2013/10/25 YITP

Multi-Pomeron Repulsion and Mass of Hyperon-Mixed Neutron Star

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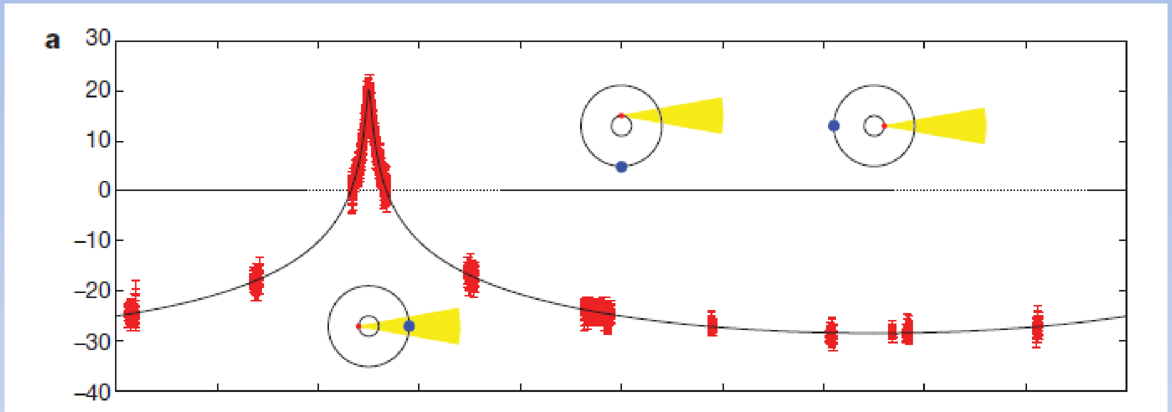
2010 PSR J1614-2230 $(1.97 \pm 0.04) M_\odot$



Shapiro delay measurement

2013 PSR J0348-0432 $(2.01 \pm 0.04) M_\odot$

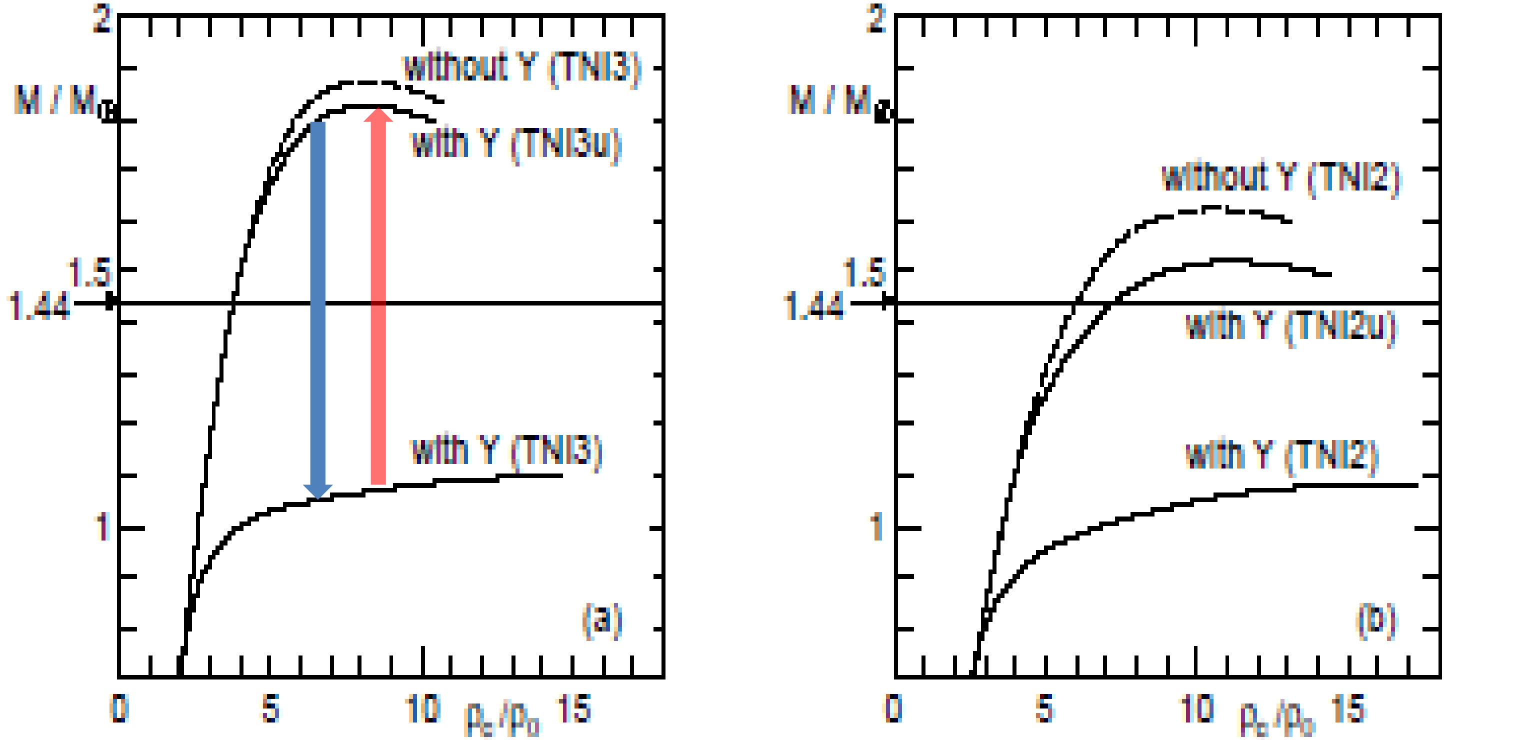


Fig. 9. The mass of a neutron star in units of the solar mass $M_{\odot}$ as functions of the central baryon density ρ_c with use of (a) TNI3 and (b) TNI2. The notation here is the same as in Fig. 8.

Softening by hyperon mixing to neutron-star matter

Massive ($2M_{\odot}$) neutron stars

Softening of EOS by hyperon mixing

Compatible ?

An idea is Universal Three-Baryon Repulsion (TBR)
by Takatsuka

Modeling of TBR in ESC = Multi-Pomeron exchange Potential

in this talk

Our strategy for neutron stars

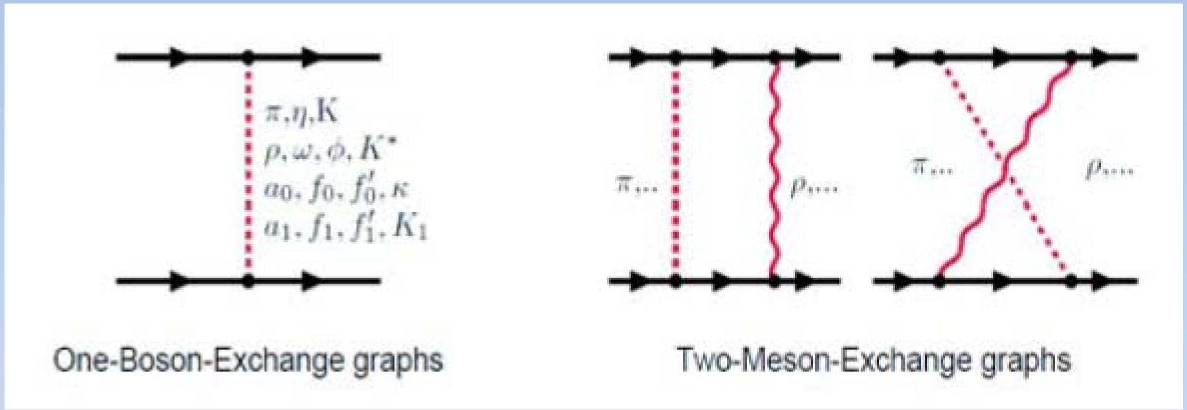
Neutron-star EOS derived from
Baryon-Baryon interaction model
in relation to Earth-based experiments

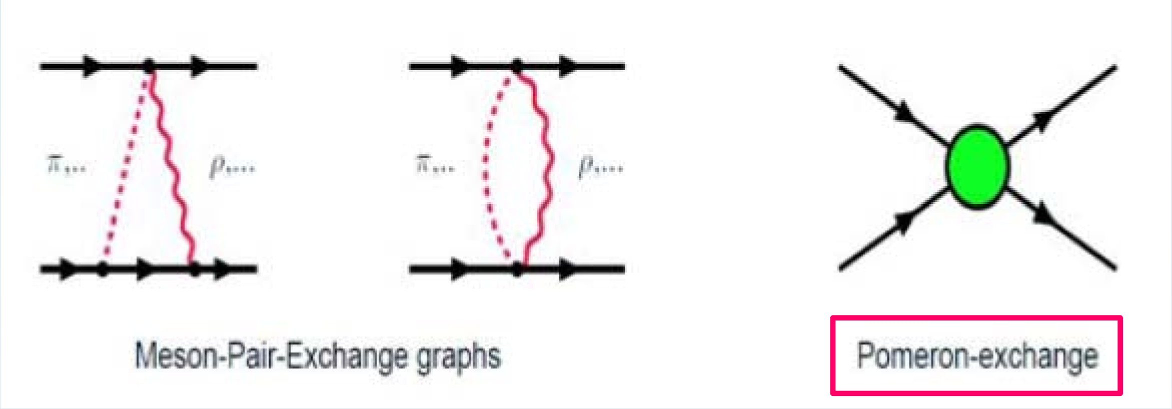
without ad hoc parameter for stiffness of EOS

on the basis of G-matrix theory

Extended Soft-Core Model (ESC)

- Two-meson exchange processes are treated explicitly
- Meson-Baryon coupling constants are taken consistently with Quark-Pair Creation model





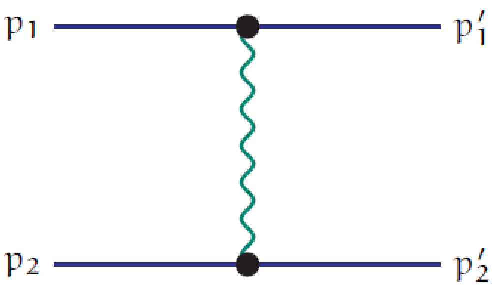
repulsive cores

Two-body Potential from Pomeron-exchange

Lagrangian & Propagator

$$\mathcal{L}_{PNN} = g_P \bar{\psi}(x)\psi(x)\, \sigma_P(x)$$

$$\Delta_F^P(k^2) = +\exp(-\mathbf{k}^2/4m_P^2)/\mathcal{M}^2$$



scaling mass $\mathcal{M} = 1\text{GeV}$

$$\begin{aligned} -iM_P(p_1', p_2'; p_1, p_2) &= (+i)^2 g_P^2\, [\bar{u}(p')u(p)]\,[\bar{u}(-p')u(-p)] \cdot i\Delta_F^P[(p'-p)^2] \\ M_P(p_1', p_2'; p_1, p_2) &= g_P^2\, [\bar{u}(p')u(p)]\,[\bar{u}(-p')u(-p)] \cdot \Delta_F^P[(p'-p)^2] \\ &\approx g p_P^2 \exp\left(-\mathbf{k}^2/4m_P^2\right)/\mathcal{M}^2 \,, \end{aligned}$$

