Equilibrium configuration and stability of magnetized neutron stars

Kenta Kiuchi (YITP)

Collaborator : Shijun Yoshida (Tohoku Univ.), Masaru Shibata(YITP)

Ref. 1) Phys.Rev.D., 86, 044012 (2012)
Ref. 2) Astron. Astrophys., 532,A30 (2011)
Ref. 3) Phys.Rev.D., 78, 024029 (2008)

YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

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Talk plan

0. Introduction

1. Stabilities of pure fields

   1.1 Pure poloidal fields (review)

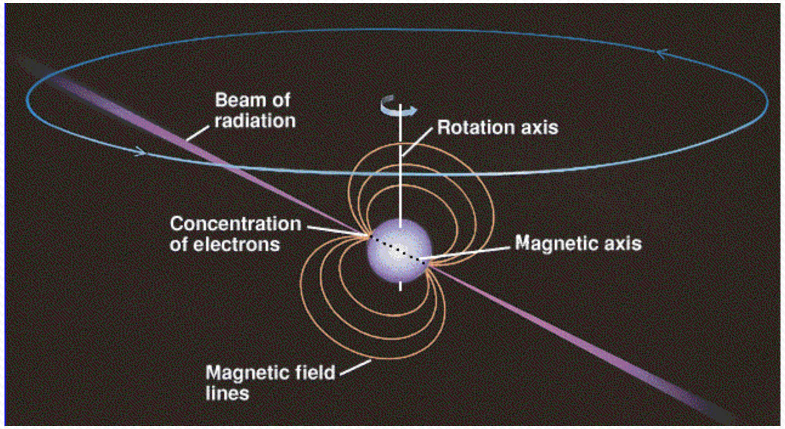
   1.2 Pure toroidal fields (based on our works)

2. Stably stratified magnetized neutron star model

3. Summary

Introduction

Observational evidence

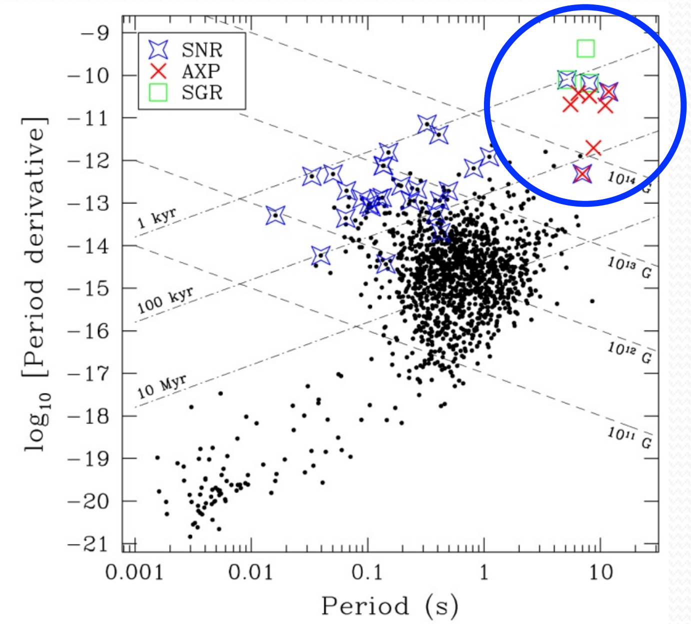


Pulsar = magnetized neutron star; Manetic dipole radiation model gives

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}I\Omega^2\right) = -4\pi^2 I\frac{\dot{P}}{P^3}$$

$$\Rightarrow \quad B_p \propto (IP\dot{P})^{1/2}R^{-3}$$

$P - \dot{P}$ Diagram



- Radio pulsars : $B \sim 10^{11-13}$G (Manchester 04)
- Millisecond pulsars : $B \sim 10^{8-9}$G
- Anomalous x-ray pulsars and soft gamma-ray repeaters : $B \sim 10^{14-15}$G(Woods & Tomposon 04)

Another important evidence   Rea et al. 04

Spectral features during SGR/AXP bursts $\Rightarrow$ $B \sim 10^{15}$G if interpreted as proton cyclotron lines $E = \bar{h}eB/m_pc = 5.79(B/10^{15})$ keV

Introduction

Microphysics in strongly magnetized stars (Harding & Lai 06)

$B \gtrsim B_{QED} = 2\pi m_e^2 c^3 / eh = 4.414 \times 10^{13}$ G

electron gyration radius = de Broglie wavelength

✓ Modification of atoms, molecules, and condensed matters

✓ Propagation of photon (X-mode & O-mode)

✓ Equation of State in outer crust : $P_e \propto B^{-2} \rho^3$ for electron

✓ Modified thermal diffusion coefficients ⇒ Thermal evolution of NS

Macrophysics in magnetized stars

✓ Fossil or Dynamo ? (Wickramasinghe & Ferrario 05, Thompson & Duncan 93)

✓ (Dynamical) Stability of magnetic fields (Parker 66, Tayler 73, Wright 73)

Introduction

✓ Magnetic fields of neutron star $\sim 10^{11\text{-}15}$G (Manchester 04, Woods & Tompson 04)

✓ Various magneto hydrodynamical (MHD) instability (Parker 66, Tayler 73, Wright 73)

