Numerical Models of Isolated Rotating Neutron Stars

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Plan

1. Stationary axisymmetric spacetimes
2. Magnetic field
3. Two Fluids
4. Equation of state

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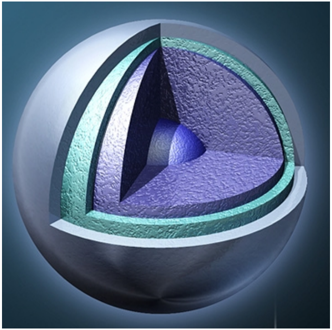
The complex physics of neutron stars

The description of neutron stars involves many different fields of physics, with overall conditions that can hardly be tested on Earth:



- cold, highly asymmetric nuclear matter,
- very strong gravitational field (last stage before black hole),
- intense magnetic field, up to $\sim 10^{17}$ G,
- rapid rotation, implying relativistic fluid velocities.

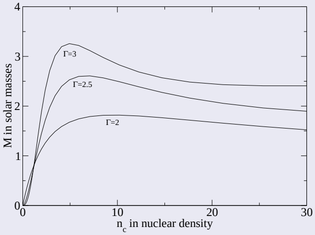
$\Rightarrow$need for theoretical models, often involving numerical simulations

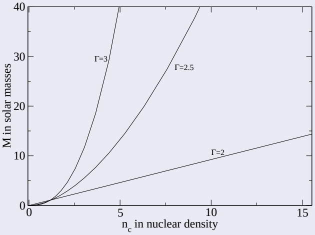
Need for GR

Influence of general relativity (GR) can be measured by the compactness ratio:

$$\mathcal{C} = \frac{GM}{Rc^2}. \qquad \mathcal{C} = 0.5 \iff \text{black hole.}$$

GR makes qualitative difference:



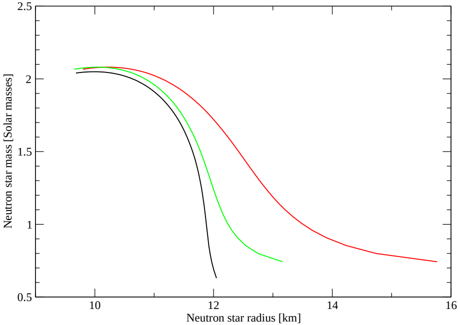


No maximal mass in Newtonian theory!

$\Rightarrow$General Relativity is absolutely necessary…

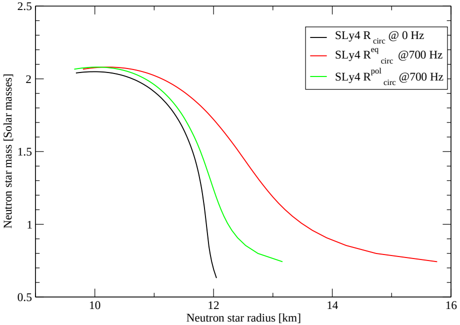
Need for rotation?

Three different EoSs . . .



Need for rotation

One Eos : SLy4 Douchin & Haensel (2001)



Rotation is important when dealing with radii. ⇒Need for precise definitions. . .

Brief history

rotating neutron star models

- Hartle & Thorne (1968) : slow rotation approximation,
- Bonazzola & Maschio (1971) : Lewis-Papapetrou coordinates,
- Wilson (1972) : differentially rotating stars
- Butterworh & Ipser (1975) : Bardeen-Wagoner formulation,
- Friedman *et al.* (1986) and Lattimer *et al.* (1990) : realistic EoSs,
- Bocquet *et al.* (1995) : (electro)magnetic field,
- ...

Some codes:

- Komatsu *et al.* (1989) $\Rightarrow$KEH,
- Bonazzola *et al.* (1993) $\Rightarrow$ `rotstar` (LORENE)
- Stergioulas & Friedmann (1995) $\Rightarrow$ `rns`
- compared in Nozawa *et al.* (1998)
- Ansorg *et al.* (2002) $\Rightarrow$ AKM

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Stationary axisymmetric spacetimes

Symmetries - Metric

To simplify the problem in GR: use of symmetries ⇒Killing vector fields:

- stationarity
- axisymmetry

In adapted coordinates, the metric depend only on (r, θ) and can take the form (quasi-isotropic gauge):

$$ds^2 = -N^2 dt^2 + A^2 \left(dr^2 + r^2 d\theta^2\right) + B^2 r^2 \sin^2\theta \left(d\varphi - \omega\, dt\right)^2 ,$$

with the requirement of circularity condition for matter:

- no meridional (e.g. convective) currents,
- no toroidal magnetic field,

This is quite different from the Schwarzschild gauge used for the TOV system, in spherical symmetry.

