Mass, Radius and Equation of State of Neutron Stars

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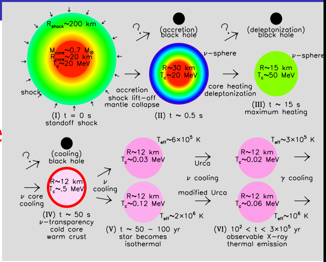
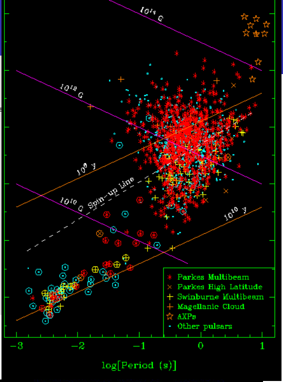
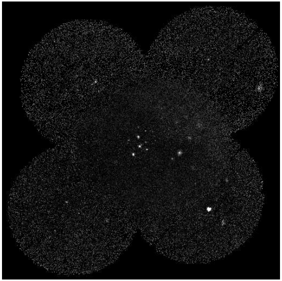
International School on Neutron Star Matter
YITP, Kyoto, Japan, 4–7 March 2014

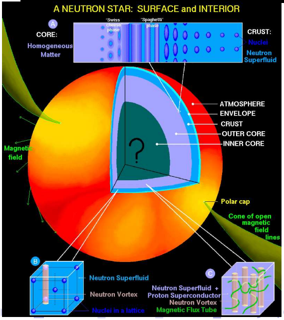
Outline

- Structure
- Dense Matter Equation of State
- Formation and Evolution
- Observational and Experimental Constraints









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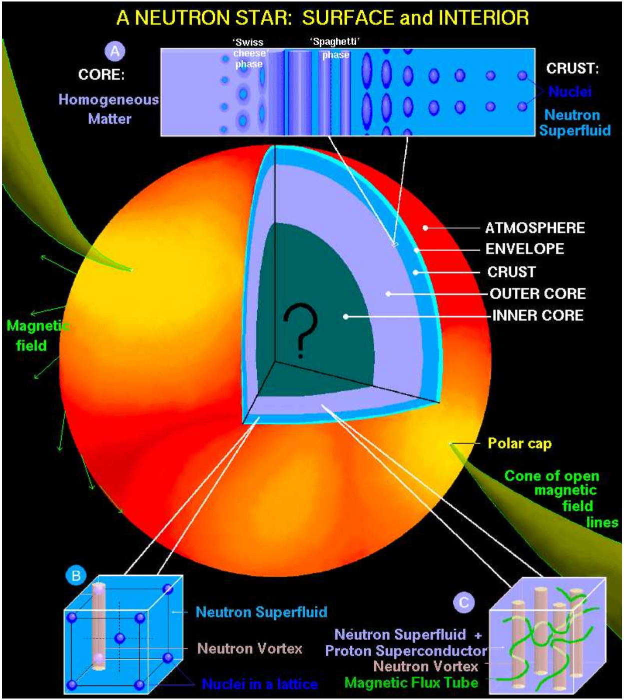
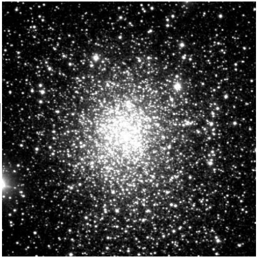


Amazing Facts About Neutron Stars

- Densest objects this side of an event horizon: 10^{15} g cm^{-3}
  Four teaspoons, on the Earth, would weigh as much as the Moon.
- Largest surface gravity: 10^{14} cm s^{-2}
  This is 100 billion times the Earth's gravity.
- Fastest spinning objects known: $\nu = 716$ Hz
  This spin rate was measured for PSR J1748-2446ad in the globular cluster Terzan 5 located 9 kpc away. (33 pulsars have been found in this cluster.) If the radius is about 15 km, the velocity at the equator is one fourth the speed of light.

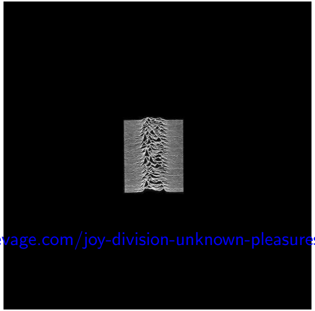


- Largest known magnetic field strengths: $B = 10^{15}$ G
- Highest temperature superconductor: $T_c = 10$ billion K
  The highest known superconductor on the Earth is mercury thallium barium calcium copper oxide ($Hg_{12}T_{l3}Ba_{30}Ca_{30}Cu_{45}O_{125}$), at 138 K.
- Highest temperature, at birth, anywhere in the Universe since the Big Bang: $T = 700$ billion K
- PSR B1508+55 has fastest measured stellar velocity in the Galaxy: 1083 km/s $= c/300$
- The only place in the universe except for the Big Bang where neutrinos become *trapped*.

Neutron Stars: History

1920 Rutherford predicts the neutron
1931 Landau *anticipates* single-nucleus stars but not neutron stars
1932 Chadwick discovers the neutron.
1934 W. Baade and F. Zwicky predict existence of neutron stars as end products of supernovae.
1939 Oppenheimer and Volkoff predict upper mass limit of neutron star.
1964 Hoyle, Narlikar and Wheeler predict neutron stars rapidly rotate.
1965 Hewish and Okoye discover an intense radio source in the Crab nebula.
1966 Colgate and White perform simulations of supernovae leading to neutron stars.
1967 C. Schisler discovers a dozen pulsing radio sources, including the Crab pulsar, using secret military radar in Alaska. X-1.
1967 Hewish, Bell, Pilkington, Scott and Collins discover "first" PSR 1919+21, Aug 6.
1968 The Crab Nebula pulsar is discovered, found to be slowing down (ruling out binary and vibrational models), and clinched the connection to supernovae.
1968 The term "pulsar" first appears in print, in the *Daily Telegraph*.

1969 "Glitches" observed; evidence for superfluidity in neutron star crust.
1971 Accretion powered X-ray pulsar discovered by Uhuru (*not* the Lt.).
1974 Hewish awarded Nobel Prize (but Bell and Okoye were not).
1974 Binary pulsar PSR 1913+16 discovered by Hulse and Taylor., orbital decay due to GR gravitational radiation
1979 Chart recording of PSR 1919+21 used as album cover for *Unknown Pleasures* by Joy Division (#19/100 greatest British album).
1982 First millisecond pulsar, PSR B1937+21, discovered by Backer et al. at Arecibo.
1992 Discovery of first extra-solar planets orbiting PSR B1257+12 by Wolszczan and Frail.
1993 Hulse and Taylor receive Nobel Prize
1998 Kouveletiou et al. discover first magnetar
2004 SGR 1806-20 flares: largest burst of energy seen in Galaxy since SN 1604, brighter than full moon in γ rays, more energy emitted than Sun in 100,000 years.



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2004 Hessels et al. discover PSR J1748-2446ad; fastest rotation rate, 716 Hz.
2005 Hessels et al. discover PSR J0737-3039, first two-pulsar binary
2013 Antoniadis et al. find most-massive PSR J0348+0432, 2.01 $M_\odot$
2013 Ransom et al. find first pulsar in triple system (two white dwarfs)

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Neutron Star Structure

Tolman-Oppenheimer-Volkov equations of relativistic hydrostatic equilibrium:

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m+4\pi r^3 p/c^2)(\epsilon+p)}{r(r-2Gm/c^2)}$$
$$\frac{dm}{dr} = 4\pi\frac{\epsilon}{c^2}r^2$$

p is pressure, ϵ is mass-energy density
Useful analytic solutions exist:

- Uniform density $\epsilon = \text{constant}$
- Tolman VII $\epsilon = \epsilon_c[1-(r/R)^2]$
- Buchdahl $\epsilon = \sqrt{pp_*} - 5p$
- Tolman IV $e^\nu = \alpha + \beta(r/R)^2$

Uniform Density Fluid

$$
\begin{aligned}
m(r) &= \frac{4\pi}{3}\epsilon r^3 = Mx^{3/2}, \qquad x \equiv \left(\frac{r}{R}\right)^2, \qquad \beta \equiv \frac{M}{R} \\
e^{-\lambda(r)} &= 1 - 2\beta x, \\
e^{\nu(r)} &= \left[\frac{3}{2}\sqrt{1-2\beta} - \frac{1}{2}\sqrt{1-2\beta x}\right]^2, \\
p(r) &= \epsilon\left[\frac{\sqrt{1-2\beta x} - \sqrt{1-2\beta}}{3\sqrt{1-2\beta} - \sqrt{1-2\beta x}}\right], \\
\epsilon(r) &= \text{constant}; \qquad n(r) = \text{constant} \\
\frac{\mathrm{BE}}{M} &= \frac{3}{4\beta}\left(\frac{\sin^{-1}\sqrt{2\beta}}{\sqrt{2\beta}} - \sqrt{1-2\beta}\right) \simeq \frac{3\beta}{5} + \frac{9}{14}\beta^2 + \cdots, \\
c_s^2 &= \infty
\end{aligned}
$$

$$p_c < \infty \Longrightarrow \beta < 4/9$$

Tolman VII

$$
\begin{aligned}
\epsilon(r) &= \epsilon_c\left[1-(r/R)^2\right] \equiv \epsilon_c[1-x] \\
e^{-\lambda(r)} &= 1-\beta x(5-3x) \\
e^{\nu(r)} &= (1-5\beta/3)\cos^2\phi, \\
p(r) &= \frac{1}{4\pi R^2}\left[\sqrt{3\beta e^{-\lambda(r)}}\tan\phi(r)-\frac{\beta}{2}(5-3x)\right], \\
n(r) &= \frac{\epsilon(r)+p(r)}{m_b}\frac{\cos\phi(r)}{\cos\phi_1} \\
\phi(r) &= \frac{w_1-w(r)}{2}+\phi_1, \qquad \phi_1=\phi(x=1)=\tan^{-1}\sqrt{\frac{\beta}{3(1-2\beta)}}, \\
w(r) &= \ln\left[x-\frac{5}{6}+\sqrt{\frac{e^{-\lambda(r)}}{3\beta}}\right], \quad w_1=w(x=1)=\ln\left[\frac{1}{6}+\sqrt{\frac{1-2\beta}{3\beta}}\right].
\end{aligned}
$$

$$
(p/\epsilon)_c=\frac{2\tan\phi_c}{15}\sqrt{\frac{3}{\beta}}-\frac{1}{3}, \qquad c^2_{s,c}=\tan\phi_c\left(\frac{1}{5}\tan\phi_c+\sqrt{\frac{\beta}{3}}\right)
$$

$$
\frac{\mathrm{BE}}{M}\simeq\frac{11}{21}\beta+\frac{7187}{18018}\beta^2+\cdots
$$

$$
p, c_s^2<\infty \Longrightarrow \phi_c<\pi/2 \Longrightarrow \beta<0.3862
$$

Buchdahl's Solution: Relativistic n=1 Polytrope

$$\epsilon = \sqrt{p_* p} - 5p$$

$$\begin{aligned}
e^{\nu(r)} &= (1-2\beta)(1-\beta-u(r))(1-\beta+u(r))^{-1}, \\
e^{\lambda(r)} &= (1-2\beta)(1-\beta+u(r))(1-\beta-u(r))^{-1}(1-\beta+\beta\cos Ar')^{-2}, \\
8\pi p(r) &= A^2 u(r)^2 (1-2\beta)(1-\beta+u(r))^{-2}, \\
8\pi\epsilon(r) &= 2A^2 u(r)(1-2\beta)(1-\beta-3u(r)/2)(1-\beta+u(r))^{-2}, \\
m_b n(r) &= \sqrt{p_* p(r)}\left(1-4\sqrt{\frac{p(r)}{p_*}}\right)^{3/2}, \qquad c_s^2(r) = \left(\frac{1}{2}\sqrt{\frac{p_*}{p(r)}}-5\right)^{-1} \\
u(r) &= \frac{\beta}{Ar'}\sin Ar' = (1-\beta)\left(\frac{1}{2}\sqrt{\frac{p_*}{p(r)}}-1\right)^{-1}, \\
r' &= r(1-2\beta)(1-\beta+u(r))^{-1}, \\
A^2 &= 2\pi p_*(1-2\beta)^{-1}, \qquad R = (1-\beta)\sqrt{\frac{\pi}{2p_*(1-2\beta)}}.
\end{aligned}$$

$$p_c = \frac{p_*}{4}\beta^2, \qquad \epsilon_c = \frac{p_*}{2}\beta(1-\frac{5}{2}\beta), \qquad n_c m_b = \frac{p_*}{2}\beta(1-2\beta)^{3/2}$$

$$\frac{\mathrm{BE}}{M} = (1-\frac{3}{2}\beta)(1-2\beta)^{-1/2}(1-\beta)^{-1} \simeq \frac{\beta}{2}+\frac{\beta^2}{2}+\cdots$$

$$c_s^2 < \infty \Longrightarrow \beta < 1/5$$

Tolman IV Variation: A Self-Bound Star (Quark Star)

$$e^{\nu(r)} = \frac{\left[1-\frac{5}{2}\beta\left(1-\frac{1}{5}x\right)\right]^2}{(1-2\beta)},$$

$$e^{\lambda(r)} = \frac{\left[1-\frac{5}{2}\beta\left(1-\frac{3}{5}x\right)\right]^{2/3}}{\left[1-\frac{5}{2}\beta\left(1-\frac{3}{5}x\right)\right]^{2/3}-2(1-\beta)^{2/3}\beta x},$$

$$4\pi pR^2 = \frac{\beta}{1-\frac{5}{2}\beta\left(1-\frac{1}{5}x\right)}\left[1-(1-\beta)^{2/3}\frac{1-\frac{5}{2}\beta(1-x)}{\left[1-\frac{5}{2}\beta\left(1-\frac{3}{5}x\right)\right]^{2/3}}\right],$$

$$4\pi\epsilon R^2 = \frac{3(1-\beta)^{2/3}\beta\left[1-\frac{5}{2}\beta(1-\frac{1}{3}x)\right]}{\left[1-\frac{5}{2}\beta\left(1-\frac{3}{5}x\right)\right]^{5/3}}, \quad m = \frac{(1-\beta)^{2/3}Mx^{3/2}}{\left[1-\frac{5}{2}\beta\left(1-\frac{3}{5}x\right)\right]^{2/3}}$$

$$c_s^2 = \frac{(2-5\beta+3\beta x)}{5(2-5\beta+\beta x)^3}\left[\frac{(2-5\beta+3\beta x)^{5/3}}{2^{2/3}(1-\beta)^{2/3}}+(2-5\beta)^2-5\beta^2 x\right],$$

$$\frac{\epsilon_{surf}}{\epsilon_c} = \left(1-\frac{5}{3}\beta\right)\left(1-\frac{5}{2}\beta\right)^{2/3}(1-\beta)^{-5/3}.$$

$$0.30 < c_{s,c}^2 < 0.44, \qquad 0.265 < \frac{\epsilon_{surf}}{\epsilon_c} < 1$$

Important Questions

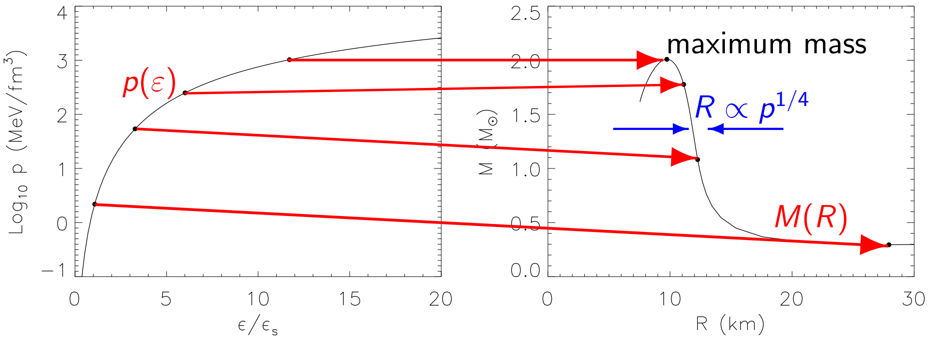
- How Does the Structure of Neutron Stars Depend On the Nucleon-Nucleon Interaction?
  - The Neutron Star Maximum Mass and Causality
  - The Neutron Star Radius and the Nuclear Symmetry Energy
  - Does Exotic Matter (Hyperons, Kaons/Pions, Deconfined Quarks) Exist in Neutron Star Interiors?
- How Do Nuclear Experiments Constrain the Nuclear Symmetry Energy and Neutron Star Radii?
  - Binding Energies
  - Heavy ion Collisions
  - Neutron Skin Thicknesses
  - Dipole Polarizabilities
  - Giant (and Pygmy) Dipole Resonances
  - Pure Neutron Matter
- What Astrophysical Constraints Exist?
  - Mass Measurements of Pulsars and X-ray Binaries
  - Photospheric Radius Expansion Bursts
  - Thermal Emission from Isolated and Quiescent Binary Sources
  - Pulse Modeling of X-ray Bursts, QPOs, Gravitational Radiation, etc.