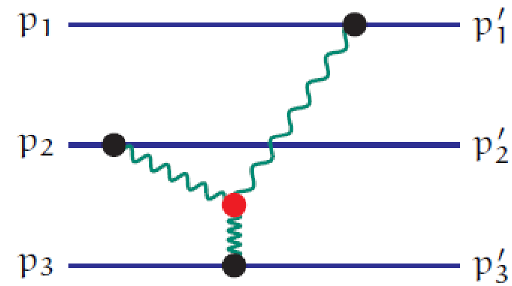
$$\begin{aligned} V_P(r) &= \int \frac{d^3k}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{x}} M_P(\mathbf{p}'-\mathbf{p})\delta(\mathbf{k}-\mathbf{p}'+\mathbf{p}) \\ &= \frac{g_P^2}{4\pi}\frac{4}{\sqrt{\pi}}\frac{m_P^3}{\mathcal{M}^2}\, \exp\left(-m_P^2 r_{12}^2\right) \,. \end{aligned}$$

Two-body repulsive core

Three(Four)-body Potential from the Triple(Quadruple)-pomeron vertex

$$\mathcal{L}_{PPP} = g_{3P}\mathcal{M}\,\sigma_P^3(x)/3!$$

$$\left[\ \mathcal{L}_{PPPP} = g_{4P}\,\sigma_P^4(x)/4!\ \right]$$



$$\begin{aligned} M_{3P}(p_1', p_2', p_3'; p_1, p_2, p_3) &= g_{3P} g_P^3\, \Pi_{i=1}^3 \left\{ [\bar{u}(p_i')u(p_i)]\, \Delta_F^P[(p_i' - p_i)^2] \right\} \\ &\approx g_{3P} g_P^3\, \Pi_{i=1}^3 \Delta_F^P[(p_i' - p_i)^2]\,. \end{aligned}$$

$$\begin{aligned} V(\mathbf{x}_1', \mathbf{x}_2', \mathbf{x}_3'; \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) &= \int \Pi_{i=1}^3 \frac{d^3 p_i'}{(2\pi)^3} \frac{d^3 p_i}{(2\pi)^3} \cdot \\ &\times \Pi_{i=1}^3 e^{-i(\mathbf{p}_i' \cdot \mathbf{x}_i' - \mathbf{p}_i \cdot \mathbf{x}_i)} \cdot \delta\left(\sum \mathbf{p}_i' - \sum \mathbf{p}_i\right) \\ &\times M_{3P}(p_1', p_2', p_3'; p_1, p_2, p_3)\,. \end{aligned}$$

$$V(\mathbf{x}_1', \mathbf{x}_2', \mathbf{x}_3'; \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \equiv V(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)\delta(\mathbf{x}_1' - \mathbf{x}_1)\delta(\mathbf{x}_2' - \mathbf{x}_2)\delta(\mathbf{x}_3' - \mathbf{x}_3) \quad (15)$$

$$V(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = p_{3P} g_P^3\, \Pi_{i=1}^3 \int \frac{d^3 k_i}{(2\pi)^3}\, \Pi_{i=1}^3 e^{-i\mathbf{k}_i \cdot \mathbf{x}_i} \cdot (2\pi)^3 \delta\left(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3\right) \cdot$$

$$\times \exp\left(-\mathbf{k}_1^2/4m_P^2\right) \exp\left(-\mathbf{k}_2^2/4m_P^2\right) \exp\left(-\mathbf{k}_3^2/4m_P^2\right) \cdot \mathcal{M}^{-5}\,,$$

Three- and Four-body repulsions with parameters g_{3P} & g_{4P}

The effective two-body potential

$$V_{\rm eff}(\mathbf{x}_1,\mathbf{x}_2) = \rho_{NM}\int d^3x_3\, V(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$$

$$\begin{aligned} V_{\rm eff}(\mathbf{x}_1,\mathbf{x}_2) &= g_{3P}g_P^3\frac{\rho_{NM}}{\mathcal{M}^5}\cdot\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}e^{-i\mathbf{k}_1\cdot\mathbf{x}_1}e^{-i\mathbf{k}_2\cdot\mathbf{x}_2}\,. \\ &\quad\times(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2)\exp\left(-\mathbf{k}_1^2/4m_P^2\right)\exp\left(-\mathbf{k}_2^2/4m_P^2\right) \\ &= g_{3P}g_P^3\frac{\rho_{NM}}{\mathcal{M}^5}\cdot\int\frac{d^3k_1}{(2\pi)^3}e^{-i\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{x}_2)}\cdot\exp\left(-\mathbf{k}_1^2/2m_P^2\right) \\ &= g_{3P}g_P^3\frac{\rho_{NM}}{\mathcal{M}^5}\cdot\frac{1}{4\pi}\frac{4}{\sqrt{\pi}}\left(\frac{m_P}{\sqrt{2}}\right)^3\exp\left(-\frac{1}{2}m_P^2r_{12}^2\right)\,. \end{aligned}$$

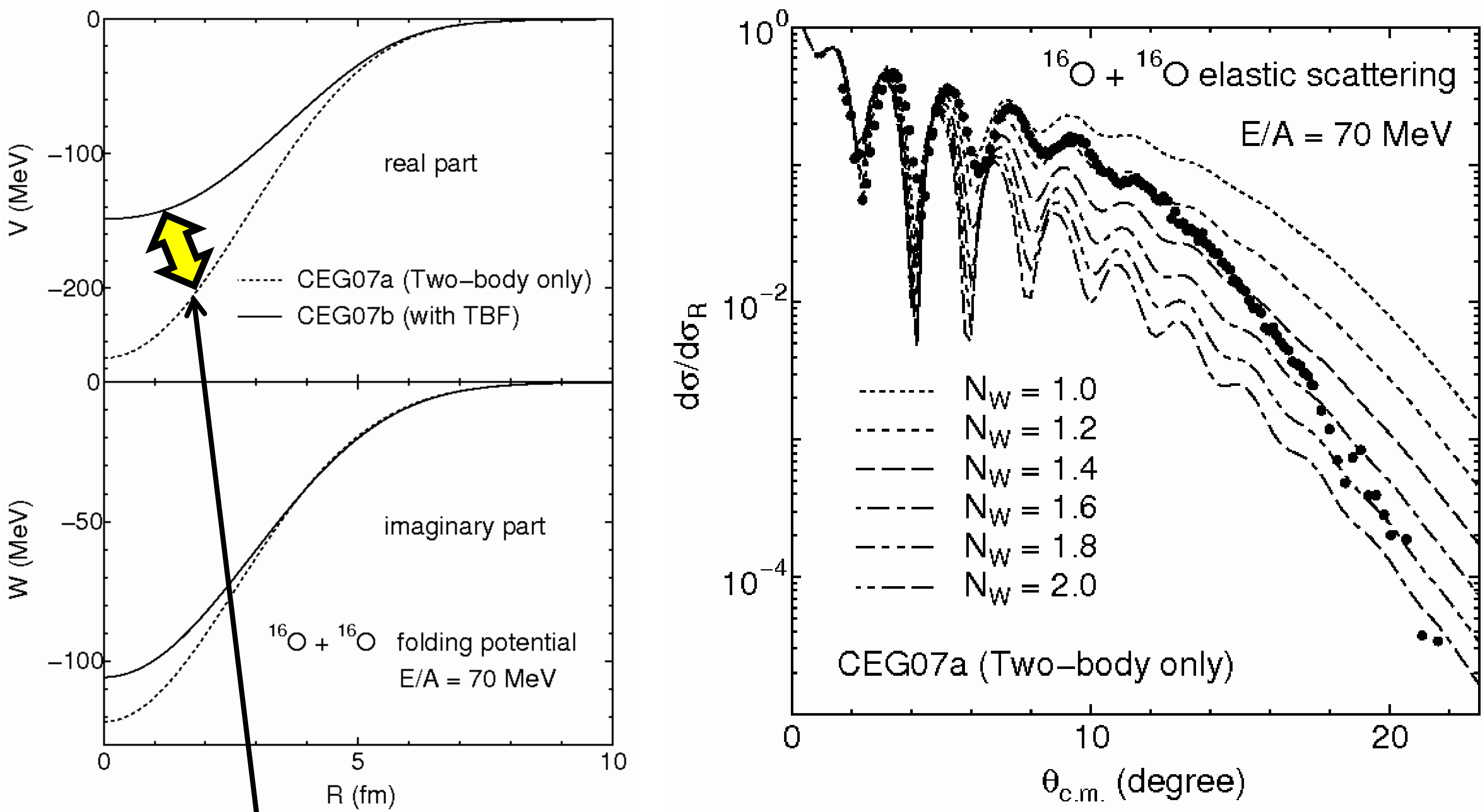
$$\left[g_{4P}g_P^4\frac{\rho_{NM}^2}{\mathcal{M}^8}\cdot\frac{1}{4\pi}\frac{4}{\sqrt{\pi}}\left(\frac{m_P}{\sqrt{2}}\right)^3\exp\left(-\frac{1}{2}m_P^2r_{12}^2\right)\right]$$

How to determine coupling constants g_{3P} and g_{4P} ?

