Hadron-Quark Crossover and Neutron Star Observations

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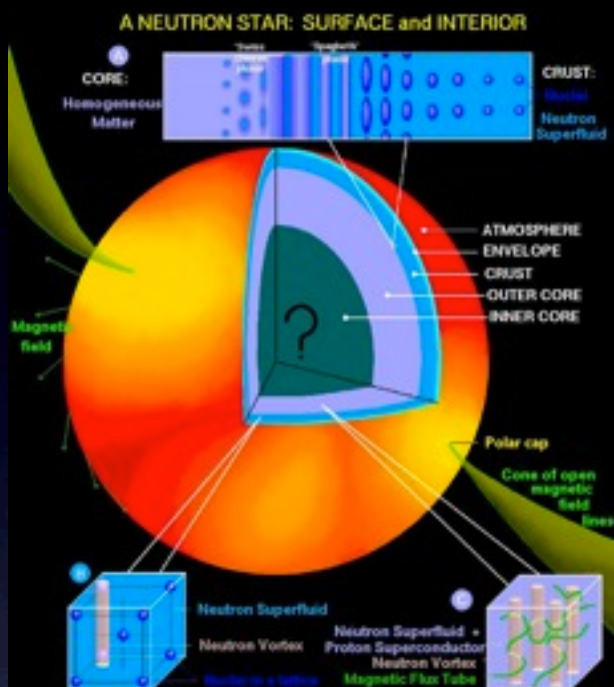
Neutron star matter in view of nuclear experiments and astronomical observations, 25th Oct., 2013

Introduction: NS observations

NS observations



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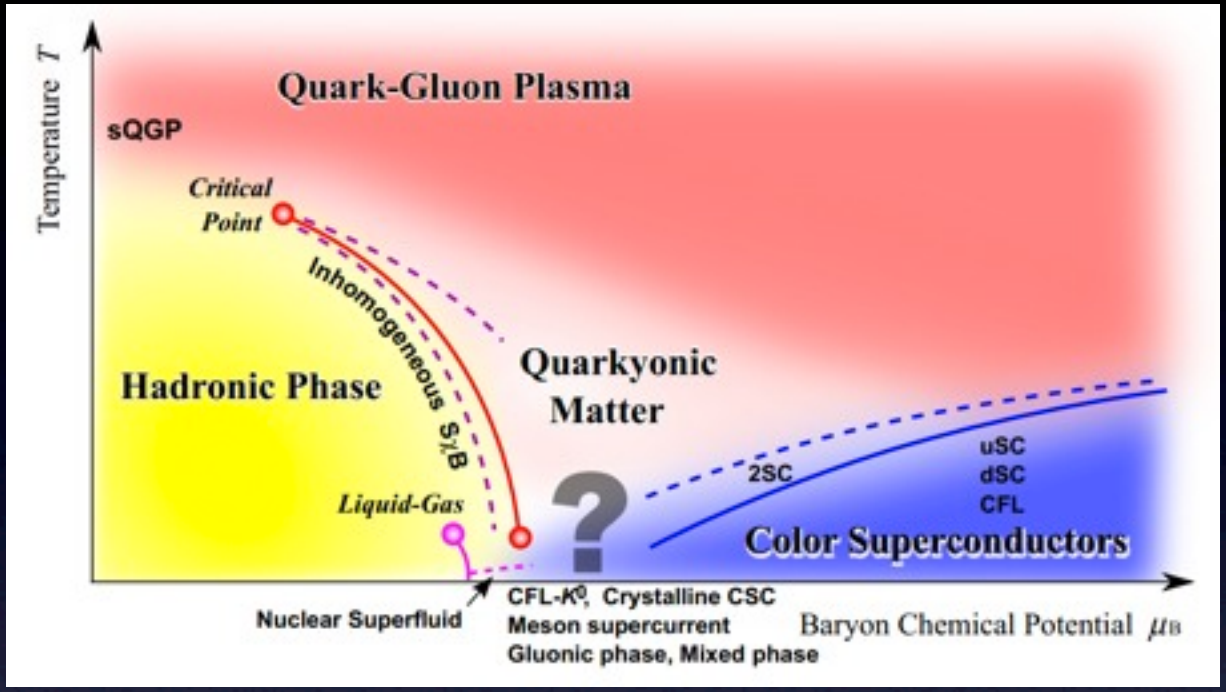
QCD phase diagram



Fukushima, Hatsuda (2010)

Mass

$(1.97 \pm 0.04)M_\odot$

Demorest *et al.* (2010)

$(2.01 \pm 0.04)M_\odot$

Antoniadis *et al.* (2013)

Cooling

Cooling of CAS-A

Heinke *et al.* (2010)

EOS

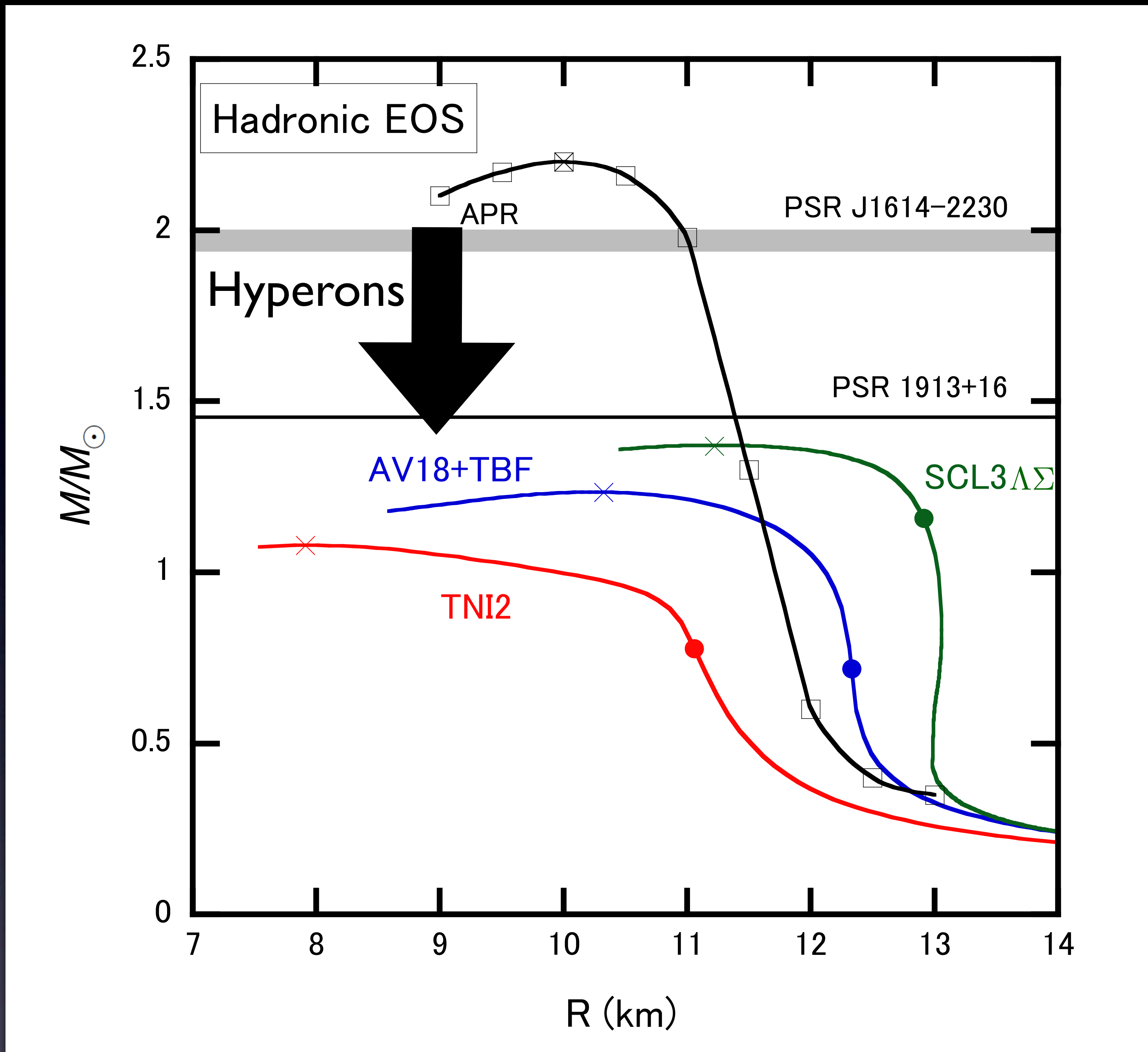
Are there any EOS which can explain $2M_\odot$ NS ?

The fate of the quark matter inside a heavy NS ?

Superfluid / Superconducting phase

Relation to nucleon and quark superfluidity inside NSs ？

Hadronic EOSs

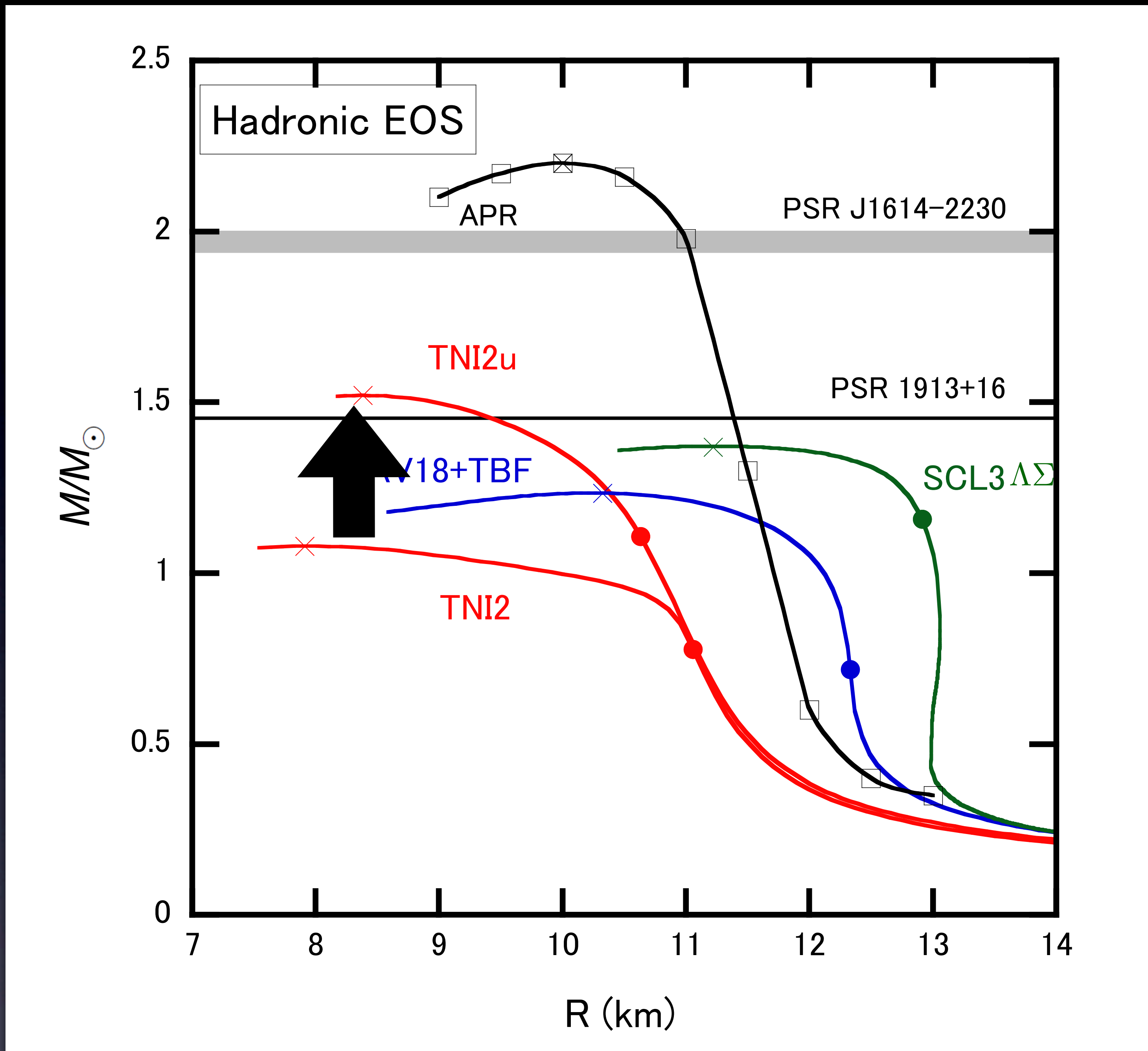


	(1) AV18+TBF	(2) TNI2	(3) SCL3ΛΣ
Method	BHF	BHF	RMF
2NF	AV18	Reid	
3NF	Yes	Yes	No
Hyperons	Yes	Yes	Yes

(1) Baldo *et al.* (2000), Schulze *et al.* (2010)
(2) Nishizaki *et al.* (2001,2002)
(3) Tsubakihara et *al.* (2010)

- Hyperons soften EOS ⟶ Maximum mass is less than $1.44M_\odot$

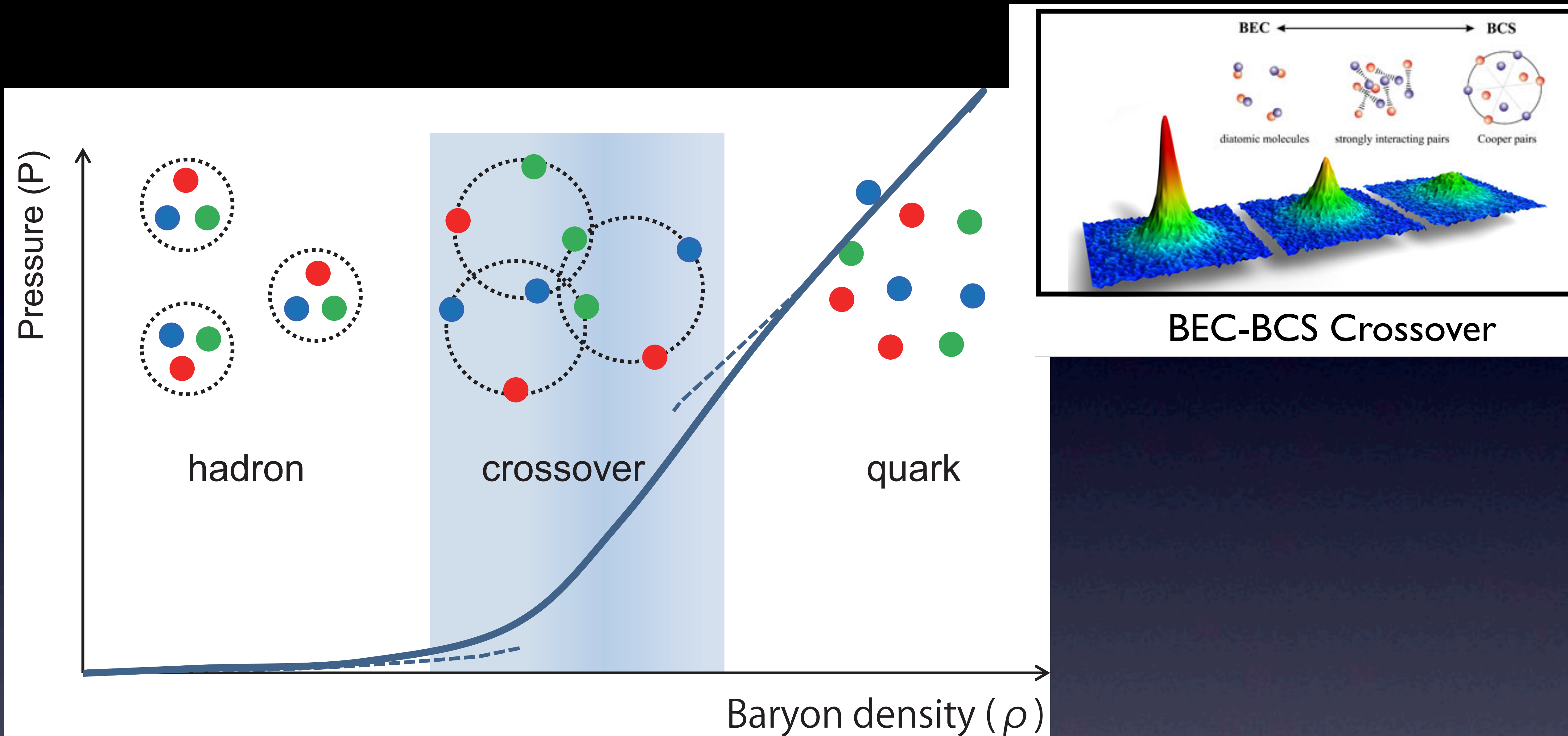
Hadronic EOSs



	TNI2	TNI2u
``NNN''	Yes	Yes
``NNY'' ``NYY'' ``YYY''	No	Yes

- Universal 3-body force stiffens EOS ⟶ Maximum mass is larger than $1.44M_\odot$
- However maximum mass cannot exceed $2M_\odot$

Hadron-Quark Crossover



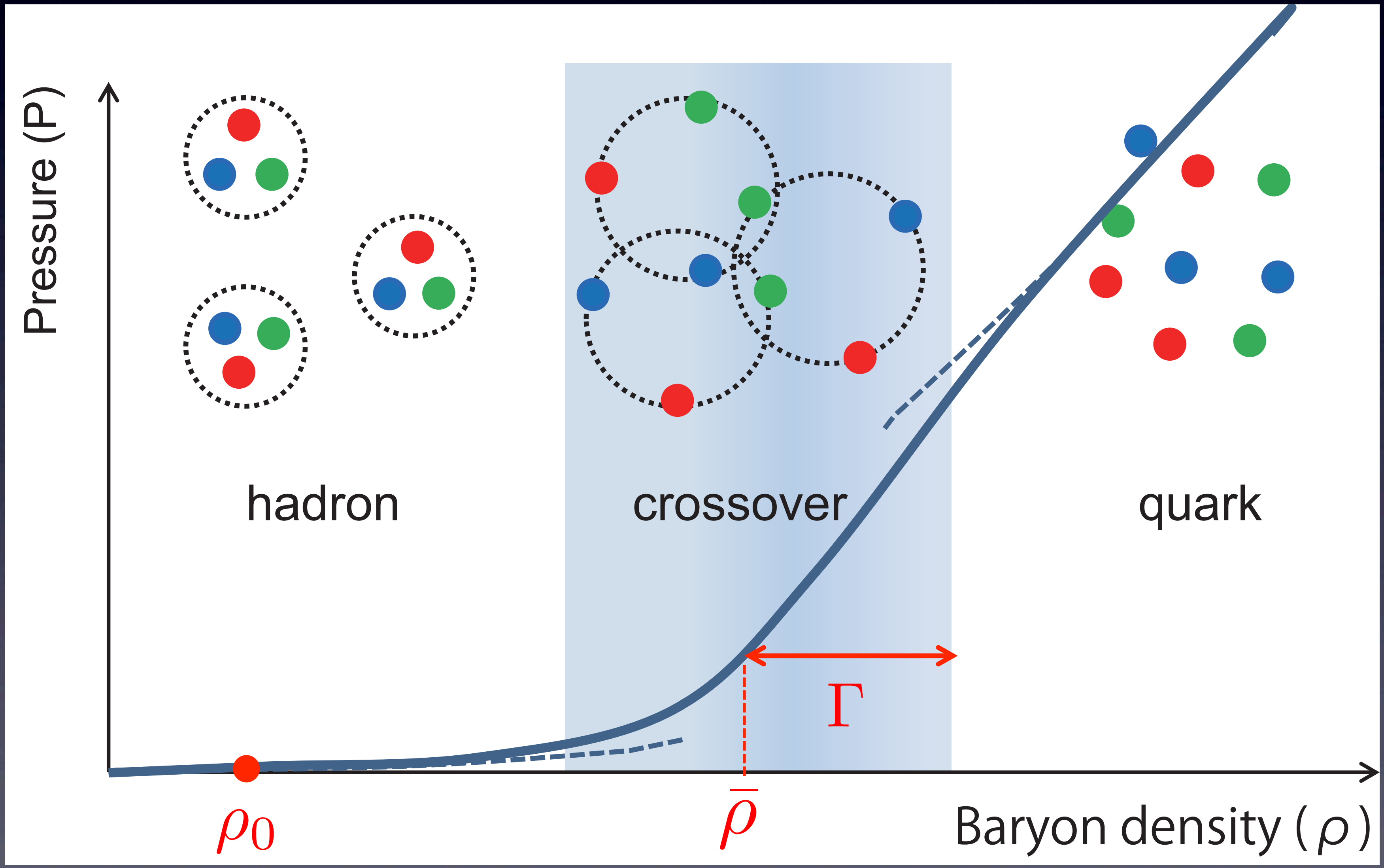
We seek the possibility of crossover

Ref.)
Baym (1979)
Celik, Karsch and Satz (1980)
Fukushima (2004)
Hatsuda, Tachibana, Yamamoto and Baym (2006)

Method of Interpolation

Phenomenological interpolation: $P(\rho)$

$$\begin{cases} P = p_H \times f_- + p_Q \times f_+ & f_\pm = \dfrac{1 \pm \tanh(\frac{\rho - \bar{\rho}}{\Gamma})}{2} \\ P = \rho^2 \dfrac{\partial(\varepsilon/\rho)}{\rho} \end{cases}$$



Condition for $\bar{\rho}$: $f_+ < 0.1$ at ρ_0 ➡ $\bar{\rho} > \rho_0 + 2\Gamma$

EOS at $\rho \gg \bar{\rho}$

(2+1)-flavor NJL Lagrangian (u,d,s, e^-, μ^-)

$$L_{NJL} = \bar{q}(i\partial\!\!\!/ - m)q + \frac{G_s}{2}\sum_{a=0}^{8}[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2] - \frac{g_v}{2}(\bar{q}\gamma^\mu q)^2 + G_D[\det\bar{q}(1+\gamma_5)q + \text{h.c.}]$$