Growth timescale of MHD instability ~ Alfvén timescael t_A

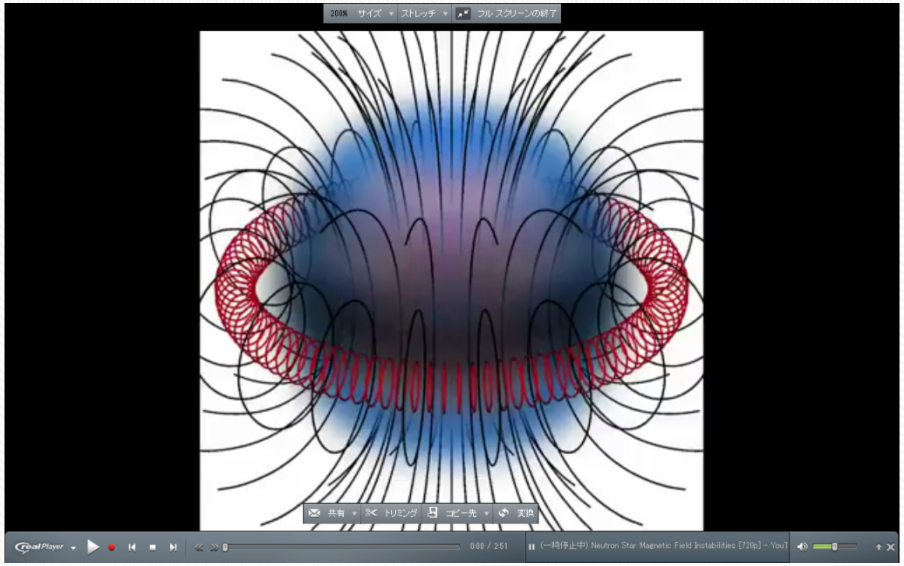
$$t_A \sim 1\mathrm{s}\left(B/10^{13}\mathrm{G}\right)^{-1}(\rho/10^{15}\mathrm{g/cm^3})^{1/2}(R/10\mathrm{km}) << t_{\mathrm{lifetime}}$$

How do magnetized neutron stars stably exist ?

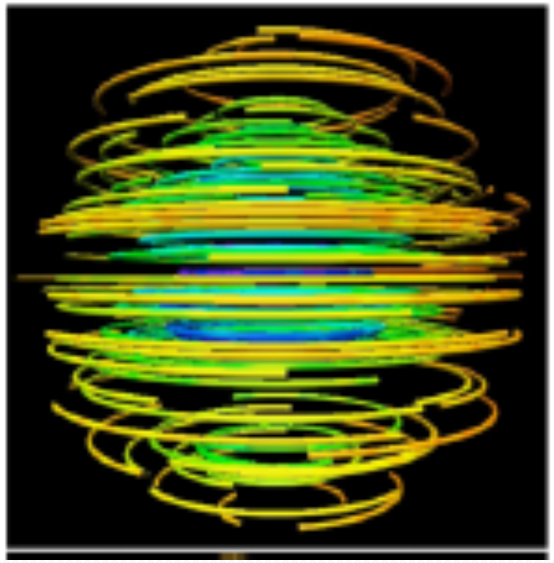
Stabilities of pure fields

Pure fields = Purely poloidal or toroidal fields

Poloidal field (Lasky+ 11)



Toroidal field (Duez+ 10)

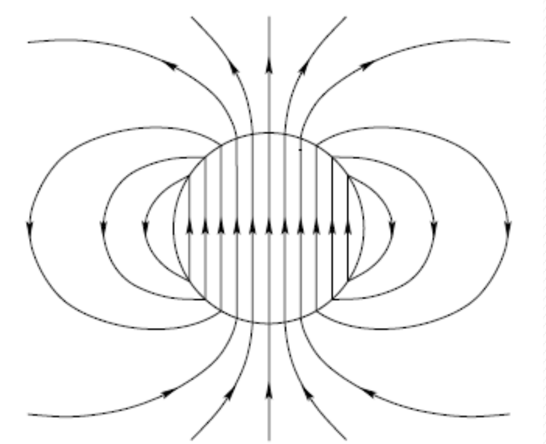


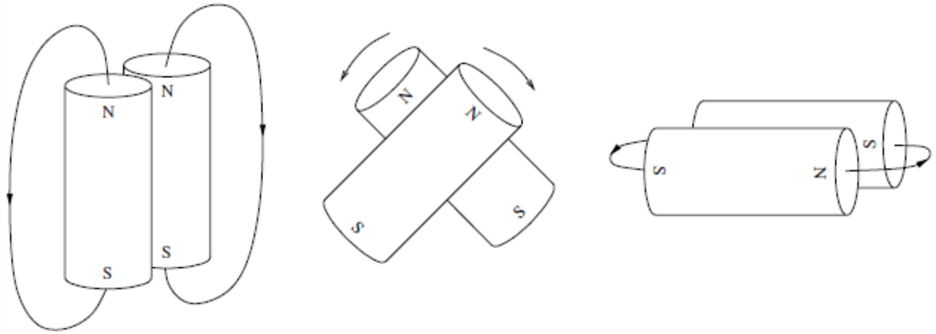
Pure poloidal fields

Field line close outside the star (Flowers-Ruderman 77)

Field lines (Braithwaite & Spuruit 06)