→ Nucleus-Nucleus scattering data

with G-matrix folding model

$^{16}O + ^{16}O$ elastic scattering E/A = 70 MeV



Effect of three-body force

$$U_{DFM} = V_{DFM} + iN_W W_{DFM}$$

T.Furumoto, Y. Sakuragi and Y. Yamamoto, Phys.Rev.C79, (2009) 011601

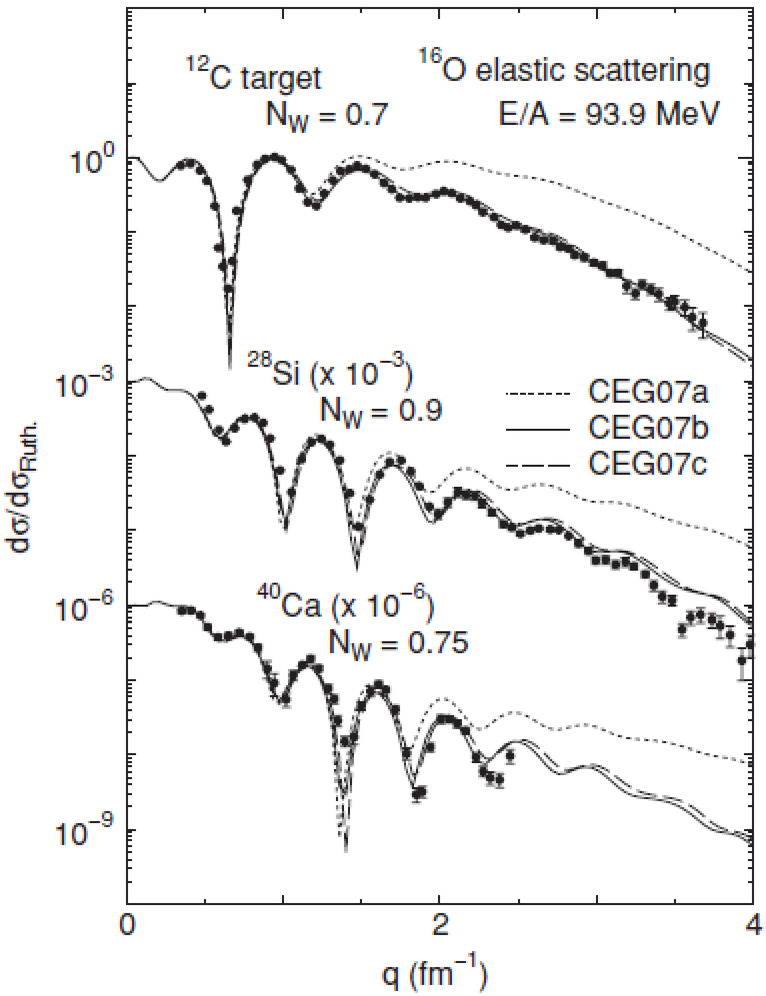


FIG. 13. Rutherford ratio of the cross sections for elastic scattering of ^{16}O by the ^{12}C, ^{28}Si, and ^{40}Ca targets at $E/A = 93.9$ MeV, calculated with the three types of complex G-matrix interactions, which are compared with the experimental data from Ref. [35]. The abscissa is the momentum transfer q defined as $q = 2k \sin \frac{\theta}{2}$, where k is the asymptotic momentum.

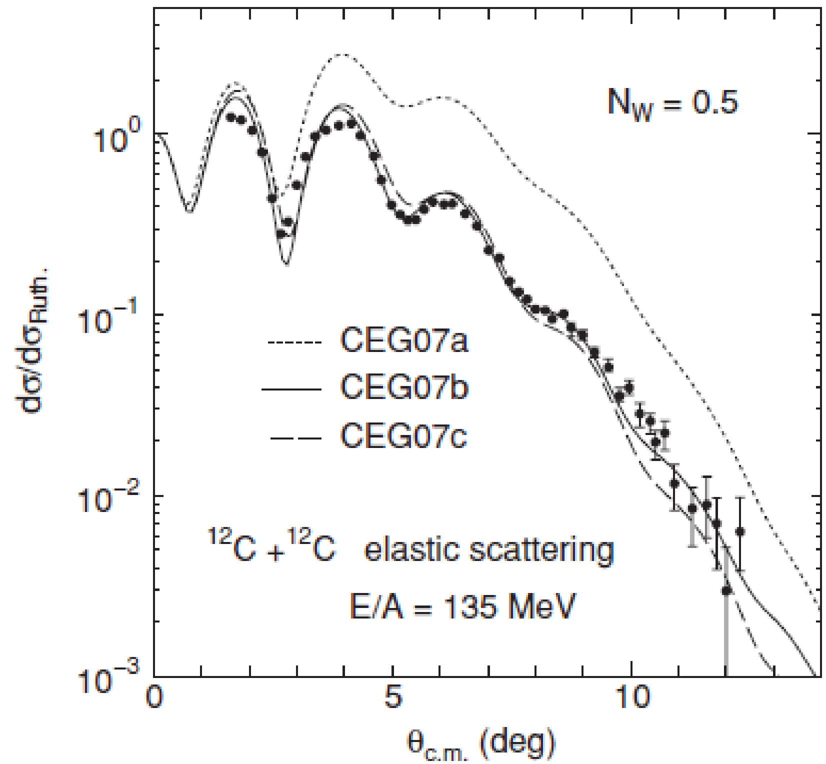


FIG. 15. Rutherford ratio for the $^{12}C + {^{12}C}$ system at $E/A = 135$ MeV calculated with the three types of complex G-matrix interactions, which are compared with the experimental data from Ref. [36].

ESC08c + MPP + TNA

repulsive attractive

phenomenological

$$V_{TNA}(r;\rho) = V_0\, \rho \exp(-\eta\rho) \exp(-(r/2.0)^2)\, (1+P_r)/2$$

V_0 and η are determined so as to reproduce saturation density/energy

MP1a : $(g_P^{(3)}, g_P^{(4)}) = (2.34,\ 30.)$ $V_0 = -32.8$ $\eta = 3.5$

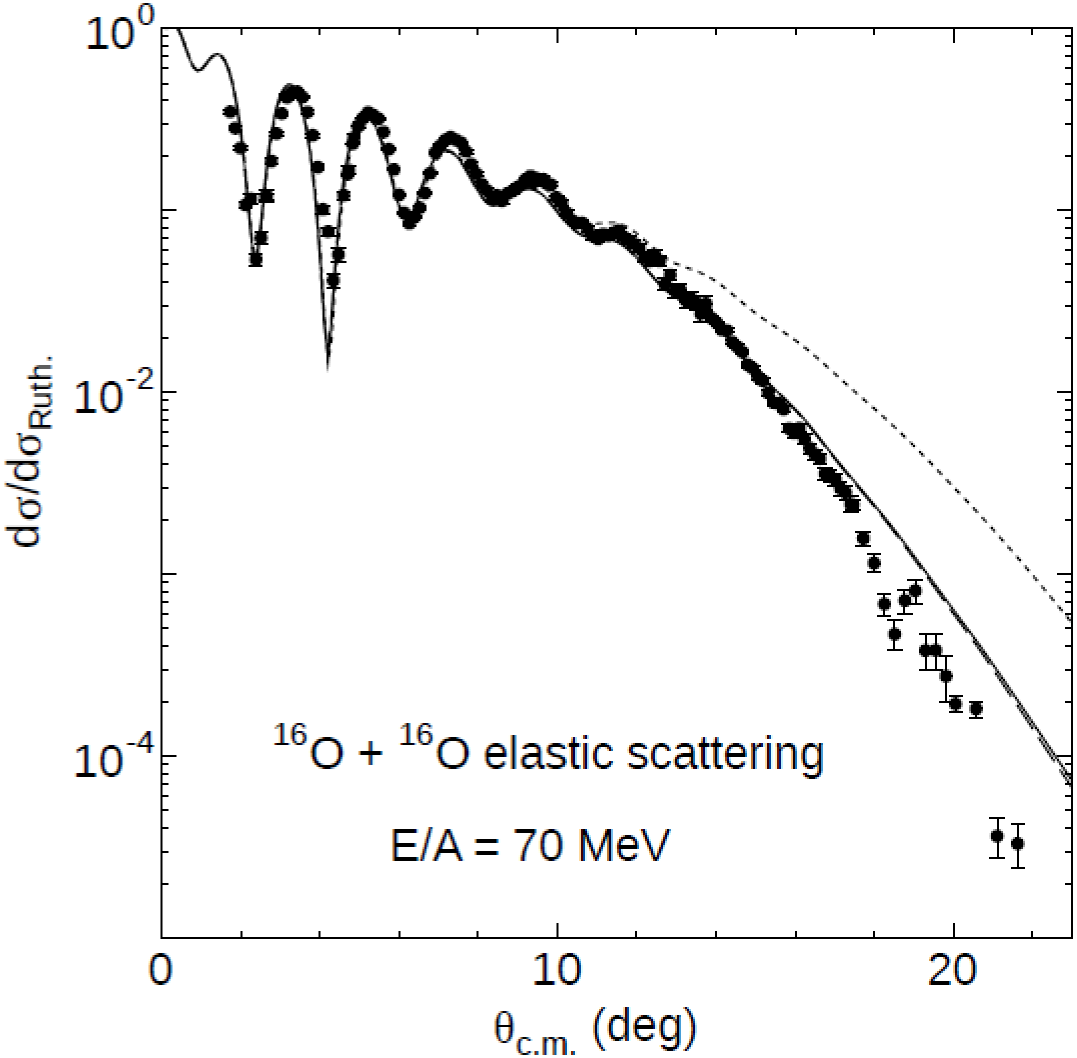
MP2a : $(g_P^{(3)}, g_P^{(4)}) = (2.94,\ 0.0)$ $V_0 = -45.0$ $\eta = 5.4$

MPP TNA

Ratio g_{4P}/g_{3P} is not determined in our analysis ---→ two versions MP1a & MP2a

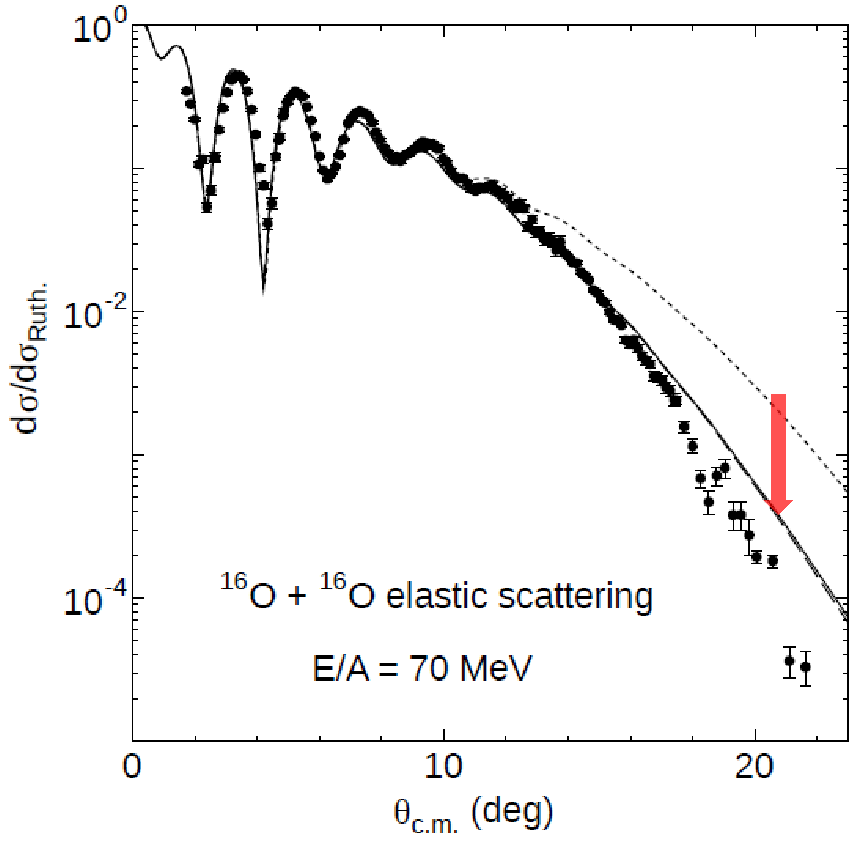
Y. Yamamoto, T. Furumoto, N. Yasutake and Th. A. Rijken: Phys. Rev. C 88 (2013) 022801(R).