Parameter set

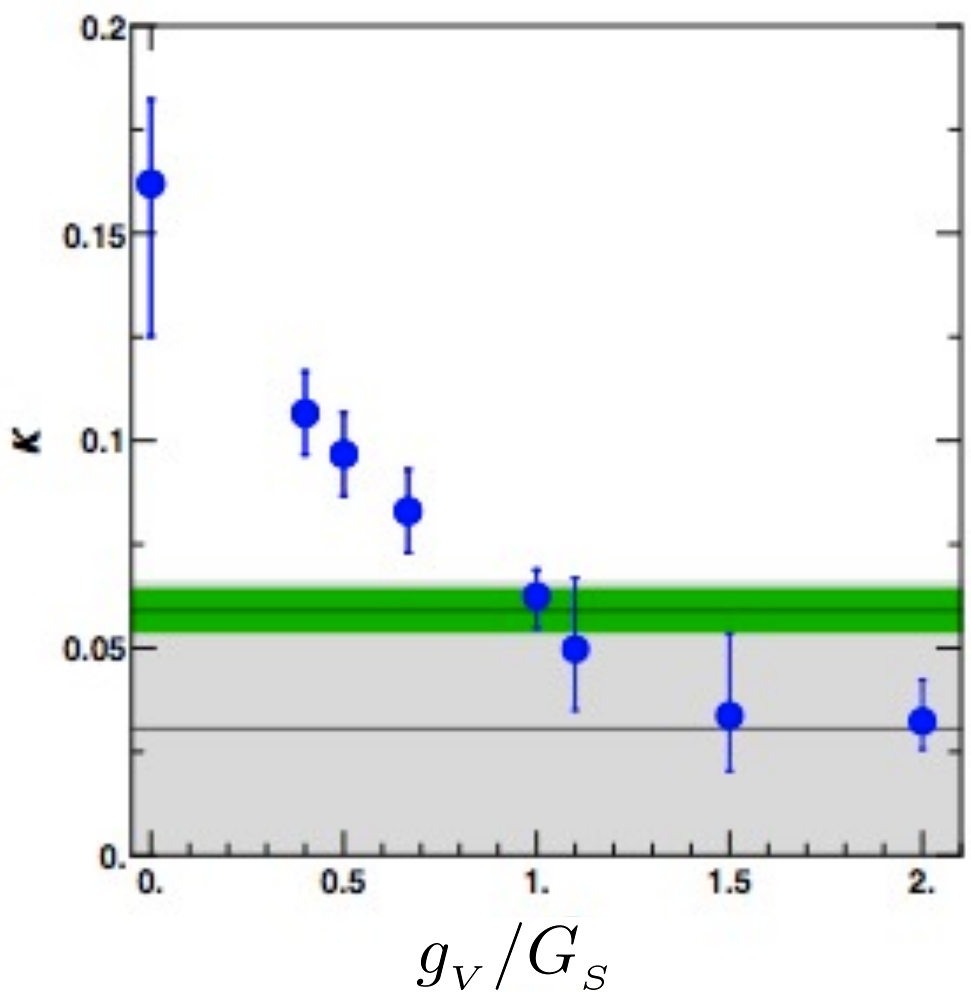
cutoff (MeV)	$G_s\Lambda^2$	$G_D\Lambda^5$	$m_{u,d}(MeV)$	$m_s(MeV)$
631.4	3.67	9.29	5.5	135.7

Hatsuda and Kunihiro (1994)

$0 \le g_v \le 1.5 G_s$

Conditions:
1. beta-equilibrium
2. charge neutrality

Recent estimate of g_V



$$\kappa = -T_c \frac{d^2 T_c(\mu)}{d\mu^2}\bigg|_{\mu^2=0}$$

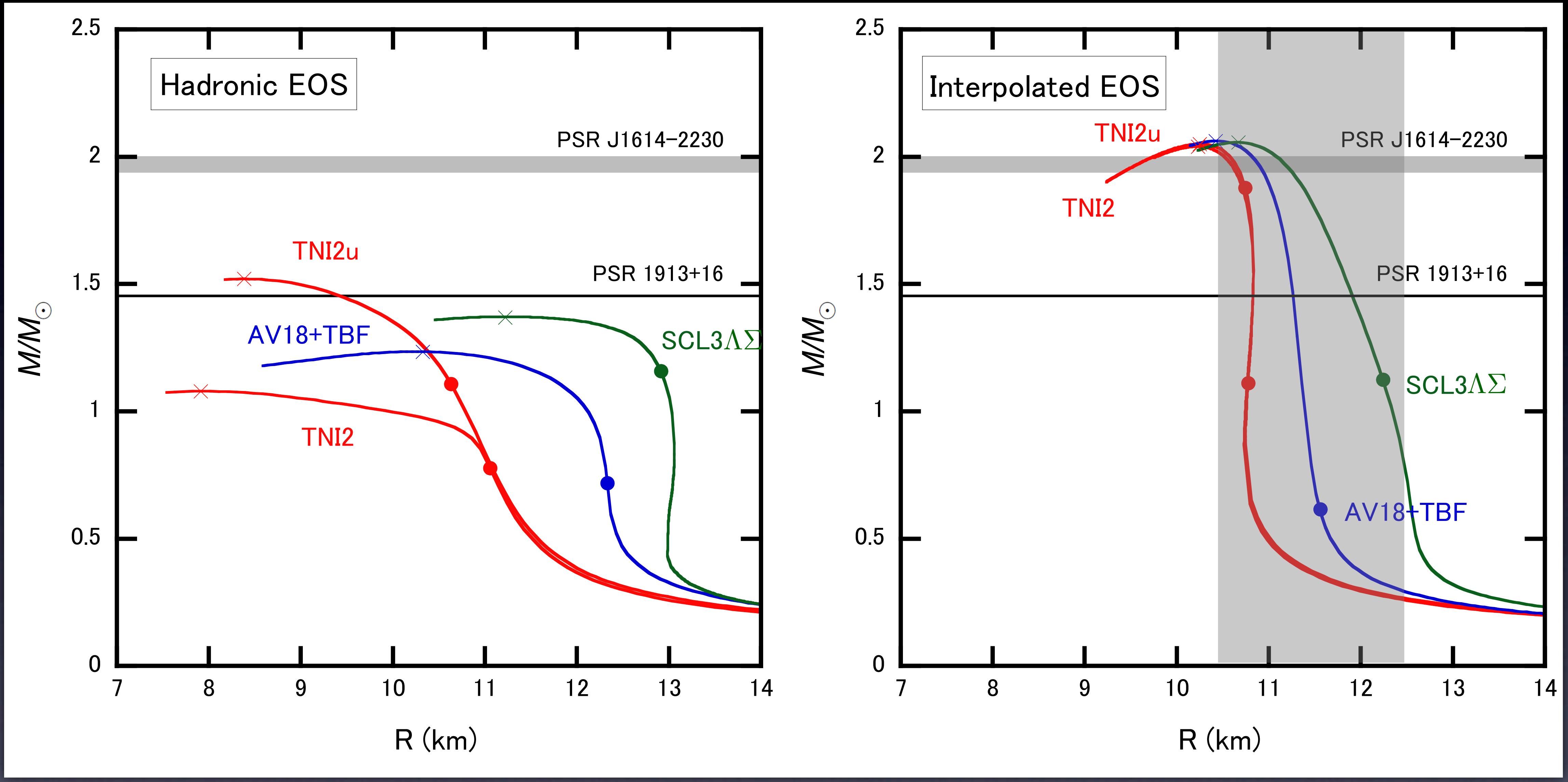
$\longrightarrow$ $g_V \sim G_S$

$g_V \ge 0$: repulsive

Bratovic *et al.*, Phys. Lett. B719 (2013)

Results (I): Effects of Q-EOS

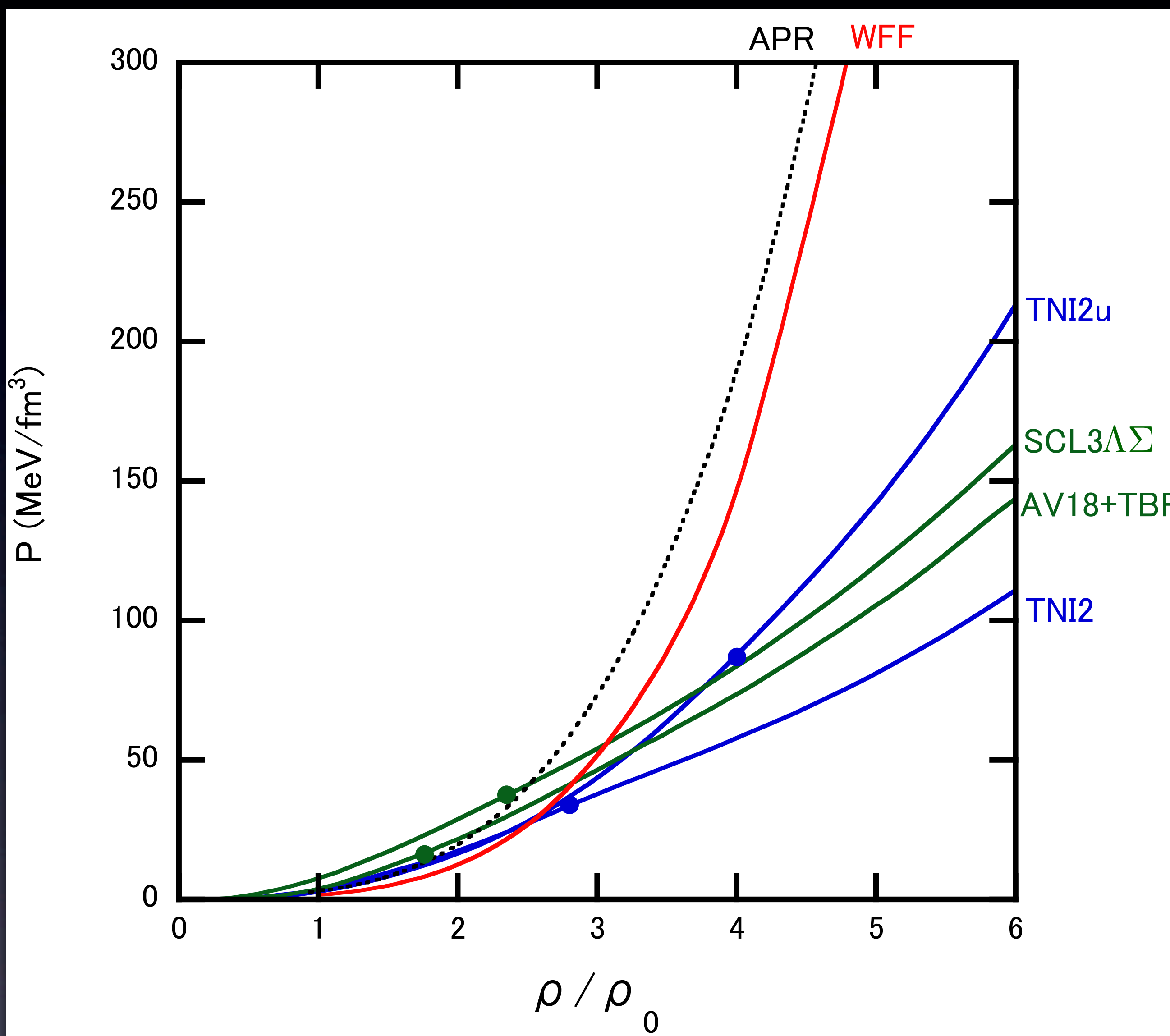
M-R relation $(\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$ $g_v = G_S$



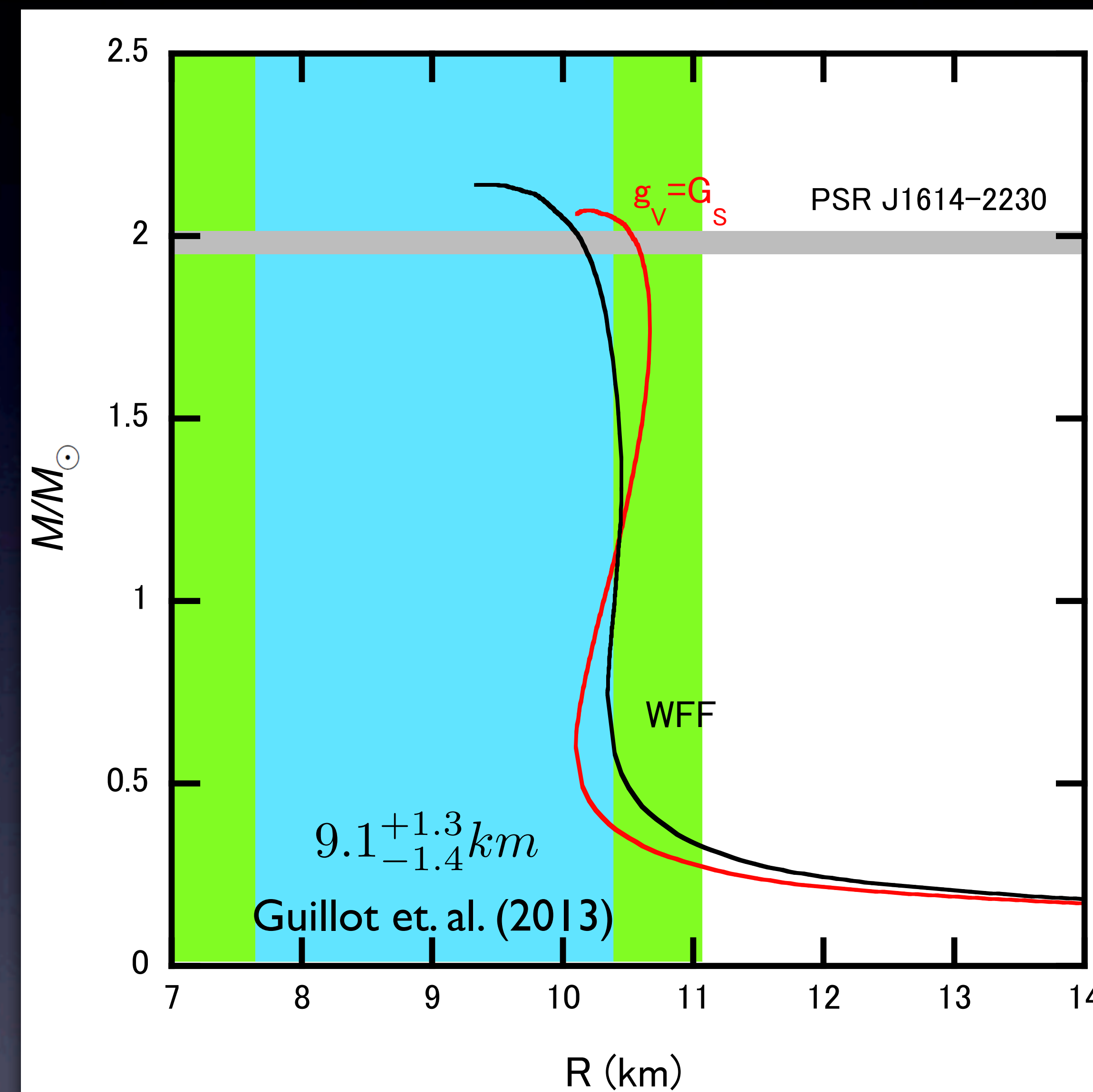
- Maximum mass exceeds 2 solar mass, no matter what kind of H-EOS is taken

Results (2): Radius

Hadronic EOSs



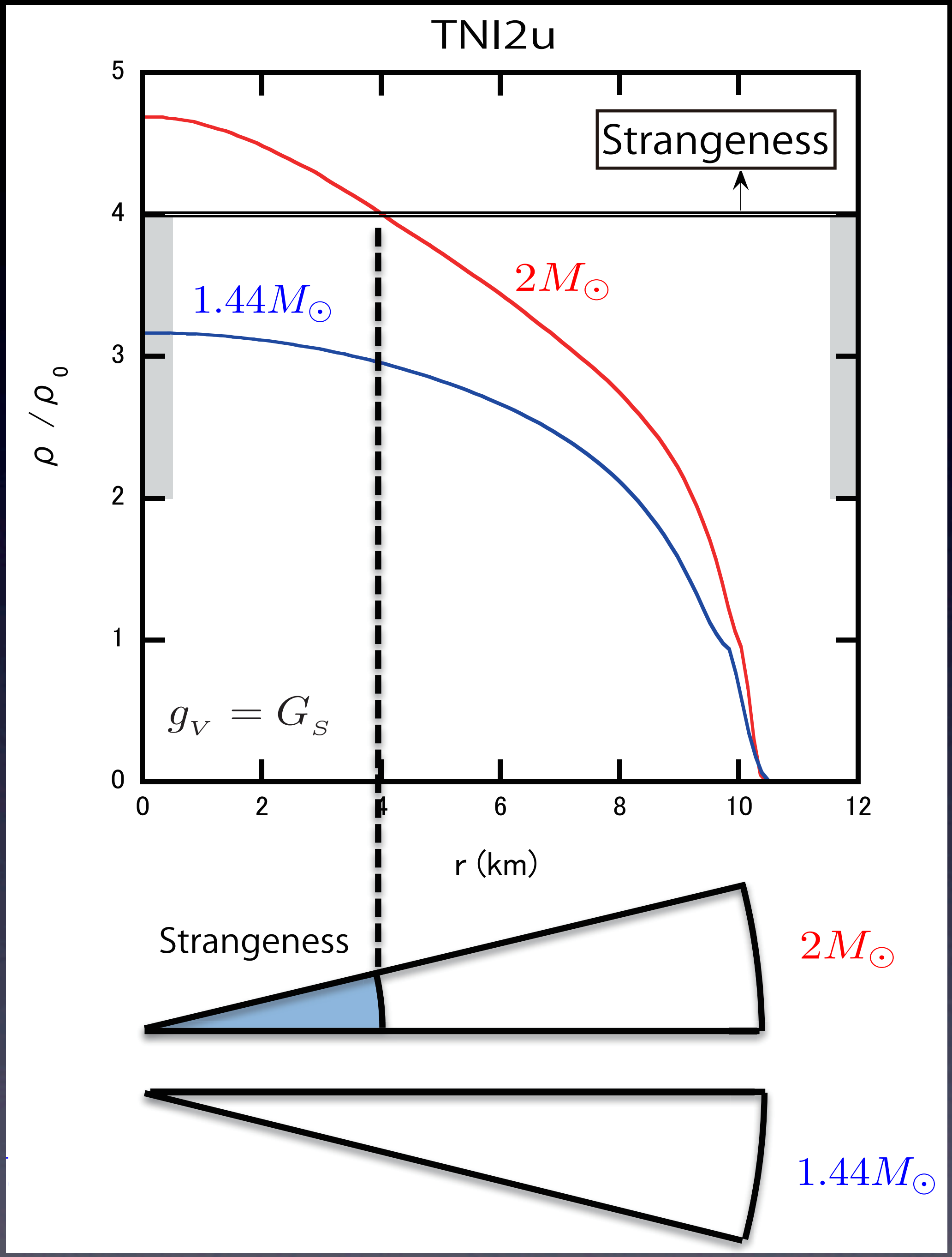
M-R relation $(\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$ $g_v = G_S$



- We use WFF EOS as hadronic EOS (Wiringa et. al., 1988)
- Radius is essentially controlled by hadronic EOS.

Results (3): Strangeness Core

$\rho - r$ relation $(\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$ $g_v = G_S$

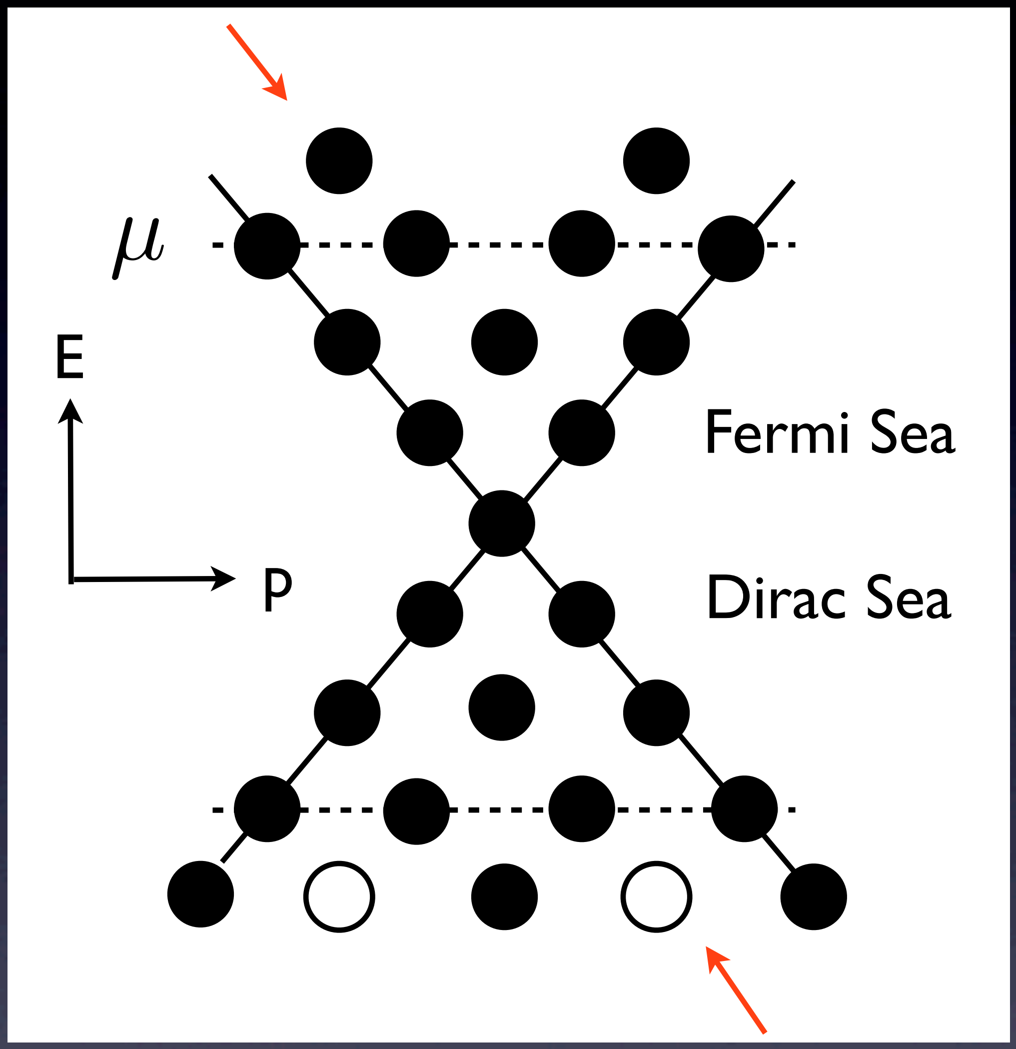


Typical NSs with universal 3-body force do not include strangeness inside themselves