Einstein equations

In quasi-isotropic gauge (+maximal slicing), Einstein equations turn into a system of four coupled non-linear elliptic PDEs:

- $\Delta N = \sigma_1$,
- $\Delta \omega = \sigma_2$,
- $\Delta (NB) = \sigma_3$,
- $\Delta (NA) = \sigma_4$.

Each σ_i contains terms involving matter and non-linear metric terms of non-compact support.
$\Rightarrow$Contrary to spherical symmetry no matching to any known vacuum solution is possible (no Birkhoff theorem).
$\Rightarrow$Only boundary condition at $r \to \infty$: flat metric.

The most widely used model for matter is that of a perfect fluid:

$$T^{\mu\nu} = (e + p)\, u^\mu u^\nu + p\, g^{\mu\nu}$$

u^μ : 4-velocity; p: pressure; e: (total) energy density.
Improvements have been implemented, taking into account:

- superfluidity, with the two-fluid approach (see later),
- electromagnetic field in a perfect conductor (see later) or superconductor (Bonazzola & Gourgoulhon 1996),
- fluid crust, with a model based only on the EoS (*e.g.* SLy4 by Douchin & Haensel 2001),
- elastic crust, with a solid-type model (Carter & Quintana 1972, Beig & Schmidt 2003, Gundlach *et al.* 2012, ...).

Equilibrium & EoS

In the stationary, axisymmetric and circular case, the conservation of stress-energy turns into a simple first integral of motion:

$$\boxed{h + \log N - \log \Gamma = \text{const.}}$$

$H = \log\left(\frac{e+p}{n_B m_0 c^2}\right)$: pseudo-enthalpy, Γ : Lorentz factor.
$\Rightarrow$ equilibrium condition.
The system is closed by the description of microscopic properties of the fluid: the equation of state:

$$\boxed{p(n_B), e(n_B) \text{ or } p(H), e(H)}$$

for cold nuclear matter at β-equilibrium.

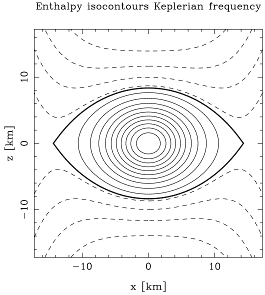
Global quantities

In general relativity, many quantities (*e.g.* mass, radius, ...) are gauge-dependent. Use of symmetries and physical definitions to get observationally relevant information:

- gravitational mass: Komar mass (stationarity) and ADM mass (asymptotic flatness) are equal in our case. They can be computed from the asymptotic behavior of N or as an integral over the support of $T^{\mu\nu}$.
  $\Rightarrow$ mass felt by a particle orbiting around the star.
- baryon mass: just counting the total number of particles... gauge independent.
- circumferential radius: ds integrated (with the metric) over a closed line (circumference), at the equator or passing through the poles.
- angular momentum (axisymmetry) : asymptotic behavior of ω

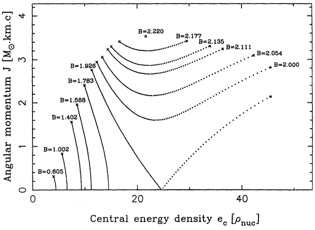
Examples

Salgado *et al.* 1994



- Rotation frequency limited by mass-shedding limit: matter leaving the star at the equator, due to centrifugal force.
- Impossible to describe in slow-rotation limit.

- Existence of supermassive sequences: for given baryon mass, there exist rotating solutions, but no static one.



Magnetic field

Motivations

Theoretical

- conservation of magnetic flux: 1 G for an O-type star of $\sim 10R_\odot \Rightarrow \sim 10^{12}$ G.
- More if magneto-rotational instability in CC-SN $\Rightarrow$magnetars.
- influence on structure?