Two bar magnets ⇒ Unstable



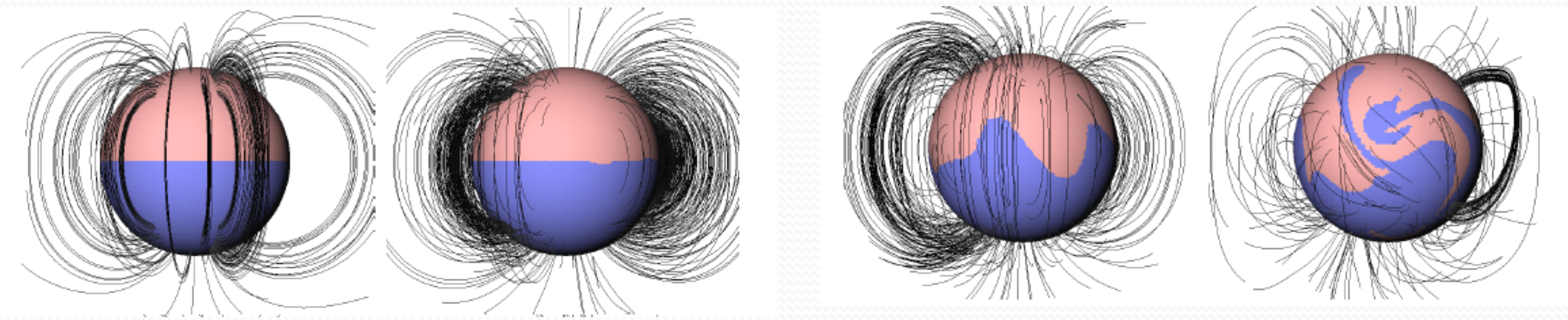


E_{mag} > E_{mag}

Non-linear simulation (Braithwaite & Spuruit 06)

t_A : Alfven crossing time

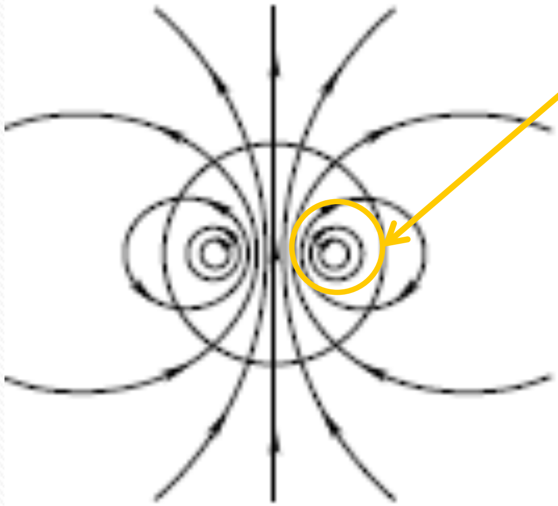
t=0 | t ~4 t_A | t ~5 t_A | t~ 5.7 t_A



Pure poloidal fields

Field line close inside the star (Wright 73, Markey and Tayler 73)

Field lines (Braithwaite & Spuruit 06)



Neutral point (B=0) is a point where the instability emerges

Dipole field is unstable, but the neutron stars is likely to have a dipole field.

Two independent Numerical Relativity simulations in 2011

- ✓ P. D. Lasky+ 11 (Univ. of Tubingen)
- ✓ Ciolfi+ 11 (AEI & Univ. of Southampton)

Pure poloidal fields

Courtesy to Paul D. Lasky

Emergence of instability near the neutral line (B=0) (Markey & Tayler 73)

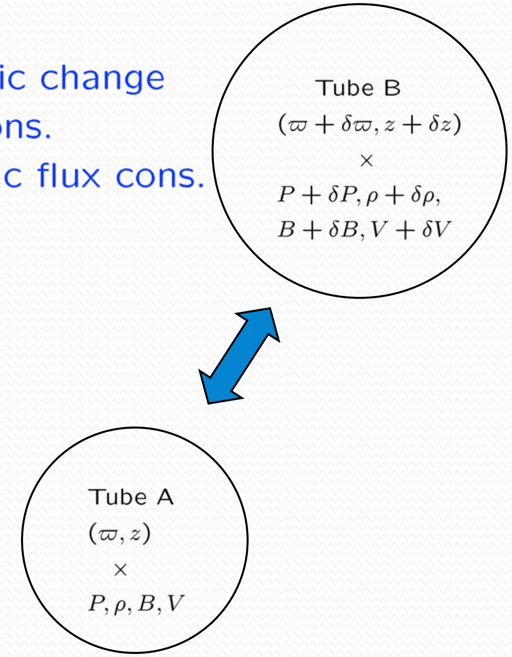
Kink instability (perp. to gravity)

Quasi-equilibrium non-axisymmetric structure

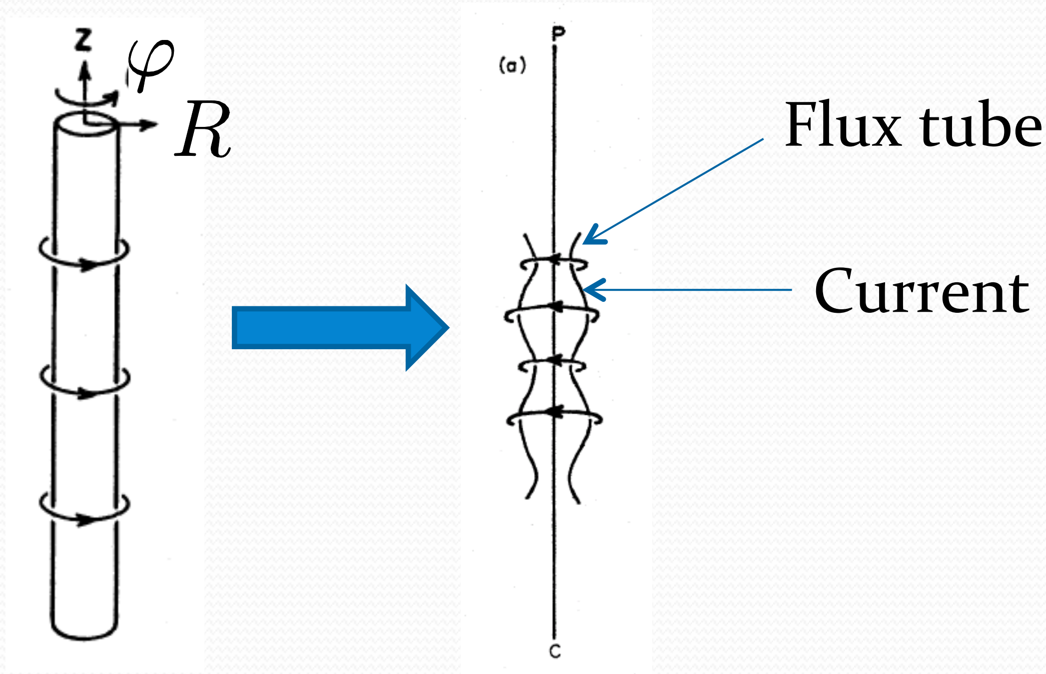
Pure toroidal fields

Interchange instability (Tayler 73)

- Adiabatic change
- Mass cons.
- Magnetic flux cons.