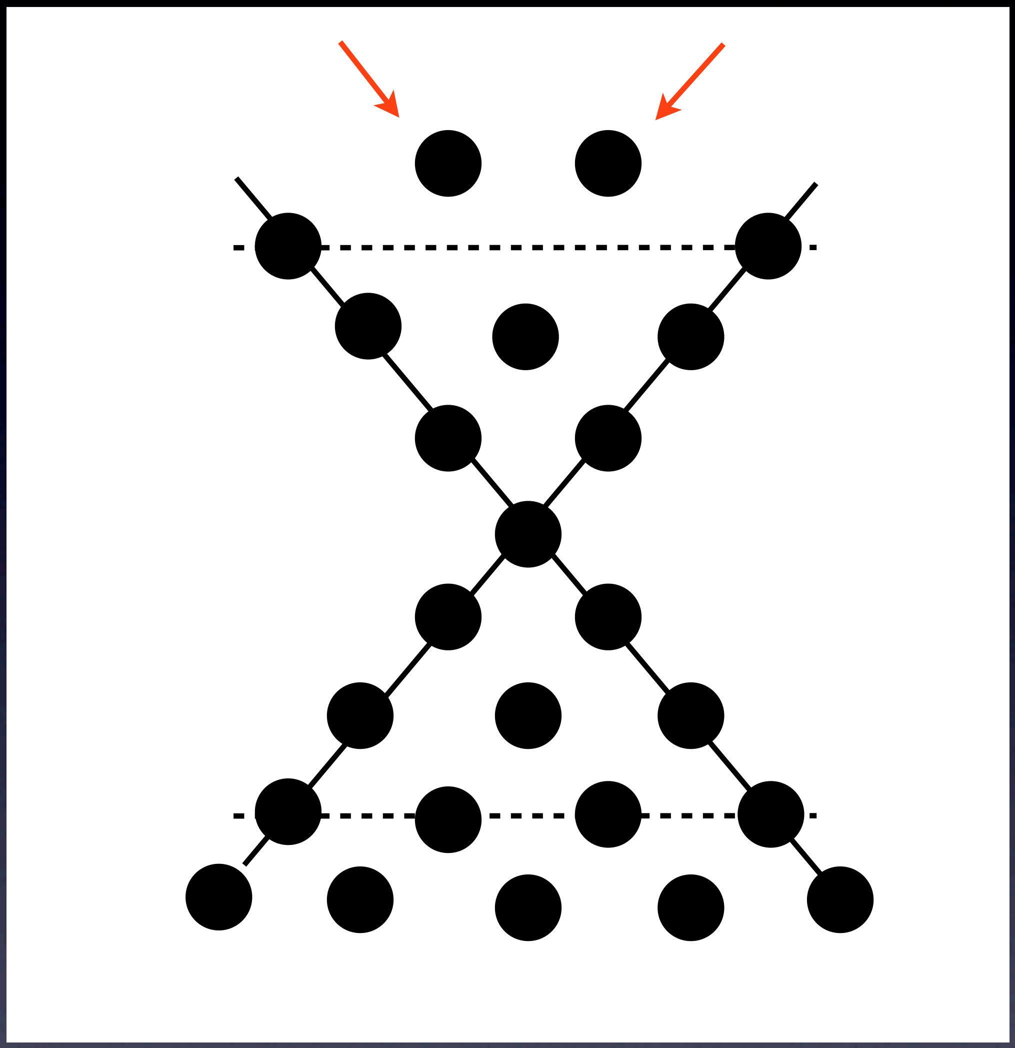
$\longrightarrow$ possibility of solving cooling problem

Color Superconductivity (CSC)

- Chiral Condensate



- Diquark Condensate



Alford

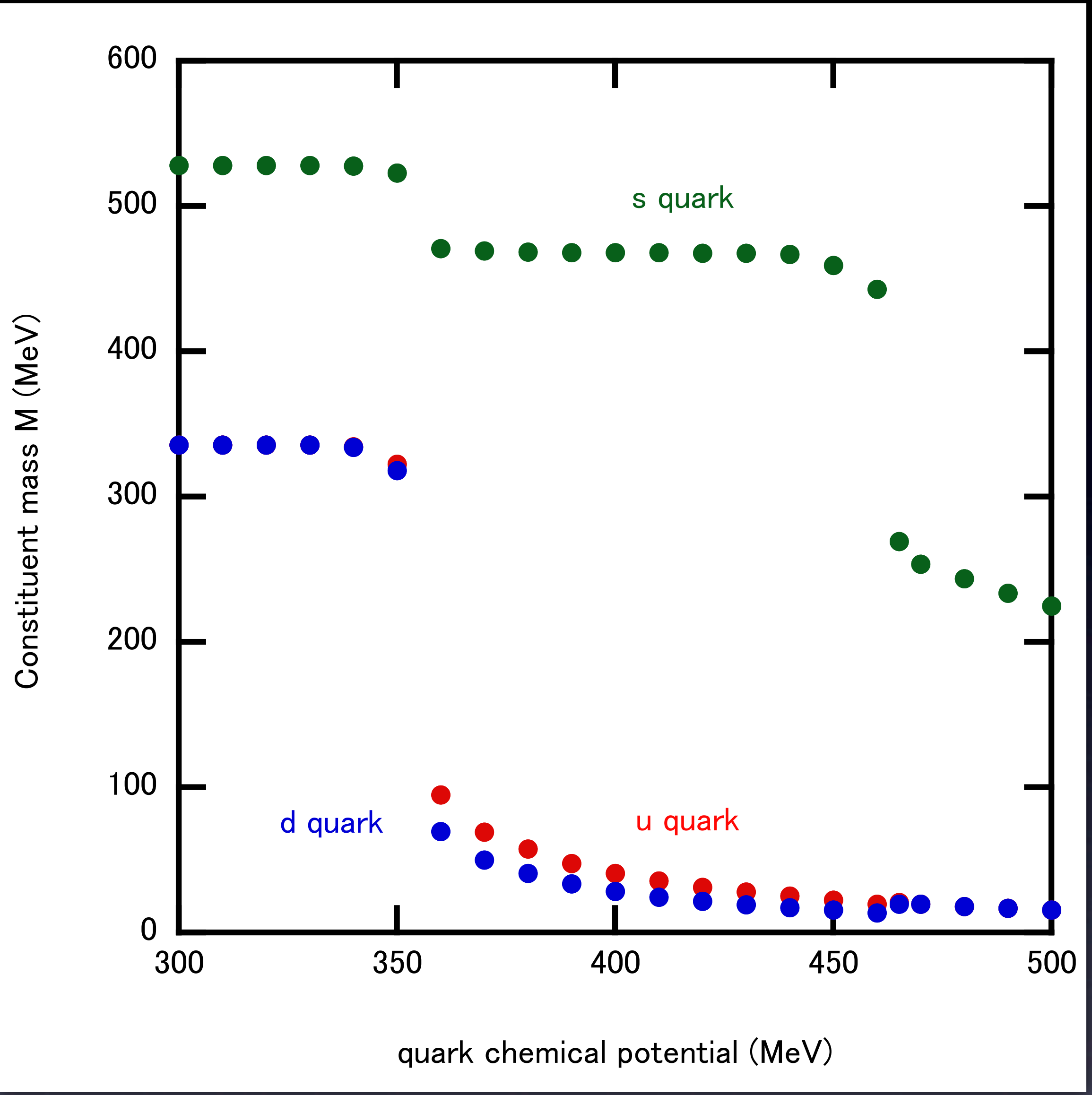
- NJL model

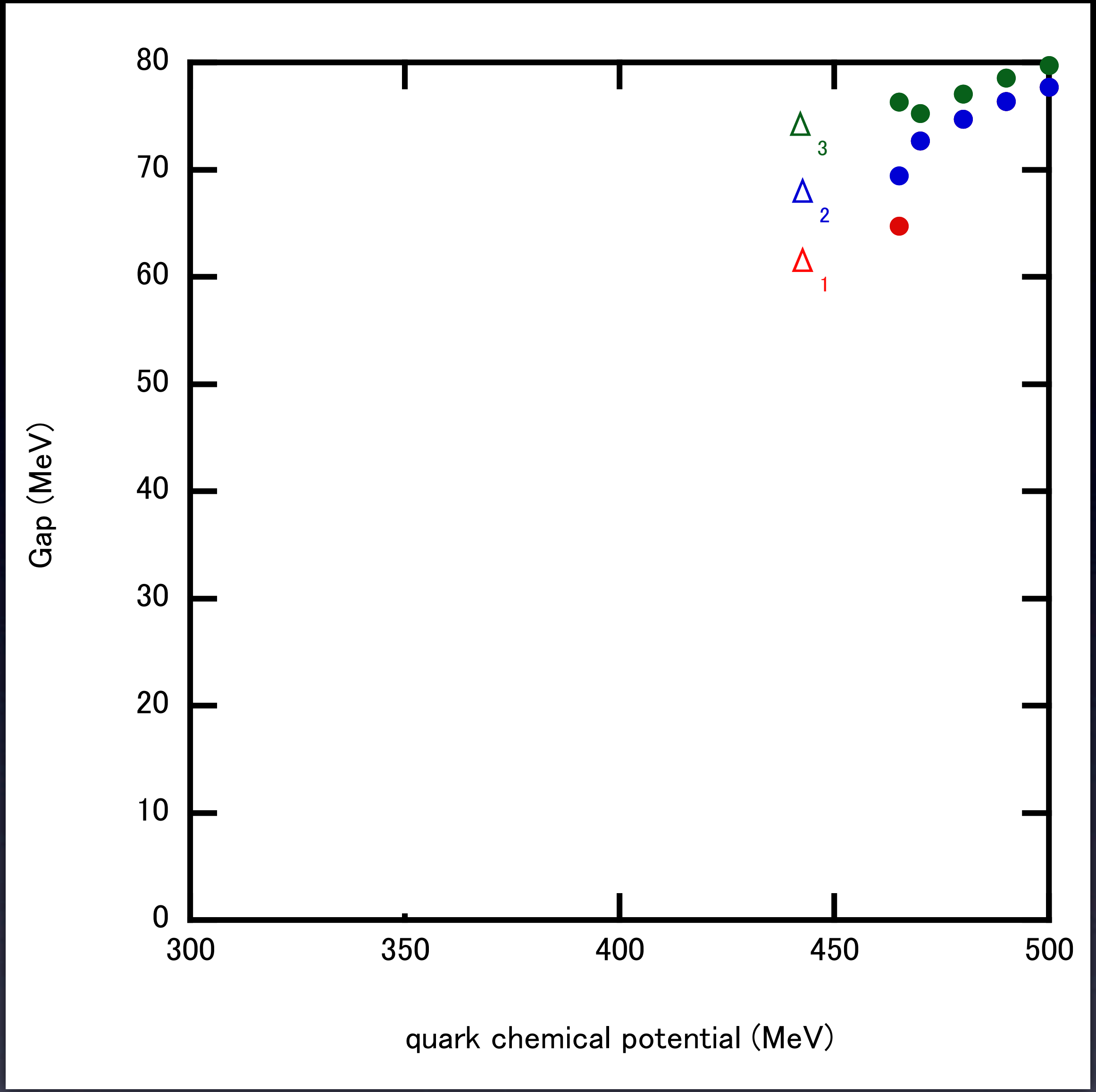
$$L_{\mathrm{CSC}} = L_{\mathrm{NJL}} + \frac{H}{2} \sum_{A=2,5,7} \sum_{A'=2,5,7} (\bar{q} i\gamma_5 \tau_A \lambda_{A'} C \bar{q}^T)(q^T C i\gamma_5 \tau_A \lambda_{A'} q) \qquad H = \frac{3}{4} G_s$$

(Fierz)

Results (4): Case I $H = \frac{3}{4}G_s$

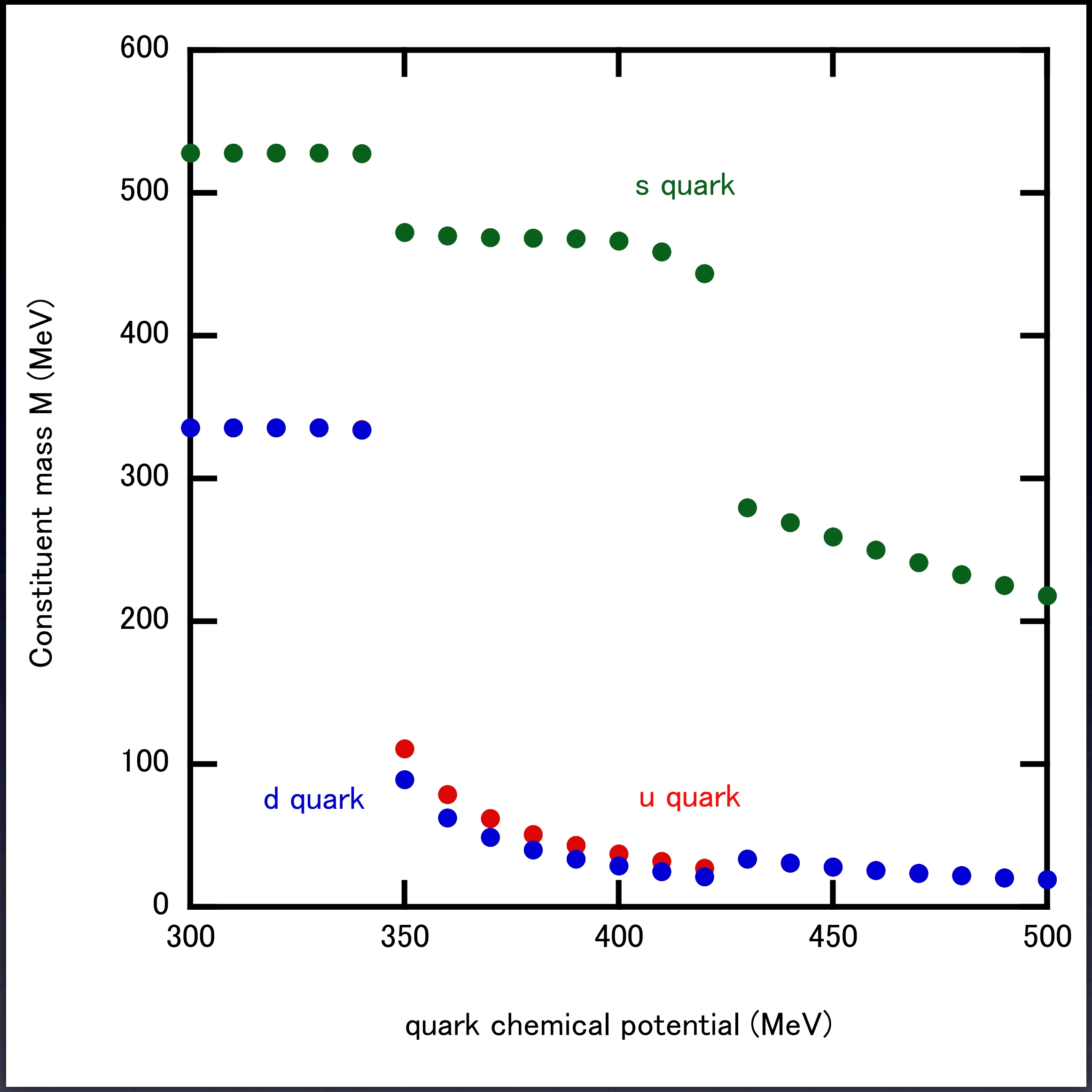
$g_v = 0$

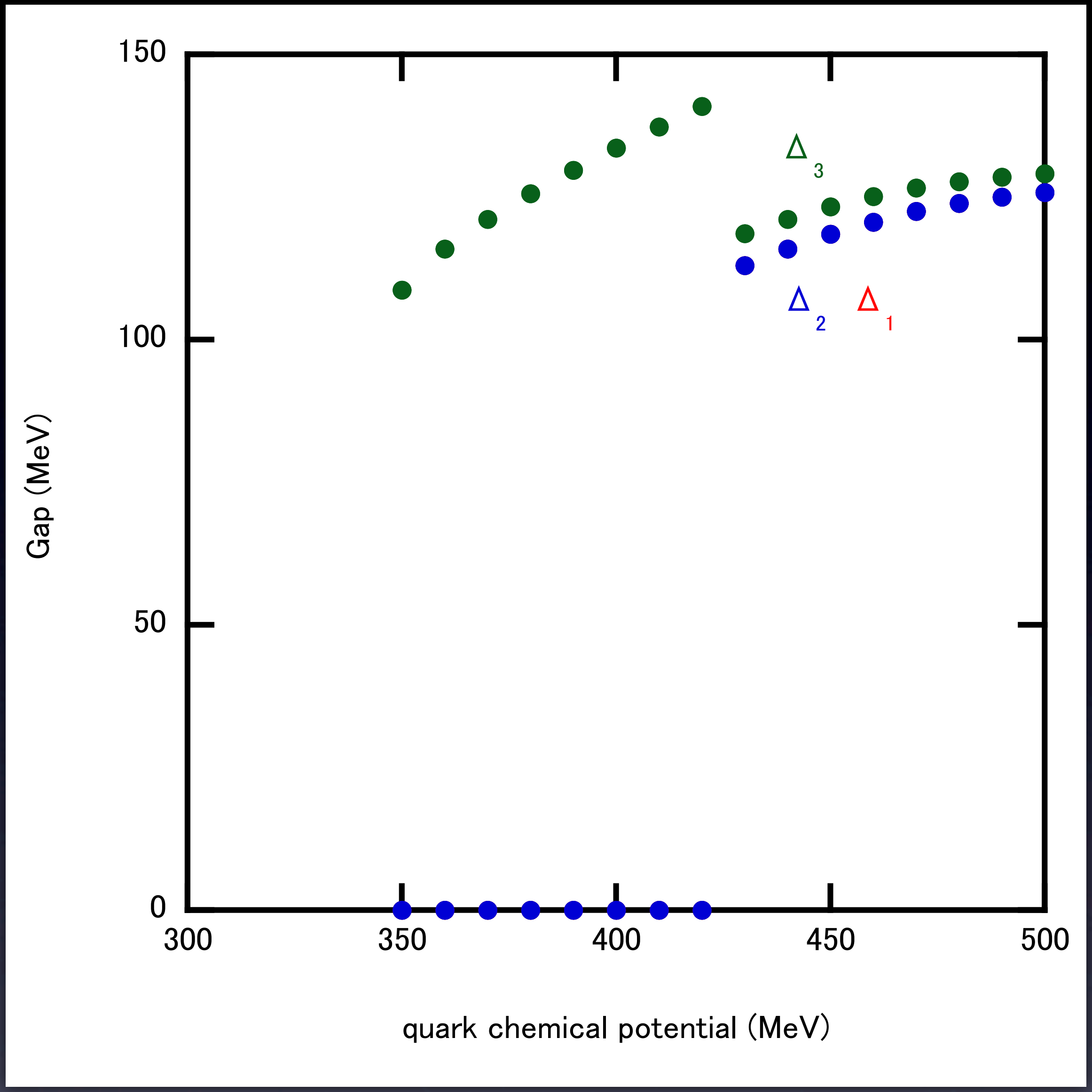




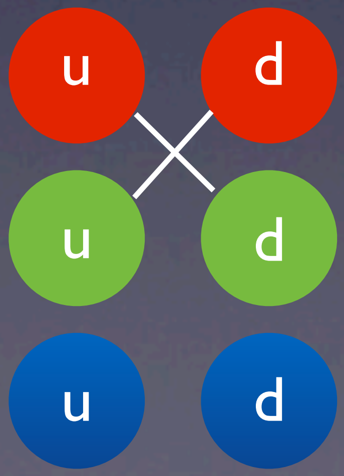
Results (5): Case 2 $H = G_s$

$g_v = 0$





2SC phase

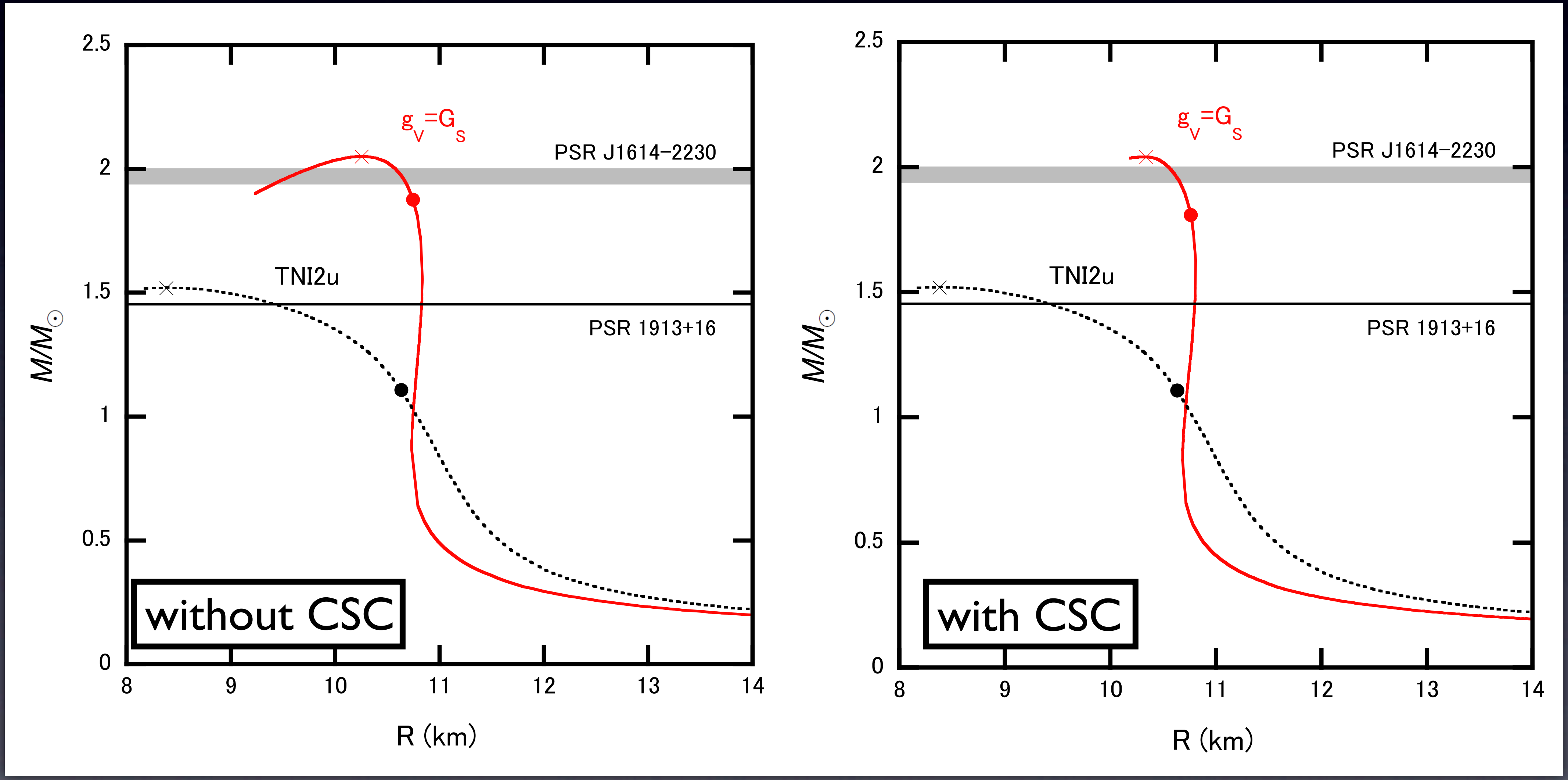


Results (6): Effects of CSC

Diquark condensation with $J^P = 0^+$

$$L_{\mathrm{CSC}} = L_{\mathrm{NJL}} + \frac{H}{2} \sum_{A=2,5,7} \sum_{A'=2,5,7} (\bar{q} i\gamma_5 \tau_A \lambda_{A'} C \bar{q}^T)(q^T C i\gamma_5 \tau_A \lambda_{A'} q)$$

M-R relation $(\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$ $g_v = G_S$ $H = \frac{3}{4} G_s$



• CSC softens EOS, but the effects of CSC is very small

Summary

Summary

(1) Crossover occurs at relatively low densities

(2) Quarks are strongly interacting at and above the crossover region

EOS at T=0

(A) Interpolated EOS can become stiffer due to the presence of quark matter

↓

Observation of very massive neutron star cannot exclude the existence of the quark matter core

(B) CSC phase does not have effects on the maximum mass
However, CSC may have large effect on phenomena related to transport.