Observational

- Magnetic slowdown measured through the spin-down $\dot{P}$ gives values of $B_{\rm pole}$ up to 10^{16} G,
- magnetars could represent as much as 10% of all pulsars (Muno *et al.* 2008);
- they can produce very strong X- and γ-ray bursts, from the glitch-like rearrangement of the crust, in which magnetic field is pinned (*e.g.* Dec. 2004 with SGR 1806-20).

Model

Rotating stationary axisymmetric star + :

- independent currents j^α generating the electromagnetic field (Maxwell equations),
- stationary equilibrium with Lorentz force,
- symmetry conditions $\Rightarrow$ magnetic moment aligned with the rotation axis (no emission of electromagnetic waves),
- magnetic field is supposed to be purely poloidal.

Define:

- fields, as observed by the Eulerian observer

$$
\begin{aligned}
E_\mu &= F_{\mu\nu}\, n^\nu, \\
B_\mu &= -1/2\, \epsilon_{\mu\nu\rho\sigma}\, n^\nu\, F^{\rho\sigma}.
\end{aligned}
$$

- global charge Q and magnetic dipole moment $\mathcal{M}$ deduced from asymptotic behavior of E_μ and B_μ, respectively.

Magnetic field equations

- $D_\nu F^{\mu\nu} = \mu_0 j^\mu$ can be written

$$\begin{aligned} \Delta_3 A_t &= S_1\left(j^\mu, g_{\mu\nu}, A_\mu, +\text{derivatives}\right) \\ \tilde{\Delta}_3 \left(\frac{A_\varphi}{r \sin\theta}\right) &= S_2\left(j^\mu, g_{\mu\nu}, A_\mu, +\text{derivatives}\right) \end{aligned}$$

- Lorentz force appears in the first integral of motion

$$H(r,\theta) + \nu(r,\theta) - \ln\Gamma(r,\theta) - \int_0^{A_\varphi(r,\theta)} f(x)\mathrm{d}x = 0$$

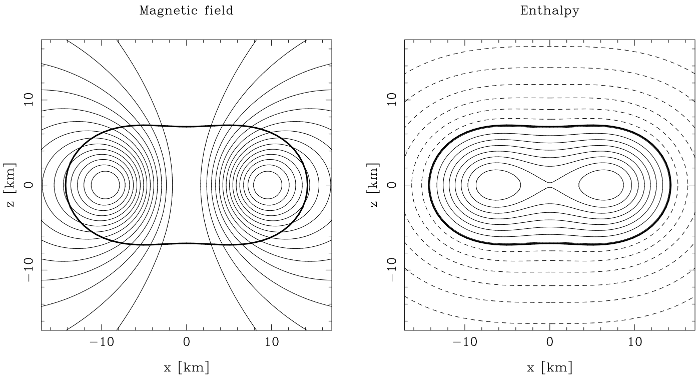
f arbitrary function such that $j^\varphi - \Omega j^t = (\varepsilon + p) f(A_\varphi)$,

- perfect conductor relation inside the star $A_t = -\Omega A_\varphi + C$.
- Non-isotropic stress-energy tensor

$$T^{\mathrm{EM}}_{\mu\nu} = \frac{1}{4\pi}\left(F_{\mu\rho}F^{\rho}_{\nu} - \frac{1}{4}F^{\rho\sigma}F_{\rho\sigma}g_{\mu\nu}\right).$$

Maximal field

Bocquet *et al.* (1995)



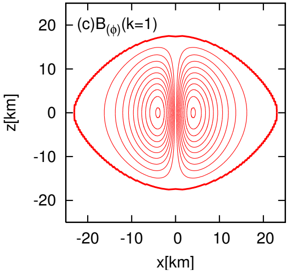
Non-rotating star with a polar magnetic field of $\sim 6 \times 10^{17}$ G.
$\Rightarrow$Stars with magnetic field can support more mass
$\Rightarrow$Negligible influence of the electric charge or different current functions f.

Poloidal / toroidal $\vec{B}$ field

see also talk by K.Kiuchi

⇒ Models with poloidal magnetic field are unstable (*e.g.* Ciolfi *et al.* 2011)

- Models with purely toroidal field have been built (Kiuchi & Yoshida 2008, Frieben & Rezzolla 2012)
- they show prolate shapes
- unstable, too ...



from Kiuchi & Yoshida (2008)

⇒ mixed configurations, with stratification. . . (Yoshida *et al.* 2012).