Schematic picture of the instability



✓ Instability criterion : $\partial_R \ln\left(\frac{B_{(\varphi)}}{\rho R}\right) > 0$ near R ~ 0

⇒ Regular field is marginally stable or unstable

✓ m=0 mode (axisymetric mode) ⇒'varicose' or 'sausage' instability

Pure toroidal fields (KK 08)

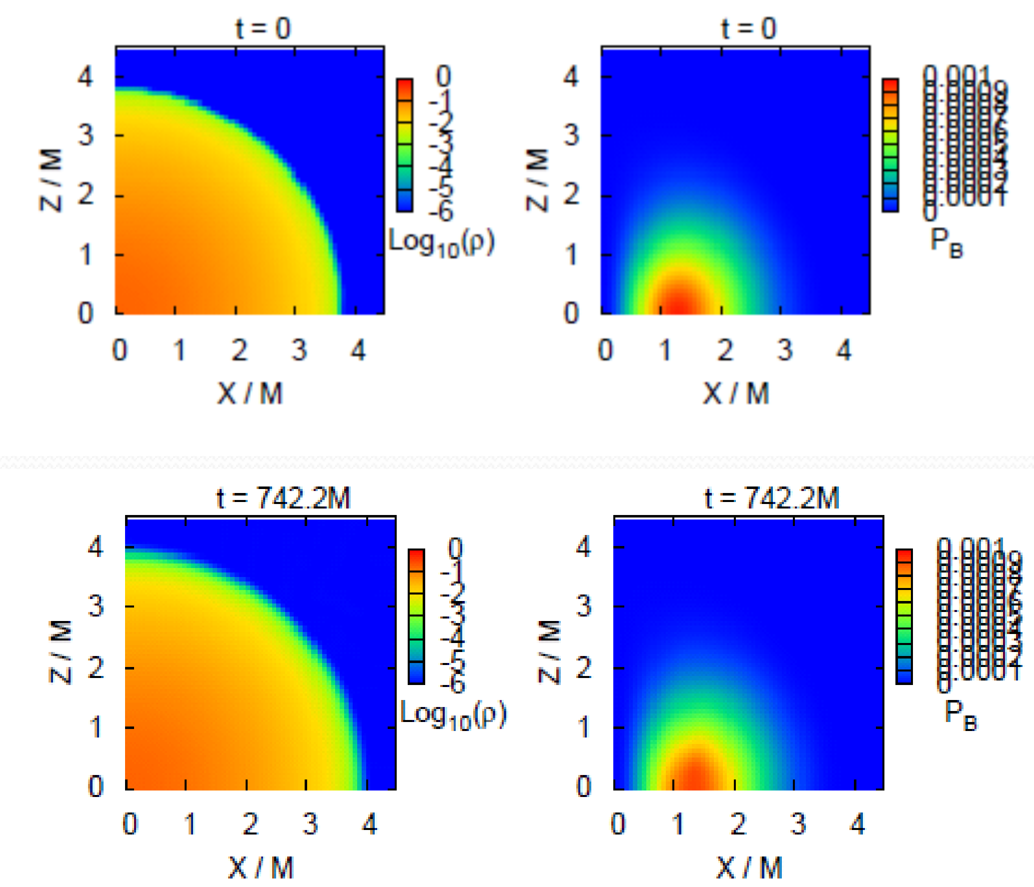
$$\partial_R \ln\left(\frac{B_{(\varphi)}}{\rho R}\right) = 0 \text{ for } B_{(\varphi)} \propto R \rightarrow \text{Marginally stable}$$