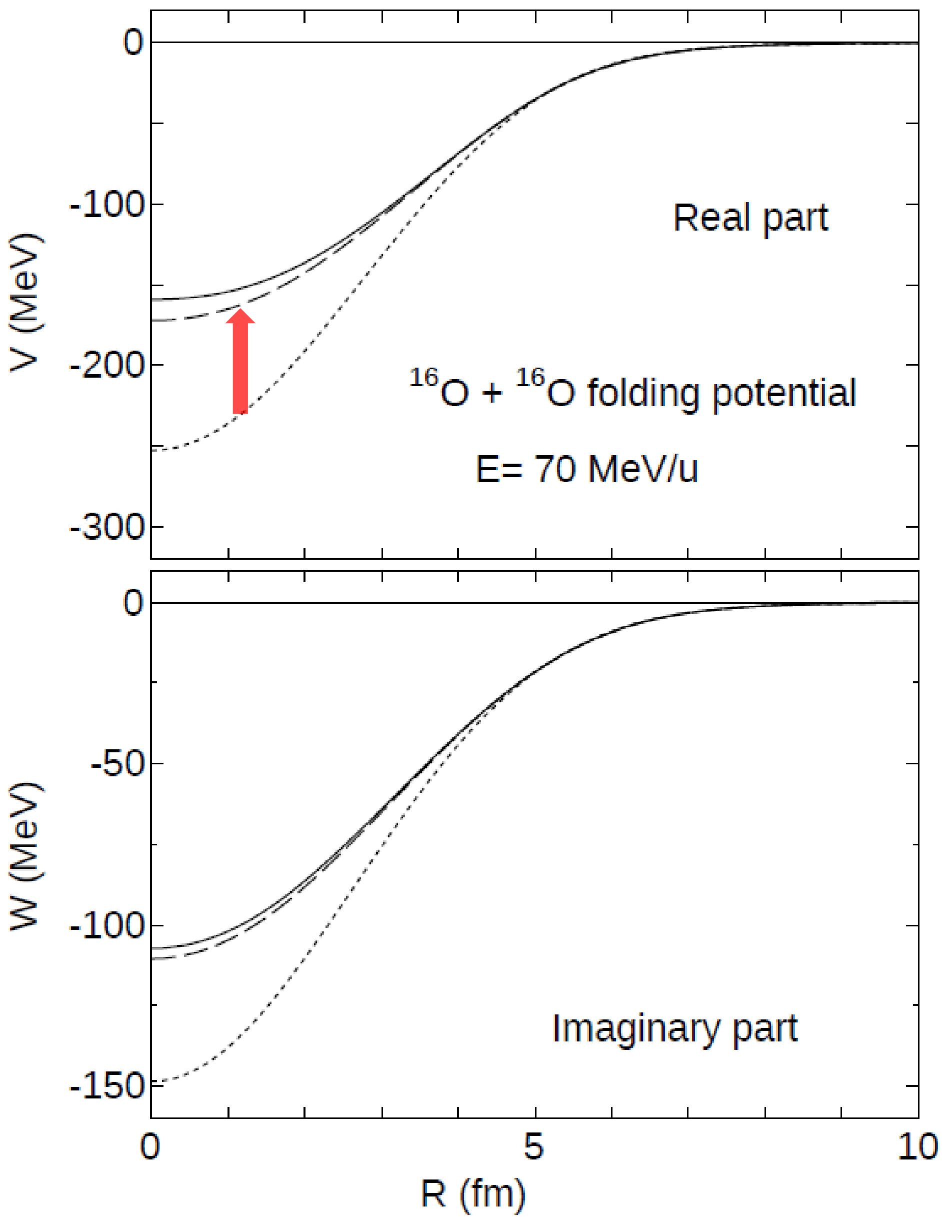
with G-matrix folding model



solid: ESC08c+MP1a+TNA
dashed: ESC08c+MP2a+TNA
dotted: ESC08c only

MP1a (MP2a): with (without) 4-body repulsion





Frozen-Density Approximation

Two Fermi-spheres separated in momentum space

can overlap in coordinate space without disturbance of Pauli principle

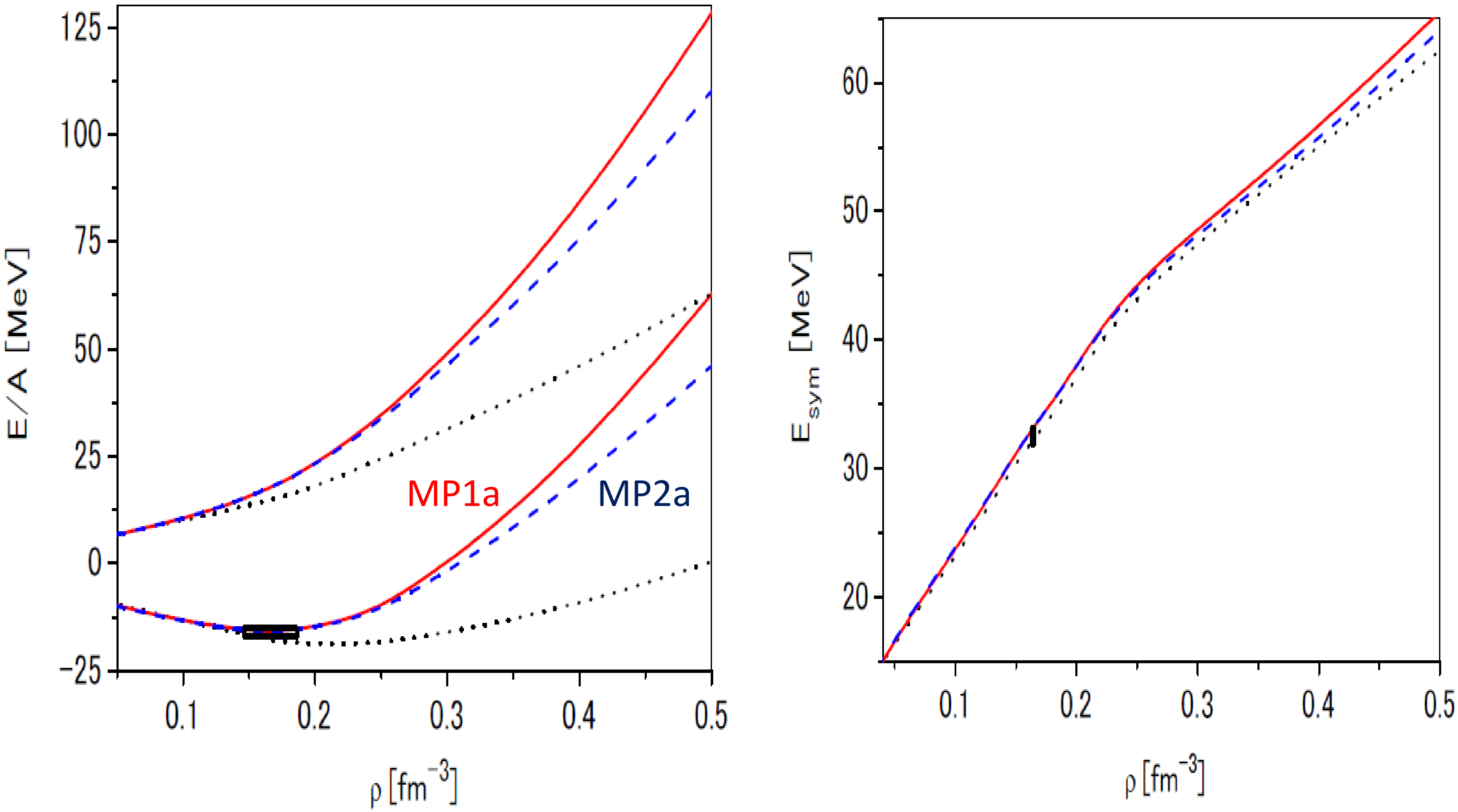
Nuclear Matter Properties are specified by

$$E_{sym}(\rho) = \frac{E}{A}(\rho, \beta = 1) - \frac{E}{A}(\rho, \beta = 0)$$

$$L = 3\rho_0 \left[\frac{\partial E_{sym}(\rho)}{\partial \rho}\right]_{\rho=\rho_0}$$

$$K = 9\rho_0^2 \left[\frac{d^2}{d\rho^2}\frac{E}{A}(\rho, \beta = 0)\right]_{\rho=\rho_0}$$

$$\beta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$



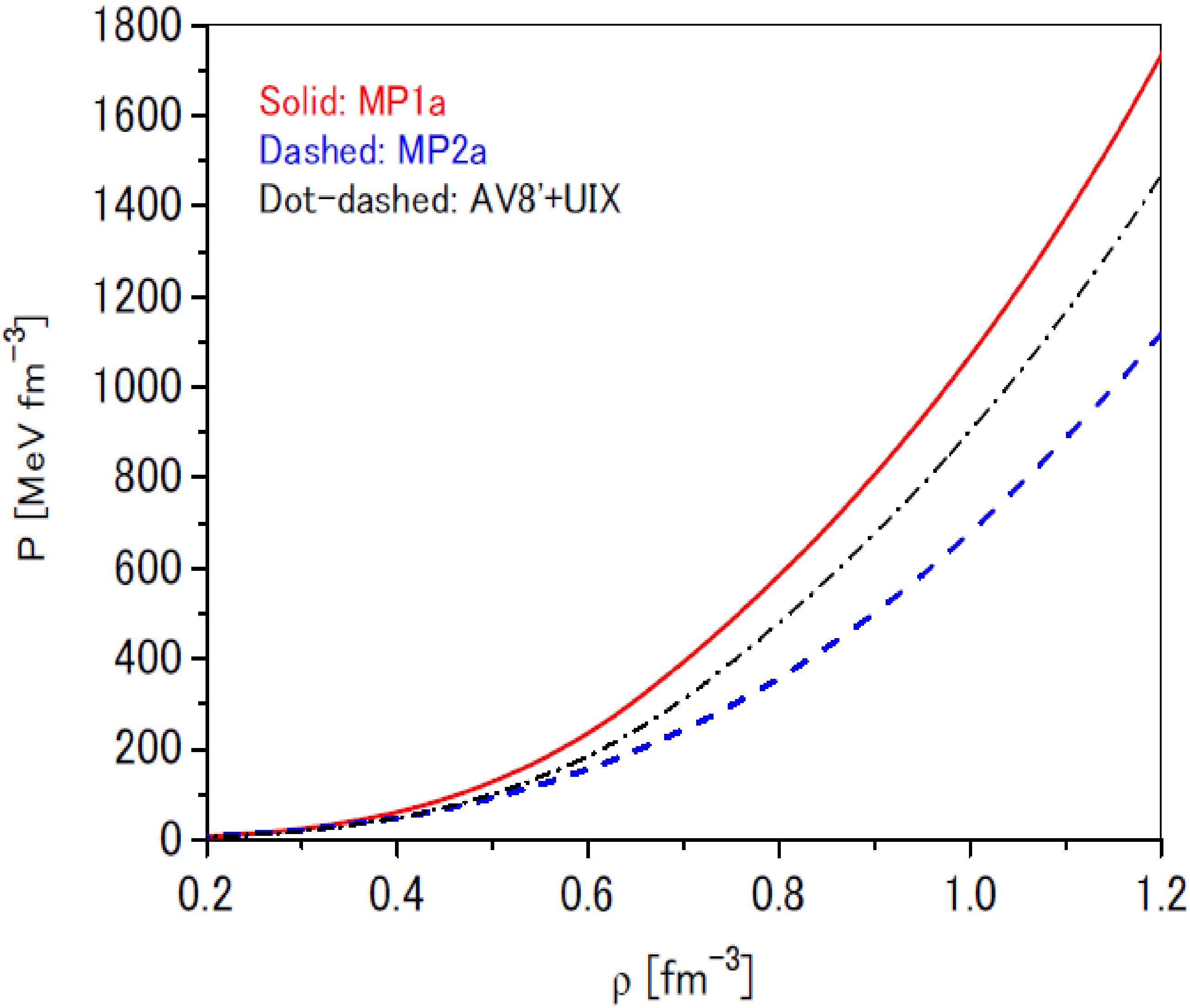
$\rho_0 = 0.164\ \mathrm{fm}^{-3}\quad E/A(\rho_0) = -15.8\ \mathrm{MeV}$ both for MP1a and MP2a

