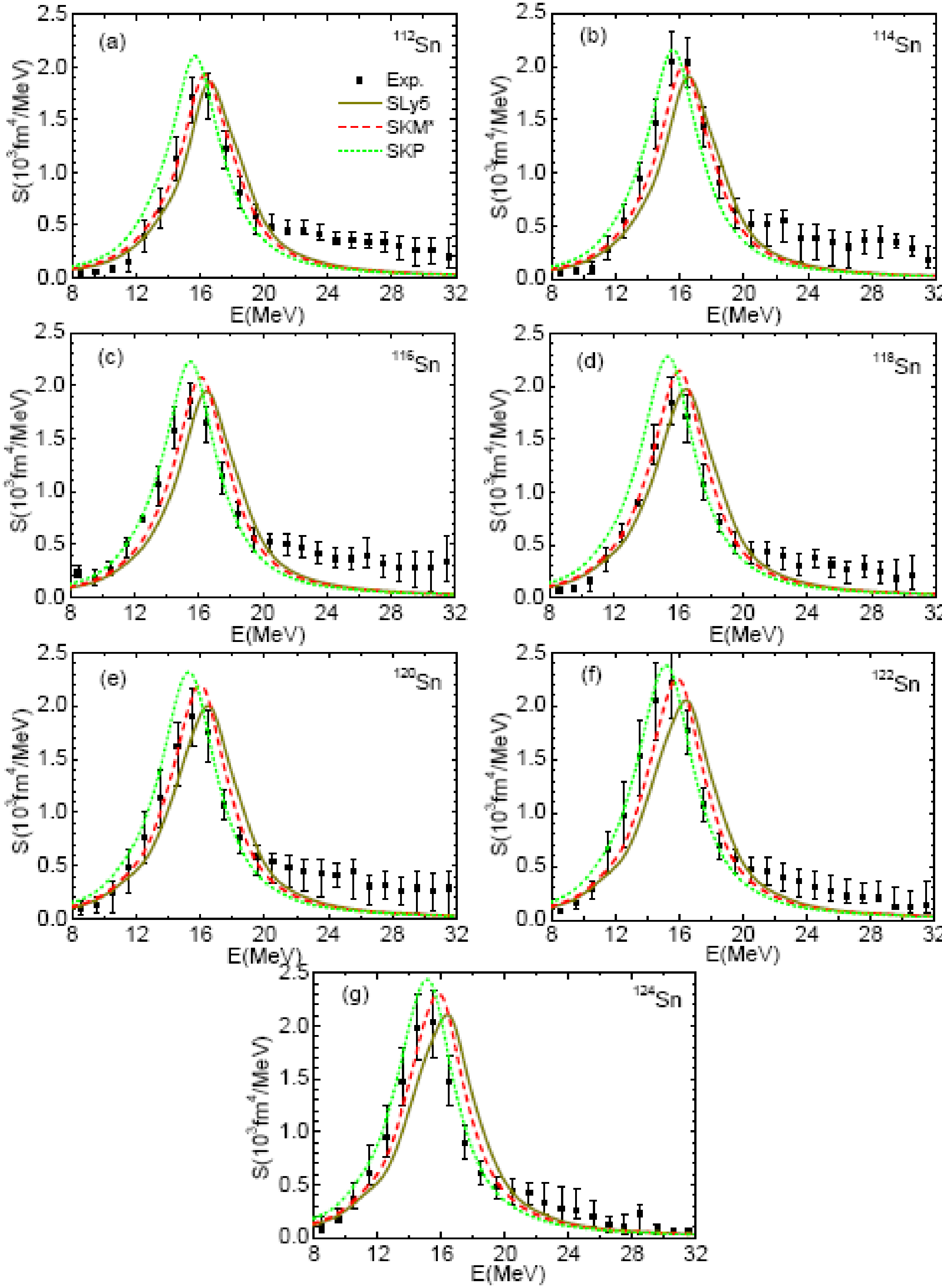
$$K_\tau = K_{sym} + 3L - \frac{27L\rho_{nm}^2}{K_\infty} \left. \frac{d^3h}{d\rho^3} \right|_{\rho=\rho_{nm}}$$