✓ Axisymmetric GRMHD simulation

Density　　Magnetic field energy density

t=0

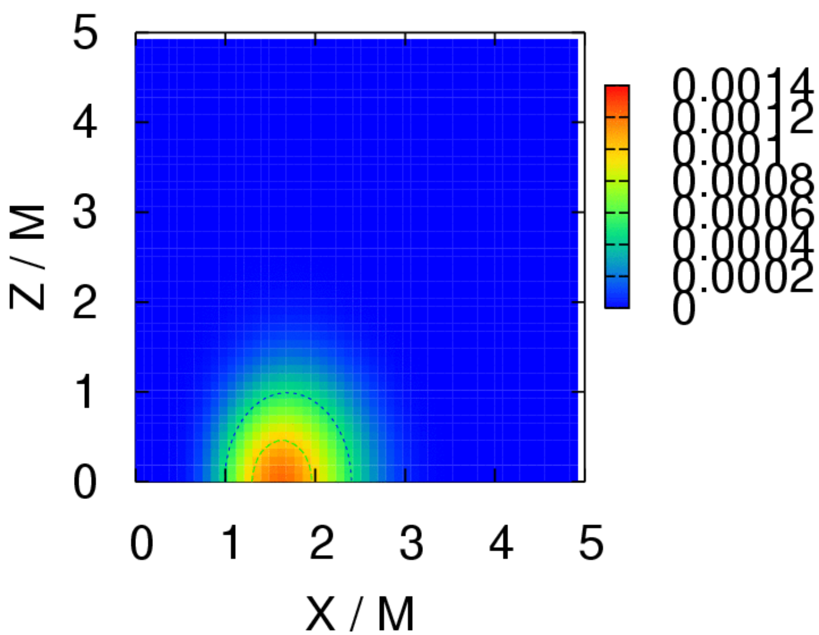
$t \approx 11 t_A$



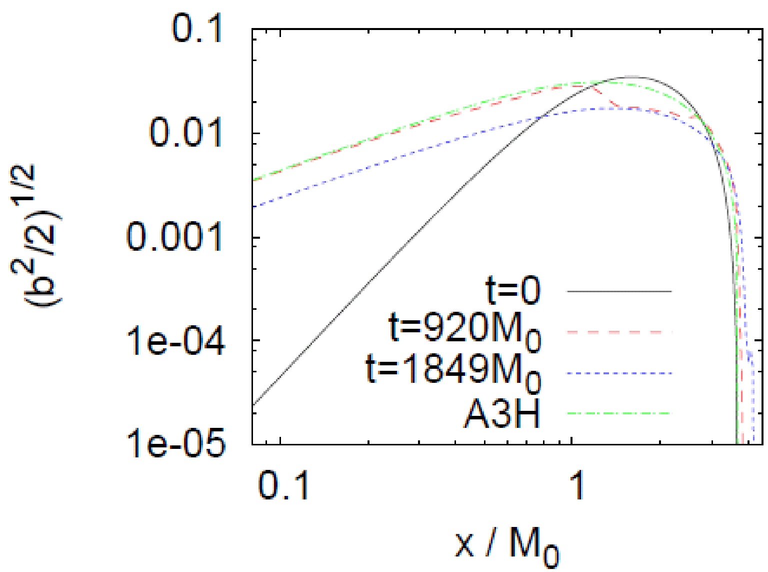
Pure toroidal fields (KK 08)

$$\partial_R \ln\left(\frac{B_{(\varphi)}}{\rho R}\right) > 0 \text{ for } B_{(\varphi)} \propto R^3 \rightarrow \text{Unstable}$$

Magnetic field energy density



Snapshot

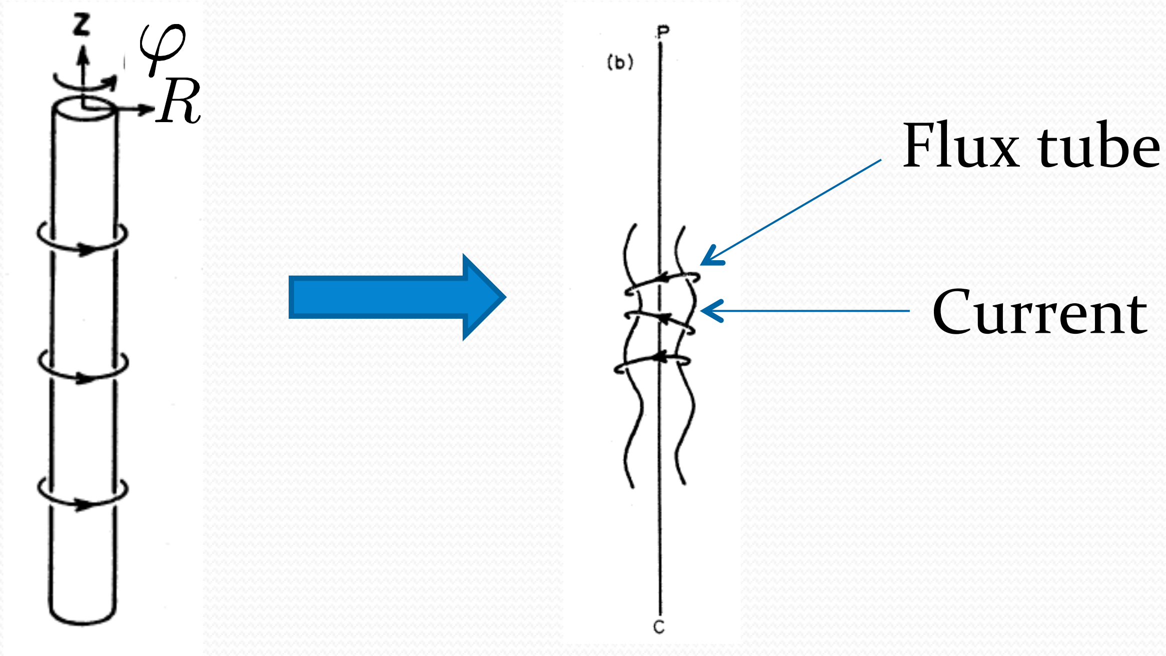


Final profile is $B_{(\phi)} \propto R$

Pure toroidal fields

Tayler instability (Tayler 73)

Schematic picture of the instability



- ✓ Instability criterion : $-\frac{2}{R}\partial_R \ln\left(B_{(\varphi)}R\right) + \frac{m^2}{R^2} < 0$ near R ~ 0

$\Rightarrow$ m=1 mode is most unstable for any regular fields

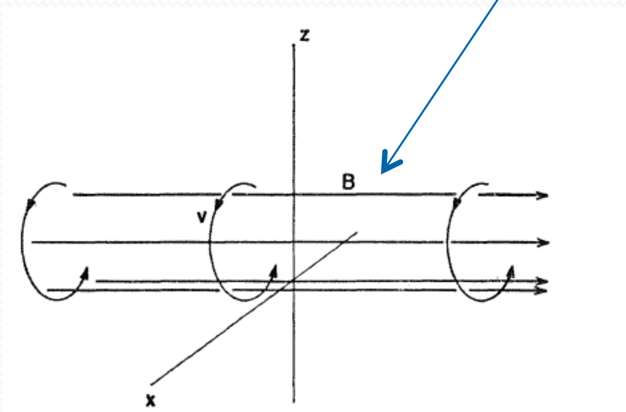
- ✓ m=1 mode $\Rightarrow$'kink' instability