＊ Other Characteristics:

1. Radius is essentially controlled by hadronic EOS
2. Interpolated EOS with the repulsive 3-body force among nucleons and hyperons have a impact on the cooling problem of neutron star with hyperon core

＊ Perspective

1. Cooling with 2SC+X phase by using our hadron-quark crossover model.
2. Constraints on the EOS from other observables such as neutron star radius.

Thank you !

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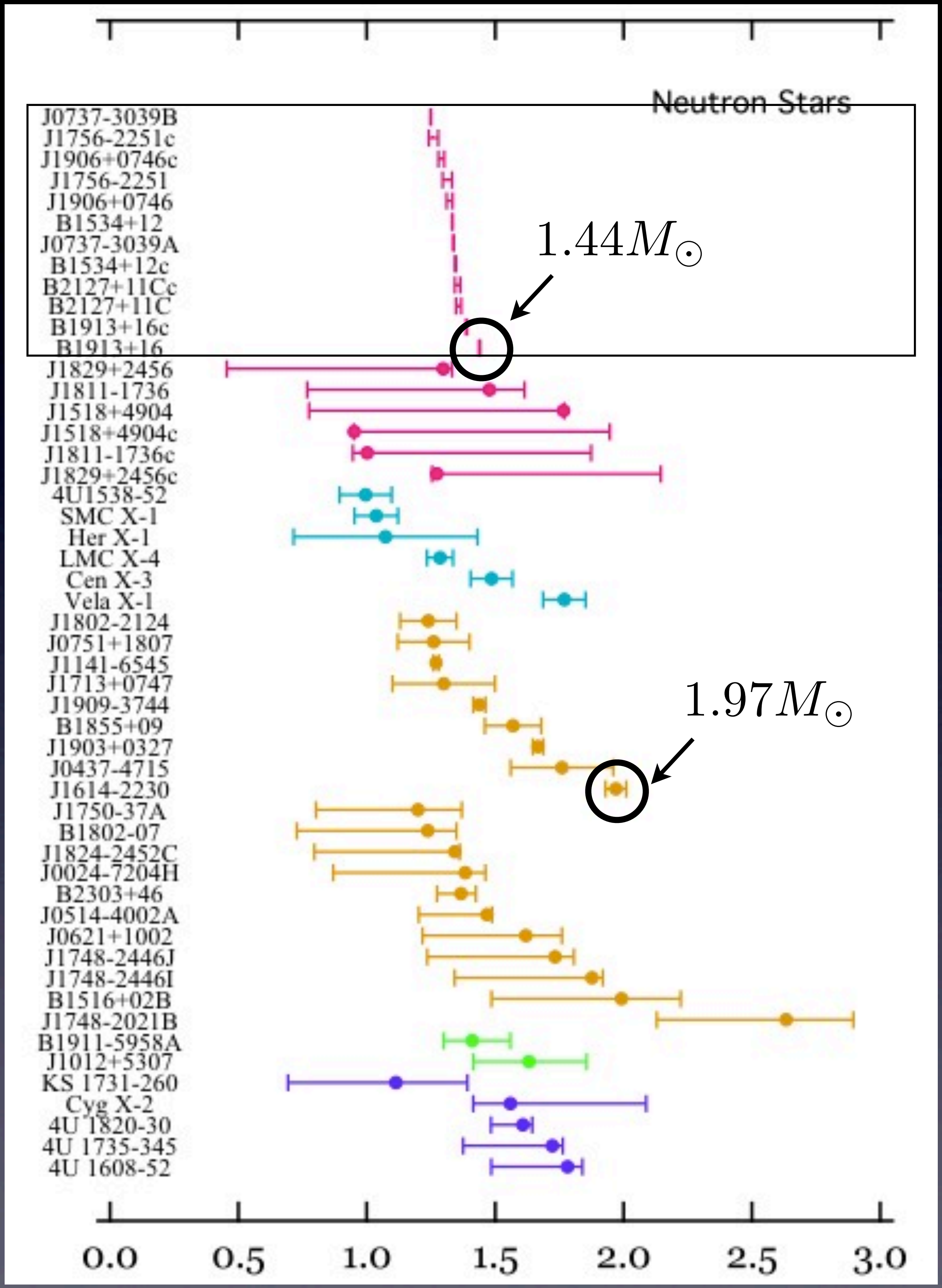
Introduction: Massive Neutron Star



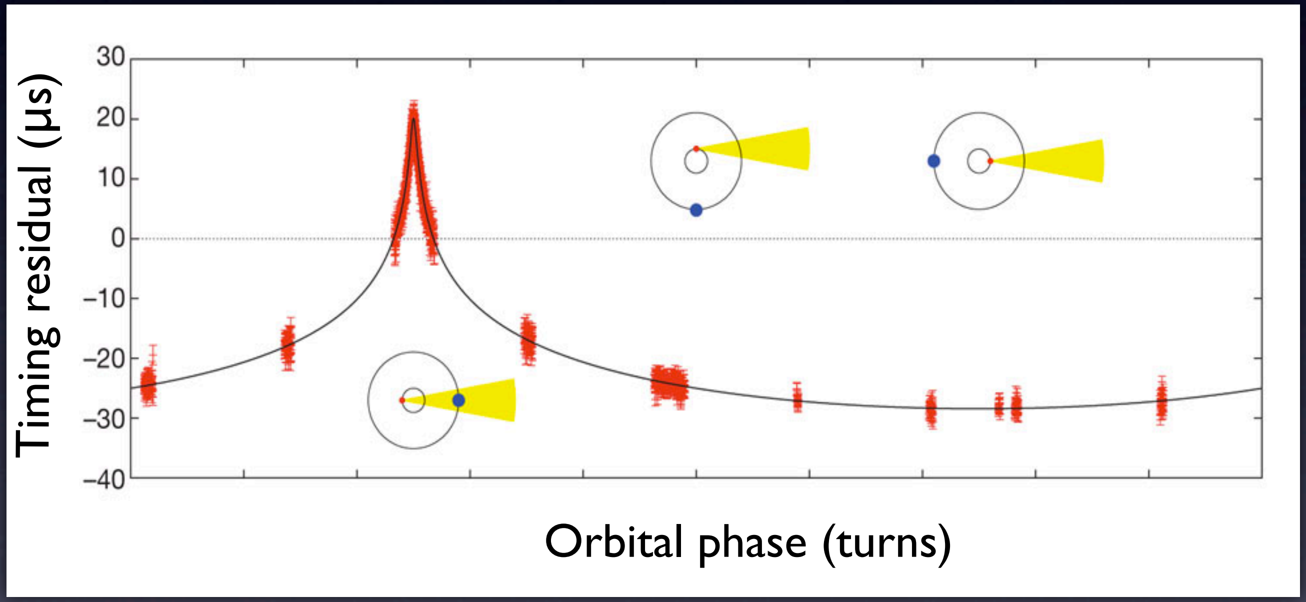
Ozel *et al.* (2012)

Typical value of the observed mass for double NS binaries $\sim 1.4 M_\odot$

↓

In 2010, NS (PSR J1614-2230, NS-WD binary) with $M = (1.97 \pm 0.04) M_\odot$ was found

Shapiro delay



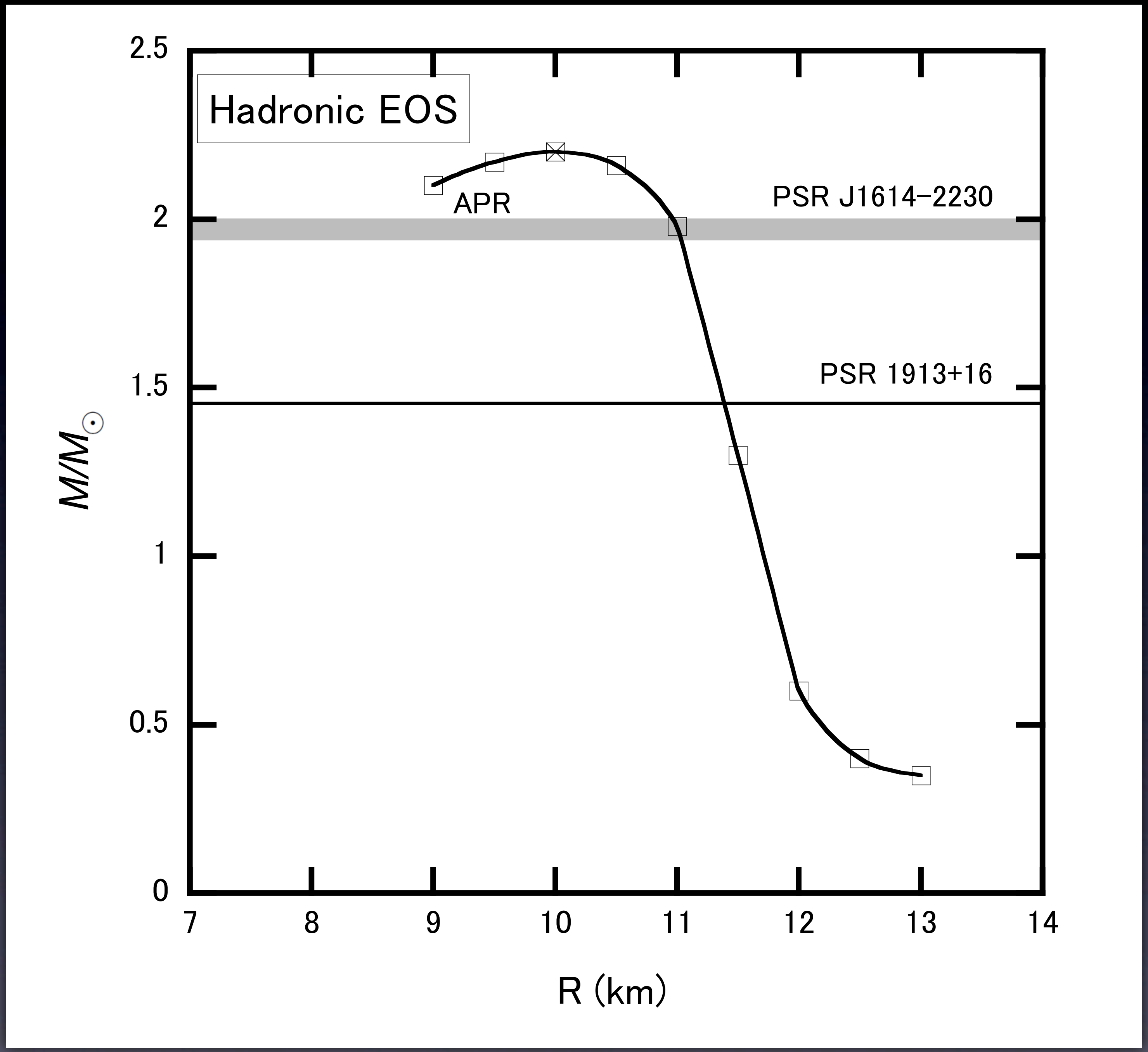
Demorest *et al.* (2010)

Key Questions:

Any EOS which can explain 2 $M_\odot$ NS ?

The fate of the quark matter inside a heavy NS ?

Introduction: Hadronic EOSs



	APR
Method	Variational
2NF	AV18
3NF	Yes
Hyperons	No

Akmal *et al.* (1998)

EOS at $\rho \gg \bar{\rho}$

(2+1)-flavor NJL Lagrangian (u,d,s, e^-, μ^-)

$$L_{NJL} = \bar{q}(i\partial\!\!\!/ - m)q + \frac{G_s}{2}\sum_{a=0}^{8}[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2] - \frac{g_v}{2}(\bar{q}\gamma^\mu q)^2 + G_D[\det\bar{q}(1+\gamma_5)q + \mathrm{h.c.}]$$

$\downarrow$

$$\Omega = -\frac{T}{V}\ln Z$$

$$\begin{cases} M_i = m_i - 2G_s\langle\bar{q}_i q_i\rangle - 2G_D\langle\bar{q}_j q_j\rangle\langle\bar{q}_k q_k\rangle \\ \mu_i \to \mu_i^{\mathrm{eff}} \equiv \mu_i - g_v \sum_i \langle q_i^\dagger q_i\rangle \end{cases}$$

$$= \Omega_q(M, \mu^{\mathrm{eff}}) + \Omega_l + G_s\sum\langle\bar{q}_i q_i\rangle^2 + 4G_D\langle\bar{q}_i q_i\rangle\langle\bar{q}_j q_j\rangle\langle\bar{q}_k q_k\rangle - \frac{1}{2}g_v\left(\sum_i\langle q_i^\dagger q_i\rangle\right)^2$$

$$\Omega_q(\mu^{\mathrm{eff}}) = -T\sum_i\sum_l\int\frac{d^3p}{(2\pi)^3}\mathrm{Tr}\ln\left(\frac{1}{T}S_i^{-1}(i\omega_l, \vec{p})\right),$$

$$S_i^{-1} = p - \mu^{\mathrm{eff}}\gamma^0 - M_i,\ p^0 = i\omega_l = (2l+1)\pi T$$

Gap equations: $\dfrac{\partial\Omega}{\partial\langle\bar{q}_i q_i\rangle} = 0$

Parameter sets

cutoff (MeV)	$G_s\Lambda^2$	$G_D\Lambda^5$	$m_{u,d}(MeV)$	$m_s(MeV)$
631.4	3.67	9.29	5.5	135.7

Hatsuda and Kunihiro (1994)

$0 \le g_v \le 1.5G_s$

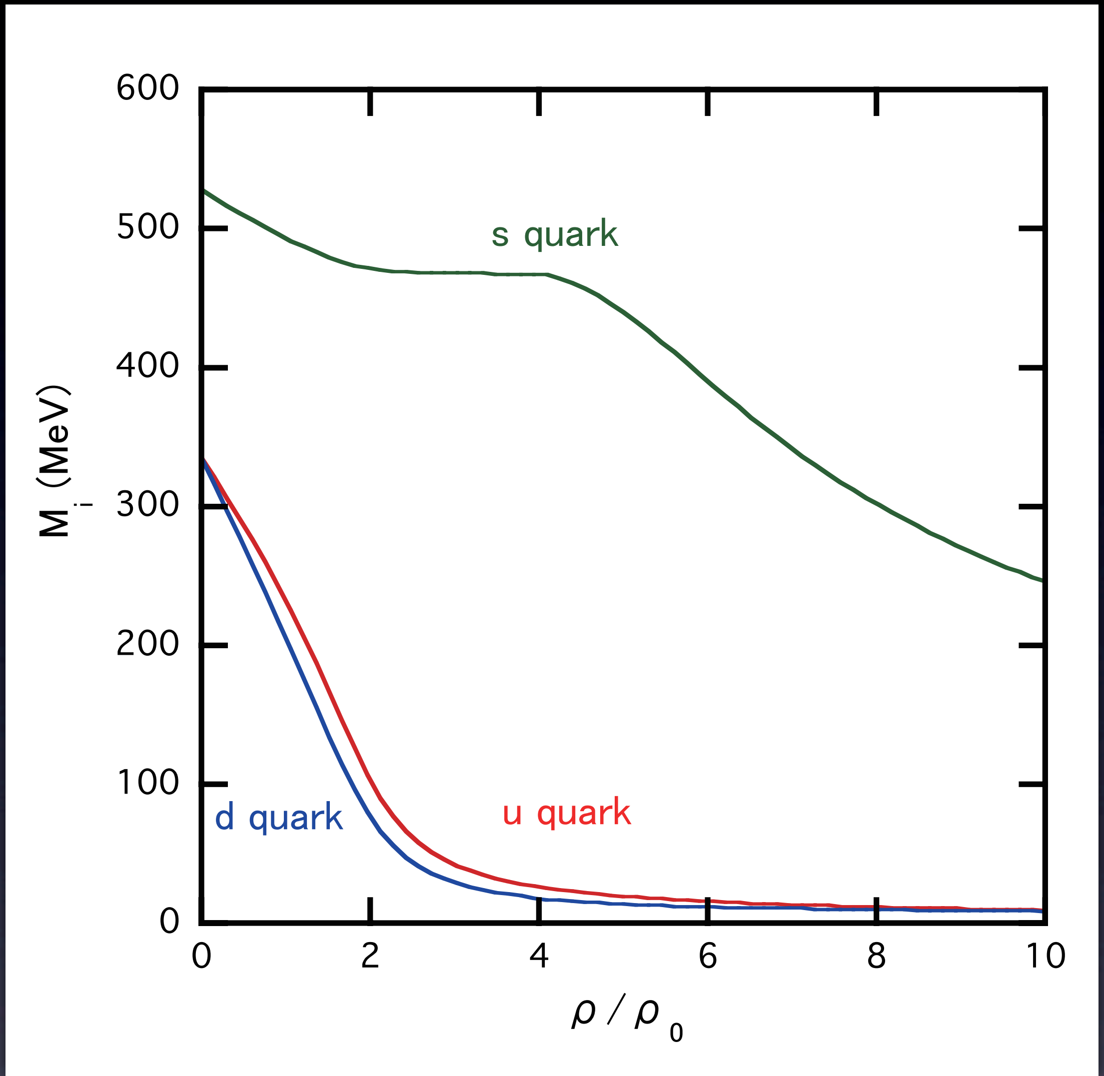
(Fierz: $G_V = 0.5G_S$)

Bratovic *et al.* (2012)

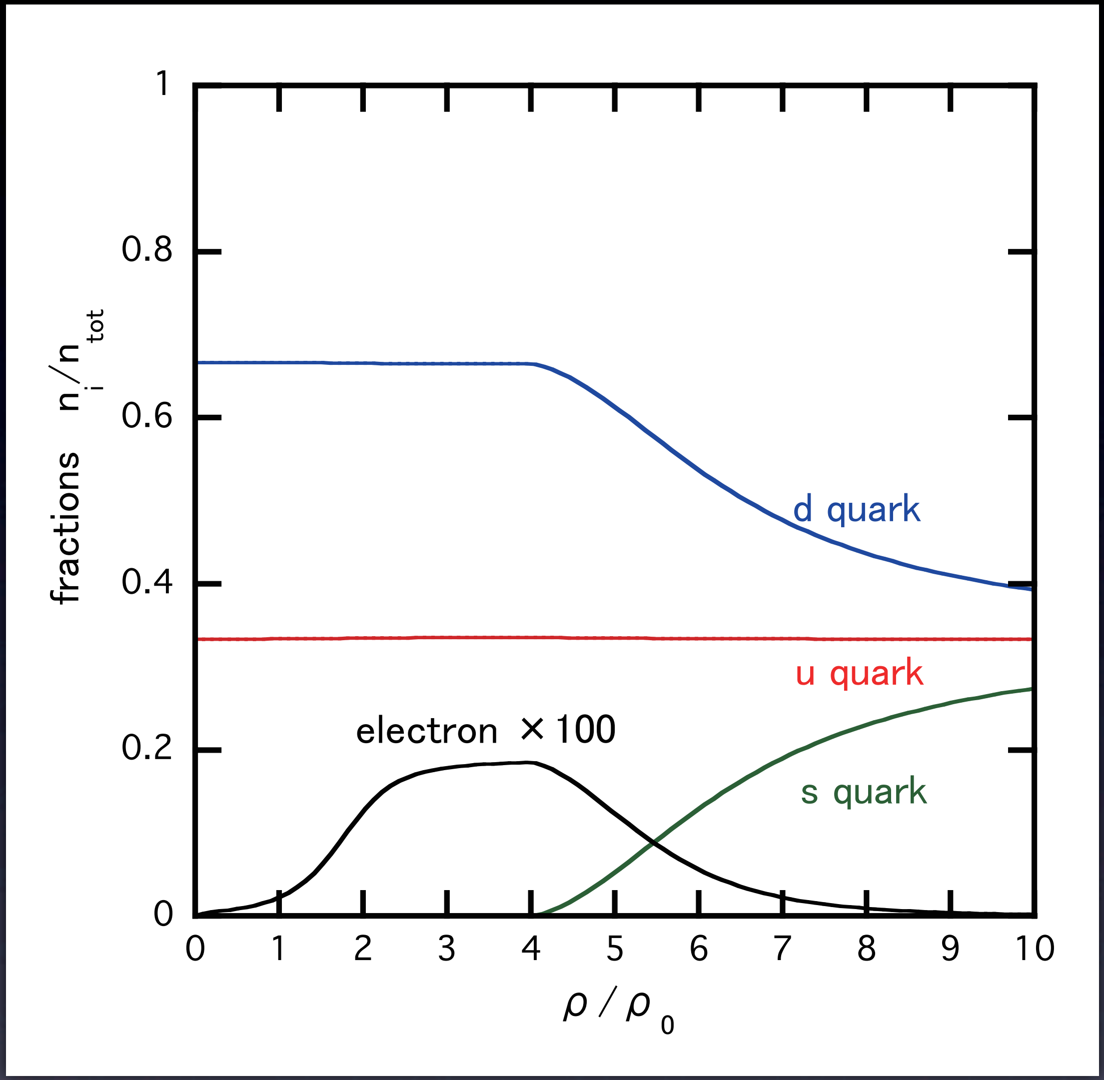
Conditions:
1. beta-equilibrium
2. charge neutrality