Results for Sn isotopes
Exp at RCNP

$$SLy5 \quad 230MeV$$

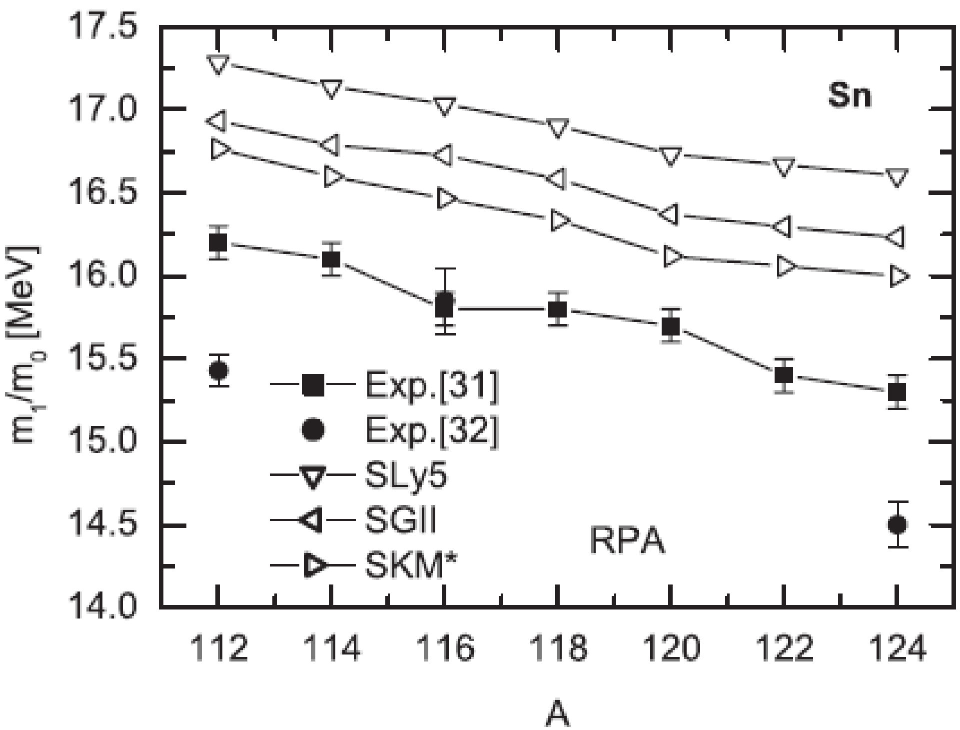
$$SKM^{*} \quad 217MeV$$

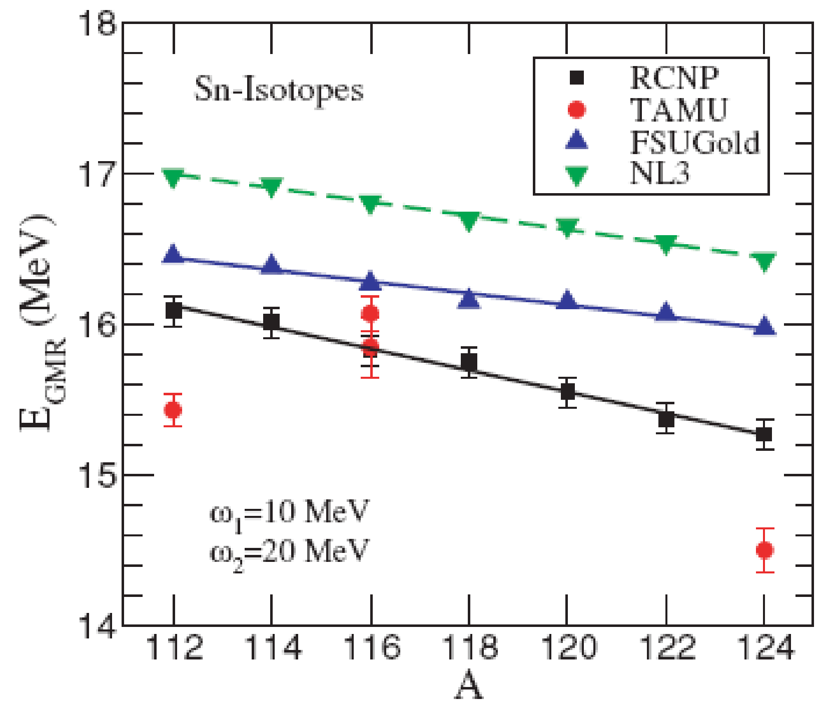
$$SKP \quad 202MeV$$



New problem is appeared.

Phys. Rev. Lett. 99, 162503 (2007).





Why Tin is so soft?

Or

Why Pb is so hard?

$SLy5 \quad 230 MeV$

$SKM^* \quad 217 MeV$

$SKP \quad 202 MeV$

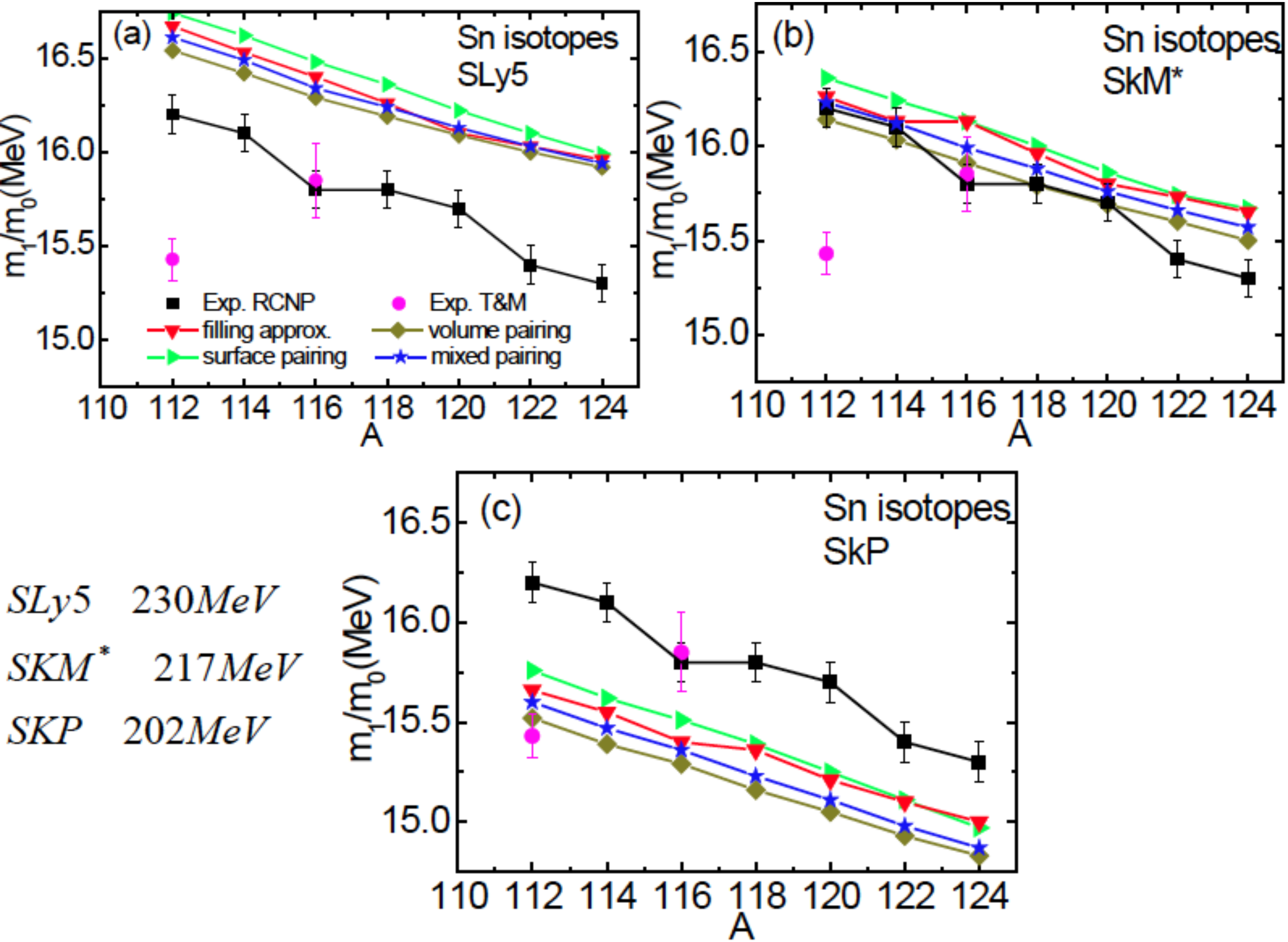
Based on the HFB+QRPA calculation, the ISGMR energies in Sn Isotopes are obtained using different Skyrme interaction, but There is No satisfied conclusion according to those calculation Because the calculations are not fully self-consistent, such as The two-body spin-orbit interaction is dropped .
J. Li et.al.,PRC78,064304(2008)

Or the HF+BCS+QRPA(QTBA). The spin-orbit interaction is dropped. V. Tselyaev, PRC 79, 034309 (2009)

T. Sil, et.al., Phys. Rev. C73, 034316 (2006). The spin-orbit residual interaction in HF+RPA produces an attractive effect on the ISGMR strength, the energies are pushed down by about 0.6MeV.
No pairing.