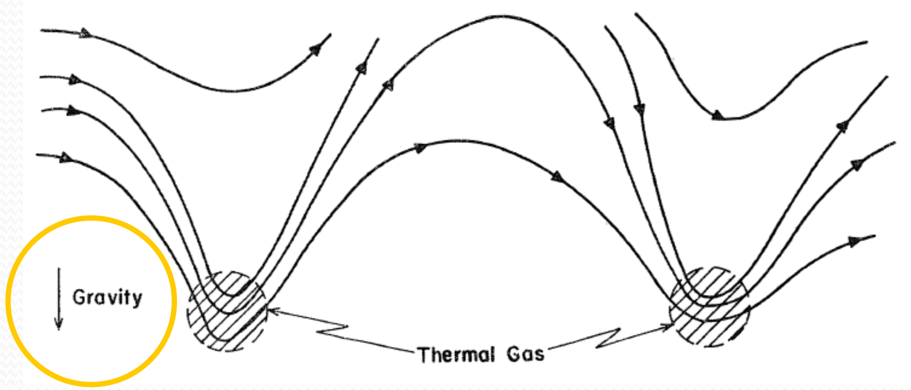
Pure toroidal fields

Parker instability (magnetic buoyancy instability) (Parker 66)

Schematic picture of instability

Field line





✓ Instability criterion : $\left(\frac{2}{R_c} - \frac{2}{R}\right) \partial_R \ln \left(B_{(\varphi)} R\right) + \frac{m^2}{R^2} < 0$

for $R > R_c = 2\, c_s^2/g$: critical radius

✓ $R > R_c$ ⇔ Magnetic hoop stress < magnetic buoyancy force

✓ m/=0 mode

Pure toroidal fields (KK et al. 11)

Set up

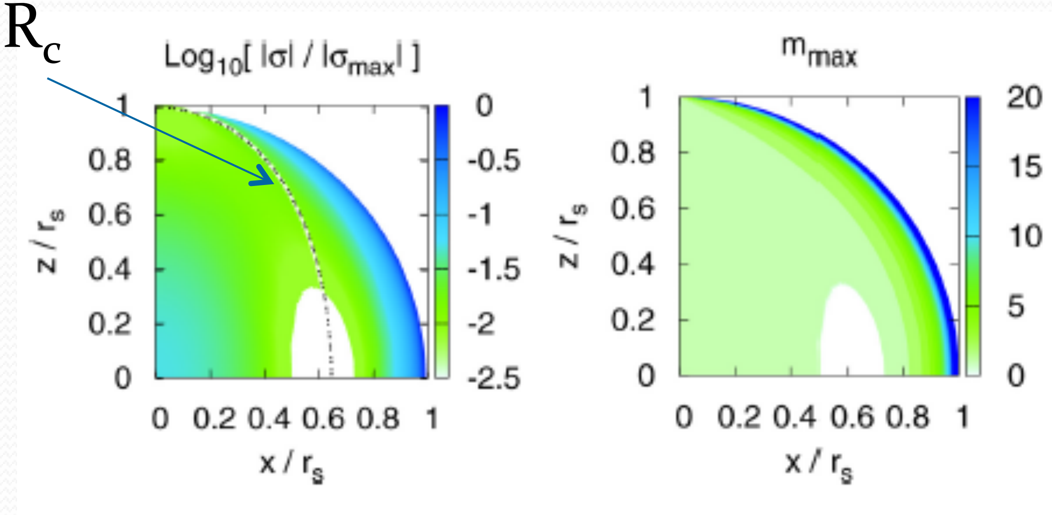
✓ Initial condition = Toroidally magnetized star in equilibrium (KK & Yoshida 08)