EOS at $\rho \gg \bar{\rho}$

Constituent mass



Number fraction



Chiral restoration

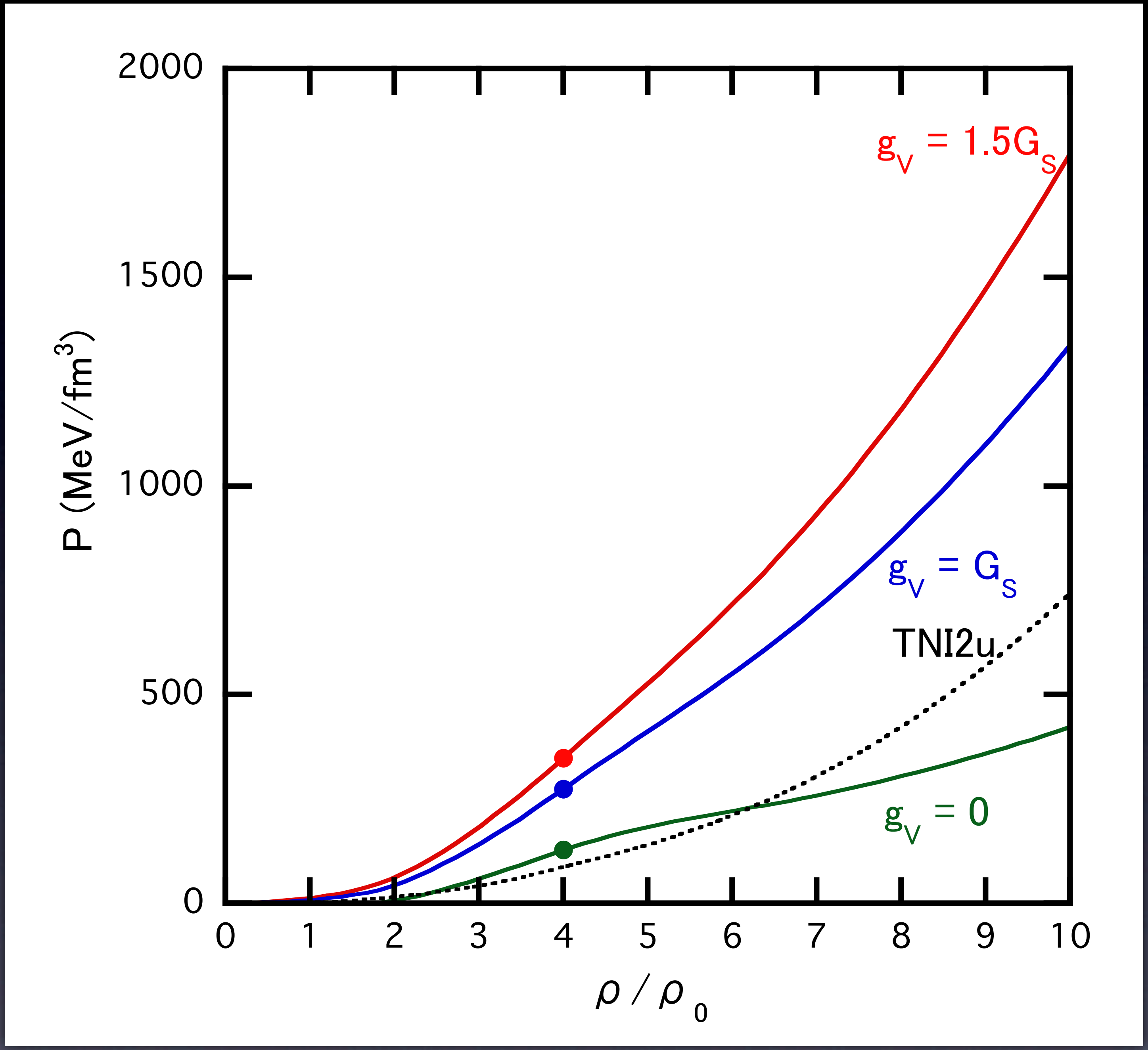
- u,d quark : low densities
- s quark : 4 ρ_0

- s quark starts to appear above 4 ρ_0
- SU(3) flavor symmetric matter at high densities
- muon does not appear due to { s quark, charge neutrality }

- figures do not depend on the magnitude of vector interaction

EOS at $\rho \gg \bar{\rho}$

Pressure P



- EOS becomes stiffer as g_v increases due to the universal repulsion

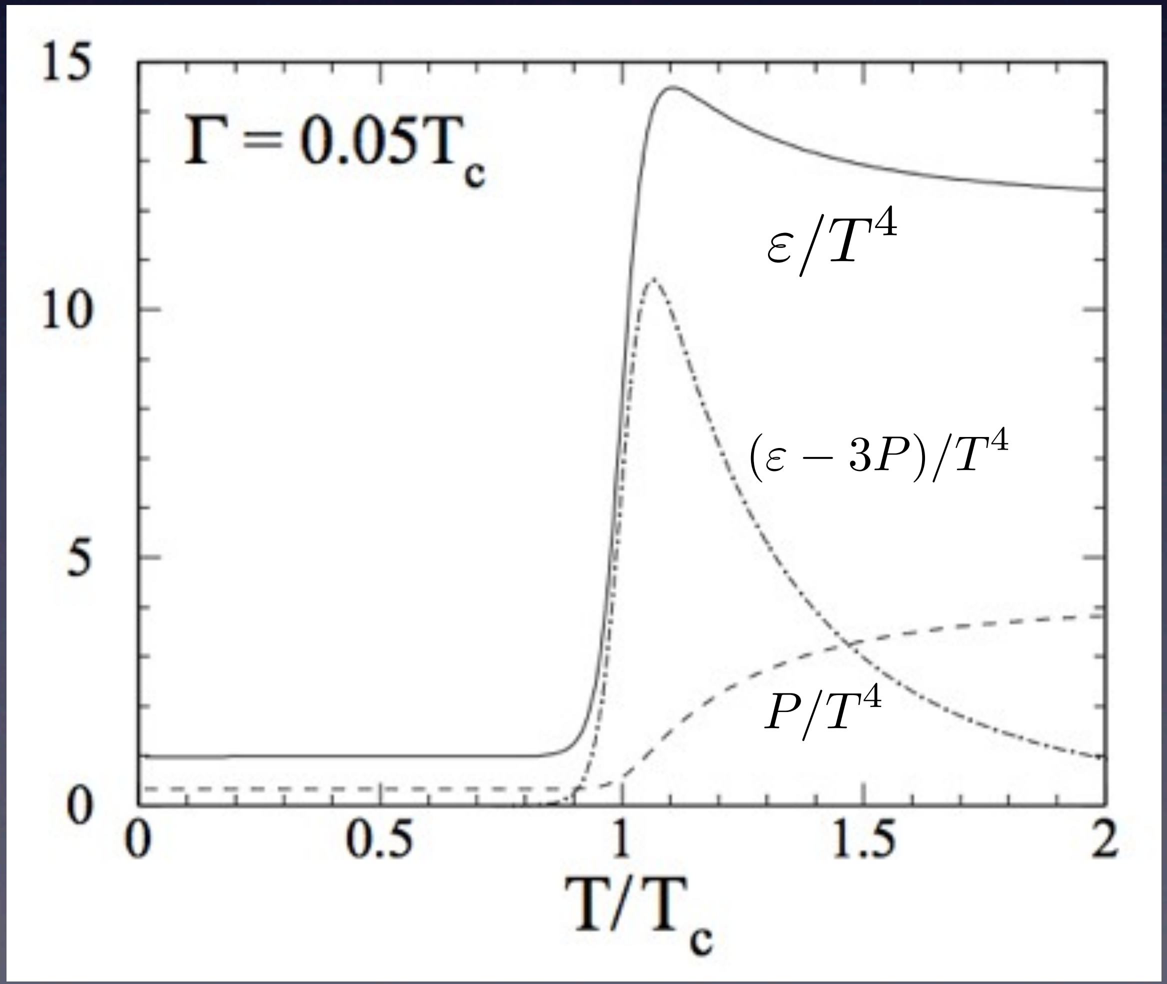
Crossover at finite temperature

- Phenomenological Interpolation: s(T)

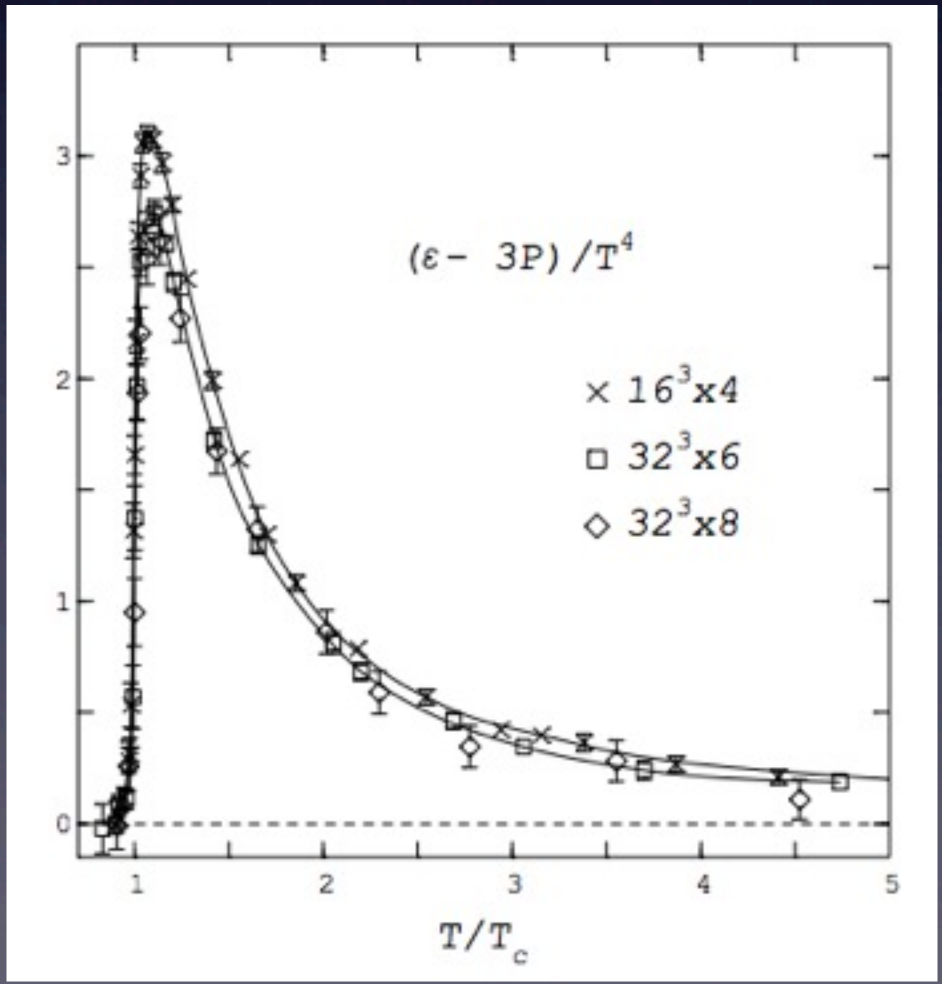
s: entropy density, T: temperature

Asakawa, Hatsuda (1995)

$$\begin{cases} s(T) = s_h(T)w_h(T) + s_q(T)w_q(T) \\ w_q(T) = \dfrac{n\left(1+\tanh\left(\frac{T-T_c}{\Gamma}\right)\right)}{m\left(1-\tanh\left(\frac{T-T_c}{\Gamma}\right)\right)+n\left(1+\tanh\left(\frac{T-T_c}{\Gamma}\right)\right)} \end{cases}$$



Phenomenological Interpolation



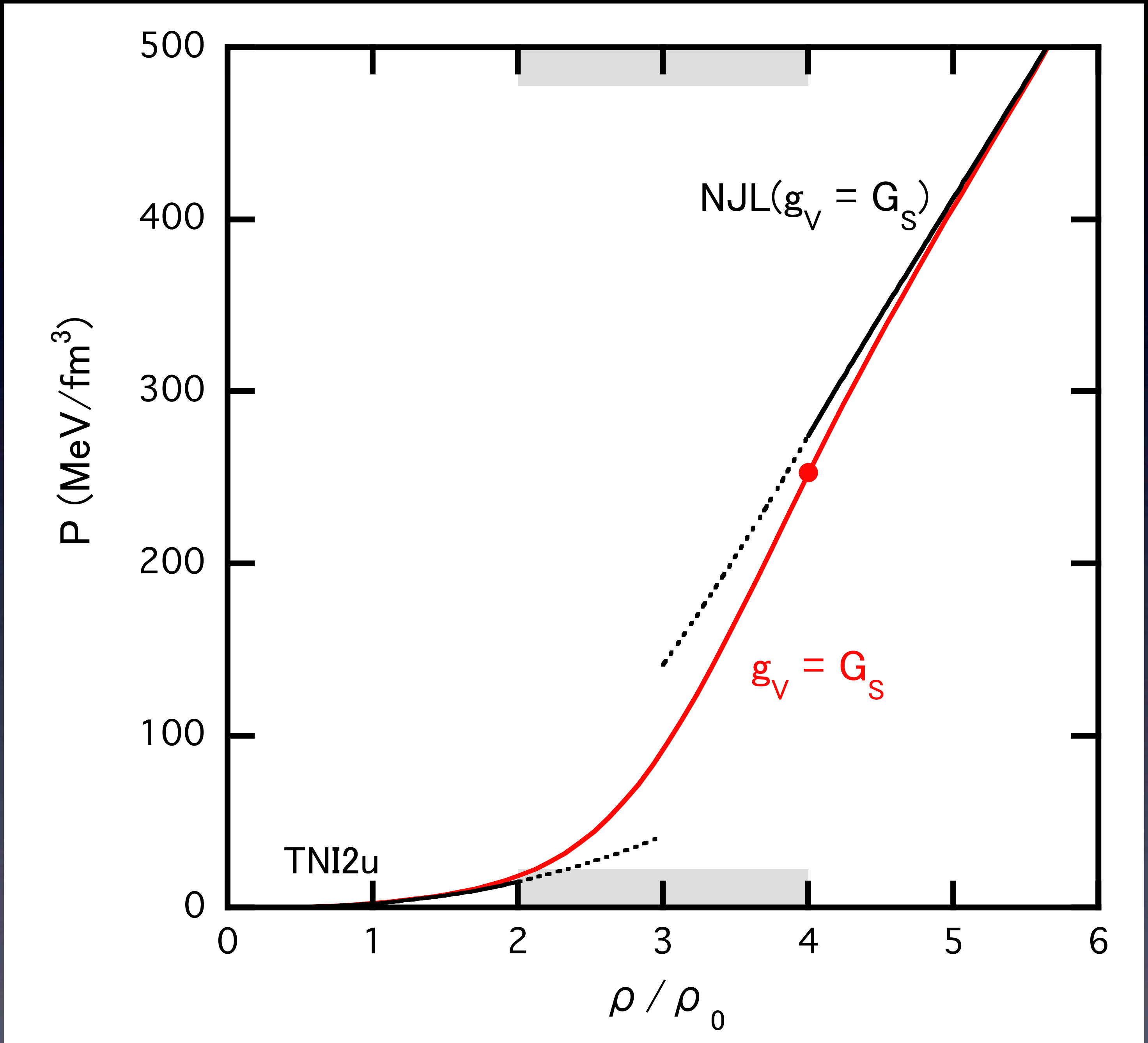
lattice QCD

Karsch (1995)

Interpolated EOS

H-EOS: TNI2u, Q-EOS: NJL

$g_v = G_S \quad (\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$



- In the crossover region, interpolated EOS is larger than H-EOS.
- Rapid stiffening of the EOS in the crossover region

Results (2): Effects of parameters

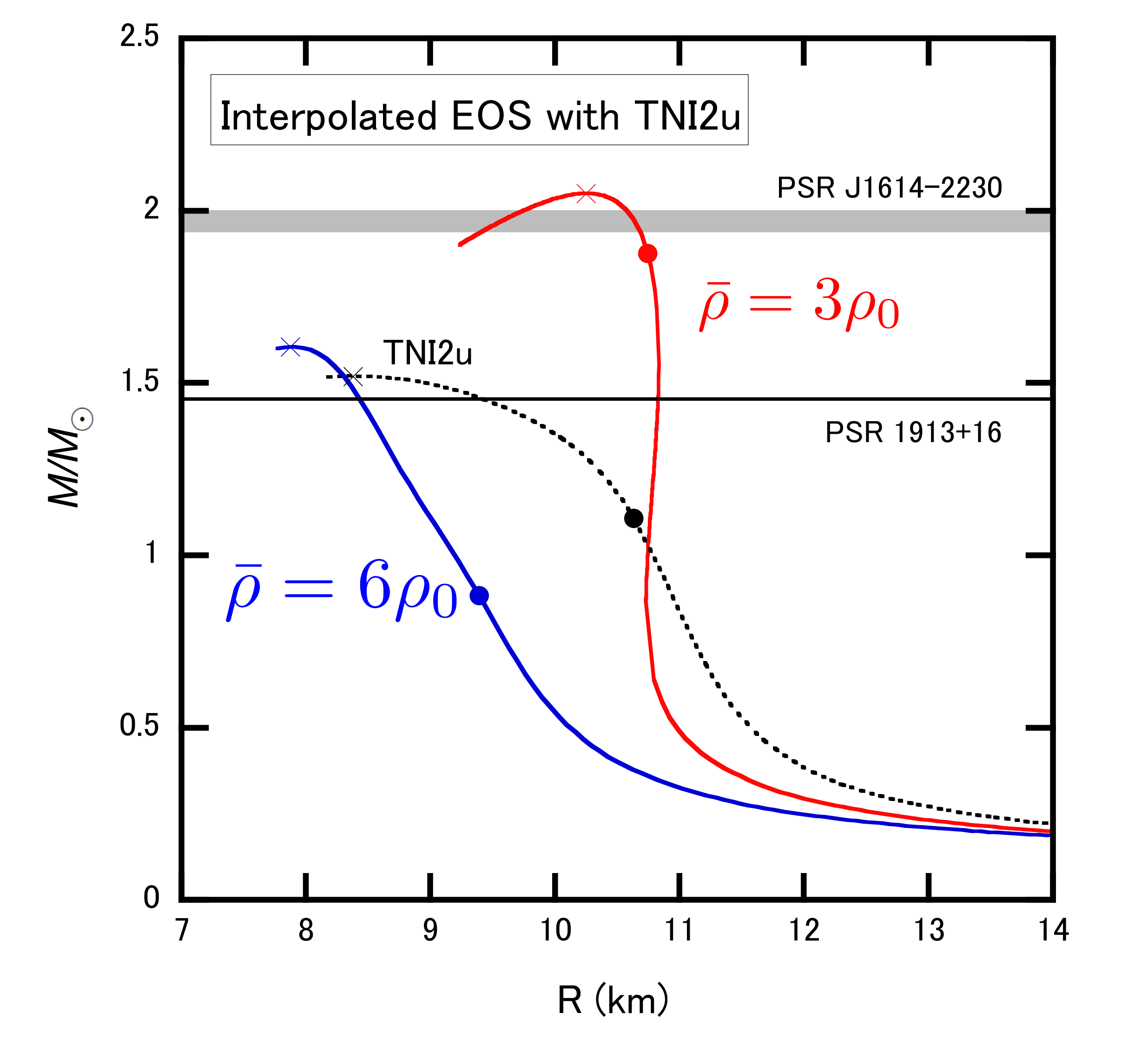
How maximum mass depends on $\bar{\rho},\ \Gamma$

$\bar{\rho}$	$\Gamma/\rho_0 = 1$		$\Gamma/\rho_0 = 2$	
	$g_v = G_s$	$g_v = 1.5G_s$	$g_v = G_s$	$g_v = 1.5G_s$
$3\rho_0$	2.05	2.17	-	-
$4\rho_0$	1.89	1.97	-	-
$5\rho_0$	1.73	1.79	1.74	1.80
$6\rho_0$	1.60	1.64	1.62	1.66

Crossover occurs at relatively low densities and quarks are strongly interacting $\longrightarrow 2M_\odot$

Results (2): Effects of Crossover Density ($\bar{\rho}$)

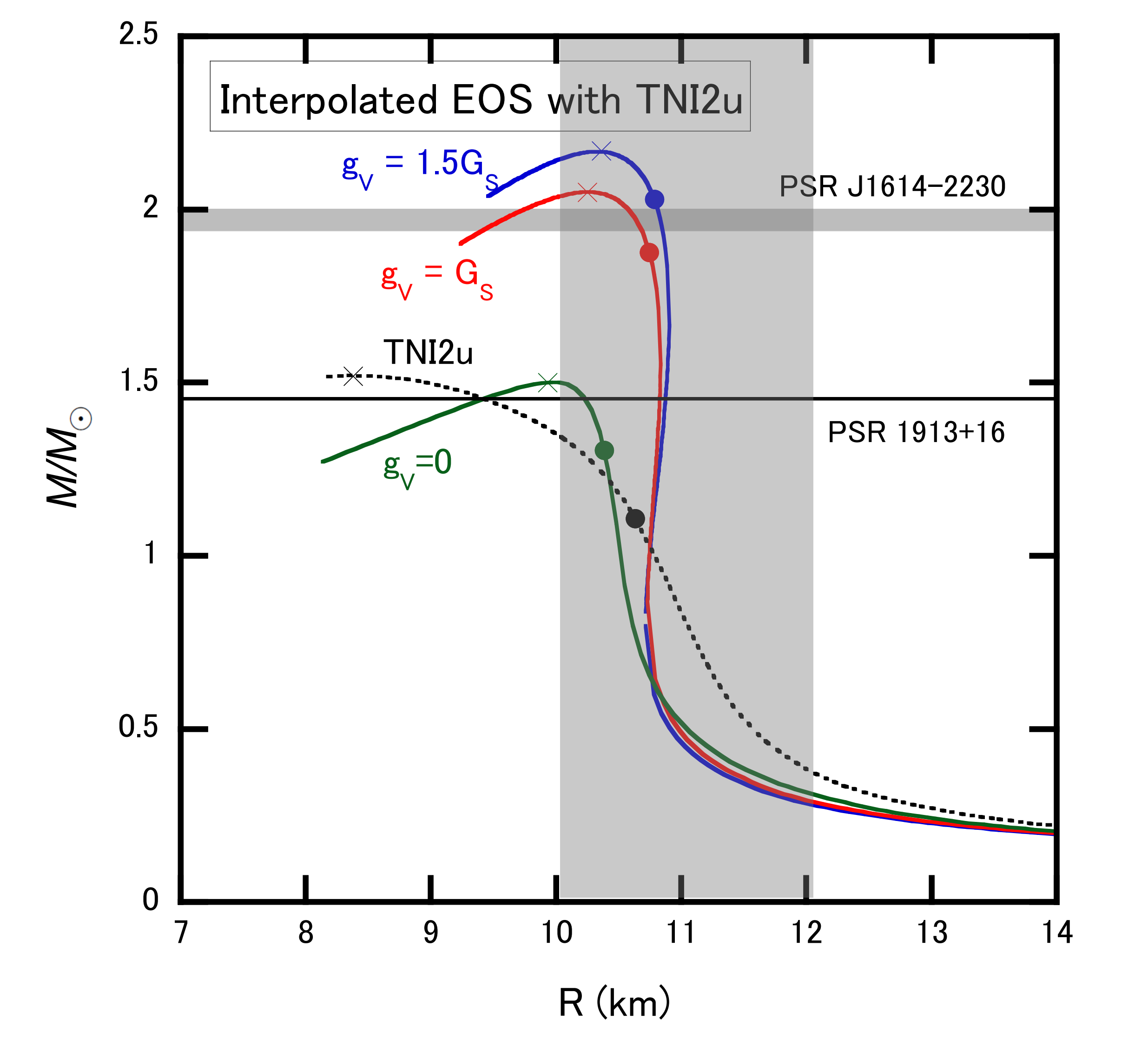
M-R relation $\Gamma = \rho_0 \quad g_v = G_S$



- Crossover occurs at relatively low densities $\longrightarrow$ $2M_\odot$

Results (3): Effects of Vector Int. (g_V)