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi p r^3/c^2)(\varepsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



Equation of State ⟵ Observations

Mass-Radius Diagram and Theoretical Constraints

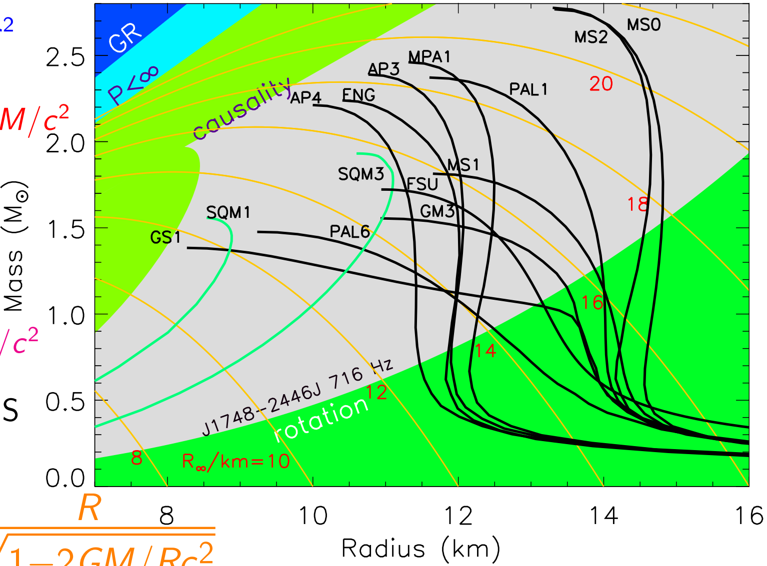
GR:
$R > 2GM/c^2$

$P < \infty$:
$R > (9/4)GM/c^2$

causality:
$R \gtrsim 2.9GM/c^2$

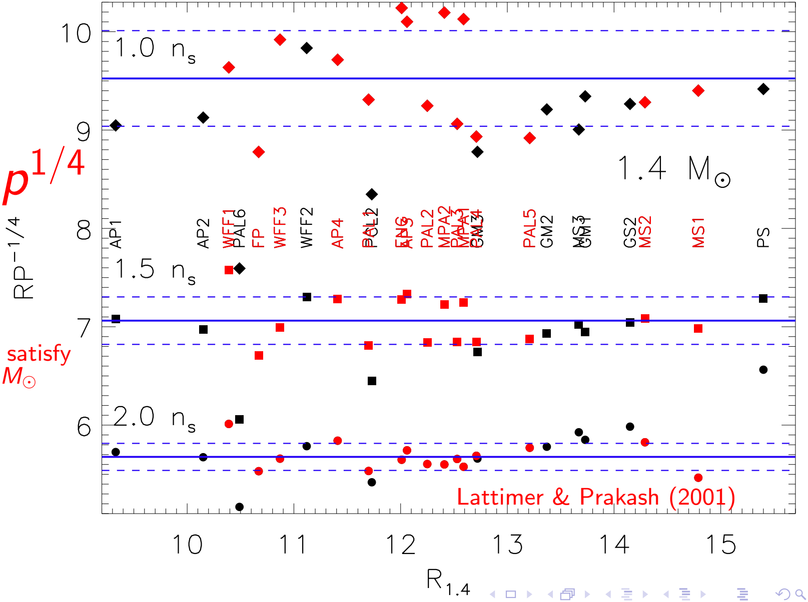
— normal NS
— SQS

— $R_\infty = \frac{R}{\sqrt{1 - 2GM/Rc^2}}$



The Radius – Pressure Correlation

$R \propto p^{1/4}$



Red points satisfy $M_{\rm max} \gtrsim 2M_\odot$

Neutron Star Structure

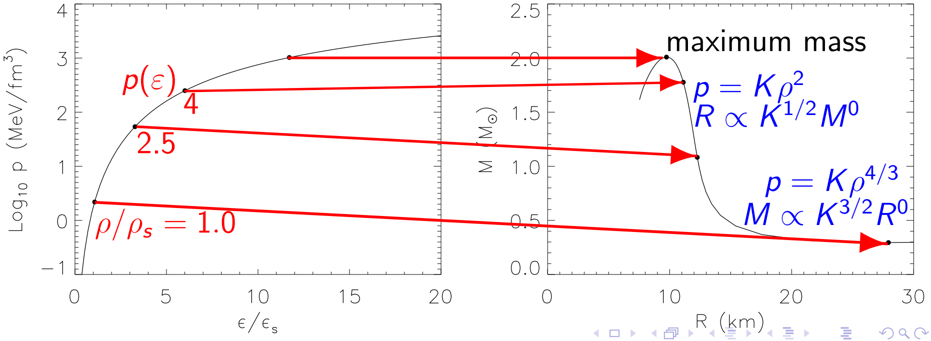
Newtonian Gravity:

$$\frac{dp}{dr} = -\frac{Gm\rho}{r^2}; \qquad \frac{dm}{dr} = 4\pi\rho r^2; \qquad \rho c^2 = \varepsilon$$

Newtonian Polytropes:

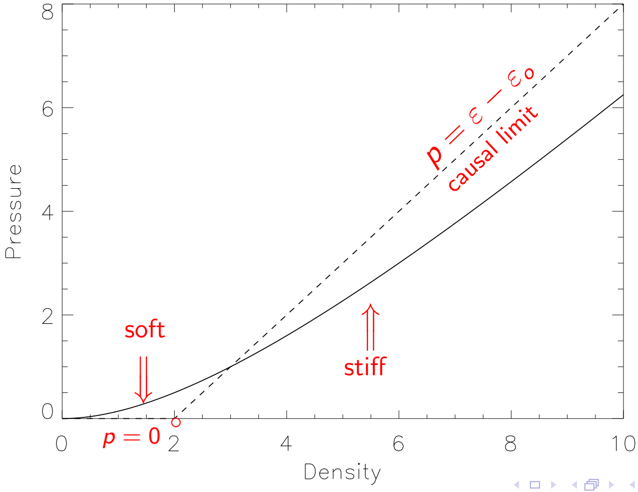
$$p = K\rho^\gamma; \qquad M \propto K^{1/(2-\gamma)} R^{(4-3\gamma)/(2-\gamma)}$$

$$\rho < \rho_s: \ \gamma \simeq \frac{4}{3}; \qquad \rho > \rho_s: \ \gamma \simeq 2$$



Extremal Properties of Neutron Stars

- The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).



ε_0 is the only EOS parameter

The TOV solutions scale with ε_0

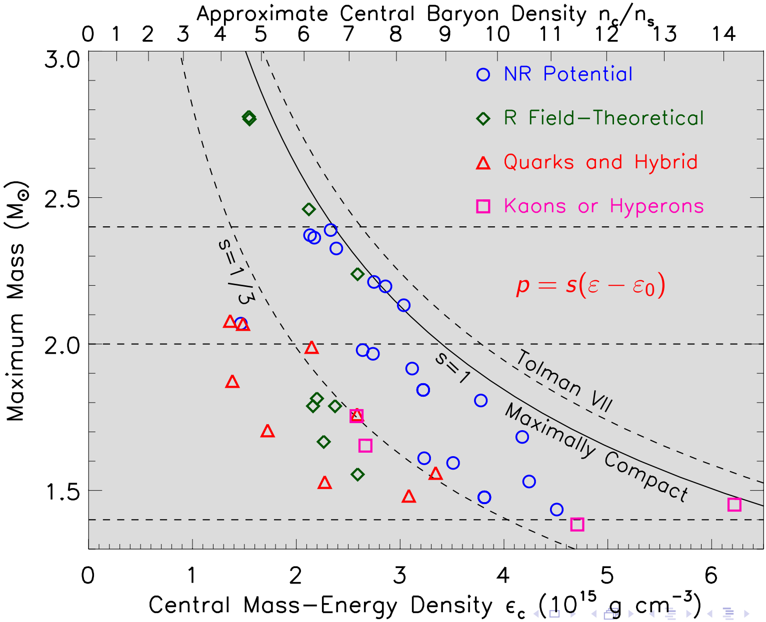
$w = \varepsilon/\varepsilon_0$
$y = p/\varepsilon_0$
$x = r\sqrt{G\varepsilon_0}/c^2$
$z = m\sqrt{G^3\varepsilon_0}/c^2$

Extremal Properties of Neutron Stars

The maximum mass configuration is achieved when $x_R = 0.2404$, $w_c = 3.034$, $y_c = 2.034$, $z_R = 0.08513$.

- $M_{max} = 4.1\ (\varepsilon_s/\varepsilon_0)^{1/2}\ \mathrm{M}_\odot$ (Rhoades & Ruffini 1974)
- $M_{B,max} = 5.41\ (m_B c^2/\mu_o)(\varepsilon_s/\varepsilon_0)^{1/2}\ \mathrm{M}_\odot$
- $R_{min} = 2.82\ GM/c^2 = 4.3\ (M/\mathrm{M}_\odot)\ \mathrm{km}$
- $\mu_{b,max} = 2.09$ GeV
- $\varepsilon_{c,max} = 3.034\ \varepsilon_0 \simeq 51\ (\mathrm{M}_\odot/M_{largest})^2\ \varepsilon_s$
- $p_{c,max} = 2.034\ \varepsilon_0 \simeq 34\ (\mathrm{M}_\odot/M_{largest})^2\ \varepsilon_s$
- $n_{B,max} \simeq 38\ (\mathrm{M}_\odot/M_{largest})^2\ n_s$
- $\mathrm{BE}_{max} = 0.34\ M$
- $P_{min} = 0.74\ (\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}\ \mathrm{ms} = 0.20\ (M_{sph,max}/\mathrm{M}_\odot)\ \mathrm{ms}$

Maximum Energy Density in Neutron Stars



Causality + GR Limits and the Maximum Mass

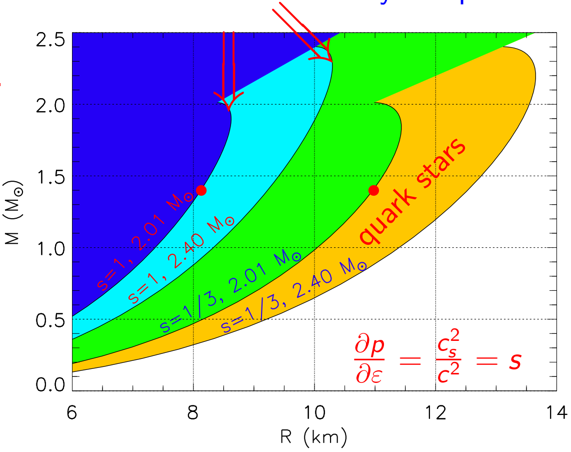
A lower limit to the maximum mass sets a lower limit to the radius for a given mass.

Similarly, a precise (M, R) measurement sets an upper limit to the maximum mass.

$1.4 M_\odot$ stars must have $R > 8.15 M_\odot$.

$1.4 M_\odot$ strange quark matter stars (and likely hybrid quark/hadron stars) must have $R > 11$ km.

$M - R$ curves for maximally compact EOS



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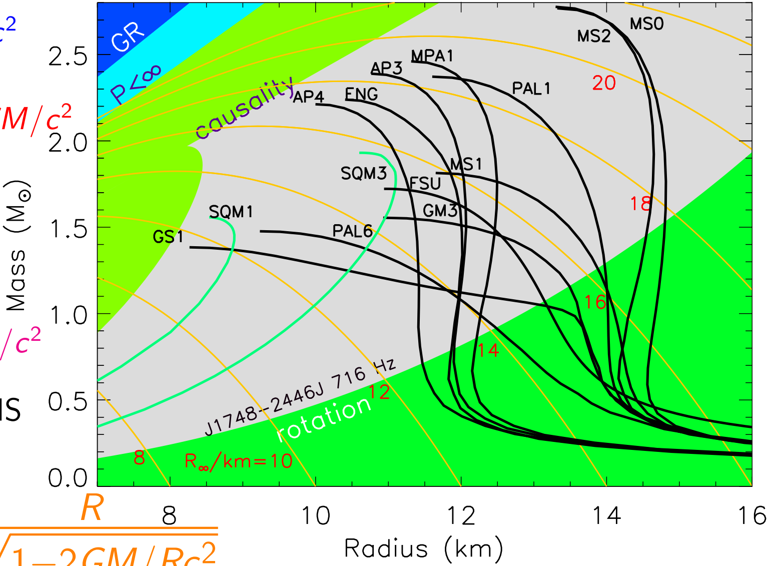


Roche Model for Stellar Rotation

(*c.f.*, Shapiro & Teukolsky 1983)

$$\rho^{-1}\nabla P = \nabla\mu = -\nabla(\Phi_G + \Phi_c)$$

$$\Phi_G \simeq -GM/r,\ \Phi_c = -\frac{1}{2}\Omega^2 r^2 \sin^2\theta$$

Bernoulli integral:

$H = \mu + \Phi_G + \Phi_c = -GM/R_p$

$\mu = \int_0^p \rho^{-1} dp = \mu_n - \mu_{n0}$

Evaluate at equator:

$$\frac{\Omega^2 R_{eq}^3}{2GM} = \frac{R_{eq}}{R_p} - 1$$

Mass-shedding limit

$\Omega^2_{shed} = GM/R^3_{eq} : R_{eq}/R_p = \frac{3}{2}$

GR: Cook, Shapiro & Teukolsky (1994): 1.43–1.51

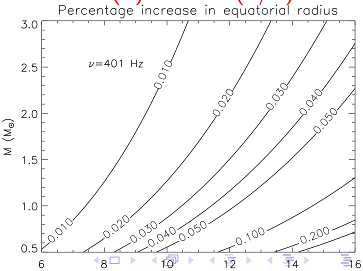
$$\Omega_{shed} = \left(\frac{2}{3}\right)^{3/2}\sqrt{\frac{GM}{R^3}}$$

$$\frac{P_{shed}}{\mathrm{ms}} \simeq 1.0\left(\frac{R}{10\ \mathrm{km}}\right)^{3/2}\left(\frac{\mathrm{M}_\odot}{M}\right)^{1/2}$$

Shape: $\dfrac{\Omega^2 R^3 \sin^2\theta}{2GM} = \dfrac{R}{R_p} - 1$

$$\frac{R_p}{R(\theta)} = \frac{1}{3} + \frac{2}{3}\cos\left(\frac{1}{3}\cos^{-1}\left[1 - 2\left(\frac{\Omega\sin\theta}{\Omega_{shed}}\right)^2\right]\right)$$