The strength function of QRPA is obtained by fully self-consistent HF+BCS+QRPA model with

Residual interaction： full Skyrme force, two-body spin-orbit, two-body Coulomb, and also the pairing in particle-particle channel



(a)
Sn isotopes
SLy5
Exp. RCNP
Exp. T&M
filling approx.
volume pairing
surface pairing
mixed pairing
(b)
Sn isotopes
SkM*
(c)
Sn isotopes
SkP
m_1/m_0(MeV)
A
SLy5 230MeV
SKM* 217MeV
SKP 202MeV

Results for Cd isotopes

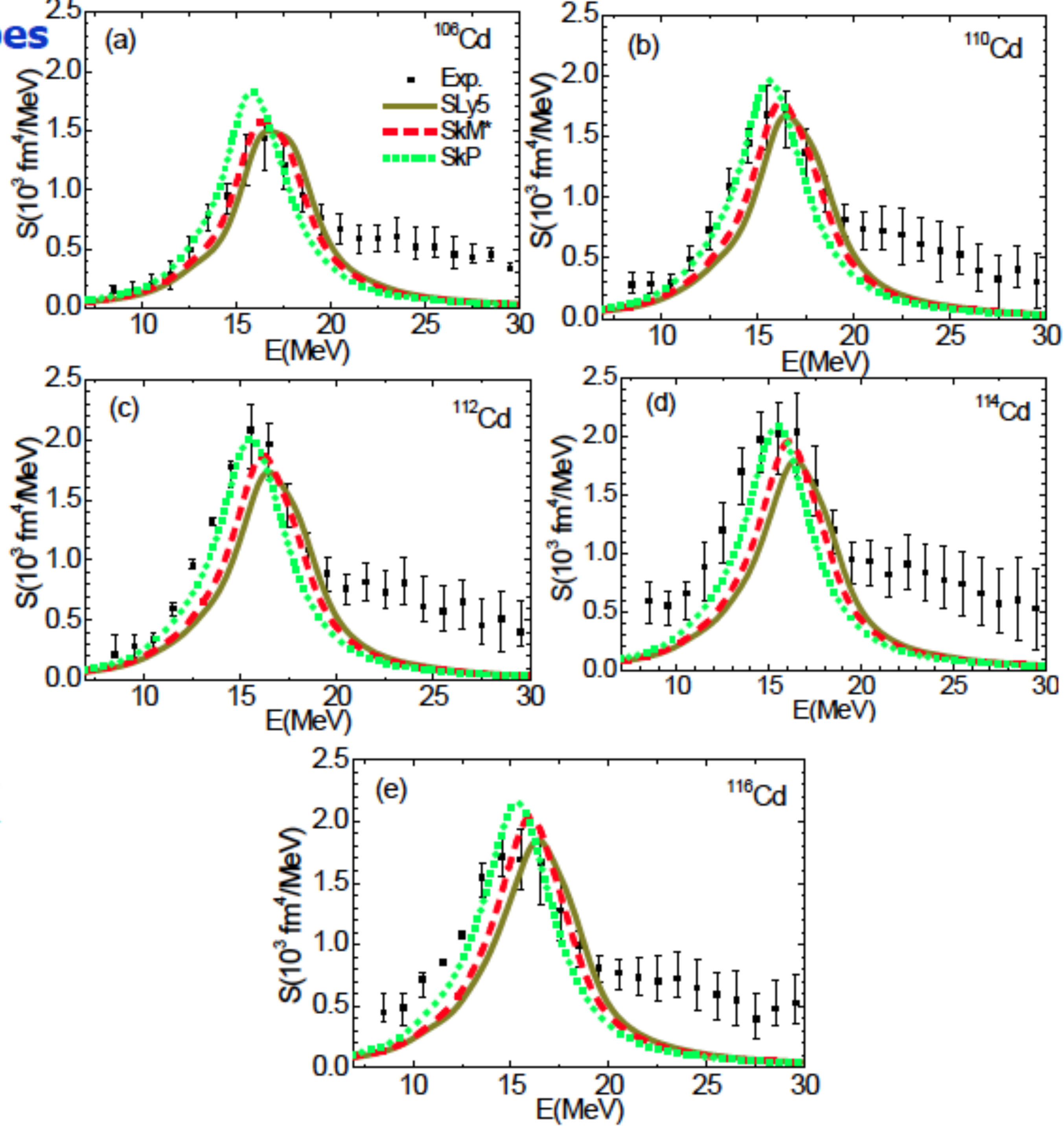
$SLy5 \quad 230 MeV$

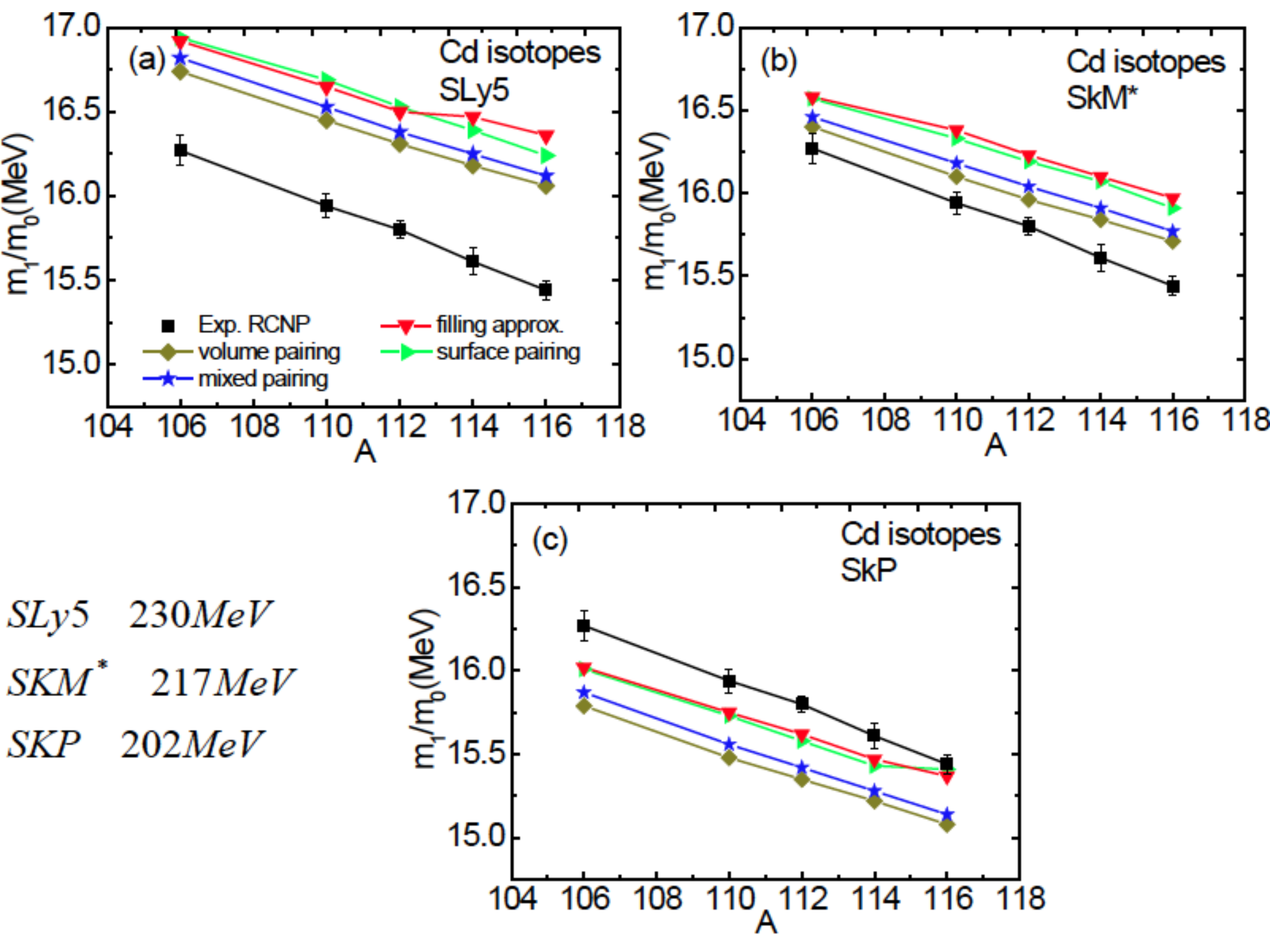
$SKM^{*} \quad 217 MeV$

$SKP \quad 202 MeV$

$HFBCS + QRPA$

mixed pairing




(a)
Cd isotopes
SLy5
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Cd isotopes
SkM*
(c)
Cd isotopes
SkP
Exp. RCNP
volume pairing
mixed pairing
filling approx.
surface pairing
m_1/m_0(MeV)
A
SLy5 230MeV
SKM* 217MeV
SKP 202MeV