M-R relation $(\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$

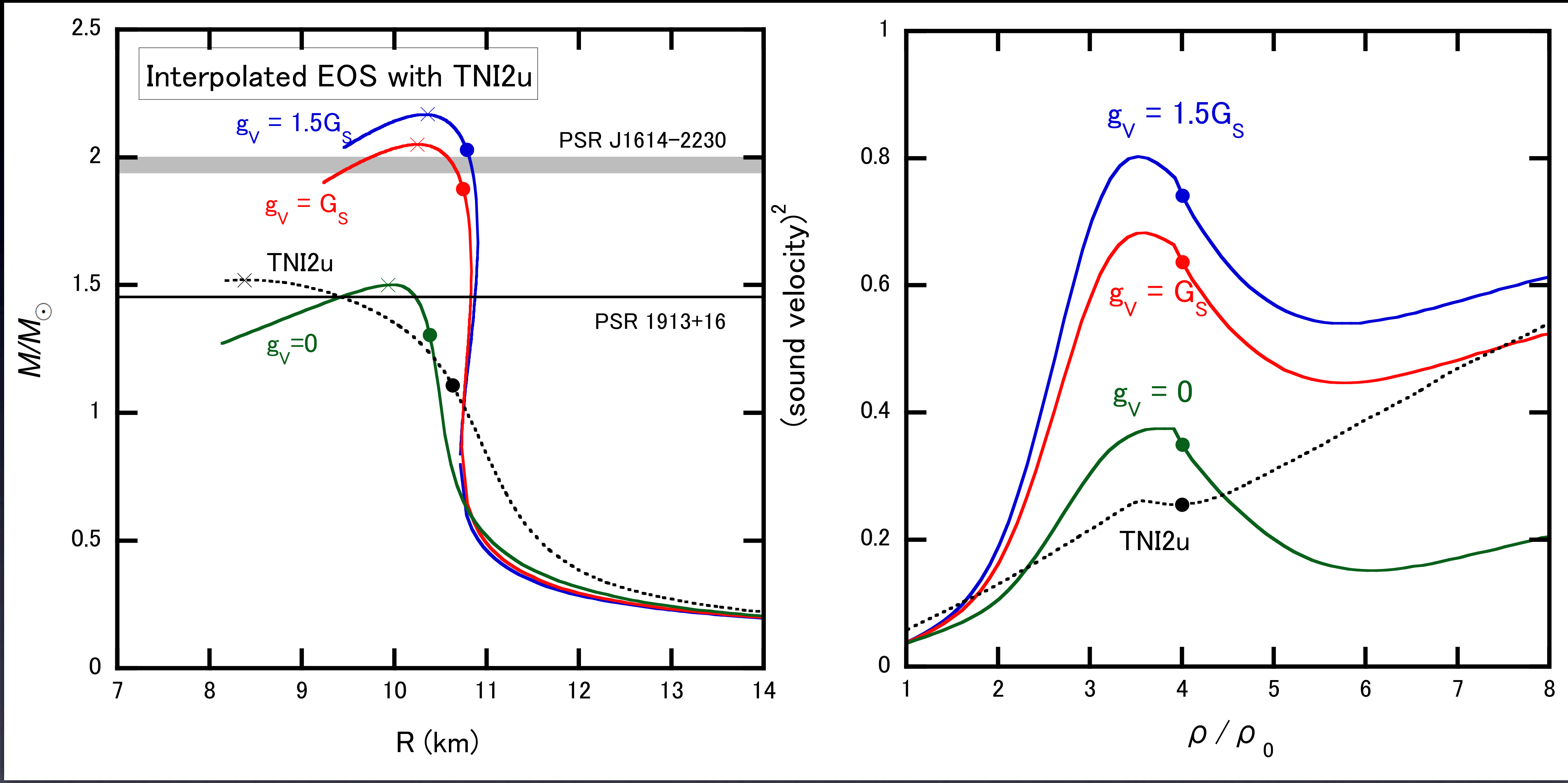


- The maximum mass exceeds $2M_\odot$ only if the vector type repulsion is as strong as the scalar interaction

- Radius: about 11km

Results (3): Sound Velocity

M-R relation $(\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$ $g_v = G_S$



- The emergence of strangeness softens EOS
- Due to the interpolation, the sound velocity increases rapidly in the crossover region

CSC Lagrangian

$$L_{\rm CSC} = L_{\rm NJL} + \frac{H}{2}\sum_{A=2,5,7}\sum_{A'=2,5,7}(\bar{q}i\gamma_5\tau_A\lambda_{A'}C\bar{q}^T)(q^TCi\gamma_5\tau_A\lambda_{A'}q)$$

$$H = \frac{3}{4}G_s$$

by Fierz

$$q^C = C\bar{q}^T \qquad \Psi = \frac{1}{\sqrt{2}}\begin{pmatrix} q \\ q^C \end{pmatrix}$$

$$\Delta_1 = -Hs_{55},\ \Delta_2 = -Hs_{77},\ \Delta_3 = -Hs_{22}$$

$$\Omega(T,\mu_{u,d,s}) = -\frac{T}{2}\sum_\ell\int\frac{d^3p}{(2\pi)^3}\mathrm{Tr}\ln\left(\frac{S^{-1}(i\omega_\ell,\mathbf{p})}{T}\right) + G_S\sum_i\sigma_i^2$$

$$+4G_D\sigma_u\sigma_d\sigma_s - \frac{1}{2}g_V\left(\sum_i n_i\right)^2 + \frac{1}{2H}\sum_{\rm color}|\Delta_c|^2$$

$$S^{-1} = \begin{pmatrix} S_{0+}^{-1} & \Phi^- \\ \Phi^+ & S_{0-}^{-1}\end{pmatrix} \qquad (\Phi^-)_{ab}^{\alpha\beta} = -\sum_{\rm color}\varepsilon^{\alpha\beta c}\varepsilon_{abc}\Delta_c\gamma_5,\quad \Phi^+ = \gamma^0(\Phi^-)^\dagger\gamma^0$$

$$S_{0\pm}^{-1} = p - M \pm \tilde{\mu}\gamma^0$$

$$\tilde{\mu} = \mu - \frac{1}{2}\mu_3 - \frac{1}{2\sqrt{3}}\mu_8$$

CSC Lagrangian (2)

Buballa (2004)

$$p = \frac{1}{4\pi^2}\sum_{i=1,36}\int_0^{\Lambda} dp p^2 \left(|\varepsilon_i| + 2T\ln\left(1+e^{-|\varepsilon_i/T|}\right)\right) - G_s\sum_i \sigma_i^2$$

$$-4G_D\sigma_u\sigma_d\sigma_s + \frac{1}{2}g_V\left(\sum_i n_i^2\right)^2 - \frac{1}{2H}\sum_{\text{color}}|\Delta_c|^2$$

Gap equations: $\frac{\partial p}{\partial \sigma_i} = \frac{\partial p}{\partial \Delta_i} = \frac{\partial p}{\partial \mu_i} = 0$

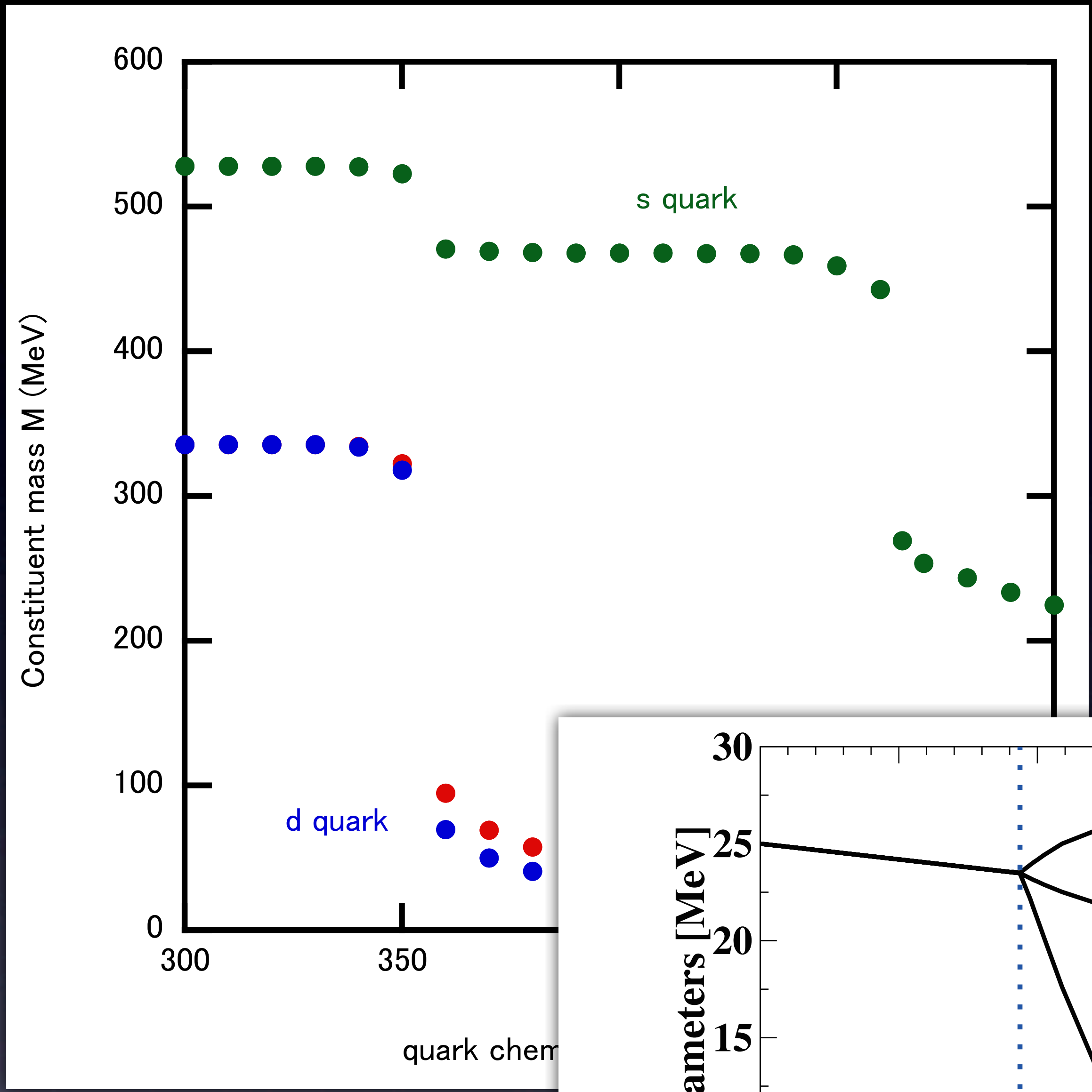
$$\begin{pmatrix} -\mu_d^r + M_d & p & 0 & -\Delta_3 \\ p & -\mu_d^r - M_d & \Delta_3 & 0 \\ 0 & \Delta_3 & \mu_u^g + M_u & p \\ -\Delta_3 & 0 & p & \mu_u^g - M_u \end{pmatrix} \begin{pmatrix} \mu_d^r - M_d & p & 0 & -\Delta_3 \\ p & \mu_d^r + M_d & \Delta_3 & 0 \\ 0 & \Delta_3 & -\mu_u^g - M_u & p \\ -\Delta_3 & 0 & p & -\mu_u^g + M_u \end{pmatrix} \begin{pmatrix} -\mu_s^r + M_s & p & 0 & -\Delta_2 \\ p & -\mu_s^r - M_s & \Delta_2 & 0 \\ 0 & \Delta_2 & \mu_u^b + M_u & p \\ -\Delta_2 & 0 & p & \mu_u^b - M_u \end{pmatrix}$$

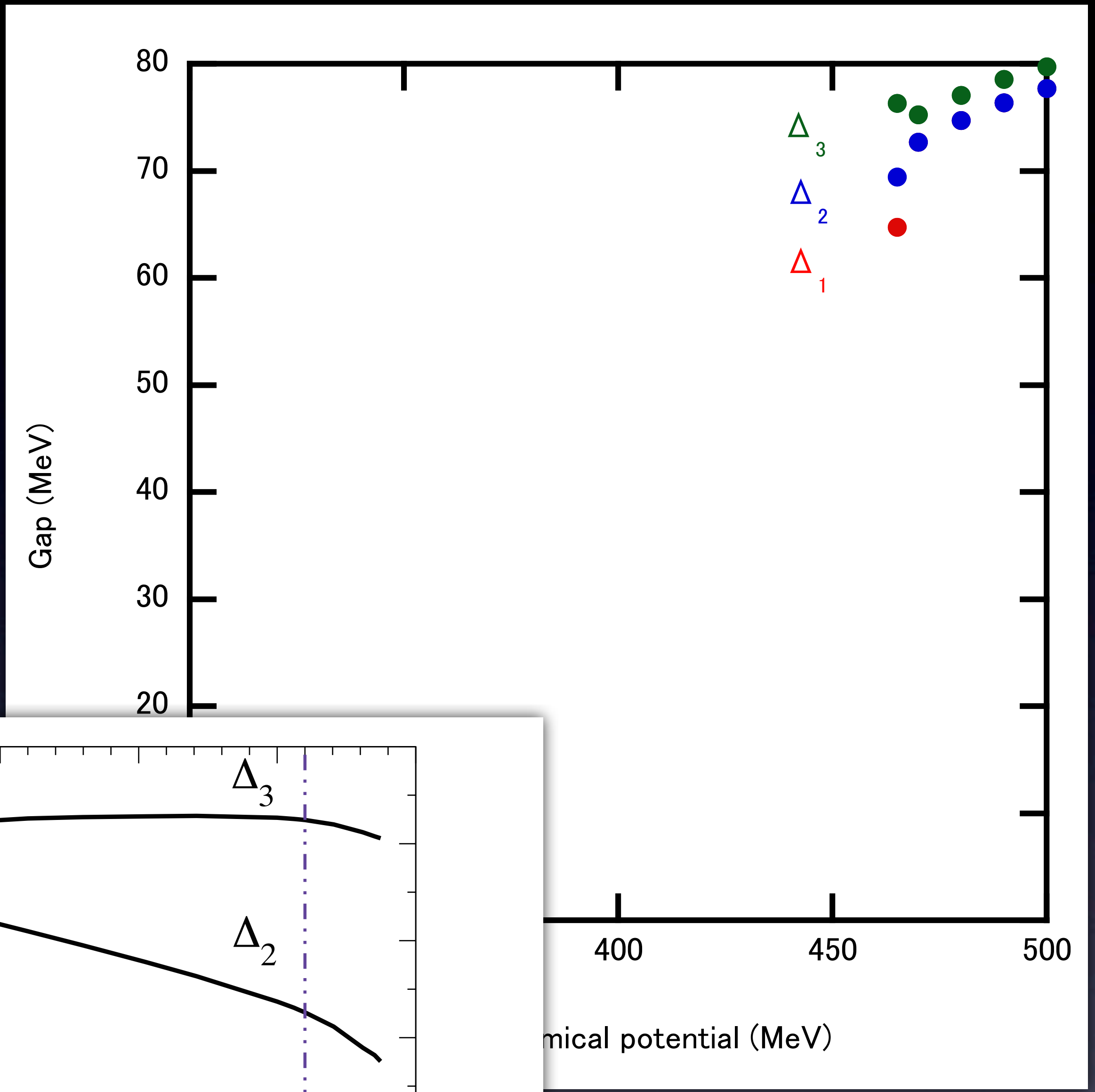
$$\begin{pmatrix} \mu_s^r - M_s & p & 0 & -\Delta_2 \\ p & \mu_s^r + M_s & \Delta_2 & 0 \\ 0 & \Delta_2 & -\mu_u^b - M_u & p \\ -\Delta_2 & 0 & p & -\mu_u^b + M_u \end{pmatrix} \begin{pmatrix} -\mu_s^g + M_s & p & 0 & -\Delta_1 \\ p & -\mu_s^g - M_s & \Delta_1 & 0 \\ 0 & \Delta_1 & \mu_d^b + M_u & p \\ -\Delta_1 & 0 & p & \mu_d^b - M_d \end{pmatrix} \begin{pmatrix} \mu_s^g - M_s & p & 0 & -\Delta_1 \\ p & \mu_s^g + M_s & \Delta_1 & 0 \\ 0 & \Delta_1 & -\mu_d^b - M_u & p \\ -\Delta_1 & 0 & p & -\mu_d^b + M_d \end{pmatrix}$$

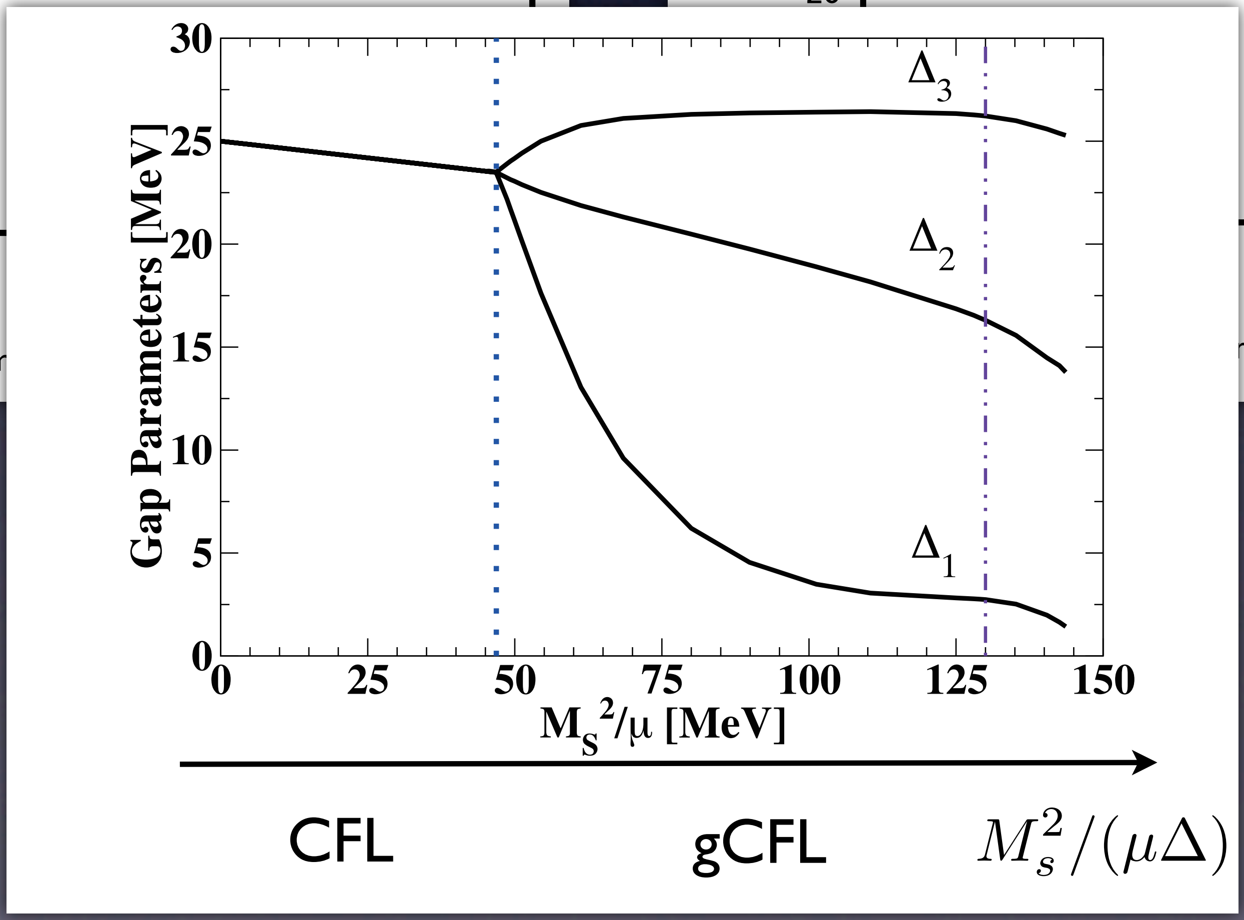
$$\begin{pmatrix}
-\mu_u^r - M_u & p & 0 & 0 & 0 & 0 & 0 & -\Delta_3 & 0 & 0 & 0 & -\Delta_2 \\
p & -\mu_u^r + M_u & 0 & 0 & 0 & 0 & \Delta_3 & 0 & 0 & 0 & \Delta_2 & 0 \\
0 & 0 & \mu_u^r - M_u & p & 0 & \Delta_3 & 0 & 0 & 0 & \Delta_2 & 0 & 0 \\
0 & 0 & p & \mu_u^r + M_u & -\Delta_3 & 0 & 0 & 0 & -\Delta_2 & 0 & 0 & 0 \\
0 & 0 & 0 & -\Delta_3 & -\mu_d^g - M_d & p & 0 & 0 & 0 & 0 & 0 & -\Delta_1 \\
0 & 0 & \Delta_3 & 0 & p & -\mu_d^g + M_d & 0 & 0 & 0 & 0 & \Delta_1 & 0 \\
0 & \Delta_3 & 0 & 0 & 0 & 0 & \mu_d^g - M_d & p & 0 & \Delta_1 & 0 & 0 \\
-\Delta_3 & 0 & 0 & 0 & 0 & 0 & p & \mu_d^g + M_d & -\Delta_1 & 0 & 0 & 0 \\
0 & 0 & 0 & -\Delta_2 & 0 & 0 & 0 & -\Delta_1 & -\mu_s^b - M_s & p & 0 & 0 \\
0 & 0 & \Delta_2 & 0 & 0 & 0 & \Delta_1 & 0 & p & -\mu_s^b + M_s & 0 & 0 \\
0 & \Delta_2 & 0 & 0 & 0 & \Delta_1 & 0 & 0 & 0 & 0 & \mu_s^b - M_s & p \\
-\Delta_2 & 0 & 0 & 0 & -\Delta_1 & 0 & 0 & 0 & 0 & 0 & p & \mu_s^b + M_s
\end{pmatrix}$$