✓ 3D GRMHD simulation

Prediction based on the liner analysis

Growth rate Unstable mode

critical radius R_c



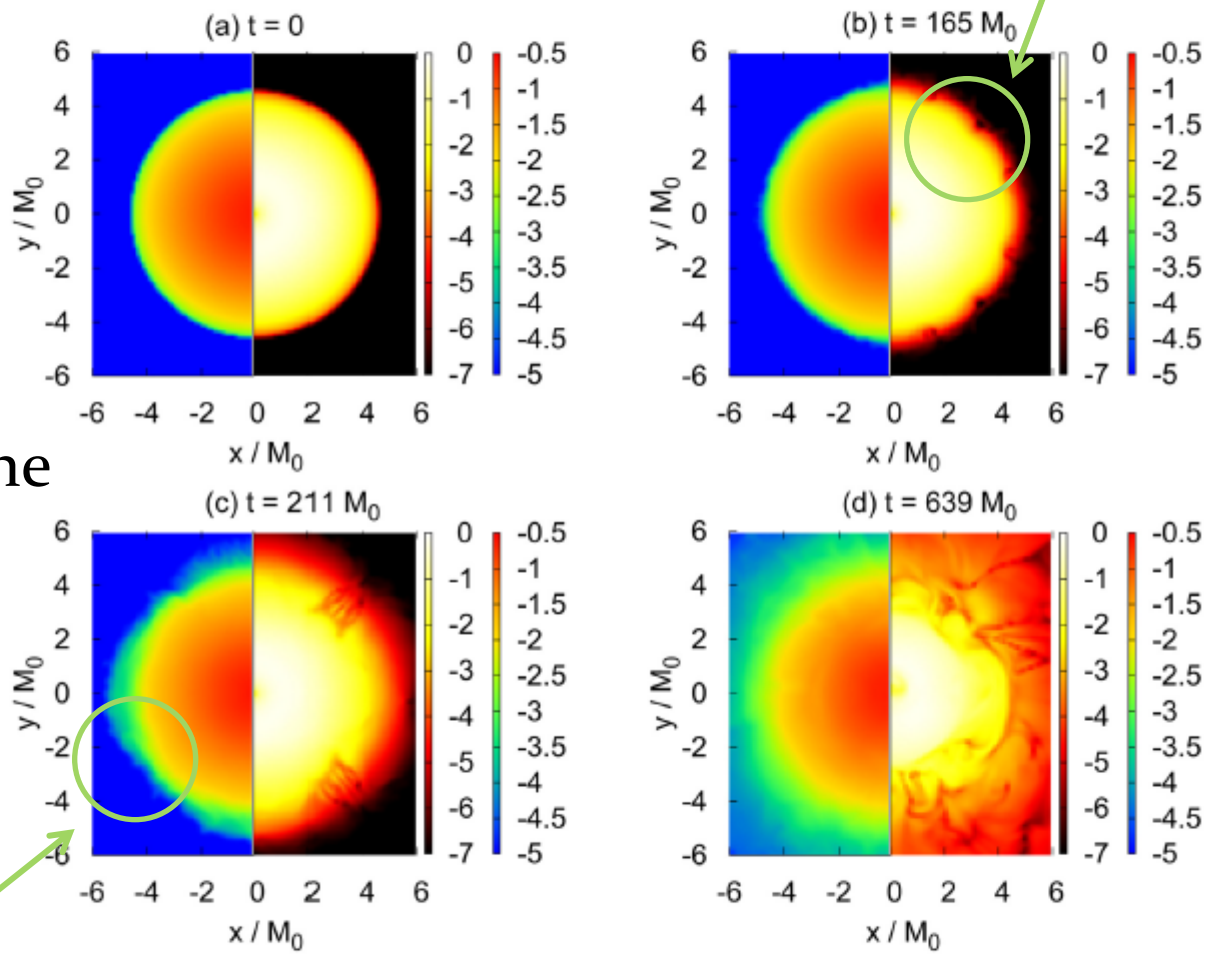
Most unstable outside R_c and high m ⇒ Parker instability

Pure toroidal fields (KK+11)

Emergence of
Parker instability

ρ $B^2/8\pi$

Equatorial plane



Matter dragged by B
(Plasma β< 1)

Turbulent filed and non-equilibrium state

Short summary of pure field instability

✓ Pure poloidal fields ⇒'Sausage' or 'kink' instability @ neutral point

✓ Pure toroidal fields ⇒ Parker instability @ surface / kink instability @ magnetic axis

✓ Non-axisymmetric instability is essential

How do you stabilize magnetic fields ?

Key ingredients

✓ Stratification against the magnetic buoyancy

✓Poloidal-Toroidal fields (Braithwaite & Spruit 04) against the 'kink' instability

✓ and more ?

Stratification

$Brunt - V\ddot{a}is\ddot{a}l\ddot{a}\ frequency$

$$N^2 \propto \left(\frac{\partial P}{\partial s}\right)_{\rho, Y_l} \left(\frac{ds}{dr}\right)_{\rm amb} + \left(\frac{\partial P}{\partial Y_l}\right)_{\rho, s} \left(\frac{dY_l}{dr}\right)_{\rm amb} > 0$$

N ~500 Hz due to the composition gradient in NSs (Reisenegger & Goldreich 92)

✓ Magnetic buoyancy can be stabilized by this mechanism

Poloidal-toroidal field (Braithwaite & Spruit 04, Duez+ 10)

Toroidal fields would suppress the kink instabiltiy @ neutral point
Poloidal fields would suppress the kink instability @ magnetic axis

Stably stratified magnetized neutron stars
(Yoshida et al. 12)

Method

✓ Solving the Grad-Shafranov equation perturbatively
(Background : TOV star) (Ioka & Sasaki 03)

✓ Stably stratified TOV stars

$P = K\rho^{\gamma}$, $\varepsilon = P/\rho/\Gamma\text{-}1$ with $\gamma \neq \Gamma$

$$d\epsilon = -Pd\left(\frac{1}{\rho}\right) + Tds,$$

$$u^{\mu}\nabla_{\mu}s = 0$$

$$\text{Barotropic EOS}: \quad P = P(\rho), \quad \epsilon = \epsilon(\rho)$$

Normally ⇒

$$s = \text{const.}$$

$$d\epsilon = -Pd\left(\frac{1}{\rho}\right),$$

$$P = P(\rho), \quad \epsilon = \epsilon(\rho),$$

$$\text{e.g.,}\ P = K\rho^{\Gamma}, \epsilon = \frac{P}{\rho(\Gamma - 1)}$$

EOS with $\gamma = \Gamma$ gives no stratified neutron star model

$P = K\rho^{\gamma}$, $\varepsilon = P/\rho/(\Gamma\text{-}1)$ with $\Gamma > \gamma \Rightarrow$ Stably stratified model

Stably stratified magnetized neutron stars

Perturbative magnetic equilibria (Grad-Shafranov Eq.)

$$\Delta_{2\mathrm{D}}\Psi + 4\pi r^2 \sin^2\theta \rho \mathrm{e}^{2\lambda} C_1 + \mathrm{e}^{2(\lambda-\nu)} L^2 \Psi = 0$$

$$\Psi = A_\varphi,\ C_1\ :\ \text{Amplitude of B},\ L\ \propto\ \text{Helicity}$$

Boundary condition

✓ Regularity @ center & pole

✓ Stellar surface (Ioka & Sasaki 03), $\Psi=0,\ \partial_r(\Psi/r^{l+1})=0$ @ $r=R$

(Confinement of B inside stars)