Summary for Incompressibility

- **We have studied the ISGMR in Cd, Sn and Pb isotopes based on the fully self-consistent HF+BCS plus QRPA calculations. The SLy5, SKM*, and SKP and different pairing interactions are used in our calculations.**

- **We found that the pairing plays a role in producing the ISGMR properties.**

- **The SLy5 interaction (K_∞=230MeV) together with the effect of pairing can give better description on ISGMR in Pb isotopes, but it has some discrepancies between experiments in Cd and Sn isotopes.**

- **SKM* (K_∞=217MeV) can produce the experimental data in Cd and Sn isotopes, but is not satisfactory to describe Pb isotopes.**

- **SKP(K_∞=202MeV) fails for all isotopes because the incompressibility is too low.**

- K_∞=(225 +/- 10)MeV is consistent with Pb, Sn and Cd data.

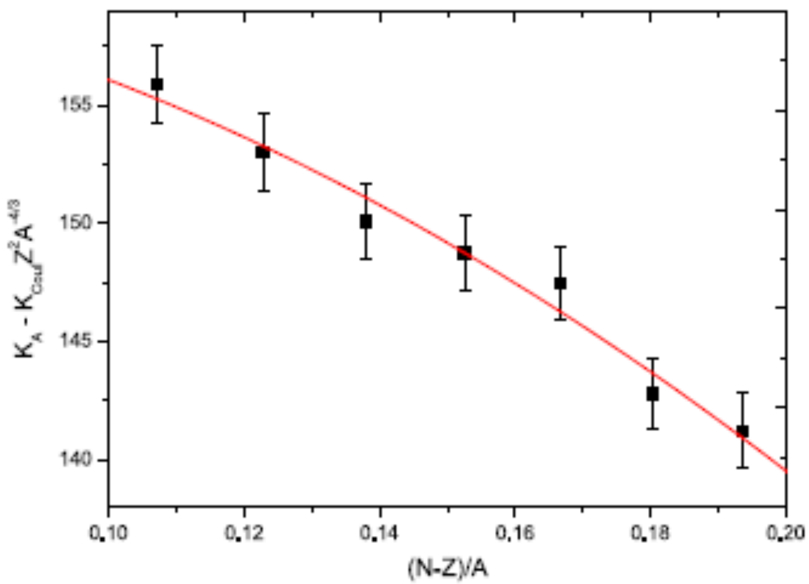
Extracting K_τ from data

$$K_A = K_\infty + K_{surf} A^{-1/3} + K_\tau \delta^2 + K_{Coul} \frac{Z^2}{A^{4/3}}$$

Using this formula globally is dangerous and should not be done (cf. M. Pearson, S. Shlomo and D. Youngblood) but one can use it locally.

K_{Coul} can be calculated and ETF calculations point to $K_{surf} \approx -K_\infty$.

$$K_A - K_{Coul} \frac{Z^2}{A^{4/3}} = K_\infty (1 - A^{-1/3}) + K_\tau \delta^2$$