Two fluids

Motivations

Theoretical

At nuclear density the critical temperature:

$$T_{\text{crit}} \simeq 0.6 \times \Delta(T = 0)$$

with $\Delta \sim 1 - 3$ MeV
$\Rightarrow$ superfluid component some minutes after their birth.

Observational

Some pulsars exhibit sudden changes in the rotation period: instead of regularly slowing down, it shows rapid speed-up.
$\Rightarrow$within the two-fluid framework:

- the outer crust (+fluid) is slowed down, not the inner fluid;
- until the stress (or interaction) between both becomes larger than some threshold.
- can turn into a "starquake".

TWO-FLUID MODEL

1. superfluid neutrons (crust and outer core).
2. protons, nuclei and electrons locked together (called "protons").

- Conserved 4-currents n_{n}^{μ} and n_{p}^{μ},
- The Lagrangian density $\Lambda = -\mathcal{E}$ depends only on the three possible scalar products between these 4-vectors.
- 4-momenta as conjugates of currents: $d\Lambda = p_{\mu}^{\mathrm{n}} dn_{\mathrm{n}}^{\mu} + p_{\mu}^{\mathrm{p}} dn_{\mathrm{p}}^{\mu}$;
- generalized pressure $\Psi = -\mathcal{E} - p_{\mu}^{\mathrm{n}} n_{\mathrm{n}}^{\mu} - p_{\mu}^{\mathrm{p}} n_{\mathrm{p}}^{\mu}$.
- Equations of motion take the integral form: $\dfrac{N}{\Gamma_{\mathrm{n}}}\mu^{\mathrm{n}} = \mathrm{const}_{\mathrm{n}}$ and $\dfrac{N}{\Gamma_{\mathrm{p}}}\mu^{\mathrm{p}} = \mathrm{const}_{\mathrm{p}}$

Equation of state

The EOS depends only on densities and "relative speed" Δ

$$x^2 = -g_{\mu\nu} n_\mathrm{n}^\mu n_\mathrm{p}^\nu, \quad \Delta^2 = \left[1 - \left(\frac{n_\mathrm{n} n_\mathrm{p}}{x^2}\right)^2\right] = \frac{(U_n - U_p)^2}{(1 - U_n U_p)^2}$$

First law of thermodynamics to define μ^n and μ^p

$$\mathrm{d}\mathcal{E} = \mu^\mathrm{n} \mathrm{d}n_\mathrm{n} + \mu^p \mathrm{d}n_\mathrm{p} + e\, \mathrm{d}\Delta^2,$$

Simple 2-fluid polytrope EoS

$$\mathcal{E} = \rho c^2 + \frac{1}{2}\kappa_\mathrm{n} n_\mathrm{n}^2 + \frac{1}{2}\kappa_\mathrm{p} n_\mathrm{p}^2 + \kappa_\mathrm{np} n_\mathrm{n} n_\mathrm{p} + \kappa_\Delta n_\mathrm{n} n_\mathrm{p} \Delta^2.$$

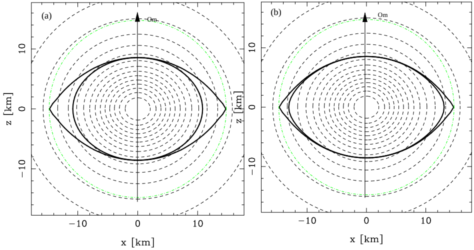
- $\rho = m_\mathrm{n} n_\mathrm{n} + m_\mathrm{p} n_\mathrm{p}$
- κ_np : symmetry energy
- κ_Δ: entrainment coefficient

$\Rightarrow$all physical features: entrainment + symmetry energy

$$\Psi = \frac{1}{2}\kappa_\mathrm{n} n_\mathrm{n}^2 + \frac{1}{2}\kappa_\mathrm{p} n_\mathrm{p}^2 + \kappa_\mathrm{np} n_\mathrm{n} n_\mathrm{p} + \kappa_\Delta n_\mathrm{n} n_\mathrm{p} \Delta^2$$

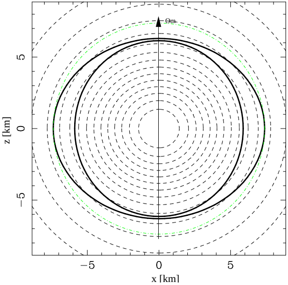
Results

Prix *et al.* (2005)



Kepler limit

no chemical equilibrium at the center, Kepler limit determined by the outer fluid, even if rotating slower.