$E_{sym}(\rho_0) = 33.1\ \mathrm{MeV}\quad L(\rho_0) = 70.4\ \mathrm{MeV}\quad K = 285\ \mathrm{MeV}$ for MP1a

$E_{sym}(\rho_0) = 33.1\ \mathrm{MeV}\quad L(\rho_0) = 69.2\ \mathrm{MeV}\quad K = 267\ \mathrm{MeV}$ for MP2a

AV8' +UIX : E_{sym}=35.1 MeV L=63.6 MeV (Gandolfi et al.)

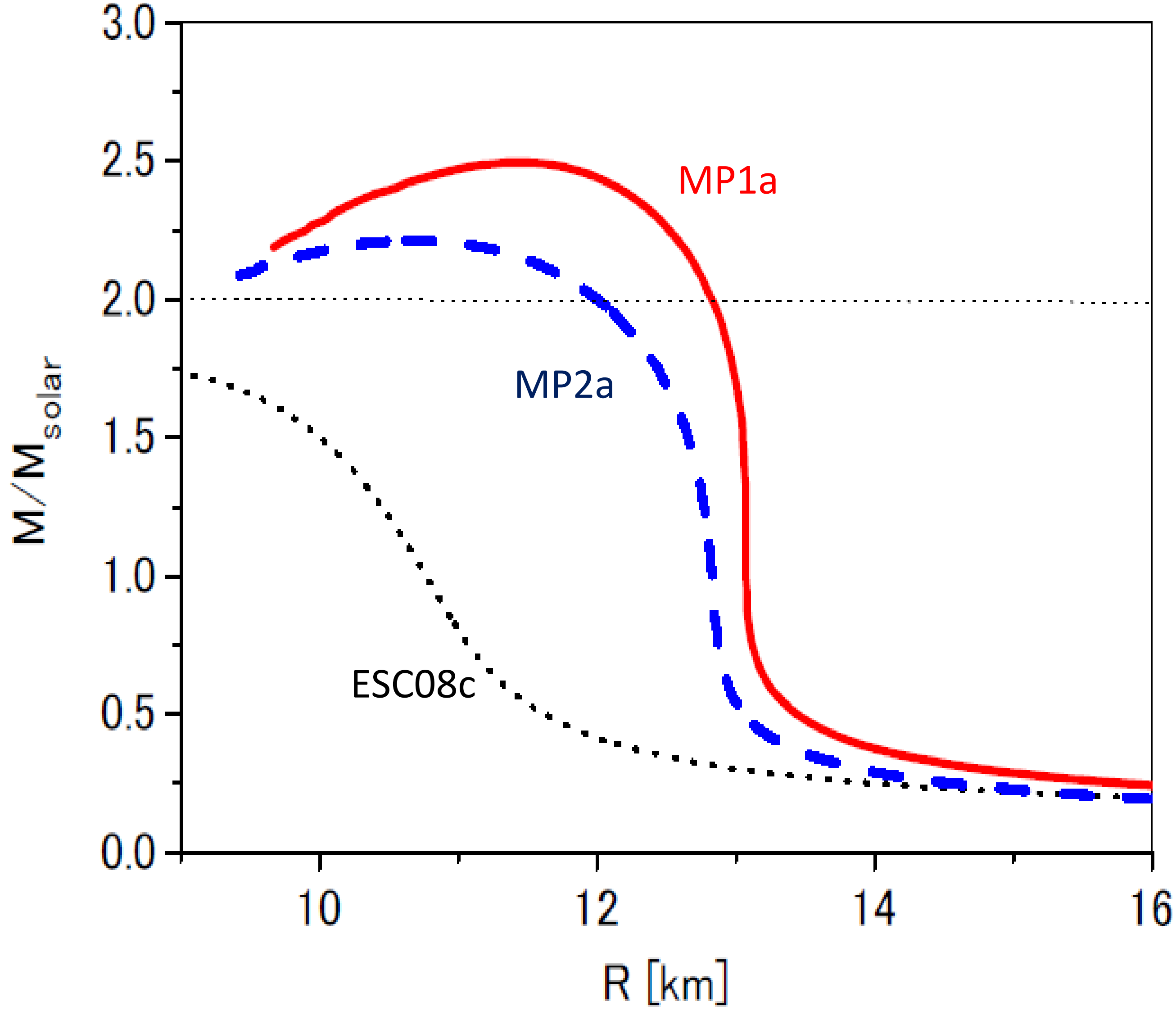
EOS (neutron matter)



Tolman-Oppenheimer-Volkoff equation

$$\frac{dP}{dr} = -\frac{GM\varepsilon}{r^2c^2}\left(1+\frac{P}{\varepsilon}\right)\left(1+\frac{4\pi r^3P}{Mc^2}\right)\left(1-\frac{2GM}{c^2r}\right)^{-1}$$

with neutron-matter EOS



Summarizing

ESC08c + MPP + TNA　　nuclear part

MPP strength determined by
analysis for $^{16}O+^{16}O$ scattering

TNA adjusted phenomenologically
To reproduce $E/A(\rho_0) = -15.8$ MeV
with $\rho_0 = 0.164\ fm^{-3}$

No ad hoc parameter for massive neutron star (stiff EOS)
on the basis of terrestrial experiments

Hyperon-Mixed Neutron-Star Matter

ESC08c defined in S=0,-1,-2 channels
MPP universal in all BB channels
TNA TBA ???

(ESC08c+MPP+TBA) model should be test in hypernuclei
hyperonic sector

Λ & Σ states based on ESC08c + MPP + TBA

TNA

Table 1: Values of U_Λ at normal density and partial wave contributions in $^{2S+1}L_J$ states for ESC08a/b/c from the G-matrix calculations with CON prescriptions (in MeV). The value specified by D gives the sum of $^{2S+1}D_J$ contributions. Contributions from S-state spin-spin interactions are given by $U_{\sigma\sigma} = (U(^3S_1) - 3U(^1S_0))/12$. m^*_Λ is defined by M^*_Λ/M_Λ, M^*_Λ being a Λ effective mass.

	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Λ	$U_{\sigma\sigma}$	m^*_Λ
ESC08c	−13.6	−25.3	2.7	0.2	1.4	−3.2	−1.6	−39.4	1.29	0.73
ESC08c$^+$	−13.3	−25.4	3.4	0.4	2.2	−1.7	−2.7	−37.2	1.22	0.70
ESC08a	−13.2	−27.7	2.7	0.0	1.3	−3.7	−1.6	−42.1	1.00	0.73
ESC08b	−13.0	−25.4	2.4	−0.2	1.4	−4.3	−1.7	−40.9	1.12	0.75
NSC97e	−13.5	−29.6	1.9	0.4	2.9	−1.4	−1.3	−40.5	0.92	0.69
NSC97f	−15.0	−26.6	2.2	0.4	3.7	−0.9	−1.3	−37.5	1.54	0.68

ESC08c$^+$ = ESC08c + MP1a + TBA

$U_\Lambda(\rho_0)$ is conceptually different from U_{WS} (−28 MeV) !!
use G-matrix folding model

Y-nucleus folding potential derived from YN G-matrix interaction $G(r; k_F)$

$$
\begin{aligned}
U_Y(\mathbf{r},\mathbf{r}') &= U_{dr} + U_{ex} \\
U_{dr} &= \delta(\mathbf{r}-\mathbf{r}') \int d\mathbf{r}'' \rho(\mathbf{r}'') V_{dr}(|\mathbf{r}-\mathbf{r}''|; \langle k_F \rangle) \\
U_{ex} &= \rho(\mathbf{r},\mathbf{r}') V_{ex}(|\mathbf{r}-\mathbf{r}'|; \langle k_F \rangle) \\
V_{dr} &= \frac{1}{2(2t_Y+1)(2s_Y+1)} \sum_{TS} (2T+1)(2S+1)[G_{TS}^{(+)} + G_{TS}^{(-)}] \\
V_{ex} &= \frac{1}{2(2t_Y+1)(2s_Y+1)} \sum_{TS} (2T+1)(2S+1)[G_{TS}^{(+)} - G_{TS}^{(-)}]
\end{aligned}
$$