Limit: $\dfrac{R_p}{R(\theta)} = \dfrac{\sin(\theta)}{3\sin(\theta/3)}.$



The Equation of State in Neutron Stars

- The EOS of an ideal Fermi gas
- Thermodynamic relations
- Degeneracy
- Limiting cases
- Interactions
- Properties of bulk nucleonic matter
- Phase coexistence
- Nuclear symmetry energy
- Finite nuclei – the liquid droplet model
- Nuclei in dense matter

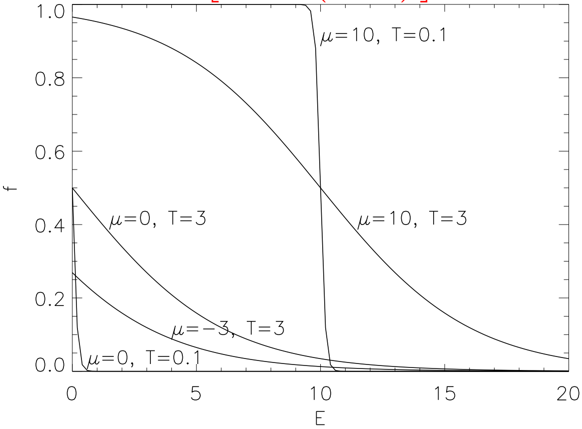
EOS of an Ideal Fermion Gas

Single particle energy

$$E^2 = m^2c^4 + p^2c^2$$

Occupation index

$$f = \left[1 + \exp\left(\frac{E-\mu}{T}\right)\right]^{-1}$$



Thermodynamics

Number and energy densities

$$n = \frac{g}{h^3}\int_0^\infty f d^3p; \qquad \varepsilon = \frac{g}{h^3}\int_0^\infty E f d^3p; \qquad \mu = \left(\frac{\partial F}{\partial n}\right)_T$$

Landau quasi-particle entropy formula:

$$ns = -\frac{g}{h^3}\int_0^\infty \left[f \ln f + (1-f)\ln(1-f)\right] d^3p$$

Pressure

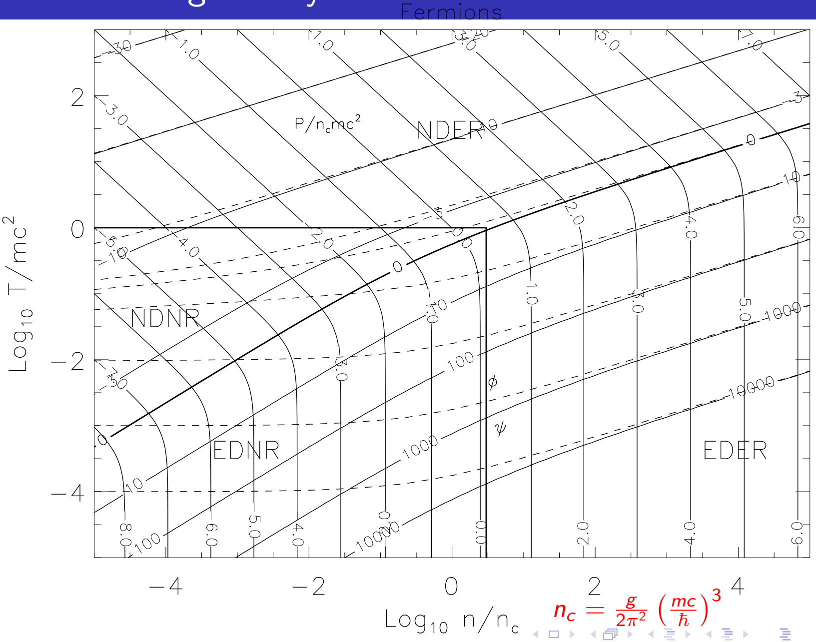
$$P = \frac{g}{3h^3}\int_0^\infty p\frac{\partial E}{\partial p} f d^3p = n^2\left(\frac{\partial(F/n)}{\partial n}\right)_T = \mu n - \varepsilon + nTs = \mu n - F$$

$$n = \left(\frac{\partial P}{\partial \mu}\right)_T; \qquad ns = \left(\frac{\partial P}{\partial T}\right)_\mu$$

Degeneracy parameters

$$\phi = \frac{\mu}{T} = \psi + \frac{mc^2}{T}; \qquad \left(\frac{\partial P}{\partial T}\right)_\phi = n(s+\phi); \qquad \left(\frac{\partial P}{\partial \psi}\right)_T = n(s+\psi)$$

Pressure and Degeneracy Parameters



$$n_c = \frac{g}{2\pi^2}\left(\frac{mc}{\hbar}\right)^3$$

Limits

Non-relativistic $p << mc$

$x = p^2/(2mT)$, $\psi = (\mu - mc^2)/T$

$$n = \frac{g(2mT)^{3/2}}{4\pi^2\hbar^3}\int_0^\infty \frac{x^{1/2}}{1+e^{x-\psi}dx} = \frac{g(2mT)^{3/2}}{4\pi^2\hbar^3}F_{1/2}(\psi)$$

$$\varepsilon = nmc^2 + \frac{gT(2mT)^{3/2}}{4\pi^2\hbar^3}F_{3/2}(\psi)$$

$$P = \frac{2}{3}(\varepsilon - nmc^2), \qquad s = \frac{5F_{3/2}(\psi)}{3F_{1/2}(\psi)} - \psi$$

Relativistic $p >> mc$

$$n = \frac{g}{2\pi^2}\left(\frac{T}{\hbar c}\right)^3 F_2(\phi)$$

$$\varepsilon = \frac{gT}{2\pi^2}\left(\frac{T}{\hbar c}\right)^3 F_3(\phi)$$

$$P = \frac{\varepsilon}{3}, \qquad s = \frac{4F_3(\phi)}{3F_2(\phi)} - \phi$$

Limits

$$\left\{ \begin{array}{ll} n & = \frac{g}{6\pi^2}\left(\frac{\phi T}{\hbar c}\right)^3\left[1+\left(\frac{\pi}{\phi}\right)^2+\cdots\right], \qquad s=\frac{\pi^2}{\phi}+\cdots \\ P & = \frac{\varepsilon}{3} = \frac{n\phi T}{4}\left[1+\left(\frac{\pi}{\phi}\right)^2+\cdots\right] \propto n^{4/3} \end{array} \right\} \text{EDER}$$

$$\left\{ \begin{array}{ll} n & = \frac{g}{6\pi^2}\left(\frac{2m\psi T}{\hbar^2}\right)^{3/2}\left[1+\frac{1}{8}\left(\frac{\pi}{\psi}\right)^2+\cdots\right], \qquad s=\frac{\pi^2}{2\psi}+\cdots \\ P & = \frac{2}{3}(\varepsilon - nmc^2) = \frac{2n\psi T}{5}\left[1+\frac{1}{2}\left(\frac{\pi}{\psi}\right)^2+\cdots\right] \propto n^{5/3} \end{array} \right\} \text{EDNR}$$

$$\left\{ \begin{array}{ll} n & = \frac{g}{\pi^2}\left(\frac{T}{\hbar c}\right)^3 e^{\phi}, \qquad s = 4-\phi \\ P & = \frac{\varepsilon}{3} = nT \end{array} \right\} \text{NDER}$$

$$\left\{ \begin{array}{ll} n & = g\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2} e^{\psi}; \qquad s = \frac{5}{2}-\psi \\ P & = \frac{2}{3}(\varepsilon - nmc^2) = nT \end{array} \right\} \text{NDNR}$$

Limit of Relativistic Gas With Pairs in Thermal Equilibrium

$$
\begin{aligned}
\mu &= \mu^+ = -\mu^- = \phi T, \\
n &= n^+ - n^- = \frac{g}{6\pi^2}\left(\frac{\mu}{\hbar c}\right)^3\left[1+\left(\frac{\pi T}{\mu}\right)^2\right], \\
P &= P^+ + P^- = \frac{\varepsilon}{3} = \frac{g\mu}{24\pi^2}\left(\frac{\mu}{\hbar c}\right)^3\left[1+2\left(\frac{\pi T}{\mu}\right)^2+\frac{7}{15}\left(\frac{\pi T}{\mu}\right)^4\right], \\
s &= s^+ + s^- = \frac{gT}{6n}\left(\frac{\mu}{\hbar c}\right)^2\left[1+\frac{7}{15}\left(\frac{\pi T}{\mu}\right)^2\right].
\end{aligned}
$$

Cubic Solution:

$$
\begin{aligned}
\mu &= r - q/r, \qquad r = \left[\left(q^3+t^2\right)^{1/2}+t\right]^{1/3}, \\
t &= \frac{3\pi^2}{g} n(\hbar c)^3, \qquad q = \frac{(\pi T)^2}{3}.
\end{aligned}
$$

Non-Relativistic Fermi Gases With Interactions

$$\varepsilon = \sum_t \frac{\hbar^2}{2m_t^*}\tau_t + U(n_n, n_p)$$

Single particle energy, $t = n, p$

$$\epsilon_t = m_t c^2 + \frac{p^2}{2m_t^*(n_n, n_p)} + V_t(n_n, n_p, \tau_n, \tau_p)$$

$$\frac{\hbar^2}{2m_t^*} = \frac{\delta\varepsilon}{\delta\tau_t}, \qquad V_t = \frac{\delta\varepsilon}{\delta n_t} = \frac{\hbar^2}{2}\sum_s \tau_s \frac{\partial (m_s^*)^{-1}}{\partial n_s} + \frac{\partial U}{\partial n_t}$$

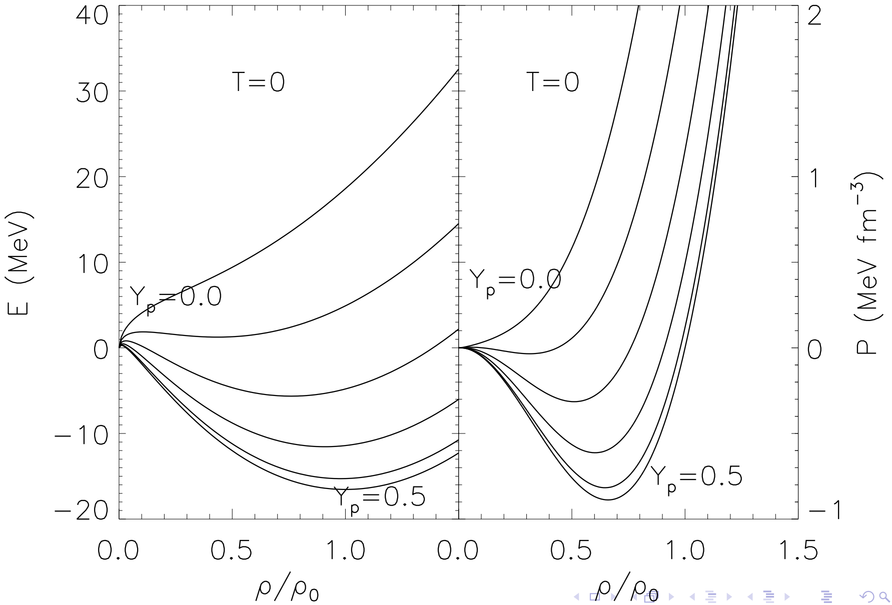
Number and kinetic densities

$$n_t = \frac{1}{2\pi^2\hbar^2}\left(\frac{2m_t^* T}{\hbar^2}\right)^{3/2} F_{1/2}(\eta_t); \qquad \tau_t = \frac{1}{2\pi^2\hbar^2}\left(\frac{2m_t^* T}{\hbar^2}\right)^{5/2} F_{3/2}(\eta_t)$$

$$\eta_t = \frac{\mu_t - m_t c^2 - V_t}{T}; \qquad f_t = \left[1 + \exp\left(\frac{\epsilon_t - \mu_t}{T}\right)\right]^{-1}$$

$$P = \sum_t \left[n_t V_t + \frac{\hbar^2 \tau_t}{3m_t^*}\right] - U, \qquad ns = \sum_t \left[\frac{5\hbar^2\tau_t}{6m_t^* T} - n_t\eta_t\right]$$

Bulk Matter Energy and Pressure



Schematic Free Energy Density

n: number density; x: proton fraction; T: temperature
$n_s \simeq 0.16 \pm 0.01$ fm^{-3}: nuclear saturation density
$B \simeq 16 \pm 1$ MeV: saturation binding energy
$K \simeq 240 \pm 20$ MeV: incompressibility parameter
$S_v \simeq 30 \pm 6$ MeV: bulk symmetry parameter
$L \simeq 60 \pm 60$ MeV: symmetry stiffness parameter
$a \simeq 0.065 \pm 0.010$ MeV^{-1}: bulk level density parameter

$$
\begin{aligned}
F(n,x,T) &= n\left[-B + \frac{K}{18}\left(1-\frac{n}{n_s}\right)^2 + S_v\frac{n}{n_s}(1-2x)^2 - a\left(\frac{n_s}{n}\right)^{2/3}T^2\right] \\
P &= n^2\frac{\partial(F/n)}{\partial n} = \frac{n^2}{n_s}\left[\frac{K}{9}\left(\frac{n}{n_s}-1\right) + S_v(1-2x)^2\right] + \frac{2an}{3}\left(\frac{n_s}{n}\right)^{2/3}T^2 \\
\mu_n &= \frac{\partial F}{\partial n} - \frac{x}{n}\frac{\partial F}{\partial x} \\
&= -B + \frac{K}{18}\left(1-\frac{n}{n_s}\right)\left(1-3\frac{n}{n_s}\right) + 2S_v\frac{n}{n_s}(1-2x) - \frac{a}{3}\left(\frac{n_s}{n}\right)^{2/3}T^2 \\
\hat{\mu} &= -\frac{1}{n}\frac{\partial F}{\partial x} = \mu_n - \mu_p = 4S_v\frac{n}{n_s}(1-2x) \\
s &= -\frac{1}{n}\frac{\partial F}{\partial T} = 2a\left(\frac{n_s}{n}\right)^{2/3}T; \qquad \varepsilon = F + nTs
\end{aligned}
$$

Phase Coexistence

Negative pressure: matter is unstable to separating into two phases of different densities (and possibly proton fractions). Physically, this represents coexistence of nuclei and vapor. Neglecting finite-size effect, bulk coexistence approximates the EOS at subnuclear densities.