TERS
week ending
19 OCTOBER 2007

K_τ = -500 ± 50 MeV

in the Even-A $^{112-124}$Sn Isotopes
ncompressibility

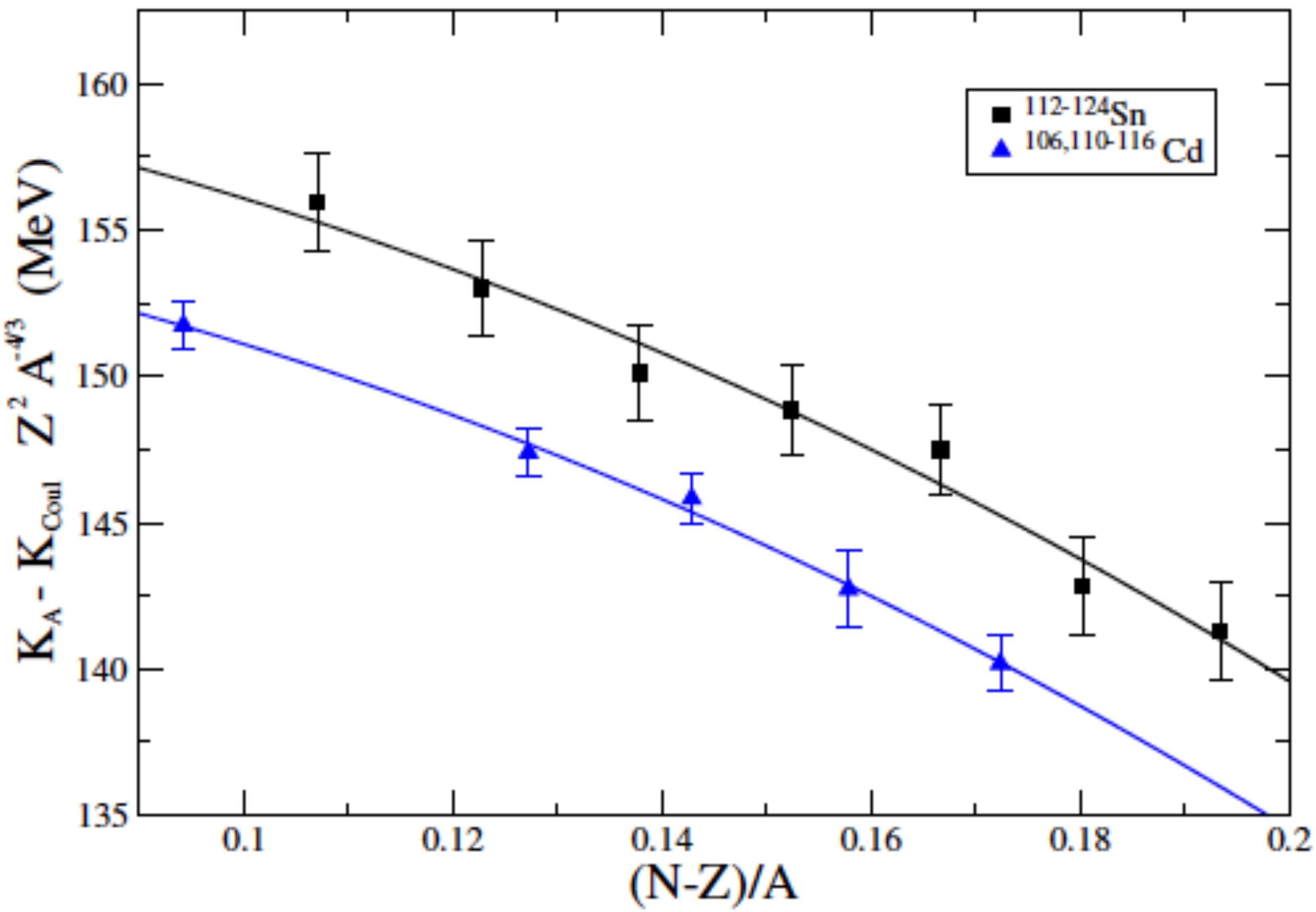
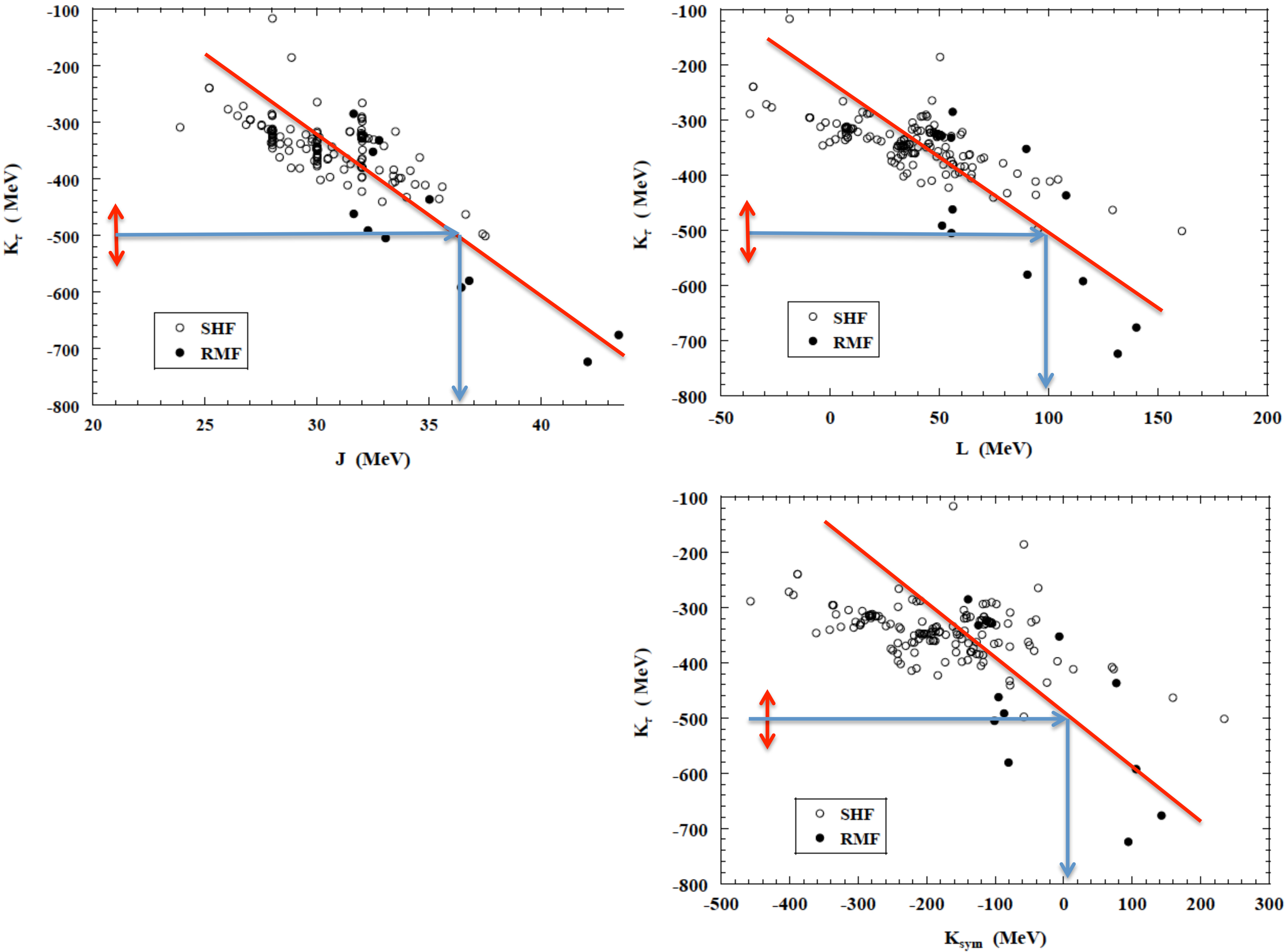


Fig. 7. The difference $K_A - K_{Coul}Z^2A^{-4/3}$ in the Sn and Cd isotopes plotted as a function of the asymmetry parameter, $(N - Z)/A$. The data are from Refs. [39,31]. The values of K_A have been derived using the customary moment ratio $\sqrt{m_1/m_{-1}}$ for the energy of ISGMR, and a value of 5.2 ± 0.7 MeV has been used for K_{Coul} (see previous Section). The solid lines correspond to K_τ = - 550 MeV.



K_τ (MeV)
J (MeV)
SHF
RMF
K_τ (MeV)
L (MeV)
SHF
RMF
K_τ (MeV)
K_sym (MeV)
SHF
RMF

Correlation between Isospin GMR and nuclear matter properties

1.Nuclear incompressibility K is determined empirically with the ISGMR in ^{208}Pb to be

K~230MeV(Skyrme,Gogny), K~250MeV(RMF).

K=(240 +/-10 +/- 10)MeV

2. Combining ISGMR data of Sn and Cd isotopes(RCNP)

K=(225 +/-10)MeV

3. $K_\tau = -(500 \pm 50)\mathrm{MeV}$ is extracted from isotope dependence of ISGMR.

4. This value provides further the isovector properties

J=(36+/-2)MeV, L=(100+/-20)MeV, Ksym= -(0+/-40)MeV

by using the mean field correlations

5. How much we can trust to extract K_{tau} by a single set od data Ni isotopes

(Maya/Ganil/Orsay) (E. Khan)

Mass model and EoS

PRL 108, 052501 (2012) PHYSICAL REVIEW LETTERS week ending 3 FEBRUARY 2012

New Finite-Range Droplet Mass Model and Equation-of-State Parameters

Peter Möller,[1,*] William D. Myers,[1] Hiroyuki Sagawa,[2] and Satoshi Yoshida[3]

Symmetry Energy

$$S(\rho) = \frac{1}{2}\frac{\partial^2(\varepsilon/\rho)}{\partial\delta^2} \quad \text{where } \delta = (\rho_n - \rho_p)/\rho$$

$$S(\rho) = J + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{1}{2}K_{sym}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2$$

$$\text{where } J = S(\rho_0),\ L = 3\rho_0 \left.\frac{\partial S}{\partial\rho}\right|_{\rho_0},\ K_{sym} = 9\rho_0^2 \left.\frac{\partial^2 S}{\partial\rho^2}\right|_{\rho_0}$$

Successive FRDM enhancements

Optimization $(1 \rightarrow 2)$

Better search for optimum FRDM parameters.