⇒ Eigenvalue problem with L

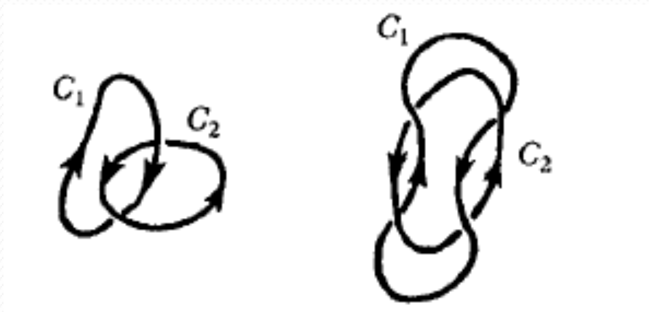
⇒ Models with discrete values of L,

i.e, L = L1, L2, L3.... (L1 < L2 < L3<)

$$H = \int \mathbf{A}\cdot\mathbf{B} d^3x$$

Fixed magnetic energy sequence

H(L1) > H(L2) > H(L3)....



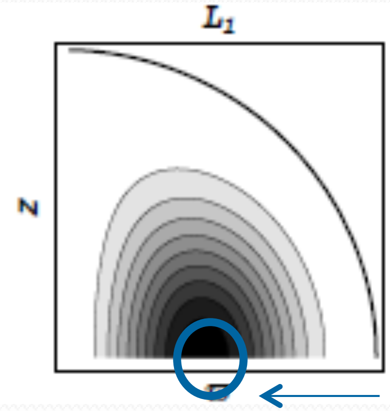
Stably stratified magnetized neutron stars

Ground state solution (L1 : E_{pol} /E_{mag}=0.37)

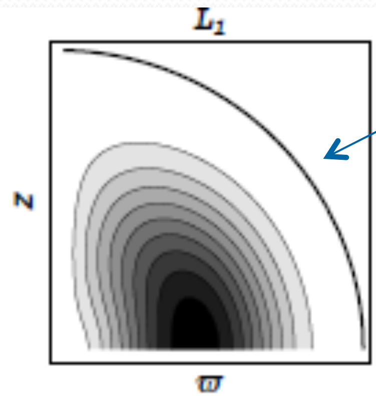
Magnetic field line

Toroidal field

White : Minimum
Black : Maximum

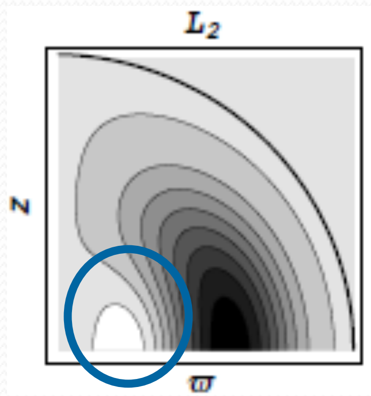


Neutral point

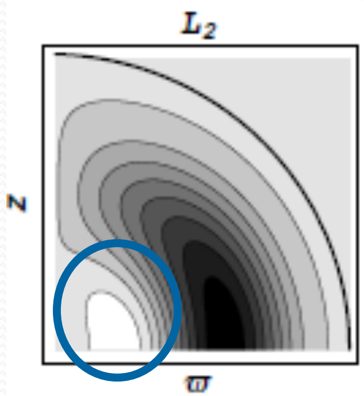


Stellar surf.

Excitation state (L2 : E_{pol} /E_{mag}=0.28)



Second neutral point

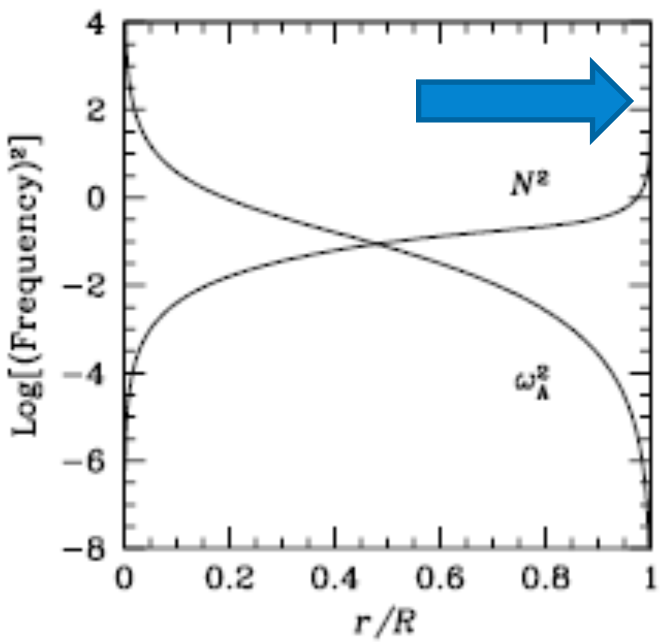


Oppositely oriented field

Stably stratified magnetized neutron stars

Stratification

Brunt-Väisälä (N^2) vs Alfvén frequency ($\omega_A{}^2$) @ $\theta=\pi/2$



It could stabilize magnetic buoyancy

$E_{mag}/|W|$ =0.025 model

Short summary of our model

✓ Poloidal-Toroidal magnetic field configuration

✓ Magnetic buoyancy force << Buoyancy force near the surface

Conjecture

The L1 model could be more stable than L2 model

Summary

✓Recent progress in the pure fields instability in NR simulations

- Pure poloidal field instability (Tubingen, AEI & Southampton)

✓ Pure toroidal field instability (YITP) => Magnetic buoyancy instability / Taylor instability

All the pure magnetic fields are unstable and the axisymmetric assumption is no longer valid

✓ Stably stratified magnetized neutron star models

They could stabilize the magnetic buoyancy and the kink instability close the magnetic pole and neutral point.