Oblate-prolate configuration

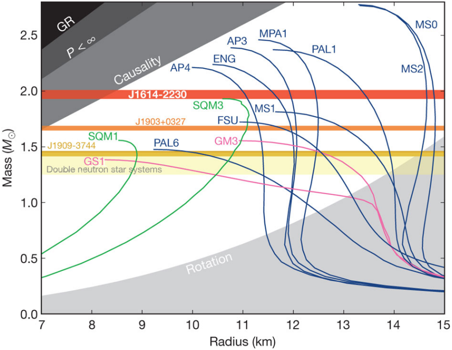
made possible by counter-rotation and the effective interaction potential, which tends to “separate” both fluids.

Equation of state

Main uncertainty

- No possibility to compute nucleon-nucleon interaction from QCD first principles;

$\Rightarrow$ use effective models calibrated on Earth-based experiments (hot, symmetric matter)



- Many models for nuclear interaction

$\Rightarrow$ quite different properties for neutron stars

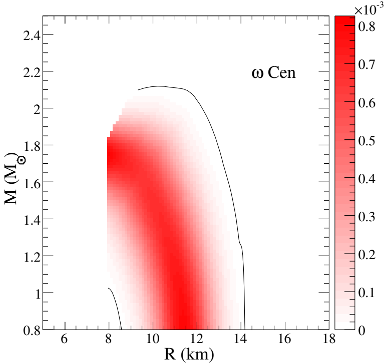
from Demorest *et al.* (2010)

following Lattimer & Prakash (2001)

Inverse problem: get information about nuclear interactions from astrophysical observations; as *e.g.* Steiner *et al.* (2010)

Inverse problem

Steiner *et al.* (2010)



Use mass and radius constraints from six neutron stars:

- 3 X-ray burst type sources
- 3 quiescent low-mass X-ray binaries

$\Rightarrow$ probability distribution in the $M - R$ plane, for each neutron star.

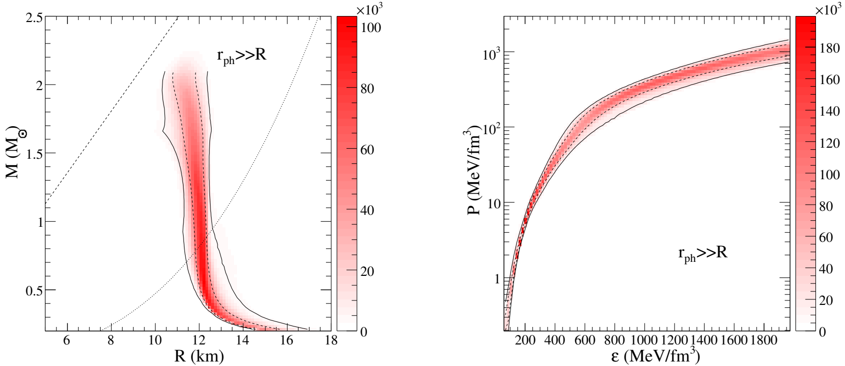
Parametrized EoS

$$\epsilon = f\left(n_B; K, K', \ldots\right)$$

$\Rightarrow$able to fit some "standard" EoSs for neutron stars, as SLy4.

Inverse problem

Steiner *et al.* (2010)



- Markov chain Monte-Carlo method within Bayesian approach
- Determination of the $M - R$ relation

⇒ Reconstruction of the EoS from the observations.

⇒ Within the parametrized model, rather soft EoS near nuclear saturation density.

Conclusions

Observation / analysis

- Very nice approach to reconstruct EoS properties from neutron star observations.
- Observations acquire much better accuracy.
- Better accuracy in the determination of the EoS?

Numerical models

- Rotation easy to take into account.
- Magnetic fluid much better understood than before.
- Superfluid models need only a realistic nuclear physics input.
- Elastic crust could soon be modelled.

$\Rightarrow$Need to take into account more refined neutron star models than TOV...see *e.g.* Cadeau *et al.* (2007)
There exist several codes able to take into account more detailed physics: interactions between groups are needed!

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