Accuracy improvement: 0.01 MeV

New mass data base (AME2003) $(2 \rightarrow 3)$

Better agreement than with AME1989.

Accuracy improvement: 0.04 MeV

Full 4D energy minimization $(3 \rightarrow 4)$

Full 4D minimization($\epsilon_2, \epsilon_3, \epsilon_4, \epsilon_6$) step=0.01.

Accuracy improvement: 0.02 MeV

Axial asymmetry $(4 \rightarrow 5)$

Also yields correct SHE gs assignments.

Accuracy improvement: 0.01 MeV

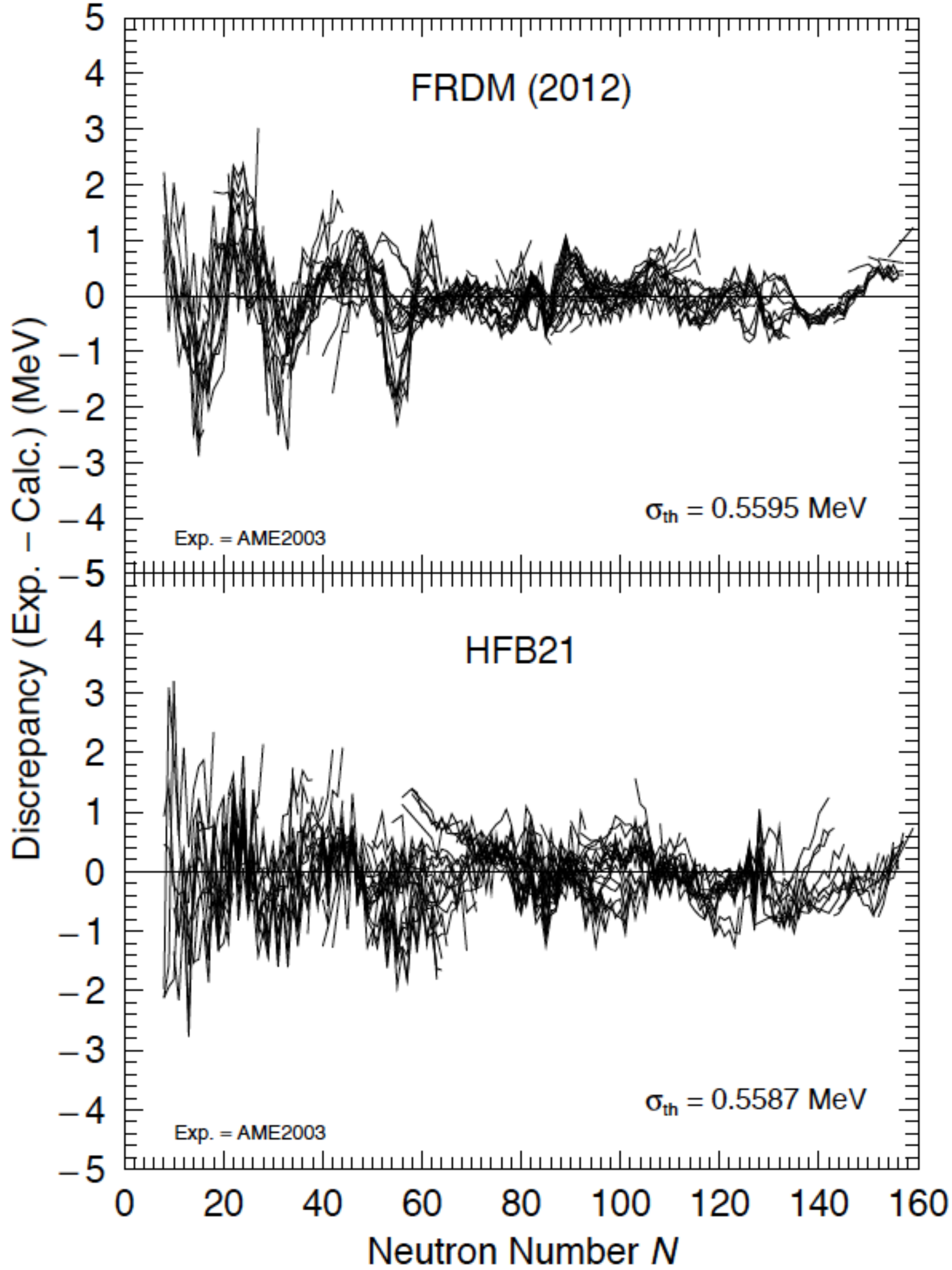
L **variation** $(5 \rightarrow 7)$

Accuracy improvement: 0.02 MeV

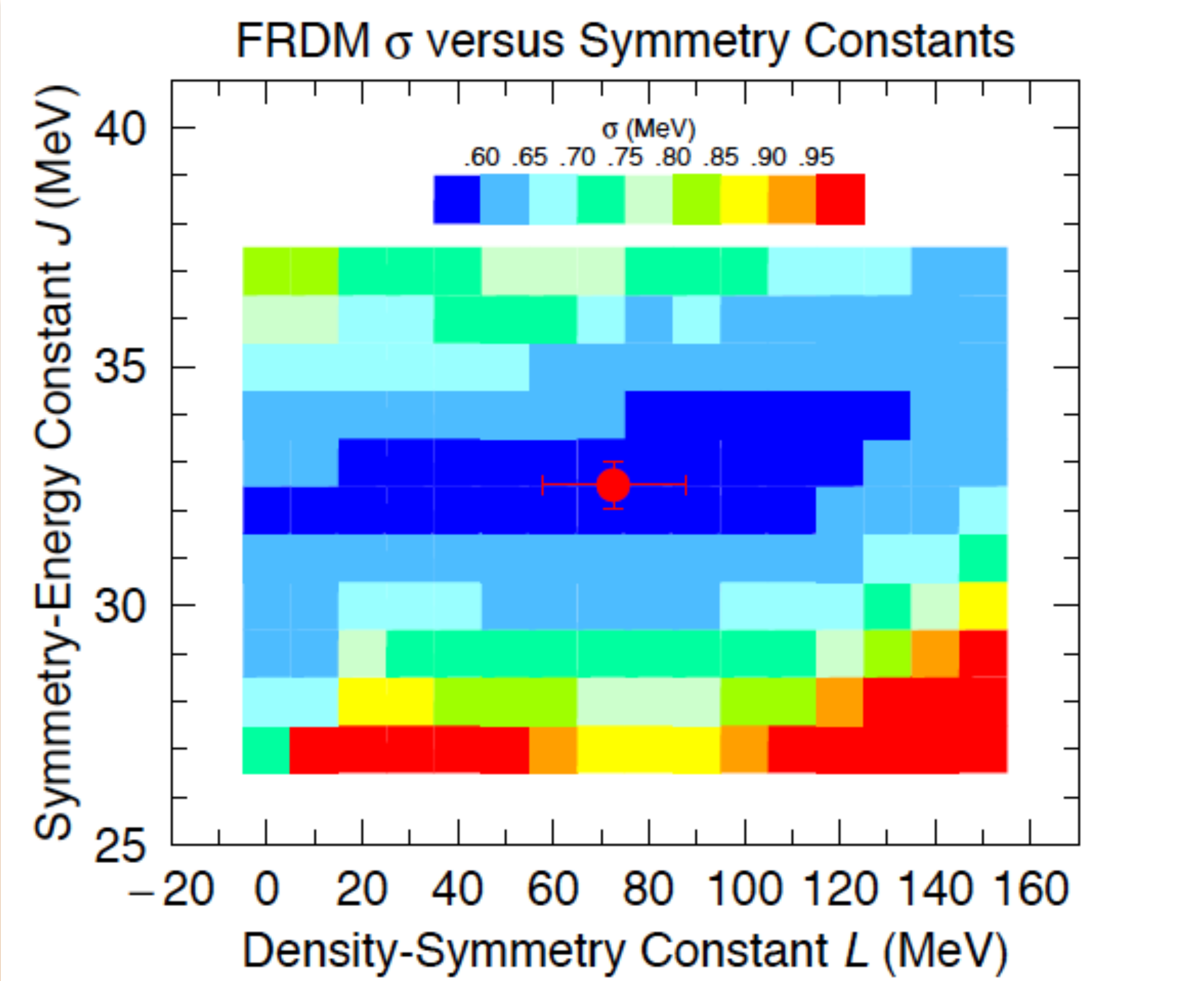
Improved gs correlation energies $(7 \rightarrow 8)$