Results (5): Case I $H = \frac{3}{4}G_s$

$g_v = 0$



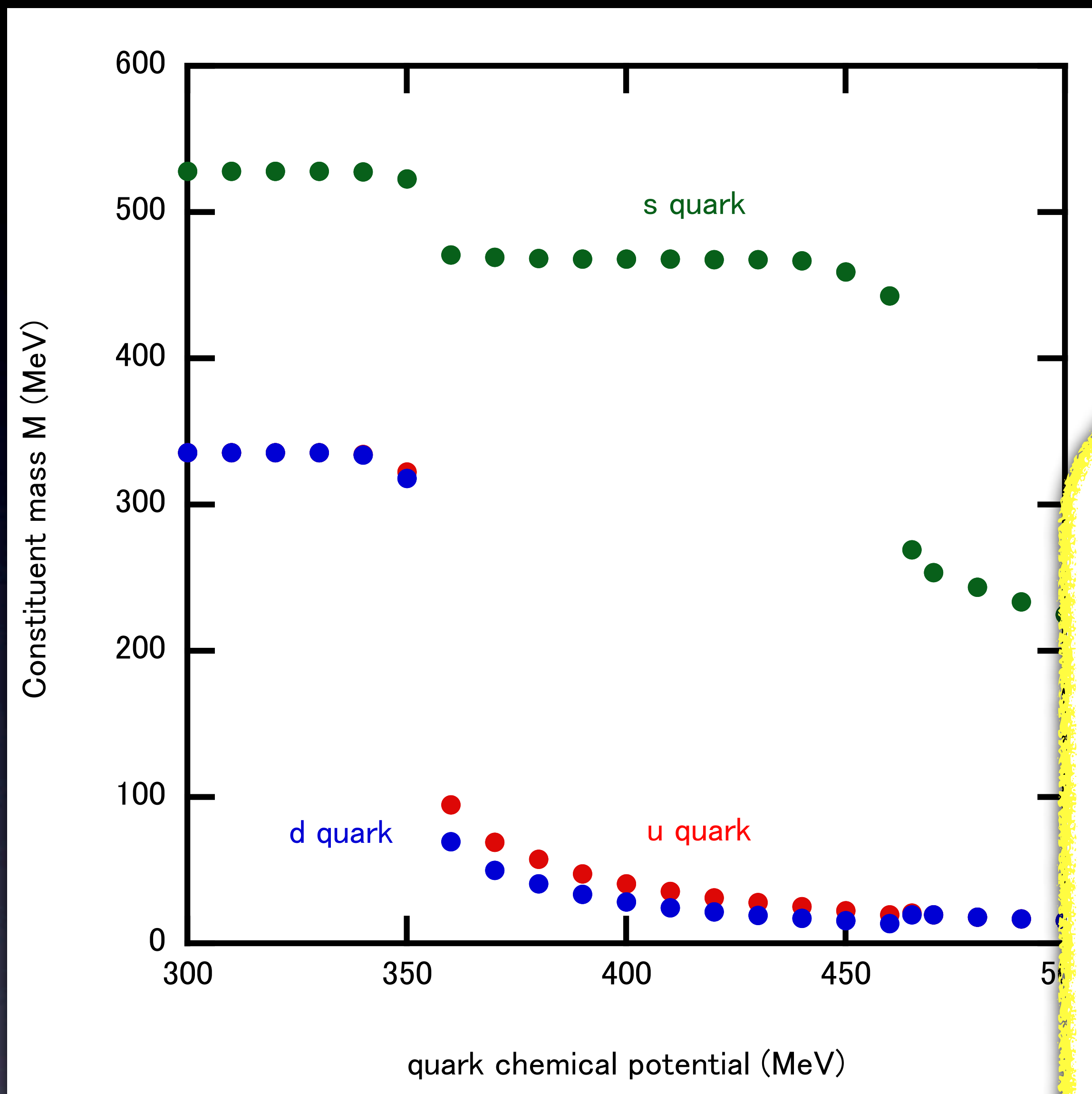


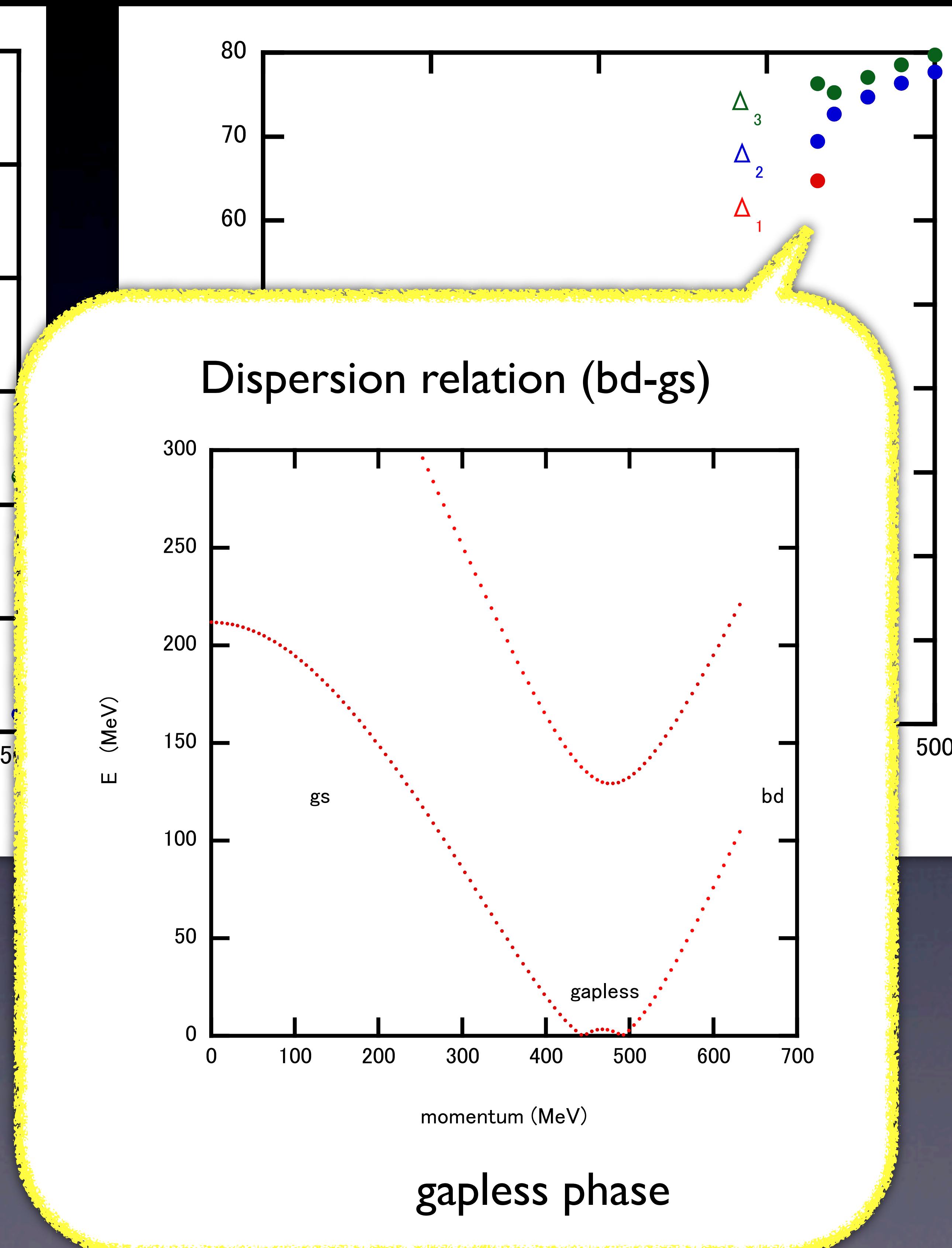


Alford (2004)

Results (5): Gap parameter

$g_v = 0$





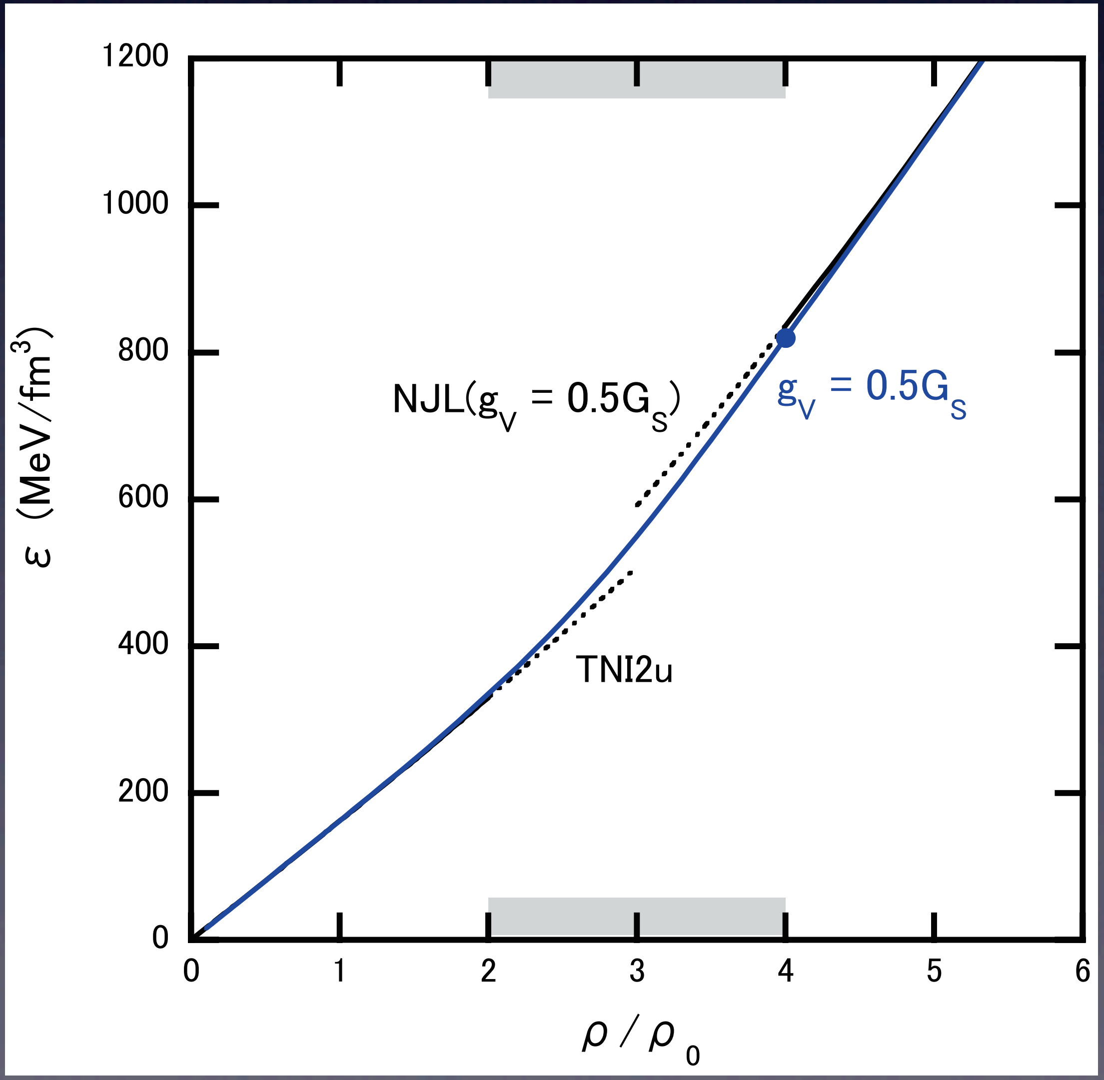
Another Interpolation

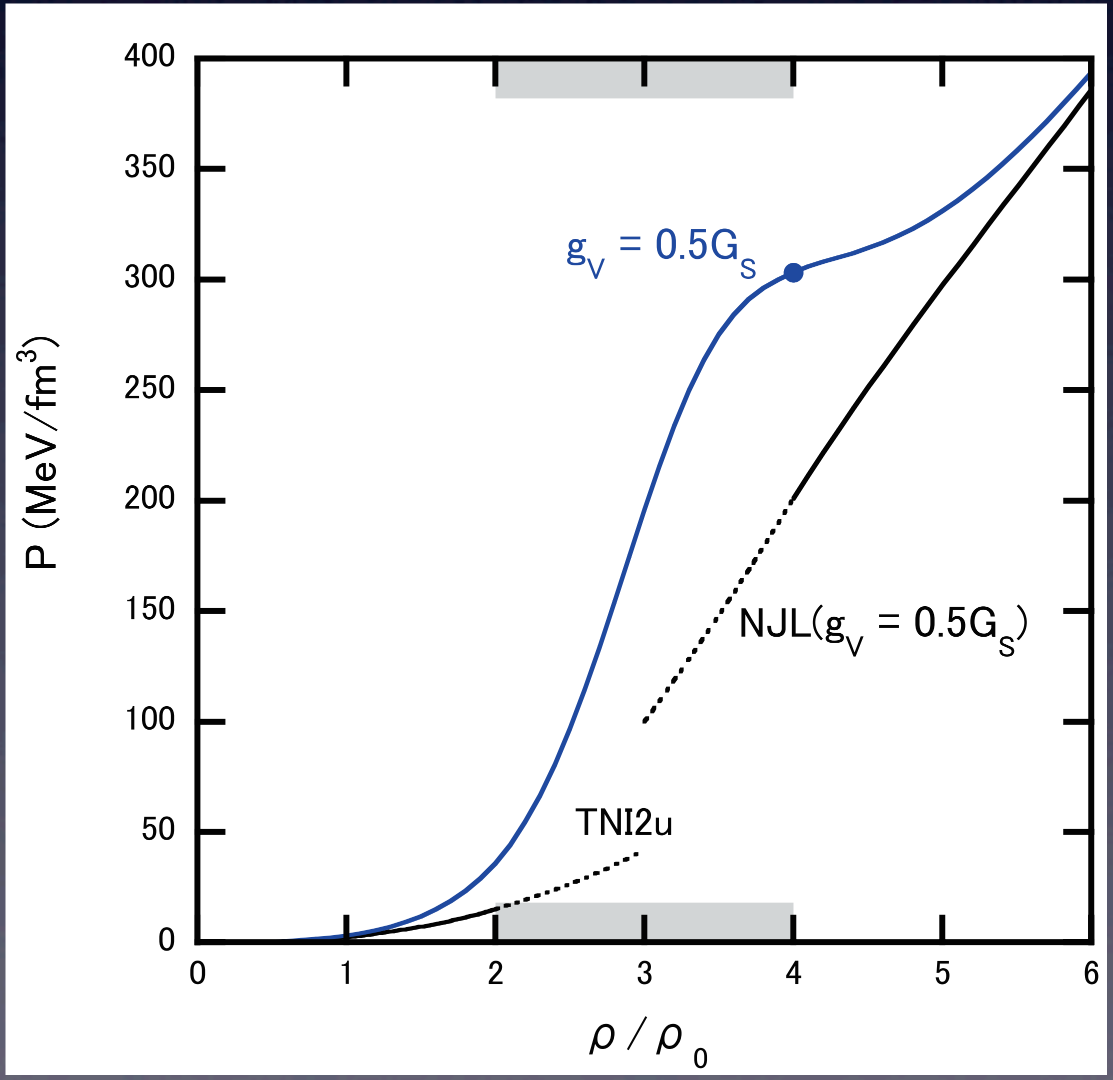
Phenomenological interpolation: $\varepsilon(\rho)$

$$\begin{cases} \varepsilon = \varepsilon_H \times f_- + \varepsilon_Q \times f_+ \qquad f_\pm = \dfrac{1 \pm \tanh(\frac{\rho - \bar{\rho}}{\Gamma})}{2} \\ P = \rho^2 \dfrac{\partial(\varepsilon/\rho)}{\rho} \end{cases}$$

H-EOS: TNI2u, Q-EOS: NJL

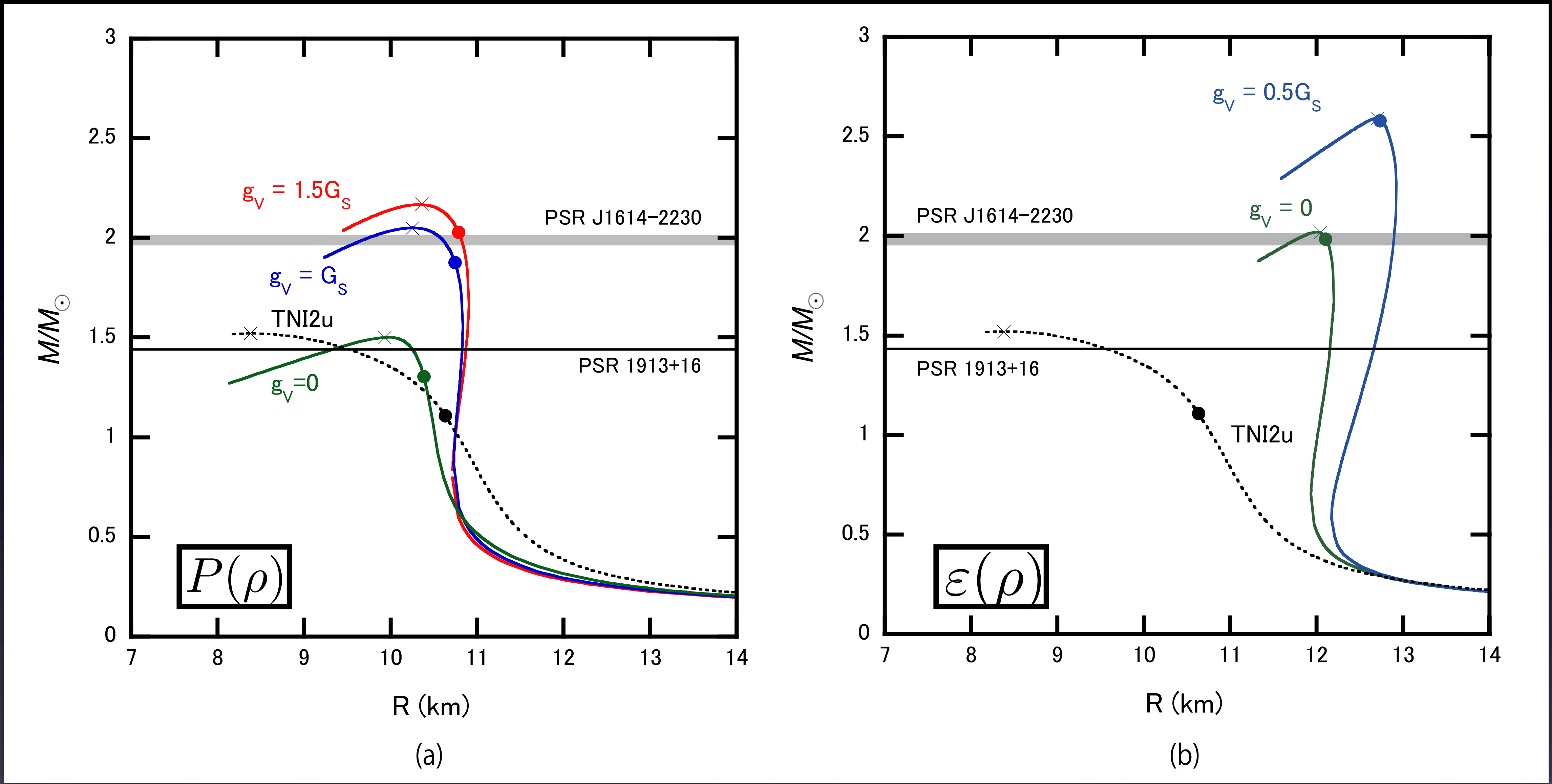
$g_v = 0.5 G_s \qquad (\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$





Results (7): Effects of Method

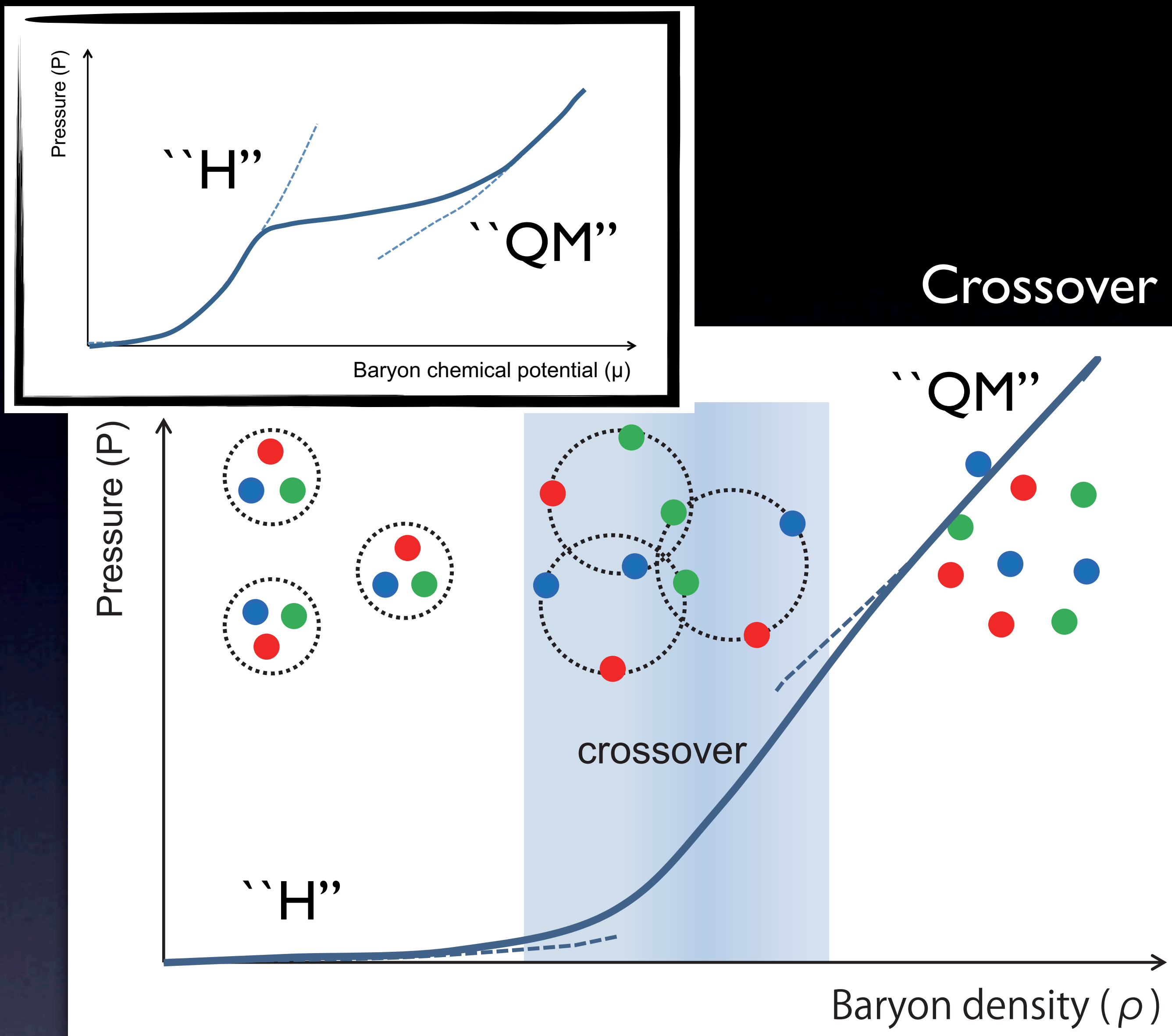
M-R relation $(\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$