G-matrix interactions

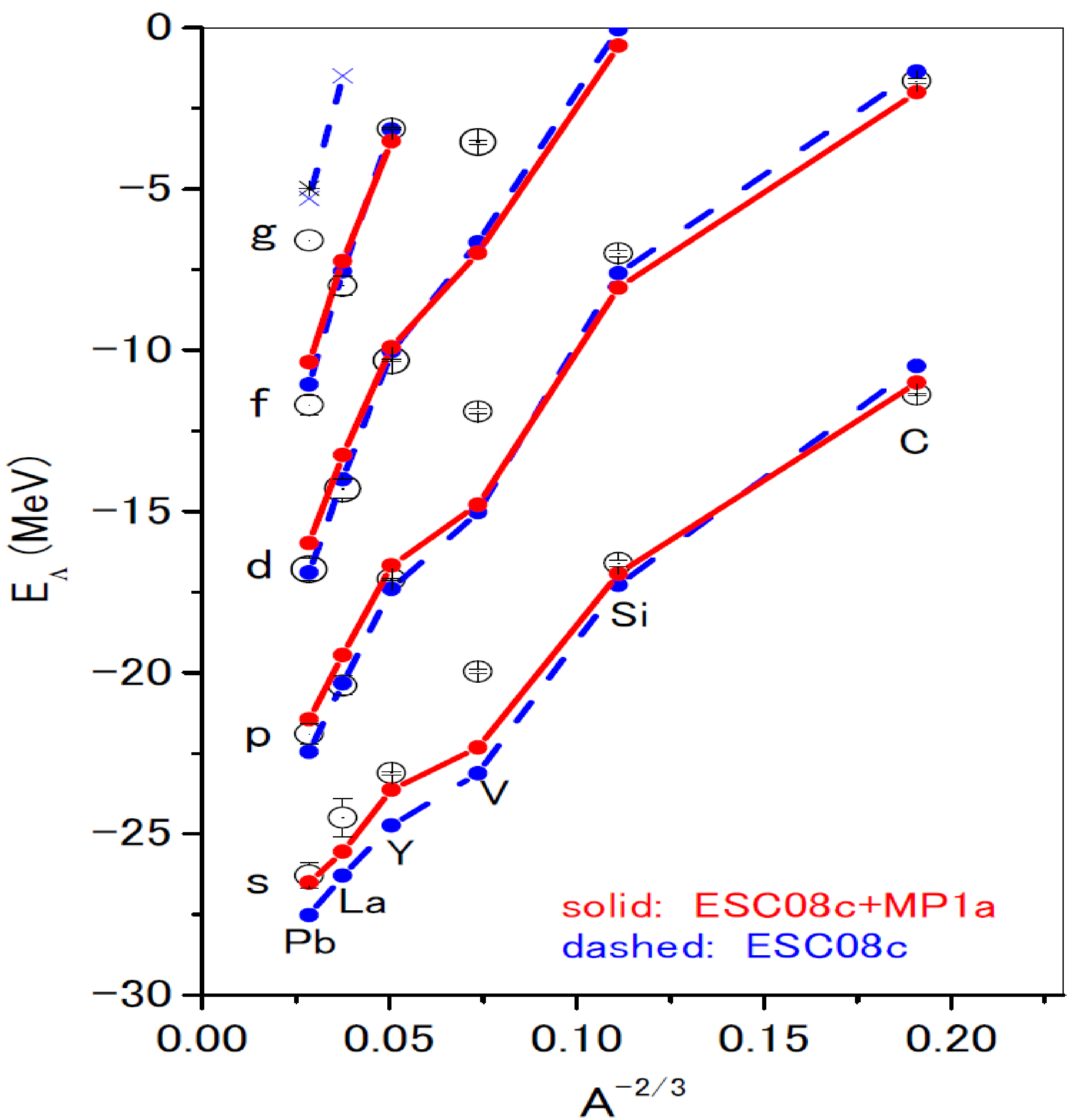
Averaged-k_F Approximation

$$\langle \rho \rangle = \langle \phi_Y(r) | \rho(r) | \phi_Y(r) \rangle$$

$$\langle k_F \rangle = (1.5\pi^2 \langle \rho \rangle)^{1/3} + \Delta k_F$$

calculated self-consistently

Mixed density $\rho(\mathbf{r}_1,\mathbf{r}_2) = \sum_j \varphi_j^*(\mathbf{r}_1)\varphi_j(\mathbf{r}_2)$ obtained from SkHF w.f.



ΔkF=-0.05

ΔkF= 0.02

Quark-Pauli effect in ESC08 models

ESC core = pomeron + ω

Repulsive cores are similar to each other in all channels

Assuming
"equal parts" of ESC and QM are similar to each other

Almost Pauli-forbidden states in [51] are taken into account by changing the pomeron strengths for the corresponding channels phenomenologically

$$g_P \longrightarrow \text{factor} * g_P$$

by Oka-Shimizu-Yazaki

Table III. $SU(6)_{fs}$-contents of the various potentials on the isospin, spin basis.

(S,I)	$V = aV_{[51]} + bV_{[33]}$
$(0,1)$	$V_{NN} = \frac{4}{9}V_{[51]} + \frac{5}{9}V_{[33]}$
$(1,0)$	$V_{NN} = \frac{4}{9}V_{[51]} + \frac{5}{9}V_{[33]}$
$(0,1/2)$	$V_{\Lambda\Lambda} = \frac{1}{2}V_{[51]} + \frac{1}{2}V_{[33]}$
$(1,1/2)$	$V_{\Lambda\Lambda} = \frac{1}{2}V_{[51]} + \frac{1}{2}V_{[33]}$
$(0,1/2)$	$V_{\Sigma\Sigma} = \frac{17}{18}V_{[51]} + \frac{1}{18}V_{[33]}$
$(1,1/2)$	$V_{\Sigma\Sigma} = \frac{1}{2}V_{[51]} + \frac{1}{2}V_{[33]}$
$(0,3/2)$	$V_{\Sigma\Sigma} = \frac{4}{9}V_{[51]} + \frac{5}{9}V_{[33]}$
$(1,3/2)$	$V_{\Sigma\Sigma} = \frac{8}{9}V_{[51]} + \frac{1}{9}V_{[33]}$

(S,I)	$V = aV_{[51]} + bV_{[33]}$
$(0,0)$	$V_{\Lambda\Lambda,\Lambda\Lambda} = \frac{1}{2}V_{[51]} + \frac{1}{2}V_{[33]}$
$(0,0)$	$V_{\Xi N,\Xi N} = \frac{1}{3}V_{[51]} + \frac{2}{3}V_{[33]}$
$(0,0)$	$V_{\Sigma\Sigma,\Sigma\Sigma} = \frac{11}{18}V_{[51]} + \frac{7}{18}V_{[33]}$
$(0,1)$	$V_{\Xi N,\Xi N} = \frac{7}{9}V_{[51]} + \frac{2}{9}V_{[33]}$
$(0,0)$	$V_{\Sigma\Lambda,\Sigma\Lambda} = \frac{2}{3}V_{[51]} + \frac{1}{3}V_{[33]}$
$(0,2)$	$V_{\Sigma\Sigma,\Sigma\Sigma} = \frac{4}{9}V_{[51]} + \frac{5}{9}V_{[33]}$
$(1,0)$	$V_{\Xi N,\Xi N} = \frac{5}{9}V_{[51]} + \frac{4}{9}V_{[33]}$
$(1,1)$	$V_{\Xi N,\Xi N} = \frac{17}{27}V_{[51]} + \frac{10}{27}V_{[33]}$
$(1,1)$	$V_{\Sigma\Lambda,\Sigma\Lambda} = \frac{2}{3}V_{[51]} + \frac{1}{3}V_{[33]}$
$(1,1)$	$V_{\Sigma\Sigma,\Sigma\Sigma} = \frac{16}{27}V_{[51]} + \frac{11}{27}V_{[33]}$

Pauli-forbidden state in $V_{[51]}$ → strengthen pomeron coupling

↓

$V_{BB} = V(pom) + w_{BB}[51] * V(PB)$

Table 1: Values of U_Σ at normal density and partial wave contributions for ESC08c models (in MeV).

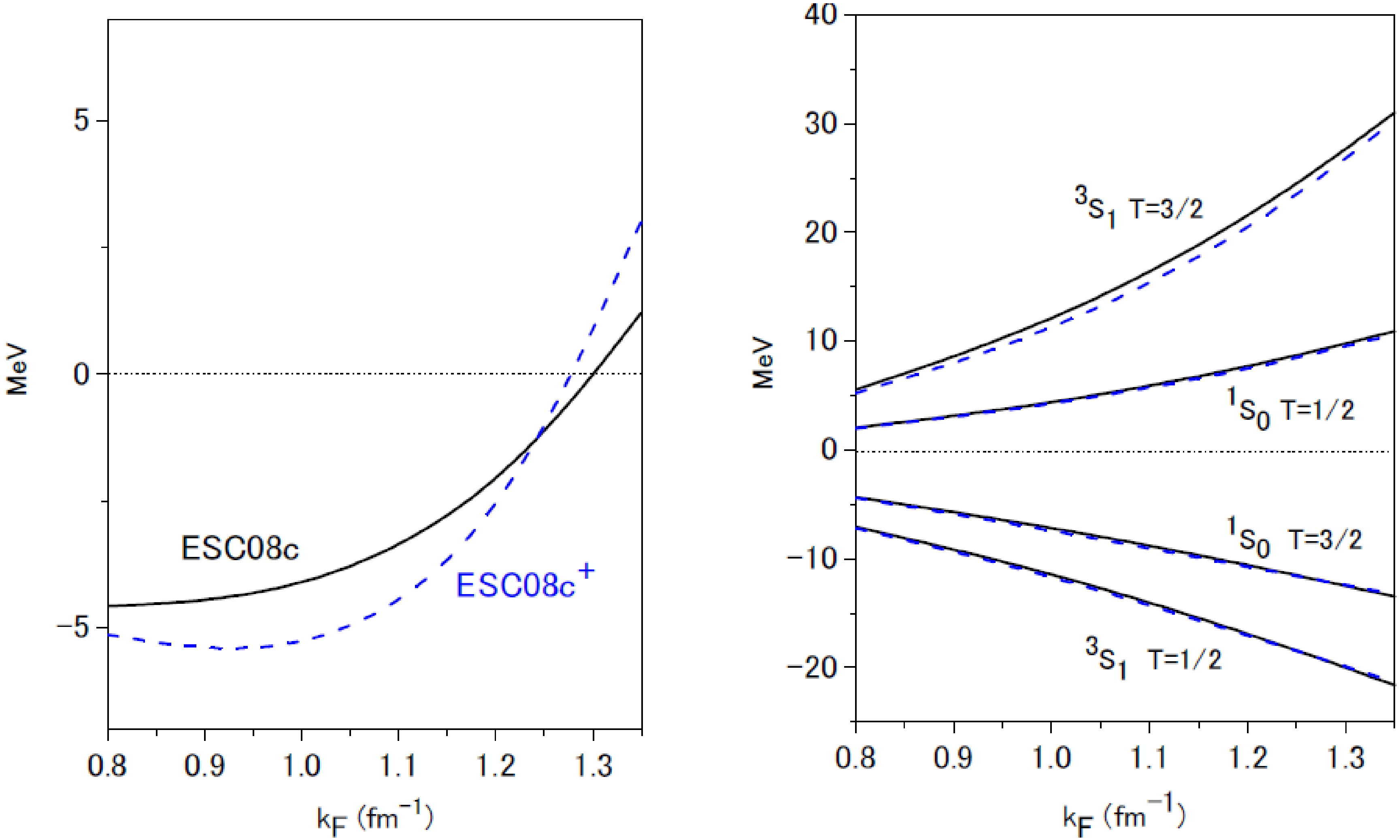
model	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Σ
ESC08c	1/2	10.9	−21.6	2.4	2.1	−6.0	−1.0	−0.7	
	3/2	−13.5	31.0	−4.7	−1.8	5.9	−1.5	−0.2	+1.2