Accuracy improvement: 0.01 MeV

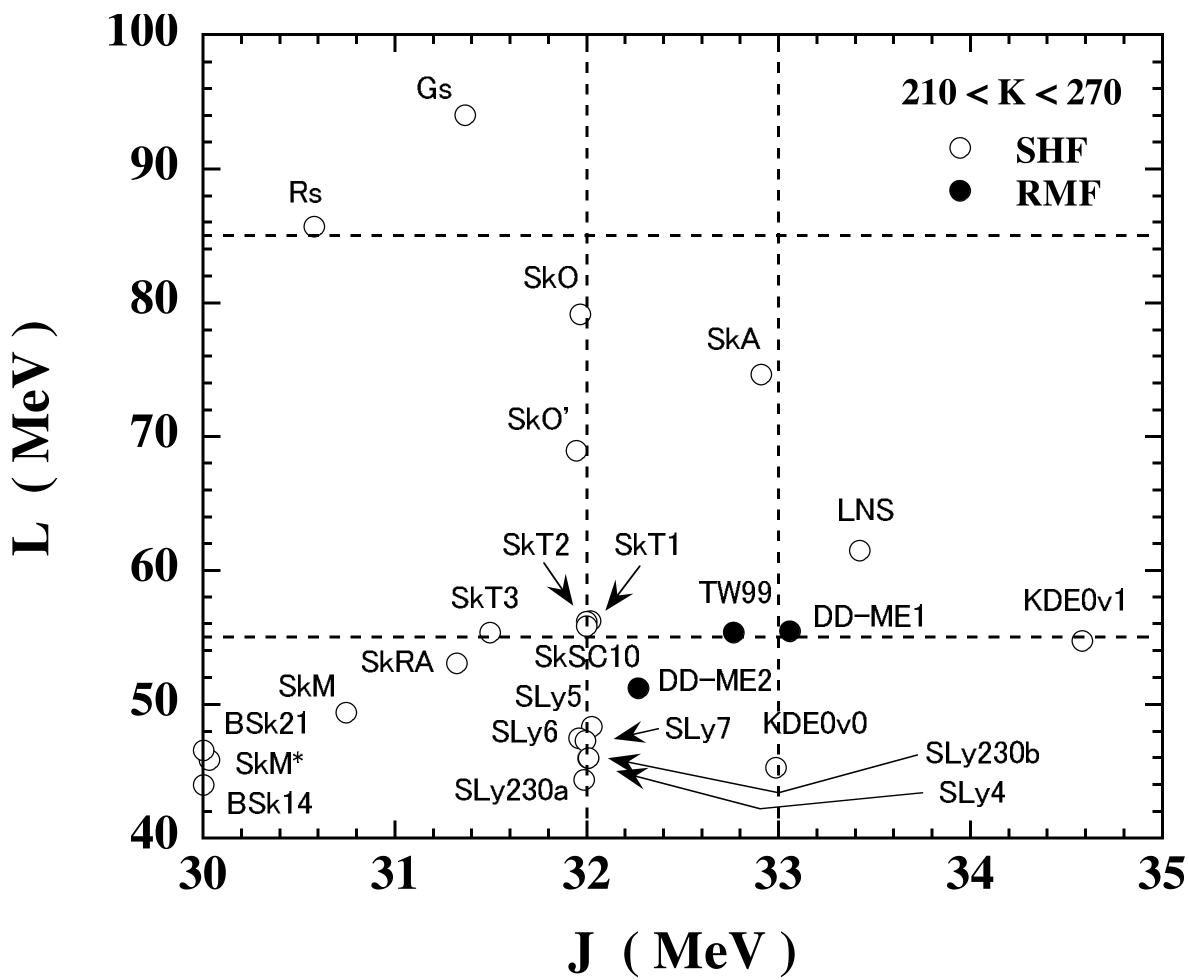
The state of the art achievement



FRDM (2012)
σth = 0.5595 MeV
Exp. = AME2003
HFB21
σth = 0.5587 MeV
Exp. = AME2003
Discrepancy (Exp. − Calc.) (MeV)
Neutron Number N

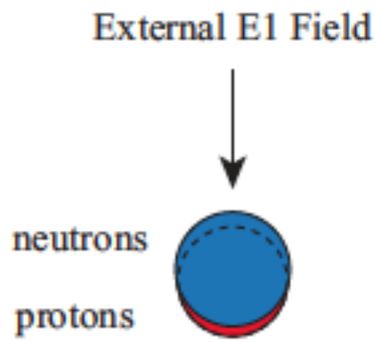


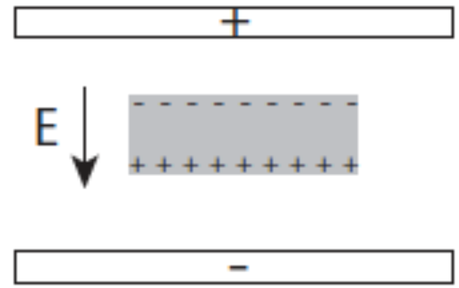
J=32.5+/-0.5MeV L=70+/-15MeV (54+/-15MeV)



J=32.5+/-0.5MeV L=70+/-15MeV (L=54+/-15MeV)

$K_{infinity}$=240+/-30MeV (94 Skyrme interactions and 7RMF Lagrangians)