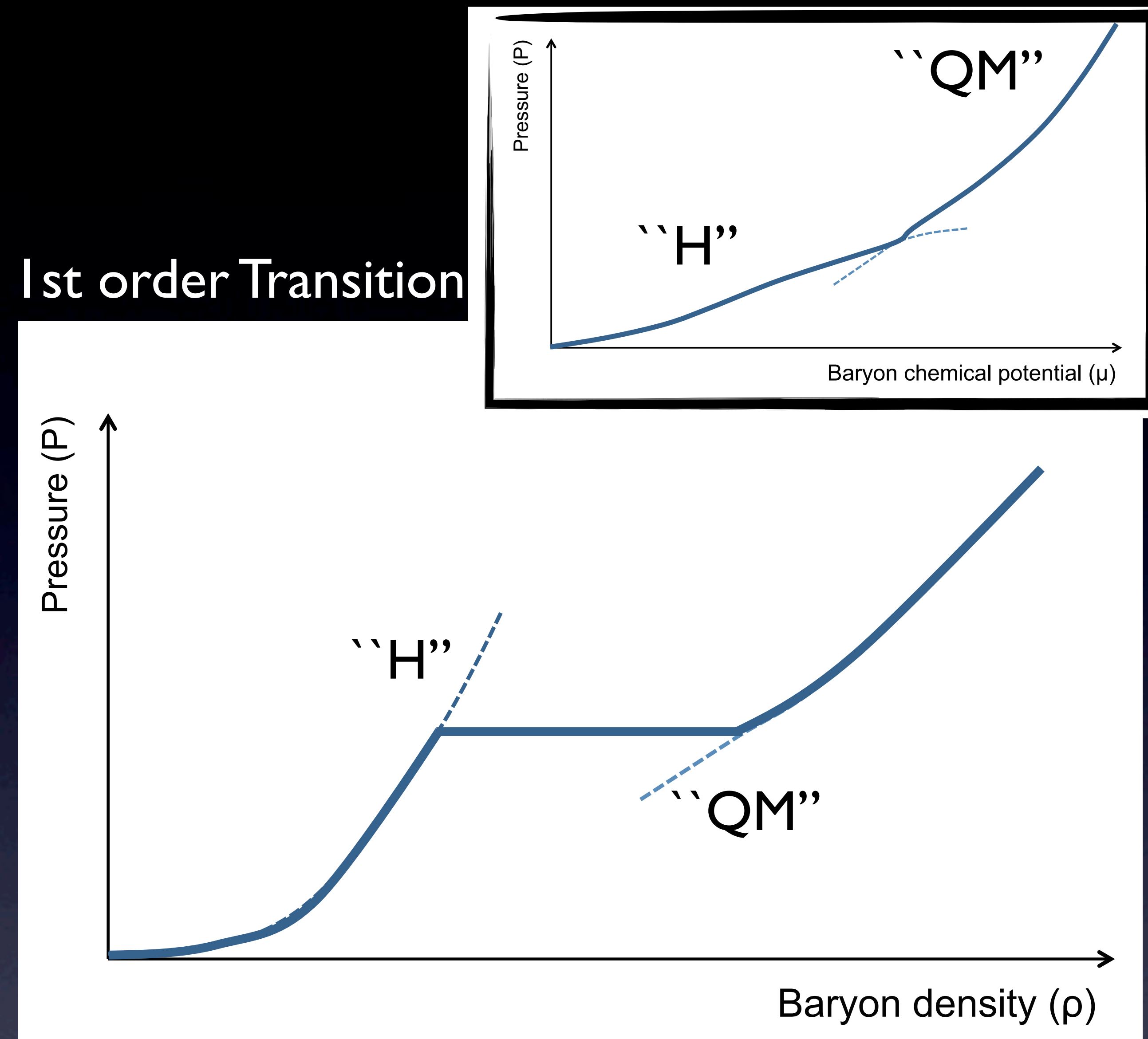
- The ε-interpolation makes EOS stiff more drastically than the P-interpolation.
- Even for $(g_v, \bar{\rho}) = (0, 3\rho_0)$, the maximum mass can exceed $1.97 M_\odot$

Crossover vs. 1st order Transition

Crossover



1st order Transition

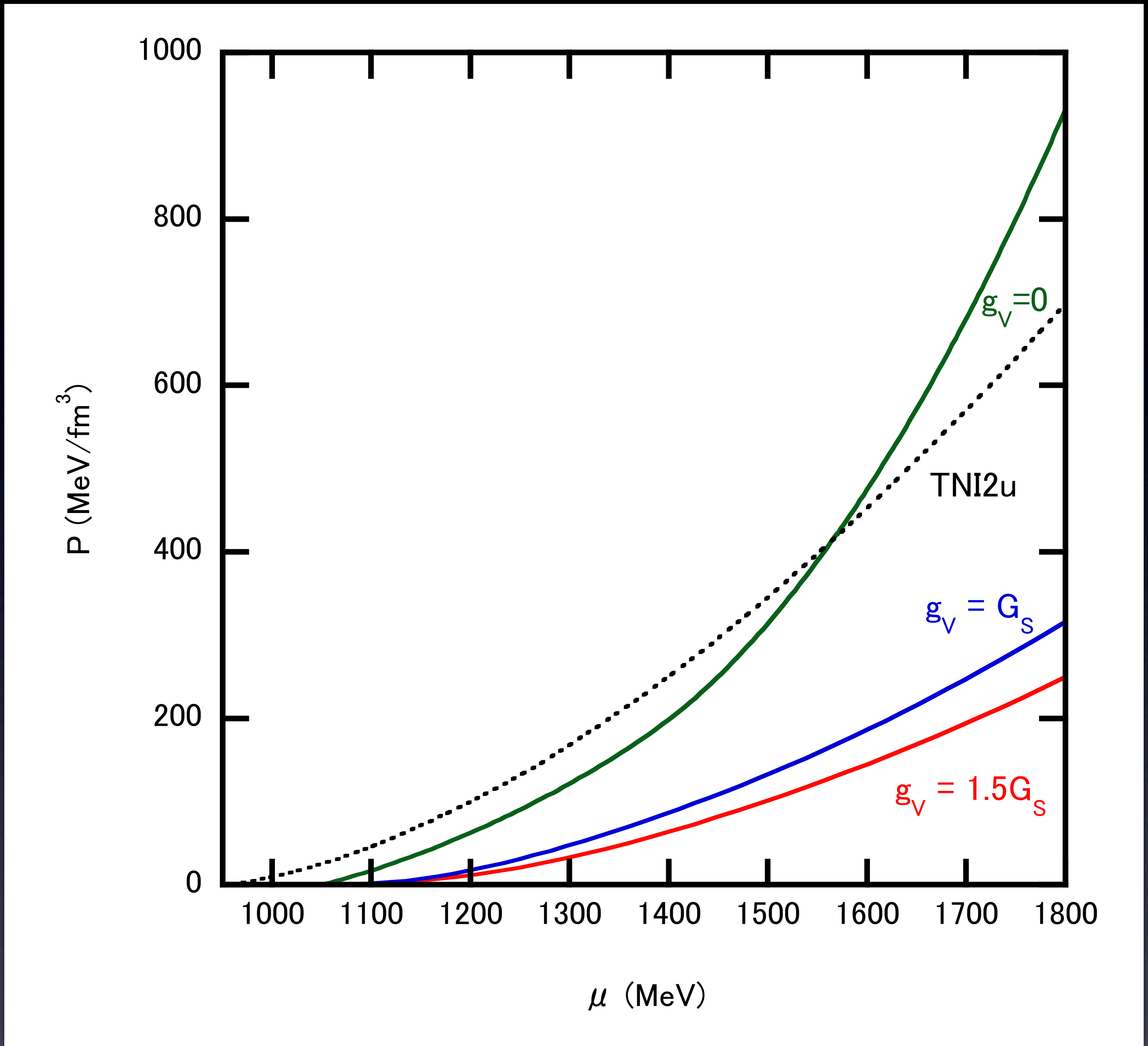


``QM” stiffens EOS ⟷ ``QM” softens EOS

$M > 2M_{\odot}$

$M < 2M_{\odot}$

Crossover vs. 1st order Transition (Example)



In the case of $g_V = 1.0, 1.5G_S$, H-EOS and Q-EOS do not cross at all densities

Neutron Star Observation

Observables:

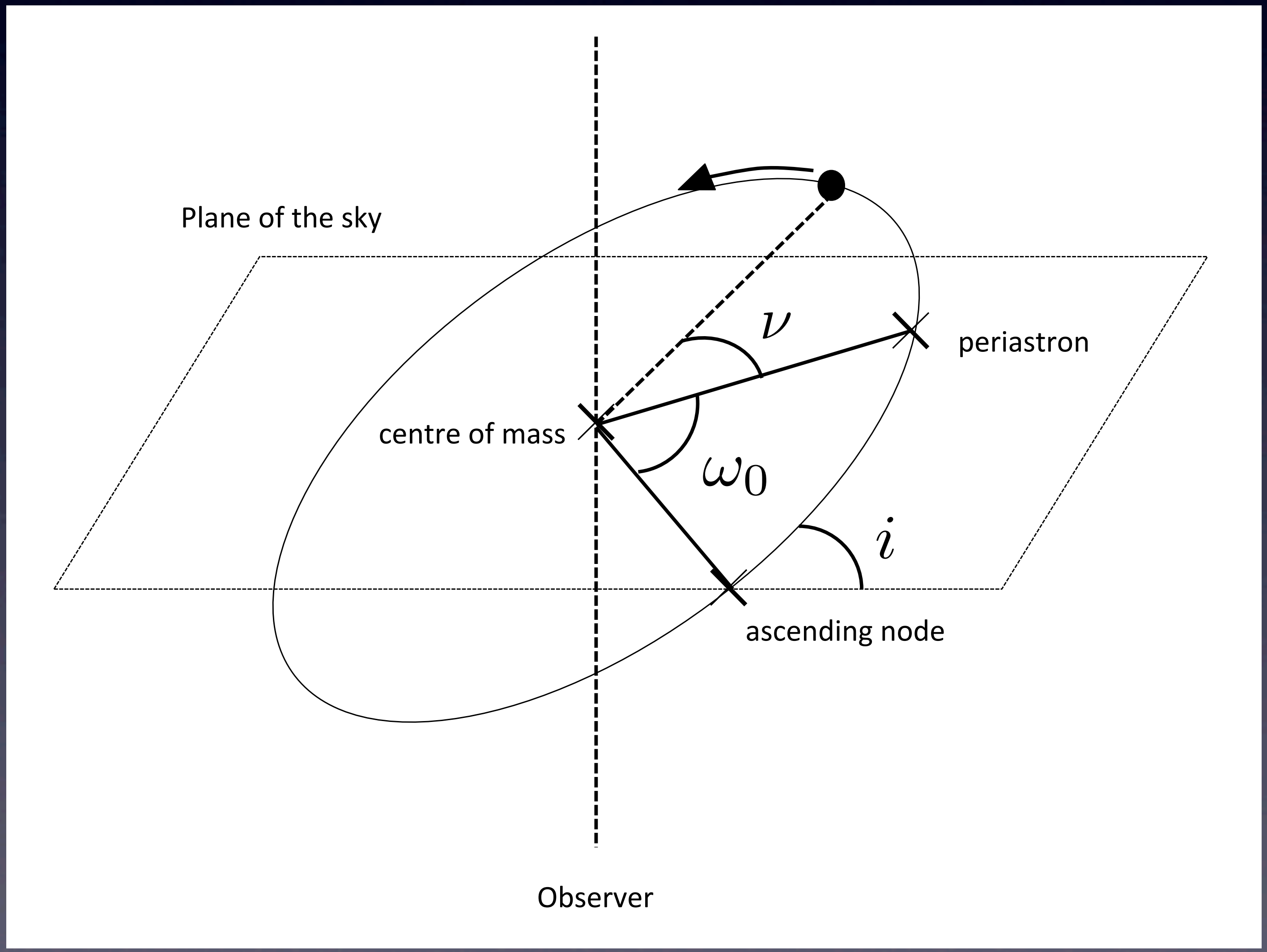
binary period P_b
projection of the pulsar's semimajor axis on the line of sight $x \equiv a\sin i/c$ → mass function $f = \frac{(m_2 \sin i)^3}{M^2}$
eccentricity e
time of periastron T_0
longitude of periastron ω_0

+

General relativity effects:

the advance of periastron of the orbit $\dot{\omega}$
Doppler + gravitational redshift γ
the orbital decay $\dot{P}_b$
range parameter r
shape parameter s

Shapiro delay: $\Delta = 2r\log\frac{1 + e\cos\nu}{1 - s\sin(\omega+\nu)}$



Mass fraction f + 2 general relativity effects

⟶ Mass estimation

Universal 3-body force

TNI model

$$v_{TNI} = v_{TNA} + v_{TNR}$$
$$= v_2 e^{-(r/\lambda_a)^2} \rho e^{-\eta_2 \rho} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)^2 + v_1 e^{-(r/\lambda_r)^2} (1 - e^{-\eta_1 \rho})$$

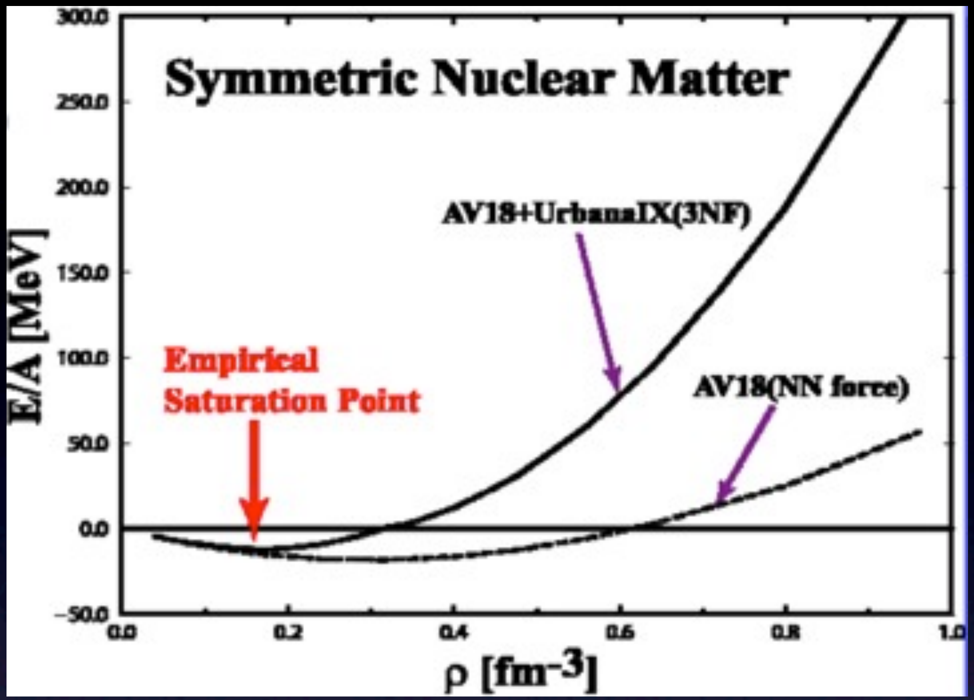
Urbana UIX model

$$v_{ijk} = v_{ijk}^{2\pi} + v_{ijk}^{R}$$
$$= A \sum_{\text{cyc}} \left(\{X_{ij}, X_{jk}\}\{\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k\} + \frac{1}{4}[X_{ij}, X_{jk}][\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k] \right) + U \sum_{\text{cyc}} T^2(r_{ij}) T^2(r_{jk})$$

$$X_{ij} = Y(r_{ij}) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + T(r_{ij}) S_{ij}$$

H-EOS: Universal 3-body force

3-body force is needed for saturation property



Akmal *et al.* (1998)

→ From the point of view of NS observation, 3-body force is needed for the stiffness of EOS

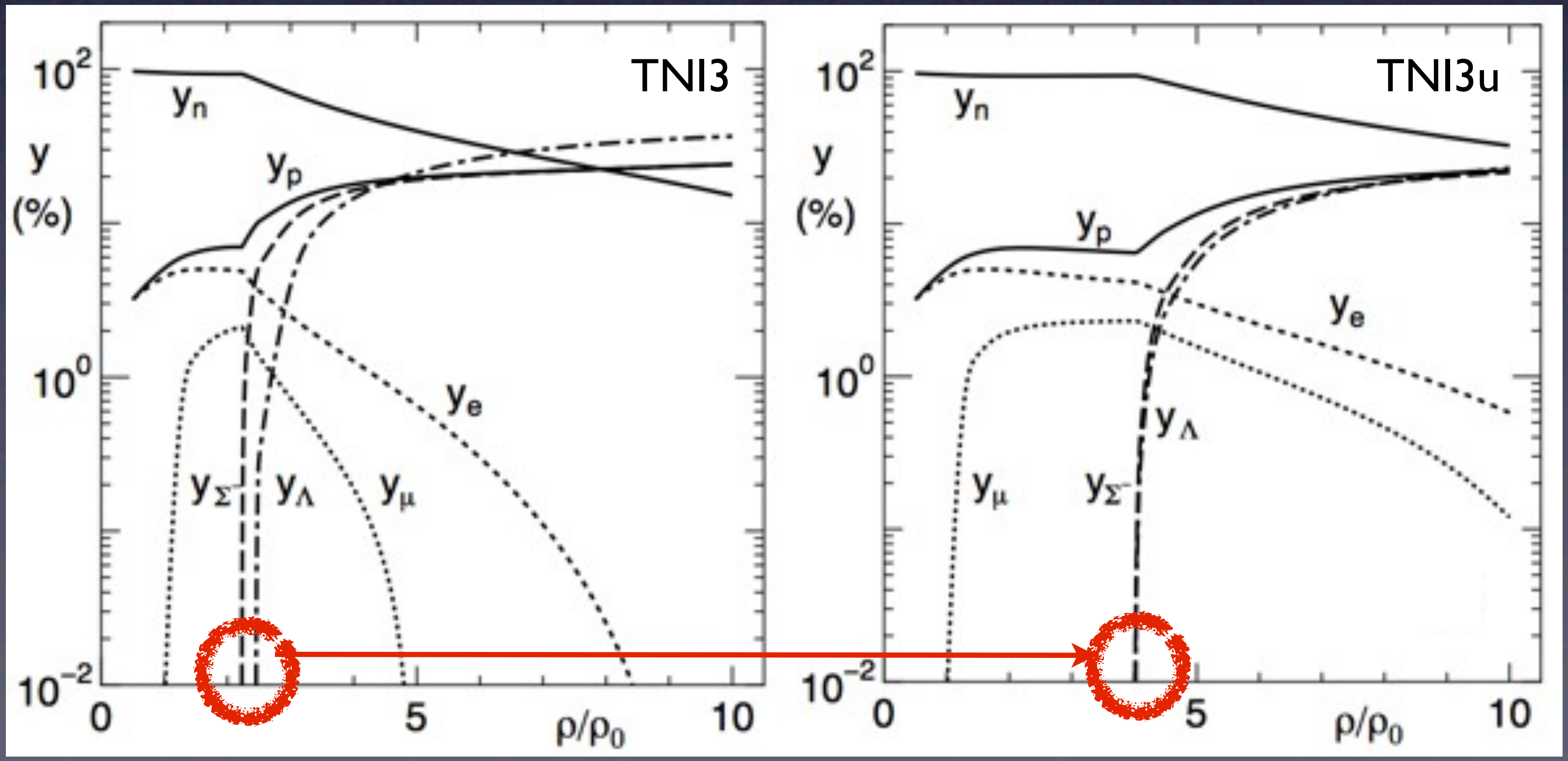
→ 3-body force between YN and YY can delay the appearance of the exotic components

Universal 3-body force

- TNI model:

G-matrix
NN : Reid soft-core potential
YN,YY: Nijmegen type-D
hard-core potential

TNI2(3):
κ=250(300)MeV

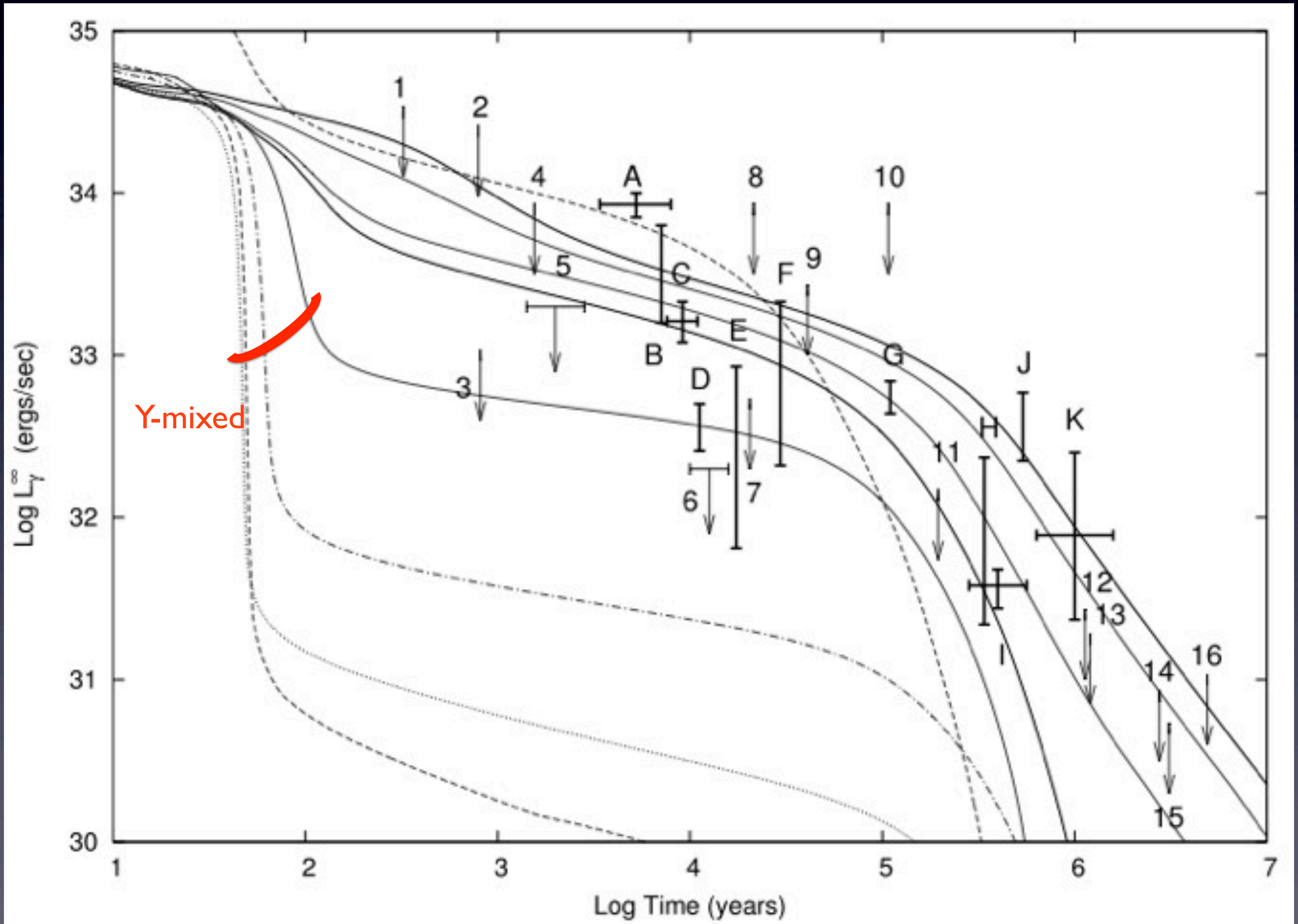


Nishizaki *et al.* (2002)

Cooling Problem

Rapid cooling is occurred by hyperons (Y-Durca)

$$\begin{cases} \Lambda \to p + l + \bar{\nu}_l,\ p + l \to \Lambda + \nu_l \\ \Sigma^- \to \Lambda + l + \bar{\nu}_l,\ \Lambda + l \to \Sigma^- + \nu_l \end{cases}$$



Tsuruta *et al.* (2009)