Pauli-forbidden state in QCM → strong repulsion in **T=3/2 $^3S_1$** state

↓

Σ^- in neutron matter

$U_\Sigma (k_F)$



Solid → Dashed : Contributions from MPP+TBA

Hyperon-mixed Neutron-Star matter with universal TBR (MPP)

EoS of n+p+Λ+Σ+e+μ system

ESC08c(YN) + MPP(YNN) +TBA(YNN)

Preliminary

Hyperon-mixed neutron matter

Starting from single particle potentials calculated with the G-matrix theory:

$$U_B(k) = \sum_{B'} U_B^{(B')}(k) \quad \text{with} \quad B, B' = n, p, \Lambda, \Sigma^-$$

$U_B^{(B')}$ means a single particle potential of B particle in B' matter

$$\begin{aligned}
\varepsilon &= \varepsilon_{mass} + \varepsilon_{kin} + \varepsilon_{pot} \\
&= 2\sum_B \int_0^{k_F^B} \frac{d^3k}{(2\pi)^3}\left[M_B - M_n + \frac{\hbar^2 k^2}{2M_B} + \frac{1}{2}U_B(k)\right]
\end{aligned}$$

$$\varepsilon_{mass} = \sum_B (M_B - M_n)\rho_B$$

$$\varepsilon_{kin} = \sum_B \frac{3}{5}\frac{\hbar^2 (k_F^B)^2}{2M_B}\rho_B = \sum_B \frac{3}{5}\frac{\hbar^2}{2M_B}(3\pi^2)^{2/3}(\rho_B)^{5/3}$$

$$\varepsilon_{pot} = 2\sum_B \int_0^{k_F^B} \frac{d^3k}{(2\pi)^3}\frac{1}{2}U_B(k) = \frac{1}{2}\sum_B \int_0^{k_F^B} \frac{k^2 dk}{\pi^2}U_B(k)$$

$$\varepsilon_{pot} = \frac{1}{2}\sum_{BB'} \mathcal{U}_B^{(B')}$$

$$\mathcal{U}_B^{(B')} = \int_0^{k_F^B} \frac{k^2 dk}{\pi^2} U_B^{(B')}(k) \qquad \mathcal{U}_B^{(B')} = \mathcal{U}_{B'}^{(B)}$$

$$\int_0^{k_F^B} \frac{k^2 dk}{\pi^2} U_B^{(B')}(k) = \int_0^{k_F^{B'}} \frac{k^2 dk}{\pi^2} U_{B'}^{(B)}(k)$$

$$\frac{\partial}{\partial \rho_B} \mathcal{U}_B^{(B')} = U_B^{(B')}(k_F^B) + \int_0^{k_F^B} \frac{k^2 dk}{\pi^2} \frac{\partial U_B^{(B')}(k)}{\partial \rho_B}$$

Chemical potential : $\mu_B = \frac{\partial \varepsilon}{\partial \rho_B}$

Chemical potential : $\mu_B = \frac{\partial \varepsilon}{\partial \rho_B}$

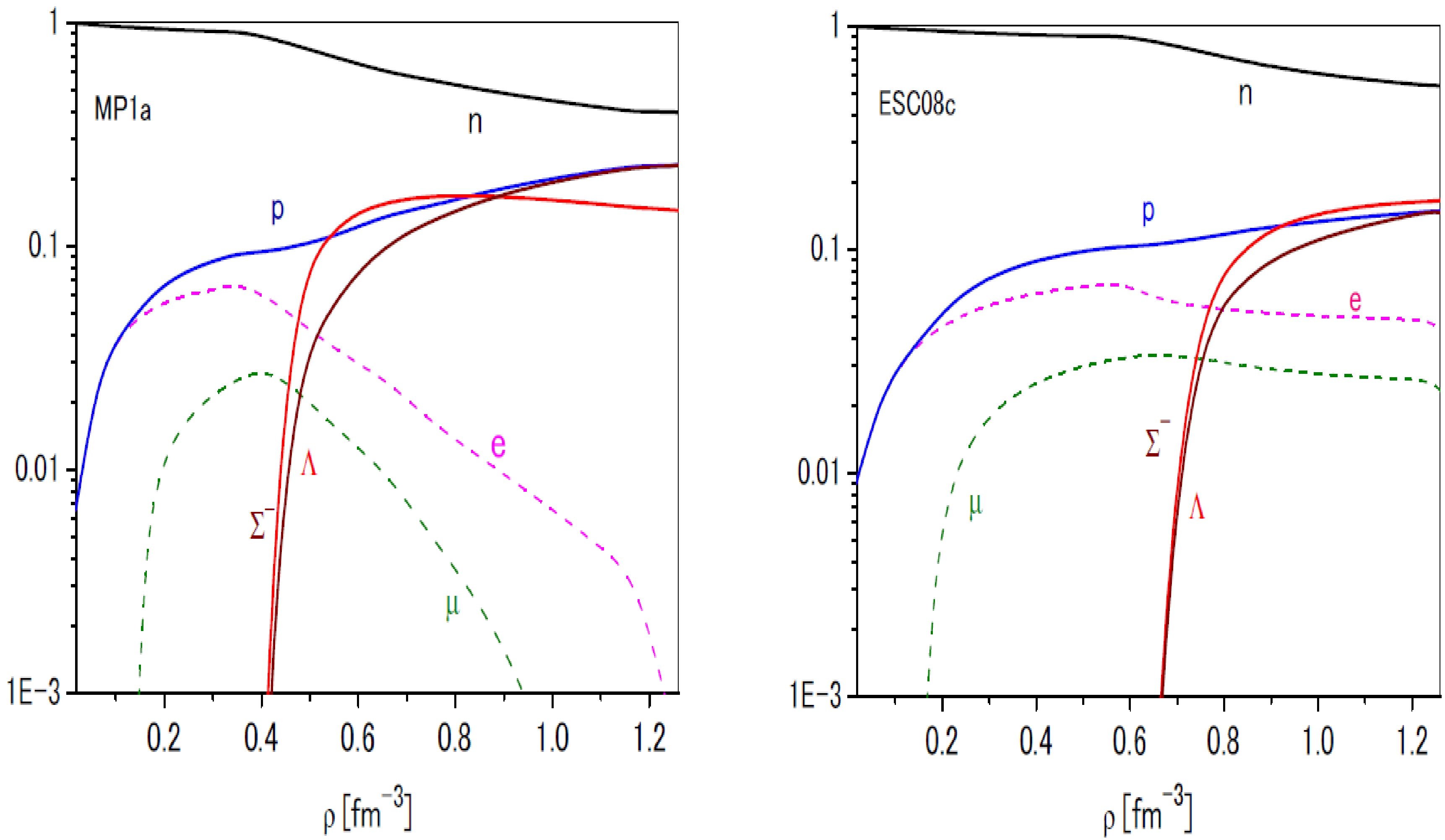
Chemical equilibrium conditions:

$$
\begin{aligned}
\mu_n &= \mu_p + \mu_e \\
\mu_e &= \mu_\mu \\
\mu_{\Sigma^-} &= \mu_n + \mu_e \\
\mu_\Lambda &= \mu_n
\end{aligned}
$$

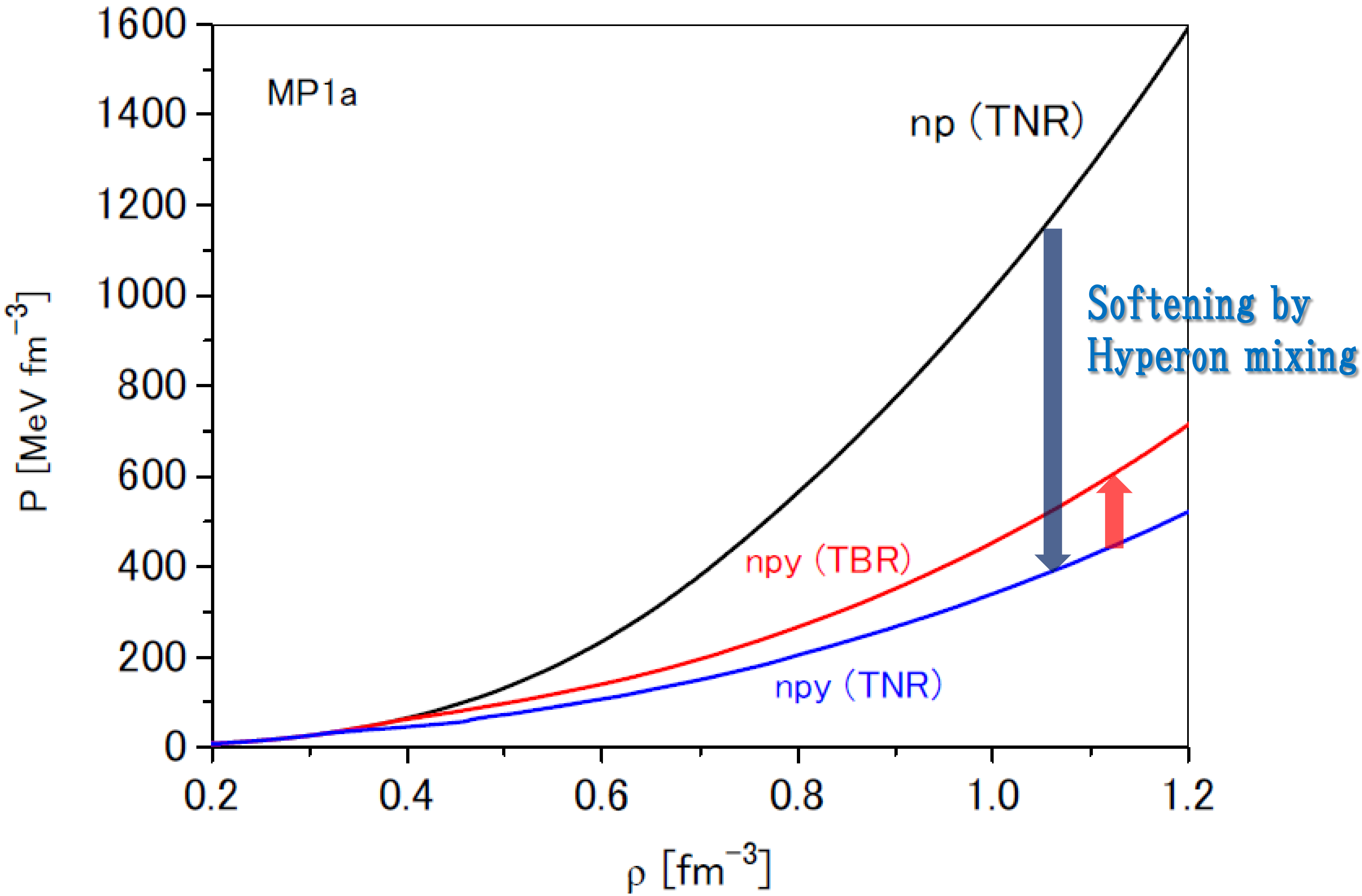
Baryon-number conservation : $y_n + y_p + y_\Lambda + y_{\Sigma^-} = 1$