Free Energy Minimization With Two Phases

$$\begin{aligned} F &= \epsilon - nTs = uF_I + (1-u)F_{II}, \\ n &= un_I + (1-u)n_{II}, \\ nY_e &= ux_I n_I + (1-u)x_{II} n_{II}. \end{aligned}$$

$$n_{II} = \frac{n - un_I}{1-u}, \qquad x_{II} = \frac{nY_e - un_I x_I}{n - un_I}$$

$$\begin{aligned} \frac{\partial F}{\partial n_I} &= u\frac{\partial F_I}{\partial n_I} + (1-u)\frac{\partial F_{II}}{\partial n_{II}}\left(\frac{-u}{1-u}\right) = u(\mu_{n,I} - \mu_{n,II}) \\ \frac{\partial F}{\partial x_I} &= u\frac{\partial F_I}{\partial x_I} + (1-u)\frac{\partial F_{II}}{\partial x_{II}}\left(\frac{-un_I}{n-un_I}\right) = un_I(\hat{\mu}_I - \hat{\mu}_{II}) \\ \frac{\partial F}{\partial u} &= F_I - F_{II} + (1-u)\left[\frac{\partial F_{II}}{\partial n_{II}}\left(\frac{n_{II}-n_I}{1-u}\right) + \frac{\partial F_{II}}{\partial x_{II}}\left(\frac{-un_I}{n-un_I}\right)\right] \end{aligned}$$

$$\Longrightarrow \mu_{nI} = \mu_{nII}\,, \qquad \mu_{pI} = \mu_{pII}\,, \qquad P_I = P_{II}$$

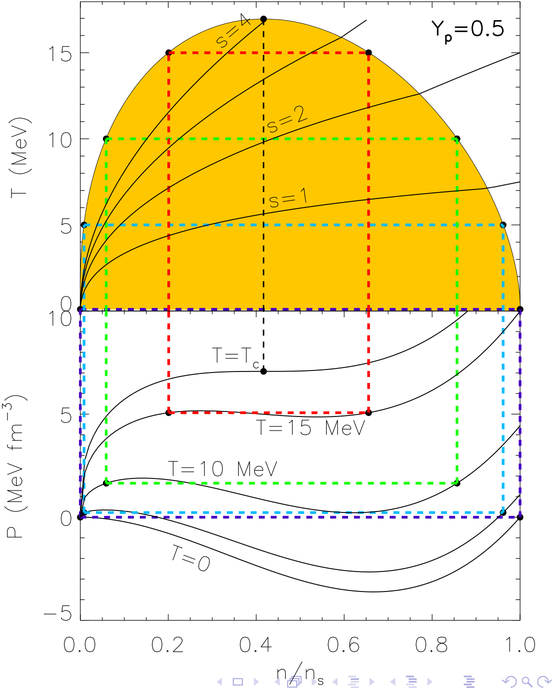
Critical Point
($Y_e = 0.5$)

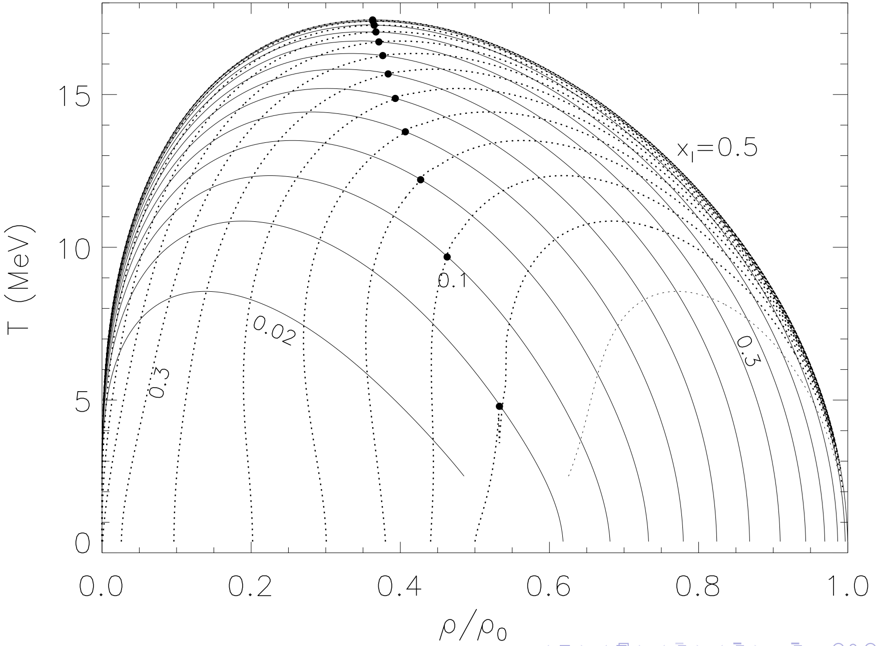
$$\left(\frac{\partial P}{\partial n}\right)_T = \left(\frac{\partial^2 P}{\partial n^2}\right)_T = 0$$

$$n_c = \frac{5}{12} n_s$$

$$T_c = \left(\frac{5}{12}\right)^{1/3} \left(\frac{5K}{32a}\right)^{1/2}$$

$$s_c = \left(\frac{12}{5}\right)^{1/3} \left(\frac{5Ka}{8}\right)^{1/2}$$





15
10
5
0
T (MeV)
x_l=0.5
0.1
0.02
0.3
0.3
0.0
0.2
0.4
0.6
0.8
1.0
ρ/ρ_0

Nuclear Symmetry Energy

Defined as the difference between energies of pure neutron matter ($x = 0$) and symmetric ($x = 1/2$) nuclear matter.

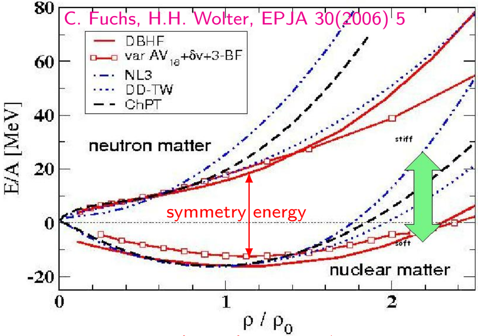
$$S(\rho) = E(\rho, x = 0) - E(\rho, x = 1/2)$$

Expanding around saturation density (ρ_s) and symmetric matter ($x = 1/2$)

$$E(\rho, x) = E(\rho, 1/2) + (1-2x)^2 S_2(\rho) + \ldots$$

$$S_2(\rho) = \mathbf{S_v} + \frac{\mathbf{L}}{3}\frac{\rho - \rho_s}{\rho_s} + \ldots$$

$$\mathbf{S_v} \simeq 31 \text{ MeV}, \mathbf{L} \simeq 50 \text{ MeV}$$



Connections to neutron matter:

$$E(\rho_s, 0) \approx S_v + E(\rho_s, 1/2) = S_v - B, \qquad p(\rho_s, 0) = L\rho_s/3$$

Neutron star matter (in beta equilibrium):

$$\frac{\partial(E + E_e)}{\partial x} = 0, \quad p(\rho_s, x_\beta) \simeq \frac{L\rho_s}{3}\left[1 - \left(\frac{LS_v}{\hbar c}\right)^3 \frac{4 - 3S_v/L}{3\pi^2\rho_s}\right]$$

Nuclear Mass Formula

Bethe-Weizsäcker (neglecting pairing and shell effects)

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2.$$

Myers & Swiatecki introduced the surface asymmetry term:

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2 - S_s I^2 A^{2/3}.$$

$a_v = B$, $a_s \simeq 18$ MeV, $a_C \simeq 0.75$ MeV, $S_s \simeq 40$ MeV, $I = (N-Z)/A$

Optimum Nucleus

$$\begin{aligned}
\frac{\partial E/A}{\partial A} &= -\frac{a_s - S_s I^2}{3} A^{-4/3} + \frac{a_C}{6} A^{-1/3}(1-I)^2 = 0 \\
\frac{\partial E/A}{\partial I} &= 2S_v I - 2S_s A^{-1/3} I - \frac{a_C}{2} A^{2/3}(1-I) = 0 \\
A &= \frac{2(a_s - S_s I^2)}{a_C(1-I)^2} \simeq \frac{45}{(1-I)^2} \simeq 60 \\
I &= \frac{a_C}{4S_v A^{-2/3} - 4S_s/A + a_C} \simeq \frac{1}{8}
\end{aligned}$$

Valley of Beta Stability: $Z \simeq \frac{A}{2} - \frac{a_C A^{5/3}}{8(S_v - S_s A^{-1/3})}$

Liquid Droplet Model

$$\begin{aligned}\mathcal{H} &= \mathcal{H}_B(n,\alpha) + n_p V_C/2 + (Q/n)\left(n'\right)^2, \\ \mathcal{H}_B(n,\alpha) &= -Bn + n(K/18)(n/n_s - 1)^2 + S_2(n)\alpha^2/n.\end{aligned}$$

$$n = n_n + n_p, \qquad \alpha = n_n - n_p; \qquad V_C = \frac{Ze^2}{R}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right)$$

Extremize total energy for fixed $A = \int n d^3r$, $\quad N - Z = \int \alpha d^3r$:

$$\frac{\delta}{\delta n}[\mathcal{H} - \mu n] = 0, \qquad \frac{\delta}{\delta \alpha}[\mathcal{H} - \bar{\mu}\alpha] = 0.$$

$$\begin{aligned}2\frac{d}{dr}\left[\frac{Q}{n}n'\right] - \frac{\partial(Q/n)}{\partial n}\left(n'\right)^2 &= \frac{\partial[\mathcal{H}_B + n_p V_c/2]}{\partial n} - \mu, \\ \mu \simeq 3Ze^2/(8R) - B, \quad Q(n')^2 &\simeq n^2(n/n_s - 1)^2, \quad n \simeq n_s/\left(1 + e^{x-y}\right), \\ r = ax, \qquad a &= \sqrt{18Q/K}, \qquad A \simeq 4\pi n_s (ay)^3/3.\end{aligned}$$

$$\begin{aligned}0 &= \frac{\partial[\mathcal{H}_B + n_p V_C/2]}{\partial \alpha} - \bar{\mu} = 2S_2\alpha/n - V_C/2 - \bar{\mu}, \\ \alpha &= \frac{n}{S_2}\left[\frac{S_v I}{H_0} + \frac{Ze^2}{8R}\left(\frac{H_2}{H_0} - \frac{r^2}{R^2}\right)\right], \qquad H_i = \int \frac{S_v}{S_2}\left(\frac{r}{R}\right)^i \frac{nd^3r}{A}.\end{aligned}$$

Liquid Droplet Model Continued

Parameters fixed by experiment: $K \simeq 240$ MeV

$$t_{90-10} = a\int_{0.1}^{0.9}\frac{dn}{n'} = 4a\ln 3 \simeq 2.3~\mathrm{fm},$$

which gives $a = 0.523$ fm, $Q = (K/18)[t_{90-10}/(4\ln 3)]^2 \simeq 3.65$ MeV fm^2. As a check, surface tension

$$\begin{aligned}\sigma_o &= \int_{-\infty}^{\infty}[\mathcal{H}-\mu n]dz = 2Q\int_{-\infty}^{\infty}\frac{(n')^2}{n}dz - \frac{Ze^2}{8R}\int_{-\infty}^{\infty}ndz \\ &\simeq \frac{Qn_s}{a} - \frac{Ze^2 n_s a}{8R^2} \simeq \left(1.17 - 0.007Z^{1/3}\right)~\mathrm{MeV~fm}^{-2}.\end{aligned}$$

Then surface energy parameter is $a_s = 4\pi r_o^2\sigma_0 \simeq 19.2$ MeV, which matches expectations from liquid droplet fits to nuclear masses.

If symmetry energy can be expanded $S_2(n) = S_v[\sum_i b_i(n/n_s)^i]^{-1}$,

$$\begin{aligned}H_i &= \frac{4\pi n_s(ay)^{3+i}}{(3+i)AR^i}\left[1-\frac{(3+i)\mathcal{T}}{y}+\cdots\right] = 3\left[\frac{1}{3+i}-\frac{\mathcal{T}}{y}+\cdots\right], \\ \mathcal{T} &= b_1 + \frac{3}{2}b_2 + \frac{11}{6}b_3 + \frac{25}{12}b_4 + \cdots, \qquad \sum_i b_i = 1.\end{aligned}$$

The total symmetry and Coulomb energies are

$$
\begin{aligned}
E_{sym} + E_C &= \int S_2 \frac{\alpha^2}{n} d^3r + \frac{1}{2}\int n_p V_C d^3 r \\
&= A I^2 \frac{S_v}{H_0} + \frac{3Z^2e^2}{5R}\left[1 + \frac{AI}{8Z}\left(1 - \frac{5H_2}{3H_0}\right)\right] \\
&= \frac{AI^2 S_v}{1 + S_s A^{-1/3}/S_v} + \frac{3Z^2e^2}{5R}\left[1 - \frac{AI}{12Z}\frac{S_s}{S_v A^{1/3} + S_s}\right]
\end{aligned}
$$

if we identify $3\mathcal{T}/y = 3a\mathcal{T}/R = -S_s A^{-1/3}/S_v$.
Static dipole polarizability is $\alpha_D = \int z\alpha_d d^3r/(2\epsilon)$, where α_d is the value of α found by extremizing $\delta\left(\int \mathcal{H} d^3r - \epsilon\int z\alpha d^3r\right)/\delta\alpha = 0$.

$$
\alpha_d = n\frac{\epsilon z + V_C/2}{2S_2}, \qquad \alpha_D = \frac{H_2 A R^2}{12 S_v} \simeq \frac{AR^2}{20S_v}\left(1 + \frac{5}{3}\frac{S_s A^{-1/3}}{S_v}\right) = 4m_{-1}.
$$

Mean square radii

$$
r^2_{n,p} = \frac{1}{2(N,Z)}\int (n \pm \alpha) r^2 d^3r = \frac{R^2}{1 \pm I}\left[\frac{3}{5} \pm I\frac{H_2}{H_0} \pm \frac{Ze^2}{8RS_v}\left(\frac{H_2^2}{H_0} - H_4\right)\right],
$$

$$
r_{np} = \frac{2r_o(1-I^2)^{-1/2}}{3(1 + S_s A^{-1/3}/S_v)}\sqrt{\frac{3}{5}}\left[I\frac{S_s}{S_v} - \frac{3Ze^2}{140 S_v r_o}\left(1 + \frac{10}{3}\frac{S_s A^{-1/3}}{S_v}\right)\right].
$$

Correlations from Experiments

The three relations for E_{sym}, α_D, and r_{np} can be combined with experimental data to give information about S_s and S_v and about L and S_v if we can relate S_s to L and S_v. This is done using $\mathcal{T}$.

$$\mathcal{T} = -\frac{r_o}{3a}\frac{S_s}{S_v} = b_1 + \frac{3}{2}b_2 + \frac{11}{6}b_3 + \cdots, \quad \frac{S_v}{S_2} = \sum_i b_i \left(\frac{n}{n_s}\right)^i, \quad \sum_i b_i = 1.$$

If $S_2 = S_v + (L/3)(n/n_s - 1)$, then $S_s \simeq La/r_o$.
If $S_2 = S_v + (L/3)(n/n_s - 1) + (K_{\rm sym}/18)(n/n_s - 1)^2$, and $K_{\rm sym} = 6L - 18S_v$ so that $S_2(0) = 0$, then

$$\frac{S_s}{S_v} \simeq \frac{3a}{2r_o}\left[1 + \frac{L}{3S_v} + \left(\frac{L}{3S_v}\right)^2\right].$$

Experiments can thus give a specific combination of values for S_v and L, i.e., a correlation in $S_v - L$ space.