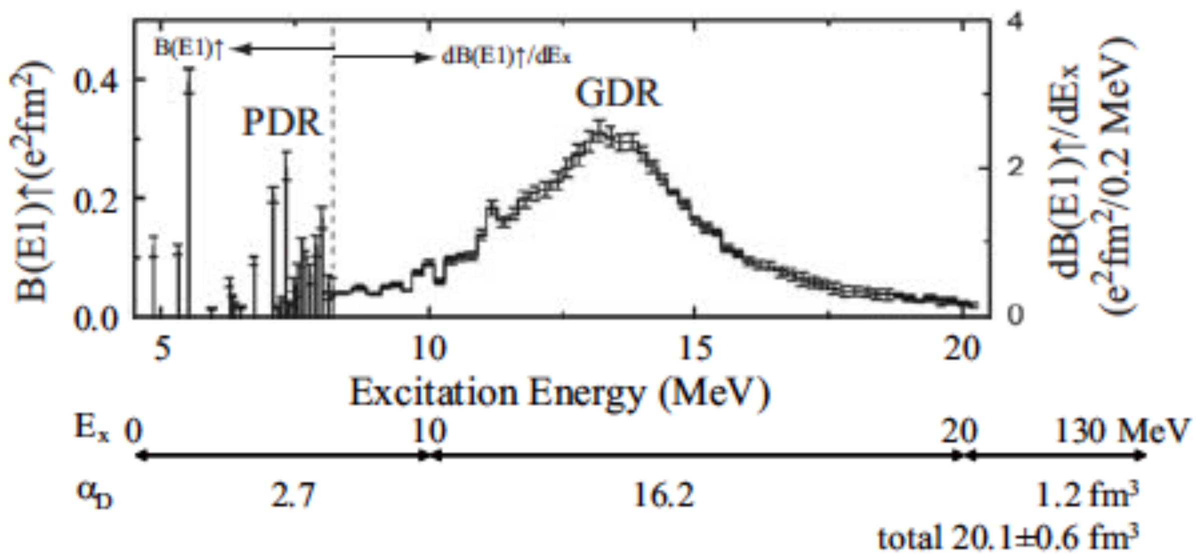




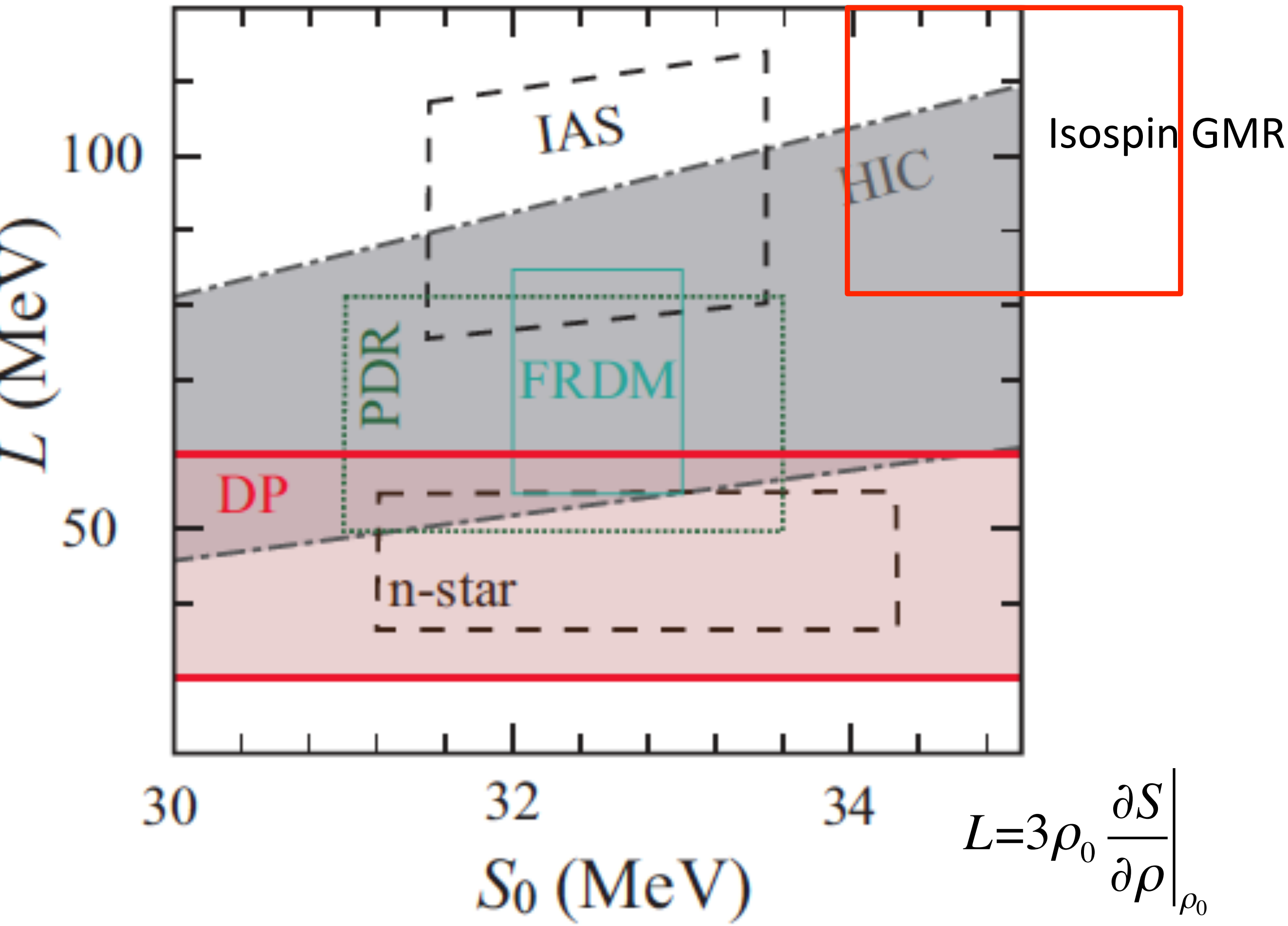
Electric Dipole Polarizability

Electric Dipole Polarization

$$\alpha_D = \frac{\hbar c}{2\pi^2}\int \frac{\sigma_{abs}}{\omega^2}d\omega = \frac{8\pi}{9}\int \frac{I(\omega)}{\omega}d\omega$$



Tamii and Zenihiro, JPS journal(2014)



Sotani (2013)
Tortional Oscillation
100<L<130MeV

$$L = 3\rho_0 \left.\frac{\partial S}{\partial \rho}\right|_{\rho_0}$$

$$S' = c\Delta r_{np}$$

$$\Delta r_{np} = 0.16 \sim 0.20\, fm \quad L = (60-80)MeV$$

$$\Delta r_{np} = (0.302 \pm 0.175)\, fm \text{ (Jefferson Lab)}$$

$$L \sim (100-120)MeV$$

Courtesy of Tamii(2013)

Summary of Symmetry Energy Studies

1. Micro-macroscopic model (FRDM) is further improved taking into account the optimization of symmetry energy coefficients J and L:

   J=32.5 +/-0.5 MeV
   L=70 +/-15 MeV
   (55+/-15 MeV with high-order effect of fluctuations)

2. Isospin dependence of GMR gives somewhat larger J and L which should be confirmed further by new experiments in RIKEN/CNS (L=100+/-20MeV which is close to torsional oscillation analysis by Sotani).

3. Effective Hamiltonians and Lagrangians (for examples, SkT1, SLy5, TW99, DD-ME2) satisfy the conditions from ISGMR and Mass model studies.