Charge neutrality : $y_p = y_{\Sigma^-} + y_e + y_\mu$

β-stable $n+p+\Lambda+\Sigma^-$ matter

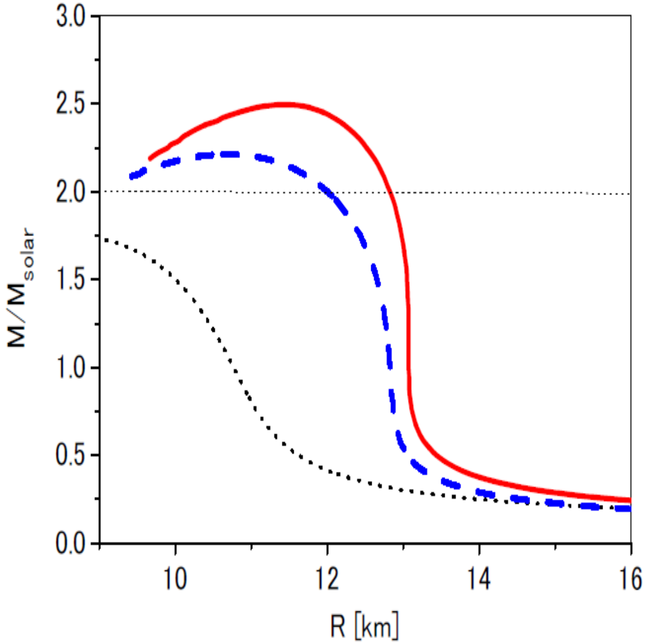


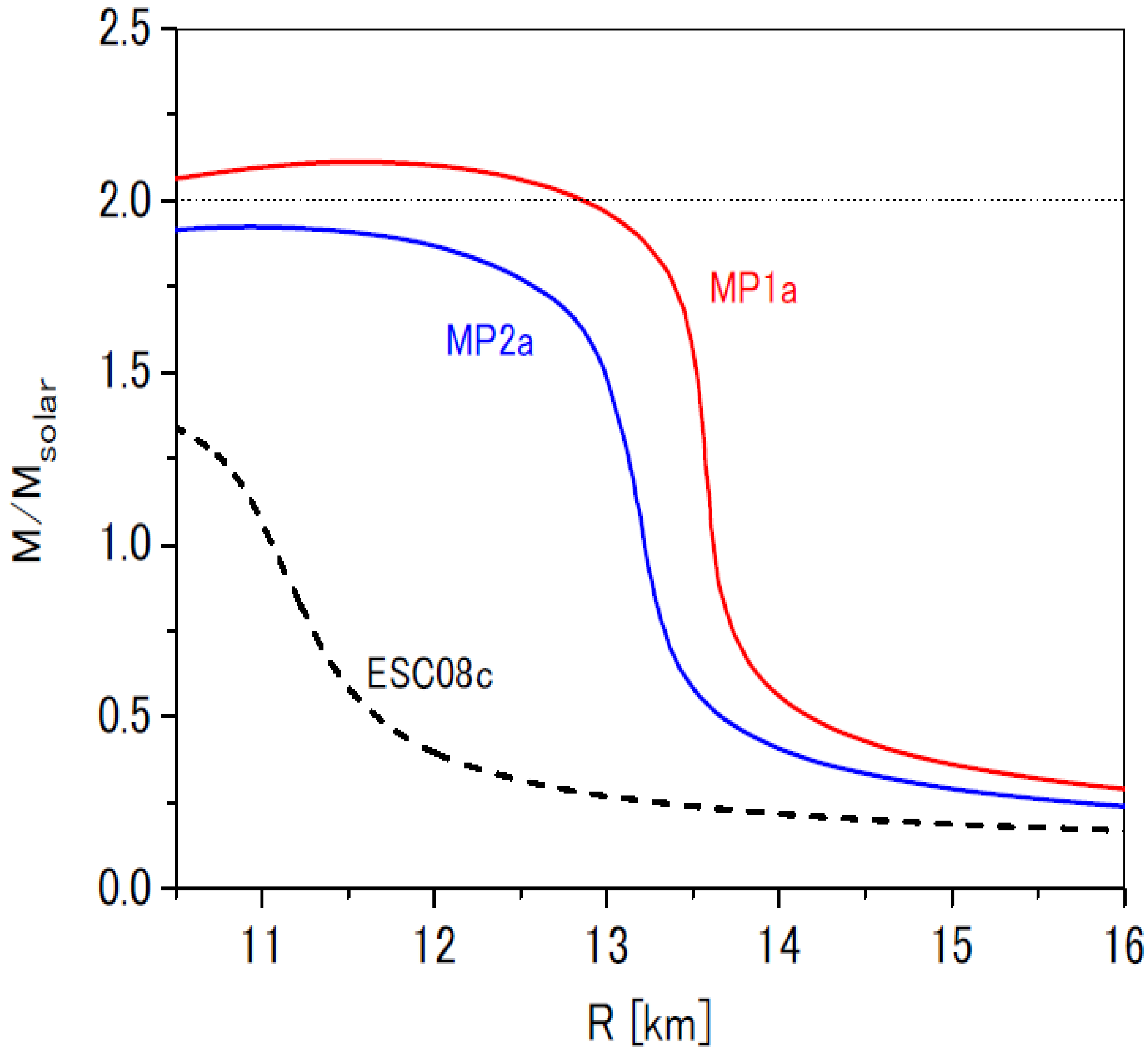
EOS



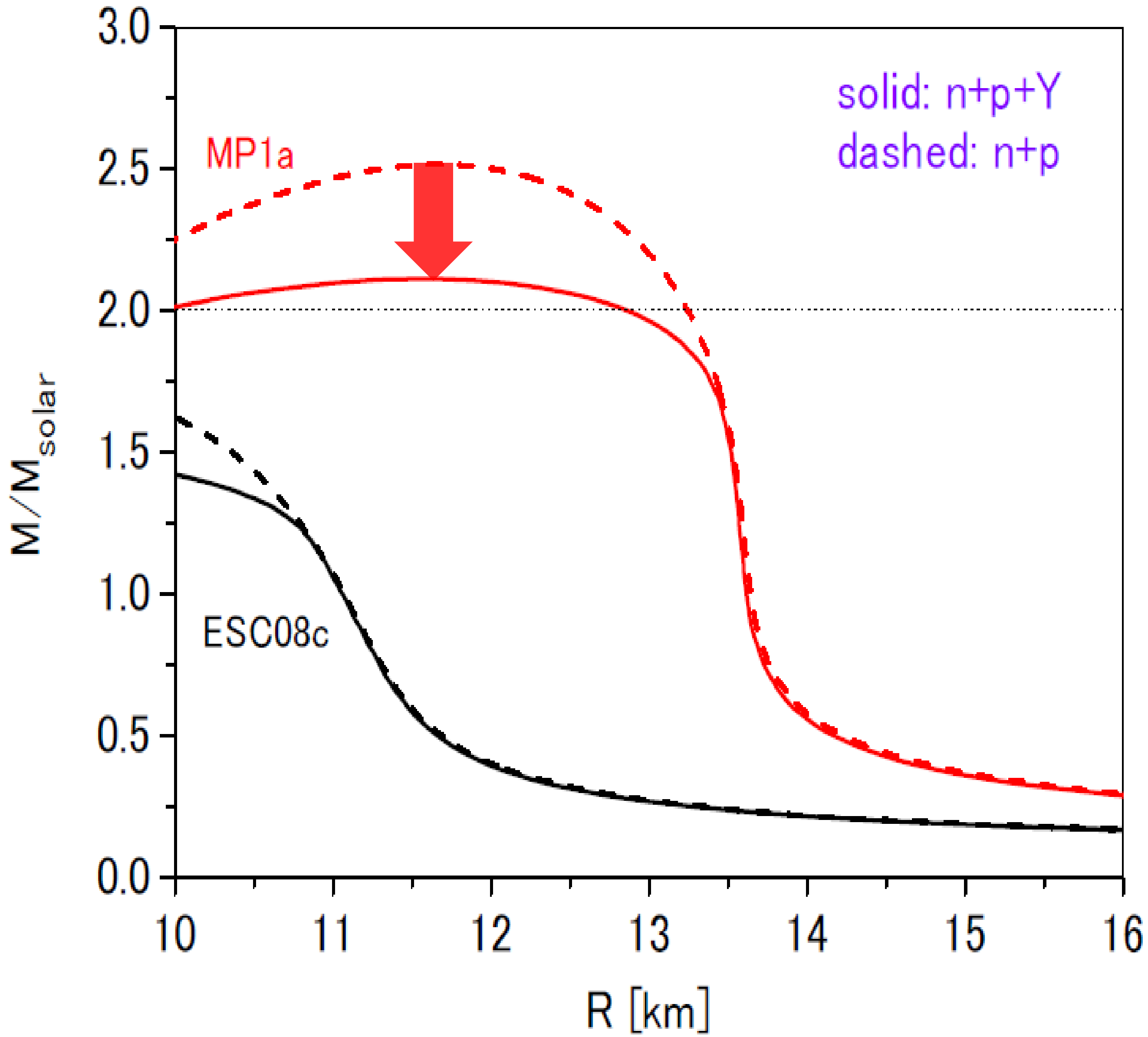
MP1a
np (TNR)
npy (TBR)
npy (TNR)
Softening by Hyperon mixing
1600
1400
1200
1000
800
600
400
200
0
P [MeV fm−3]
0.2
0.4
0.6
0.8
1.0
1.2
ρ [fm−3]

with EOS of n+p+Λ+Σ⁻ matter





Softening by hyperon mixing



Conclusion

ESC08c+MPP+TBA model

* MPP strength determined by analysis for $^{16}O+^{16}O$ scattering
* TNA adjusted phenomenologically to reproduce
  $E/A(\rho_0) = -15.8$ MeV with $\rho_0 = 0.164\ fm^{-3}$
* Consistent with hypernuclear data
* No ad hoc parameter to stiffen EOS

BB interactions based on on-Earth experiments

MP1a set including 3- and 4-body repulsions leads to massive neutron stars with $2M_{\odot}$ in spite of significant softening of EOS by hyperon mixing