- Masses: $d(S_s/S_v)/dS_v \sim 0.26$; $dL/dS_v \sim 19$
- Dipole polarizability: $d(S_s/S_v)/dS_v \sim 0.17$; $dL/dS_v \sim 14$
- Neutron skin thickness: $d(S_s/S_v)/dS_v \sim -0.08$; $dL/dS_v \sim -6$

Nuclear Binding Energies

$E_{sym}(N,Z) = I^2(S_v A - S_s A^{2/3})$

$\chi^2 = \sum_i (E_{ex,i} - E_{sym,i})^2/\mathcal{N}$

$\chi_{vv} = \frac{2}{\mathcal{N}} \sum_i I_i^4 A_i^2$

$\chi_{ss} = \frac{2}{\mathcal{N}} \sum_i I_i^4 A_i^{4/3}$

$\chi_{vs} = \frac{2}{\mathcal{N}} \sum_i I_i^4 A_i^{5/3}$

$\sigma_{S_v} = \sqrt{\frac{2\chi_{ss}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}}$

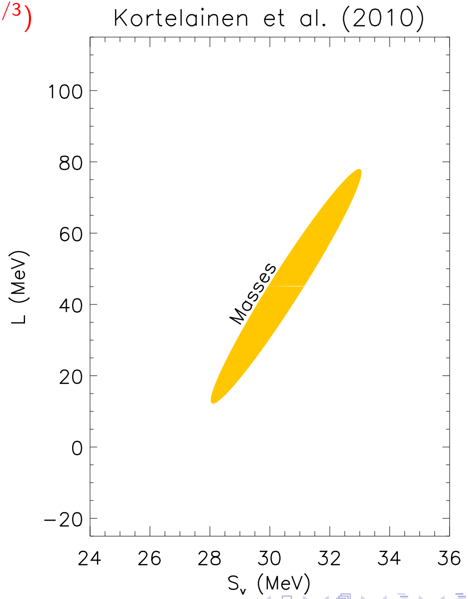
$\sigma_{S_s} = \sqrt{\frac{2\chi_{vv}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}}$

$\alpha = \frac{1}{2} \tan^{-1} \frac{2\chi_{vs}}{\chi_{vv} - \chi_{ss}}$

$r_{vs} = -\frac{\chi_{vs}}{\sqrt{\chi_{vv}\chi_{ss}}}$

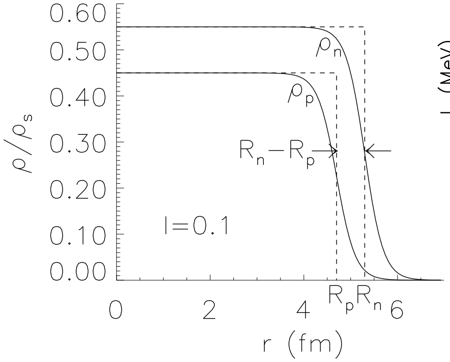
Liquid Droplet Model

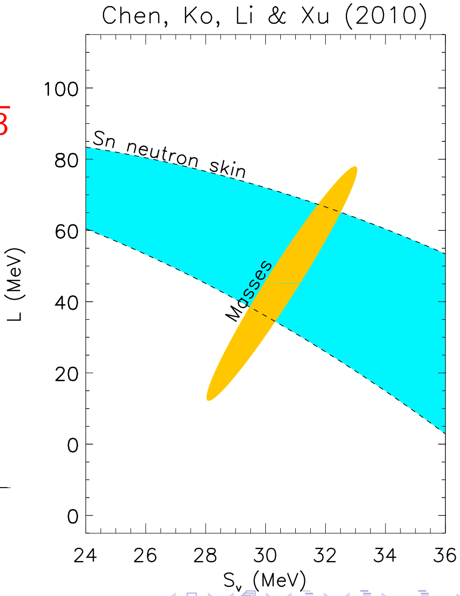
$E_{sym}(N,Z) = \frac{S_v A I^2}{1 + (S_s/S_v) A^{-1/3}}$



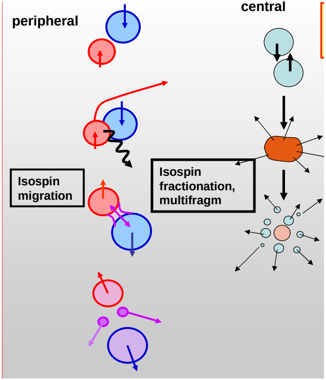
Neutron Skin Thickness

$$\frac{r_{np}}{r_o} \simeq \sqrt{\frac{4}{15}} \frac{S_s I}{S_v + S_s A^{-1/3}}$$

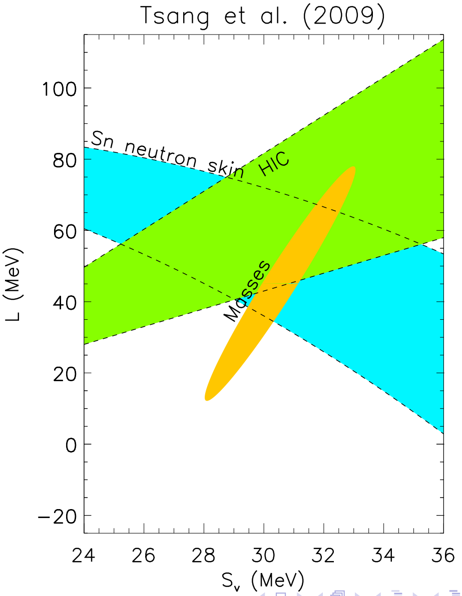




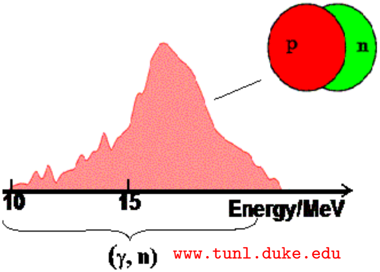
Heavy Ion Collisions



Wolter, NuSYM11

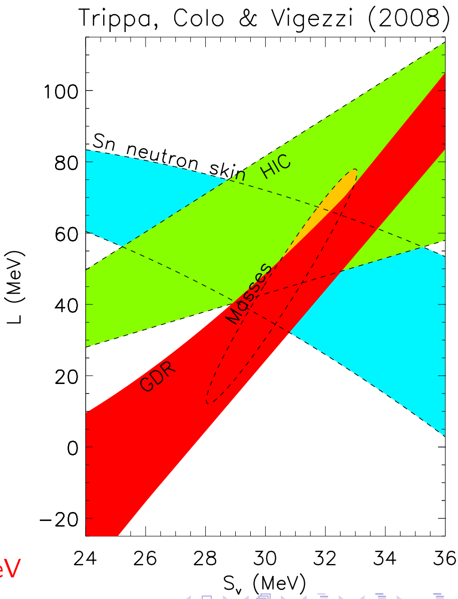


Giant Dipole Resonances



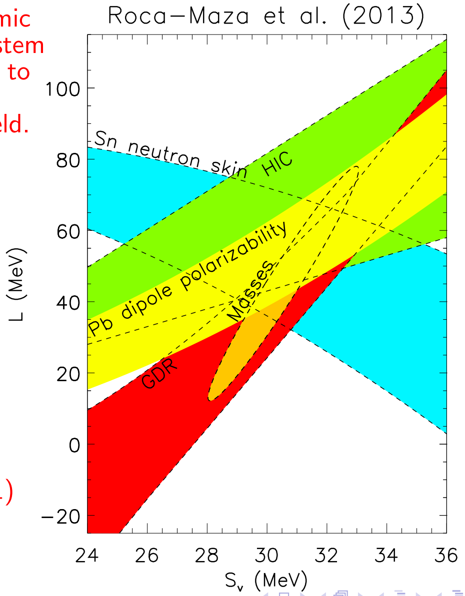
$$E_{-1} \propto \sqrt{\frac{S_v}{1+\frac{5S_s}{3S_v}A^{-1/3}}}$$

$23.3\ \mathrm{MeV} < S_2(0.1\ \mathrm{fm}^{-3}) < 24.9\ \mathrm{MeV}$



Dipole Polarizability

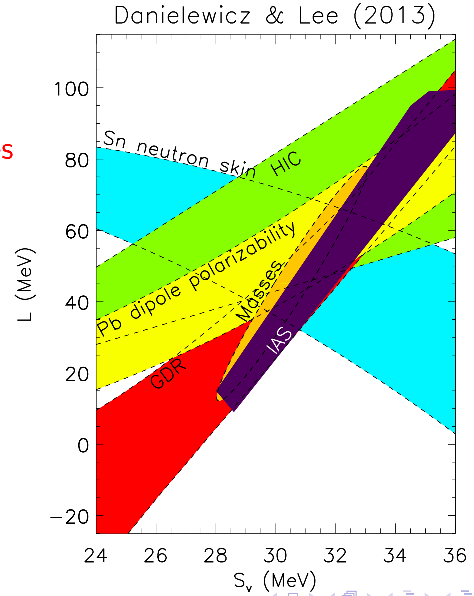
The linear response, or dynamic polarizability, of a nuclear system excited from its ground state to an excited state, due to an external oscillating dipolar field.



Data from Tamii et al. (2011)

Nuclear Experimental Constraints

Isobaric Analog States

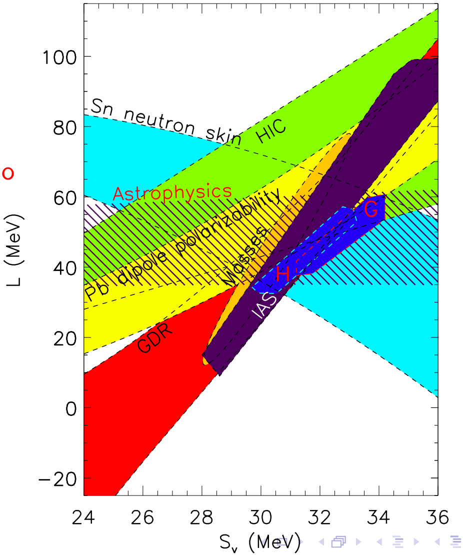


Theoretical Neutron Matter Calculations

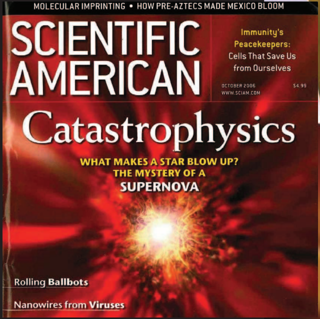
H&S: Chiral Lagrangian

GC&R: Quantum Monte Carlo

Gandolfi, Carlson & Reddy (2011);
Hebeler & Schwenk (2011)



Supernova!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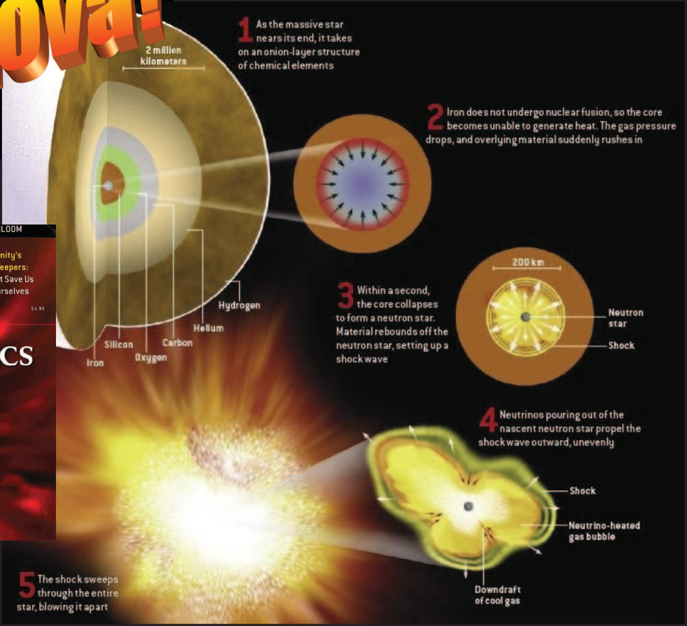

2 million kilometers
1 As the massive star nears its end, it takes on an onion-layer structure of chemical elements
Iron
Silicon
Oxygen
Carbon
Helium
Hydrogen
2 Iron does not undergo nuclear fusion, so the core becomes unable to generate heat. The gas pressure drops, and overlying material suddenly rushes in
200 km
3 Within a second, the core collapses to form a neutron star. Material rebounds off the neutron star, setting up a shock wave
Neutron star
Shock
4 Neutrinos pouring out of the nascent neutron star propel the shock wave outward, unevenly
Shock
Neutrino-heated gas bubble
Downdraft of cool gas
5 The shock sweeps through the entire star, blowing it apart

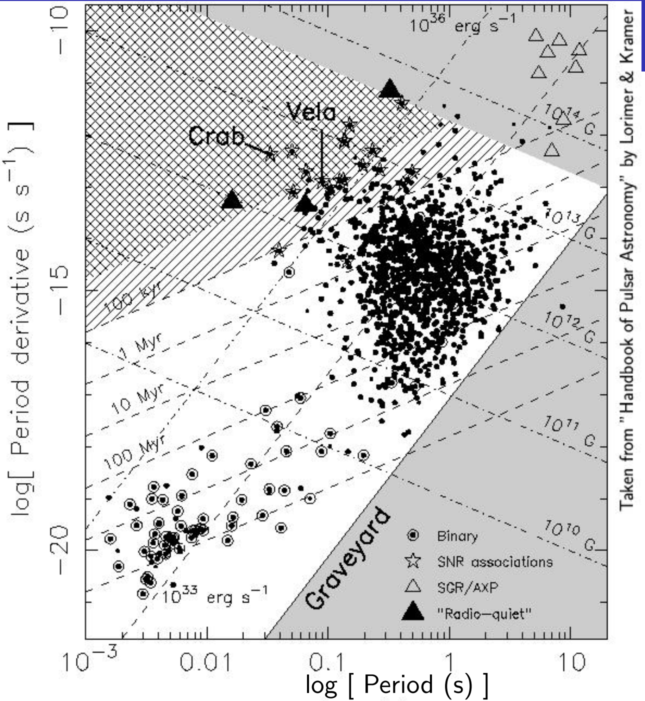
$P - \dot{P}$ Diagram

$$\tau_c = \frac{P}{2\dot{P}}$$

$$B \simeq 3 \cdot 10^{19} \sqrt{P\dot{P}} \text{ G}$$

$$-\dot{E} \simeq 10^{47} \frac{\dot{P}}{P^3} \text{ erg/s}$$

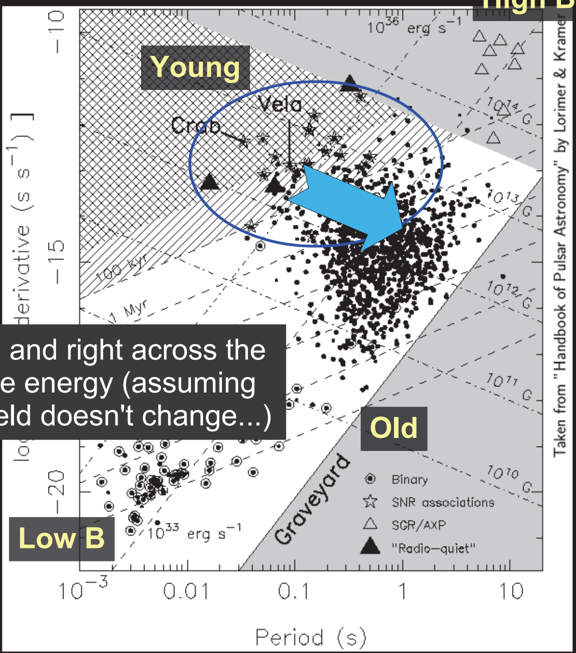
P in seconds



Pulsar Flavors

Young PSRs

(high B, fast spin, very energetic)



Pulsars move down and right across the diagram as they lose energy (assuming that the magnetic field doesn't change...)

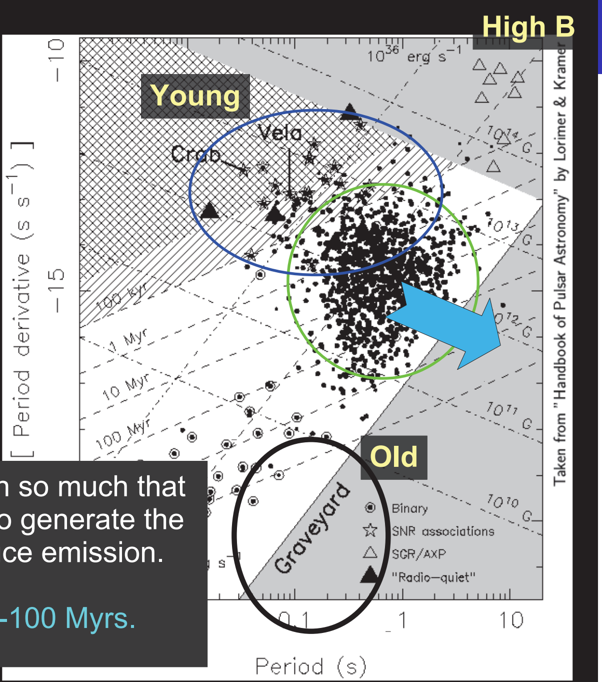
Pulsar Flavors

Young PSRs

(high B, fast spin, very energetic)

Normal PSRs

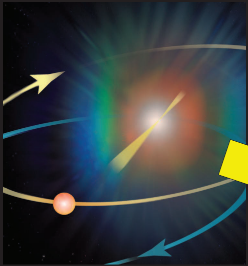
(average B, slow spin)



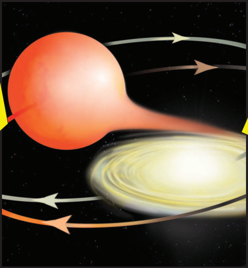
Eventually they slow down so much that there is not enough spin to generate the electric fields which produce emission.

Their lifetimes are 10-100 Myrs.

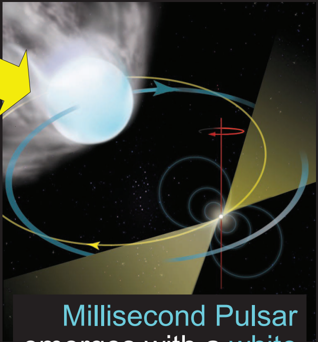
Millisecond Pulsars: via “Recycling”



Supernova produces a neutron star



Red Giant transfers matter to neutron star



Millisecond Pulsar emerges with a white dwarf companion

Alpar et al 1982
Radhakrishnan & Srinivasan 1984

Picture credits: Bill Saxton, NRAO/AUI/NSF

Pulsar Flavors

Young PSRs

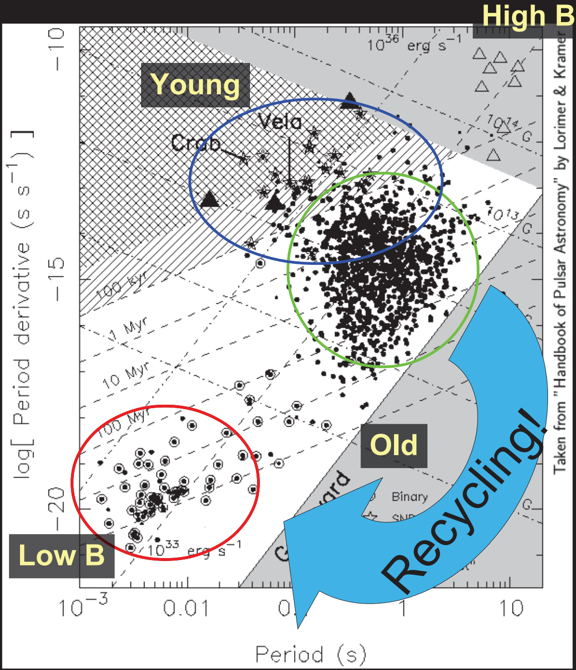
(high B, fast spin, very energetic)

Normal PSRs

(average B, slow spin)

Millisecond PSRs

(low B, very fast, very old, very stable spin, best for basic physics tests)



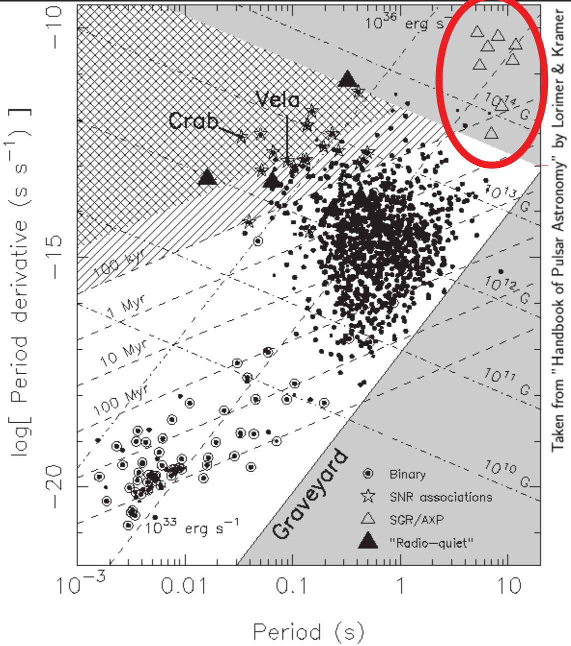
What's a Magnetar?

Neutron stars with extremely strong magnetic fields:

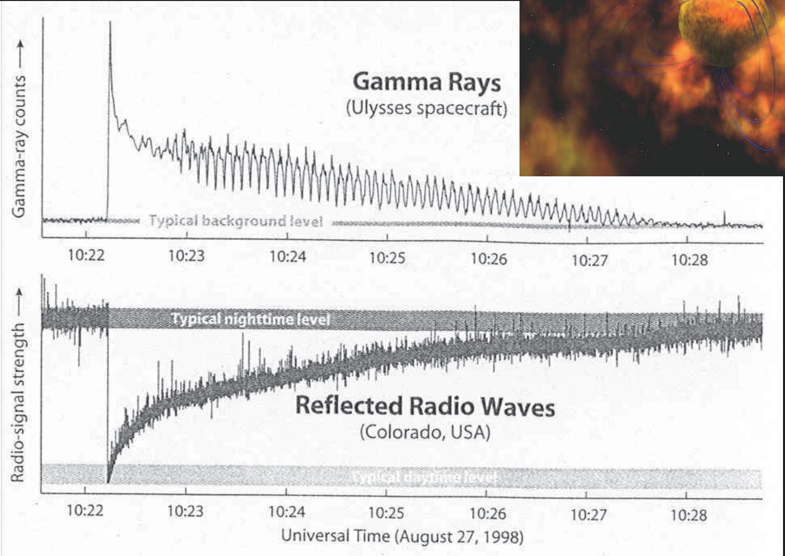
$10^{14\text{-}15}$ Gauss

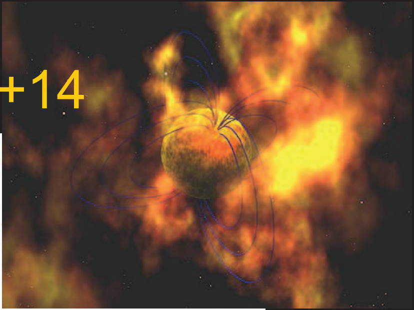
(~1000x stronger than normal PSRs)

Powered by decay of magnetic field, not rotation!



Giant X-ray Flares: Magnetar SGR 1900+14





Pulsars are Precise Clocks

PSR J0437-4715

At 00:00 UT Jan 18 2011:

P = 5.7574519420243 ms
+/- 0.0000000000001ms

The last digit changes by 1 every half hour!

This digit changes by 1 every 500 years!

This extreme precision is what allows us to ***use pulsars as tools*** to do unique physics!

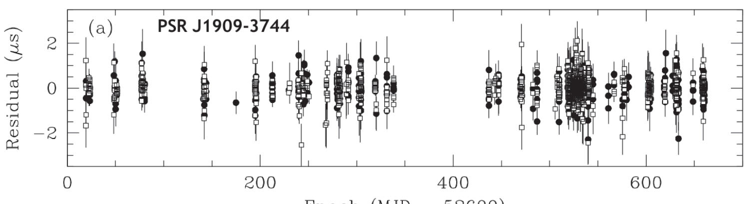
Pulsar Timing: *Pulse Phase Tracking*

Unambiguously account for every rotation of a pulsar over years

Model (prediction)

Measurement (TOAs: Times of Arrival)

Measurement - Model = Timing Residuals

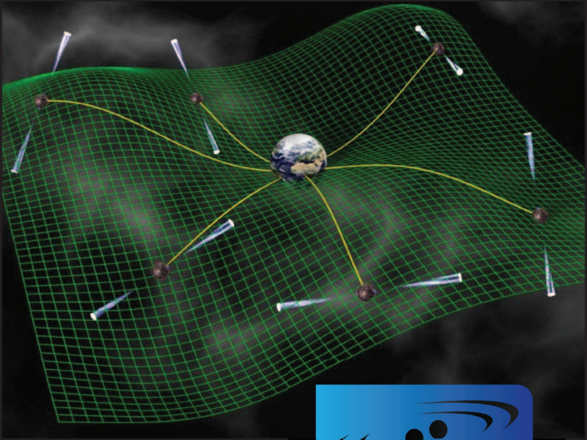


(From Jacoby et al. 2005, ApJL, 629, 113)

200ns RMS over 2 yrs

Gravitational Wave Detection with a Pulsar Timing Array

- Looking for nHz freq gravitational waves from super massive black hole binaries
- Need good MSPs
- Significance scales directly with the number of MSPs being timed.
- Must time the pulsars for 5-10 years at a precision of ~100 nanosec!
- North American (**NANOGrav**), European (EPTA), and Australian (PPTA) efforts



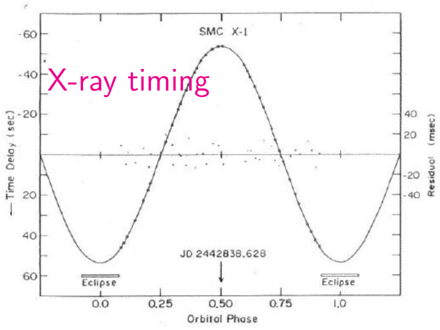
Binary Mass Measurements

Mass function

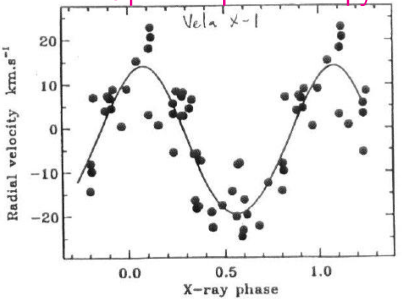
$$f(M_1) = \frac{P(v_2 \sin i)^3}{2\pi G} = \frac{(M_1 \sin i)^3}{(M_1+M_2)^2} > M_1$$

$$f(M_2) = \frac{P(v_1 \sin i)^3}{2\pi G} = \frac{(M_2 \sin i)^3}{(M_1+M_2)^2} > M_2$$

In an X-ray binary, $v_{optical}$ has the largest uncertainties. In some cases $\sin i \sim 1$ if eclipses are observed. If no eclipses observed, limits to i can be made based on the estimated radius of the optical star.



Optical spectroscopy



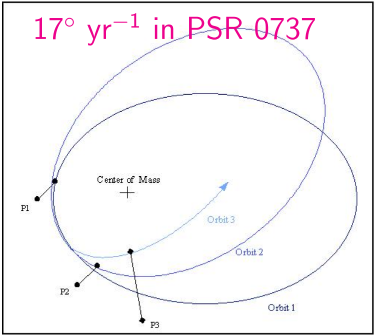
Pulsar Mass Measurements

Mass function for pulsar precisely obtained.

It is also possible in some cases to obtain the rate of periastron advance and the Einstein gravitational redshift + time dilation term:

$$\dot{\omega} = \frac{3}{1-e^2}\left(\frac{2\pi}{P}\right)^{5/3}\left(\frac{GM}{c^2}\right)^{2/3}$$

$$\gamma = \left(\frac{P}{2\pi}\right)^{1/3} e M_2 (2M_2 + M_1)\left(\frac{G}{M^2c^2}\right)^{2/3}$$

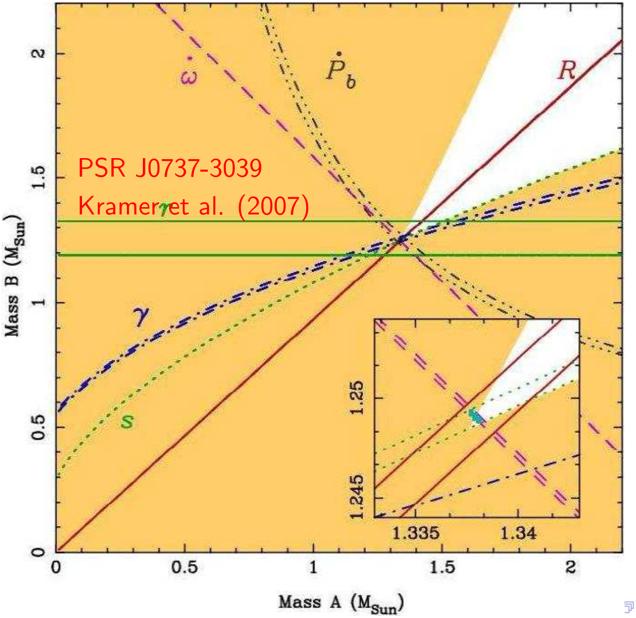


Gravitational radiation leads to orbit decay:

$$\dot{P} = -\frac{192\pi}{5c^5}\left(\frac{2\pi G}{P}\right)^{5/3}(1-e^2)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)\frac{M_1 M_2}{M^{1/2}}$$

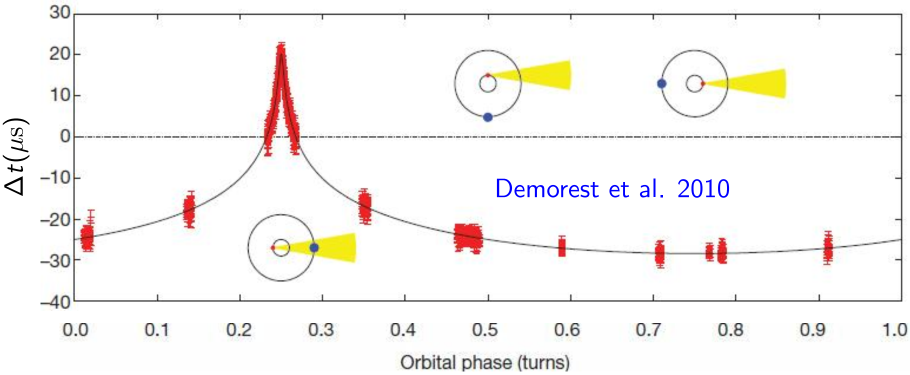
In some cases, can constrain Shapiro time delay, r is magnitude and $s = \sin i$ is shape parameter.

PSR J0737-3039



PSR J1614-2230

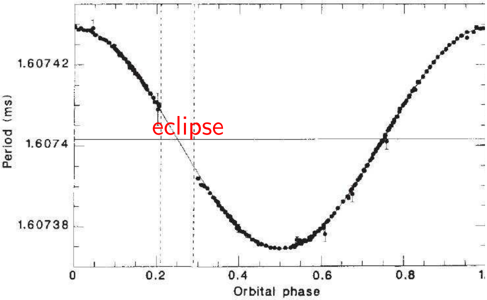
3.15 ms pulsar in 8.69d orbit with 0.5 $M_\odot$ white dwarf companion.
Shapiro delay tightly confines the edge-on inclination: $\sin i = 0.99984$
Pulsar mass is 1.97 ± 0.04 $M_\odot$
Distance > 1 kpc, $B \simeq 1.8 \times 10^8$ G

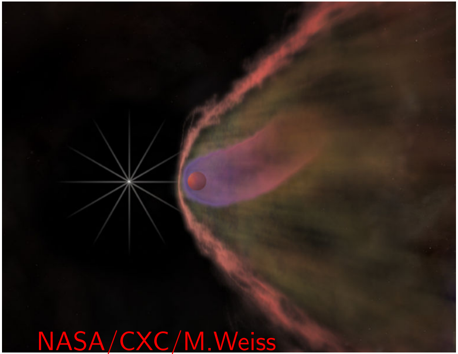


Black Widow Pulsar PSR B1957+20

1.6ms pulsar in circular 9.17h orbit with a $\sim 0.03\ M_\odot$ companion.
Pulsar is eclipsed for 50-60 minutes each orbit; eclipsing object has a volume much larger than the companion or its Roche lobe.
It is believed the companion is ablated by the pulsar leading to mass loss and an eclipsing plasma cloud. Companion nearly fills its Roche lobe.
Ablation by pulsar leads to eventual disappearance of companion.
The optical light curve does not represent the center of mass of the companion, but the motion of its irradiated hot spot.

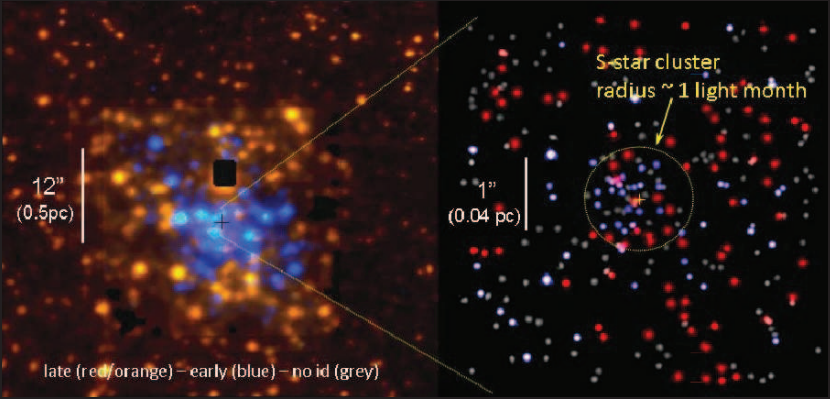
pulsar radial velocity





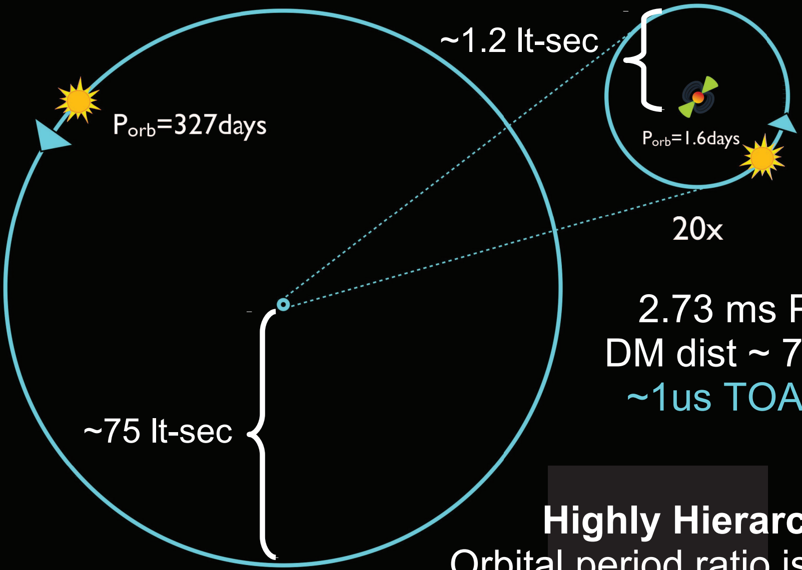
Pulsars around SgrA*?

- 100s of massive young stars within ~0.1pc
- 10s-100s of PSRs with orbits <100yrs? (e.g. Pfahl & Loeb 2004)
- PSR timing much more precise than IR imaging and astrometry



Genzel, Eisenhauer, Gillesen 2010

J0337+1715: Stellar Triple System



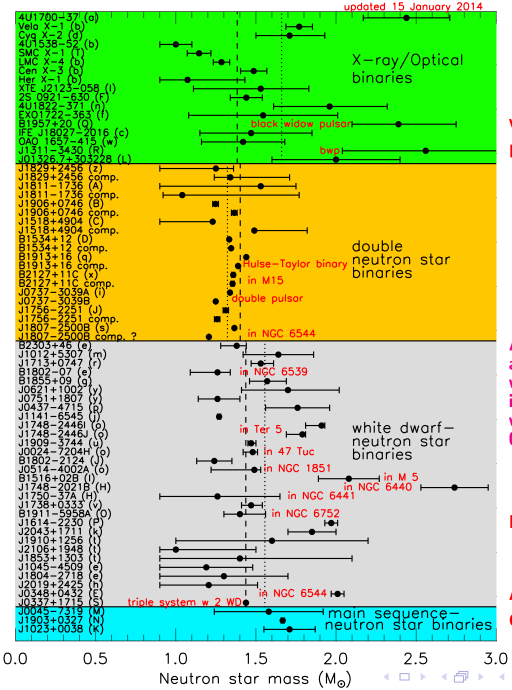
2.73 ms Pulsar
DM dist ~ 750 pc
~1us TOA / min

Highly Hierarchical:
Orbital period ratio is ~200

A fully solved system:

(thanks to Anne Archibald)

- Full three-body, high-precision model
- All masses and inclinations fully determined to high precision (10^{-4} for masses):
    - Mpsr = 1.442 Msun
    - Mc_inner = 0.198 Msun
    - Mc_outer = 0.411 Msun (another WD!)
  - Orbits inclinations are co-planar at 39.18(4) deg
  - Inner mass ratio perfectly matches optical value
  - Apsides are aligned (despite inner orbits e ~ 7×10^{-4}!)
- Osculating orbital elements are obvious



vanKerkwijk 2010

Romani et al. 2012

Although simple average mass of w.d. companions is 0.23 $M_\odot$ larger, weighted average is 0.04 $M_\odot$ smaller

Demorest et al. 2010

Antoniadis et al. 2013

Champion et al. 2008

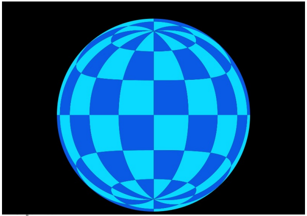
Radiation Radius

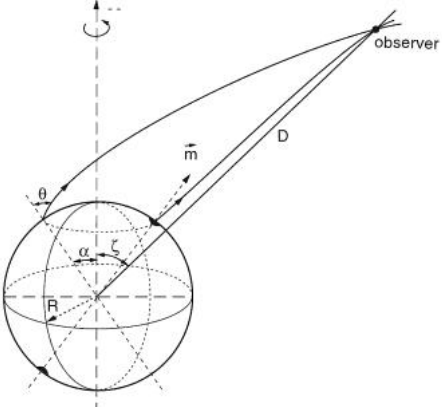
- The measurement of flux and temperature yields an apparent angular size (pseudo-BB):

$$\frac{R_\infty}{d} = \frac{R}{d}\frac{1}{\sqrt{1 - 2GM/Rc^2}}$$

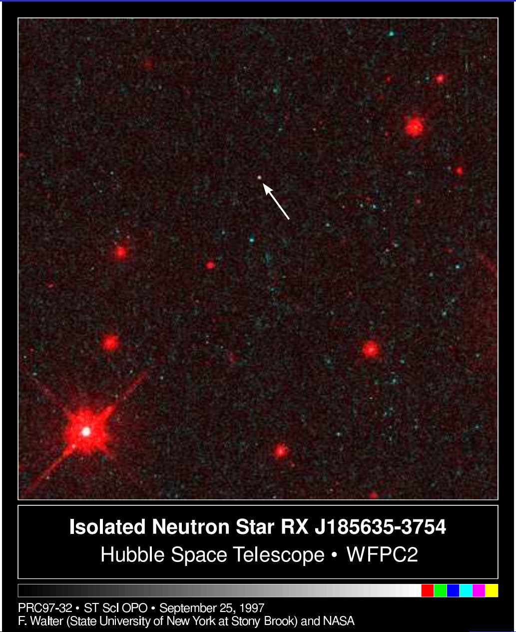
- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Nearby isolated neutron stars (parallax measurable)
- Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmosperes)
- Bursting sources in which Eddington flux is measured

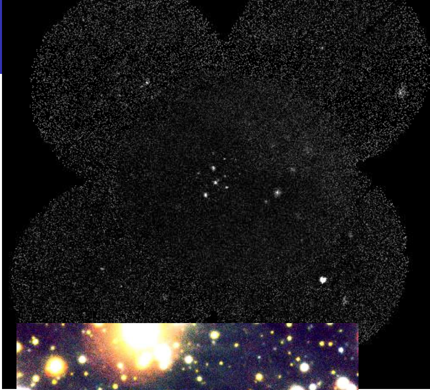
$$F_{Edd} = \frac{GMc}{\kappa D^2}\sqrt{1 - \frac{2GM}{R_{ph}c^2}}$$

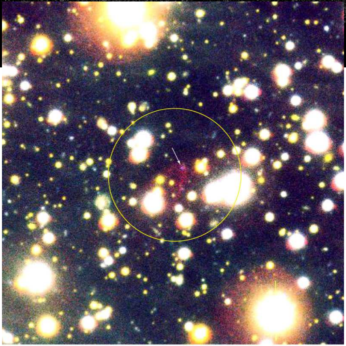




RX J1856-3754



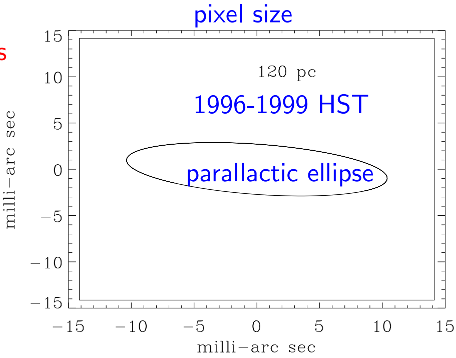




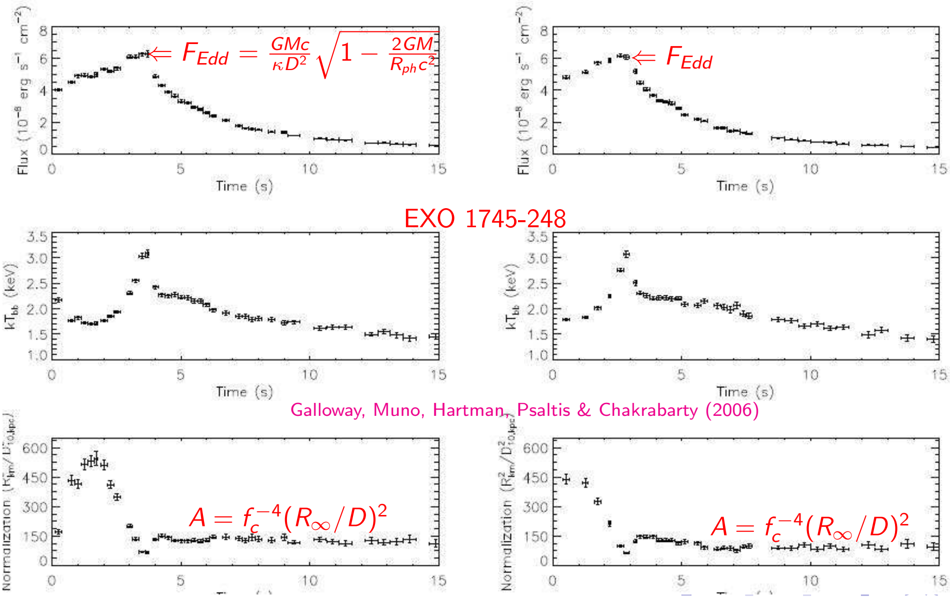
A Bowshock Nebula Near the Neutron Star RX J1856.5-3754 (Detail)
(VLT KUEYEN + FORS2)

Astrometry of RXJ 1856-3754

- Walter & Lattimer (2002) determined $D = 117 \pm 12$ pc and $v \simeq 190$ km/s from 1996-1999 HST Planetary Camera observations
- Star's age is probably 0.5 million years
- Walter, Eisenbeiβ, Lattimer, Kim, Hambaryan & Neuhäuser (2010) determined $D \simeq 115 \pm 8$ pc based on 2002-2004 HST Advanced Camera for Surveys observations (double the resolution)
- A magnetic hydrogen atmosphere model (Ho et al. 2007) suggests $M \simeq 1.29$ $M_\odot$ and $R \simeq 11.6$ km
- Redshift or gravity measurements, which would allow simultaneous M and R determinations, are not yet possible.



Photospheric Radius Expansion X-Ray Bursts



PRE Burst Models

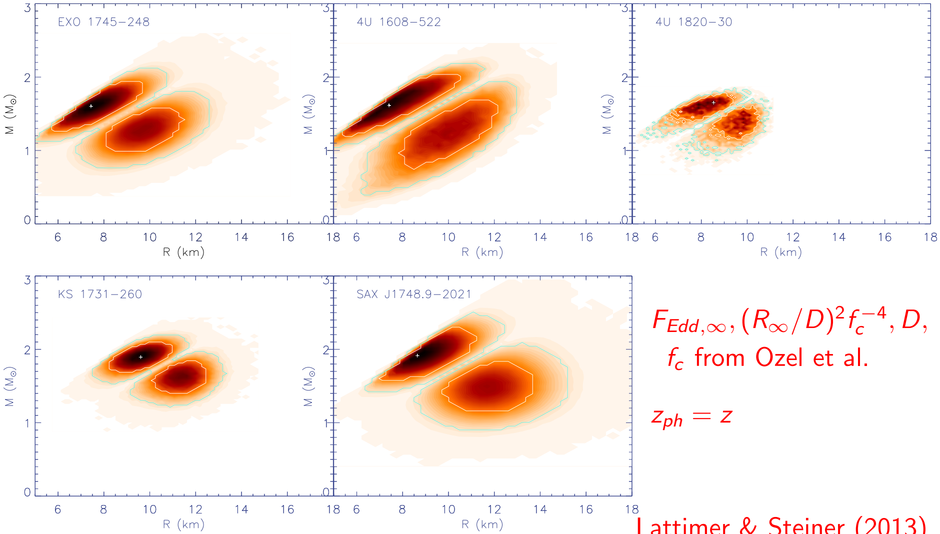
Ozel et al. $z_{\rm ph} = z$ $\beta = GM/Rc^2$ Steiner et al. $z_{\rm ph} << z$

$$
\begin{aligned}
F_{\rm Edd} &= \frac{GMc}{\kappa D}\sqrt{1-2\beta} \\
A &= \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4}\left(\frac{R_\infty}{D}\right)^2 \\
\alpha &= \frac{F_{\rm Edd}}{\sqrt{A}}\frac{\kappa D}{F_c^2 c^3} = \beta(1-2\beta) \\
\gamma &= \frac{A f_c^4 c^3}{\kappa F_{\rm Edd}} = \frac{R_\infty}{\alpha} \\
\beta &= \frac{1}{4} \pm \frac{1}{4}\sqrt{1-8\alpha} \\
\alpha &\le \frac{1}{8} \text{ required.}
\end{aligned}
$$

$$
\begin{aligned}
F_{\rm Edd} &= \frac{GMc}{\kappa D} \\
\alpha &= \beta\sqrt{1-2\beta} \\
\theta &= \cos^{-1}\left(1-54\alpha^2\right) \\
\beta &= \frac{1}{6}\left[1+\sqrt{3}\sin\left(\frac{\theta}{3}\right)\right. \\
&\quad \left. -\cos\left(\frac{\theta}{3}\right)\right] \\
\alpha &\le \sqrt{\frac{1}{27}} \simeq 0.192 \text{ required.}
\end{aligned}
$$

		α		
EXO 1745-248	4U 1608-522	4U 1820-30	KS 1731-260	SAX J1748.9-2021
0.188 ± 0.035	0.247 ± 0.058	0.235 ± 0.04	0.199 ± 0.032	0.177 ± 0.036

$M - R$ PRE Burst Estimates

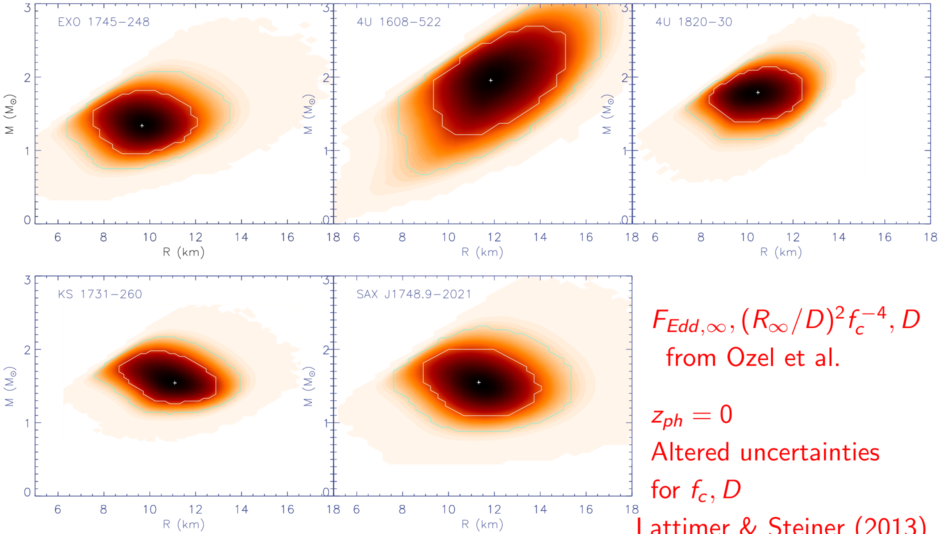


$F_{Edd,\infty}$, $(R_\infty/D)^2 f_c^{-4}$, D, f_c from Ozel et al.

$z_{ph} = z$

Lattimer & Steiner (2013)

$M - R$ PRE Burst Estimates



$F_{Edd,\infty}, (R_\infty/D)^2 f_c^{-4}, D$ from Ozel et al.

$z_{ph} = 0$

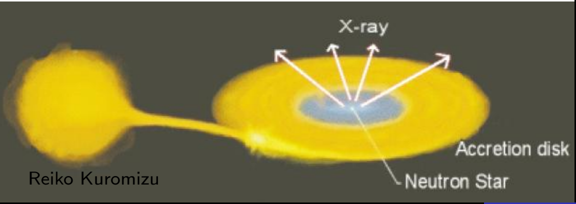
Altered uncertainties for f_c, D

Lattimer & Steiner (2013)

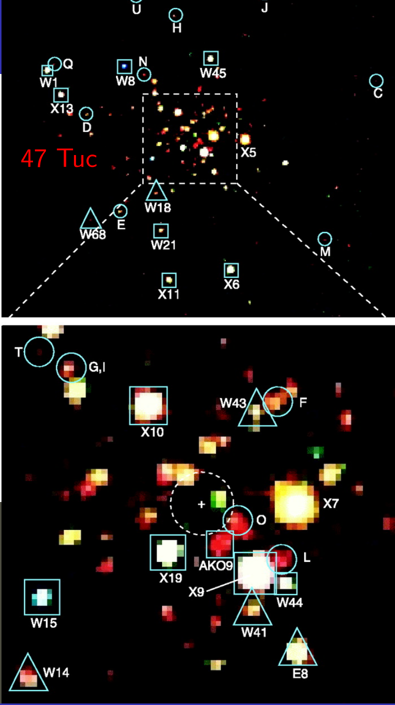
Quiescent Sources in Globulars

Hot neutron stars in globular clusters

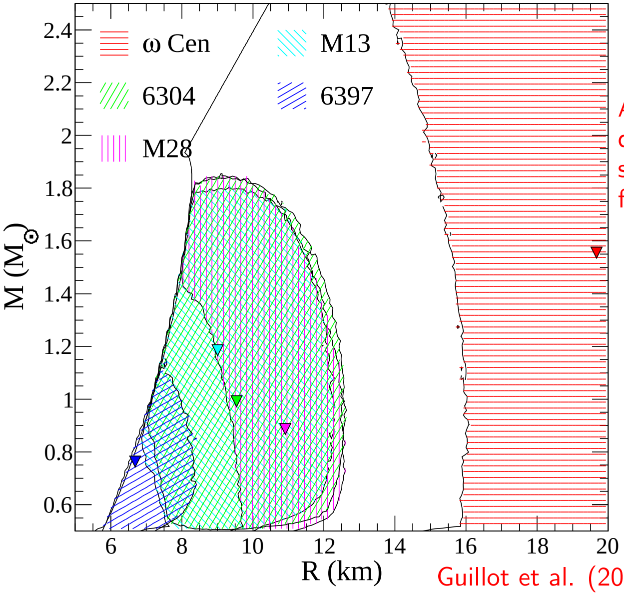
- Globular clusters evolve: more massive stars, including binaries, sink to center via long-range stellar encounters.
- Close binaries formed in encounters.
- Episodes of accretion in close binaries heats neutron stars: they are reborn.
- Following accretion, they become quiescent, low-mass X-ray sources.
- Accretion suppresses surface B fields.
- Atmospheric composition is H.



Reiko Kuromizu



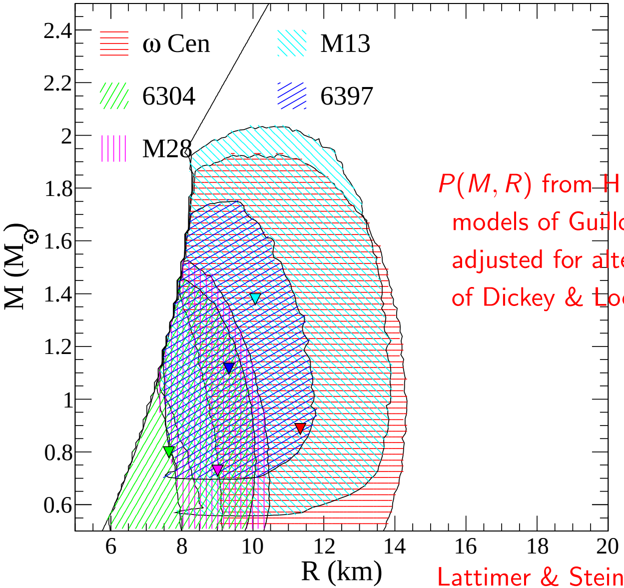
$M - R$ QLMXB Estimates



Absorption (N_H) determined self-consistently from spectra

Guillot et al. (2013)

$M-R$ QLMXB Estimates

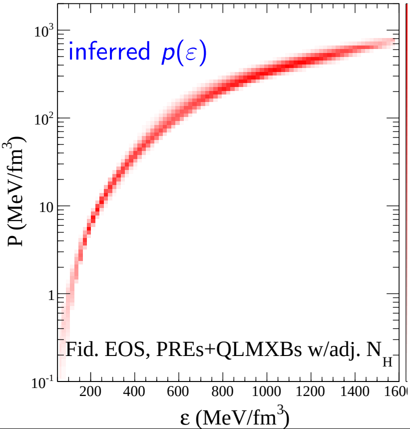


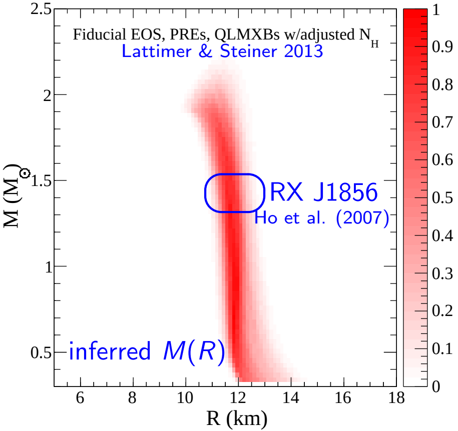
$P(M, R)$ from H atmosphere models of Guillot et al. (2013), adjusted for alternate N_H values of Dickey & Lockman (1990)

Lattimer & Steiner (2013)

Bayesian TOV Inversion

- $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- Polytropic EOS: $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1; $\varepsilon > \varepsilon_2$: n_2
- EOS parameters $K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$ uniformly distributed
- $M_{\max} \geq 1.97$ $M_\odot$, causality enforced
- All 10 stars equally weighted





Astronomy vs. Astronomy vs. Physics

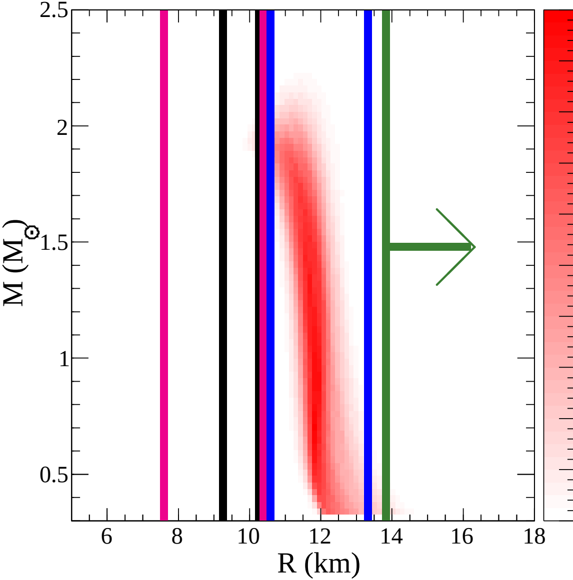
Ozel et al., PRE bursts $z_{\rm ph}$ z: $R = 9.74 \pm 0.50$ km.

Suleimanov et al., long PRE bursts: $R_{1.4} \gtrsim 13.9$ km

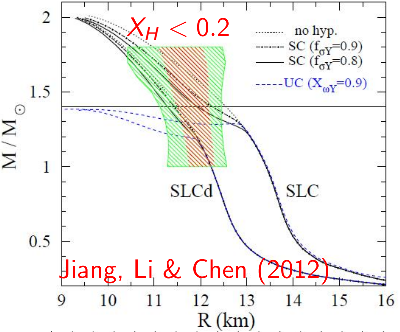
Guillot et al. (2013), all stars have the same radius, self N_H: $R = 9.1^{+1.3}_{-1.5}$ km.

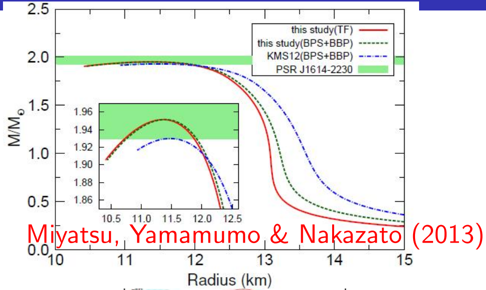
Lattimer & Steiner (2013), TOV, crust EOS, causality, maximum mass $> 2M_\odot$, $z_{\rm ph} = z$, alt N_H.

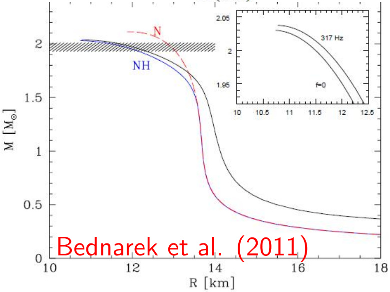
Lattimer & Lim (2013), nuclear experiments: 29 MeV $< S_v <$ 33 MeV, 40 MeV $< L <$ 65 MeV, $R_{1.4} = 12.0 \pm 1.4$ km.

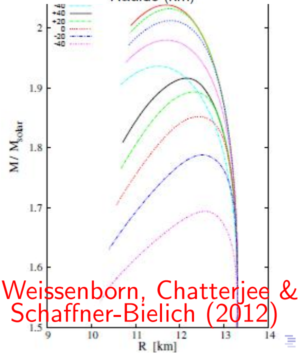


Can Hyperons Appear in Abundance in Neutron Stars?

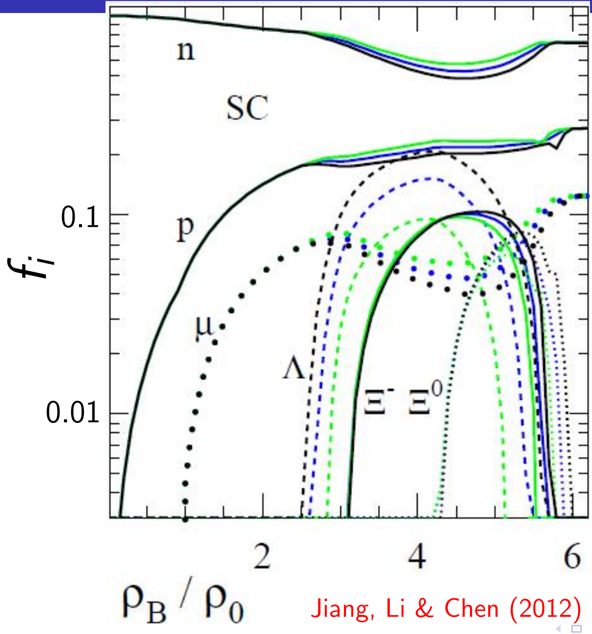






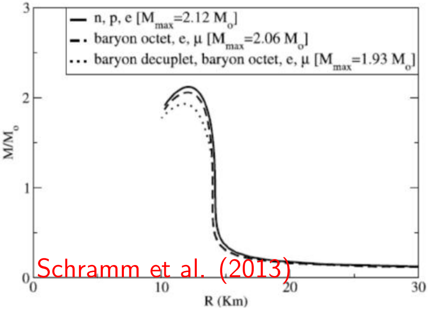


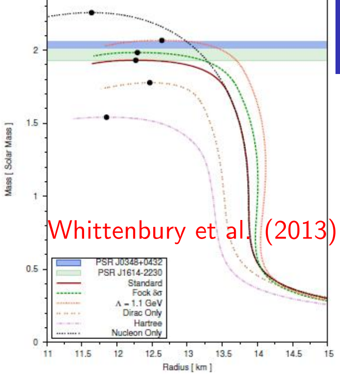
Hyperon Stars with Small Radii Have Few Hyperons

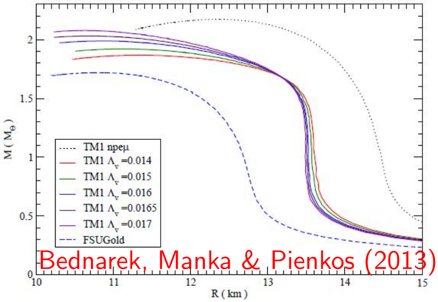


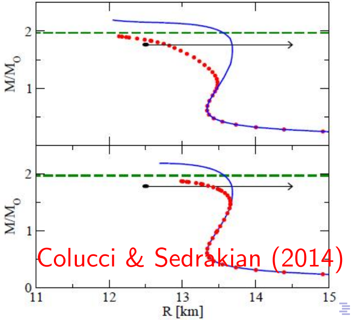
Jiang, Li & Chen (2012)

More Hyperon Stars

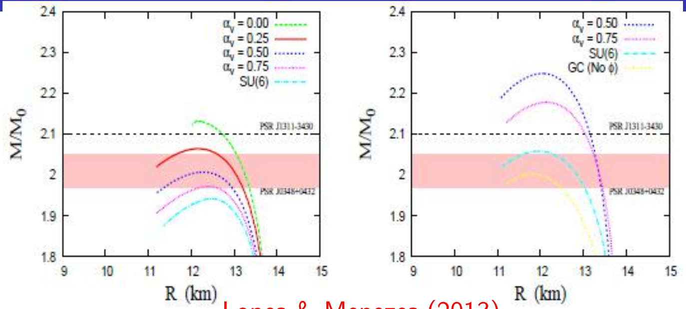




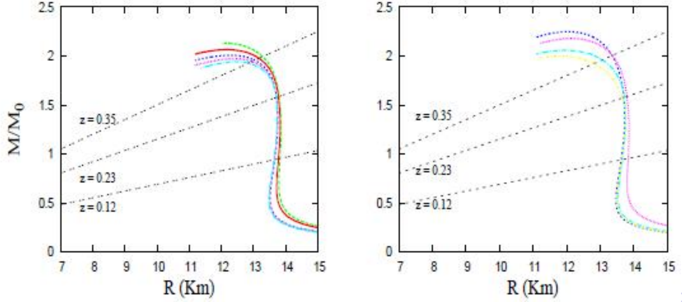




Still More Hyperon Stars

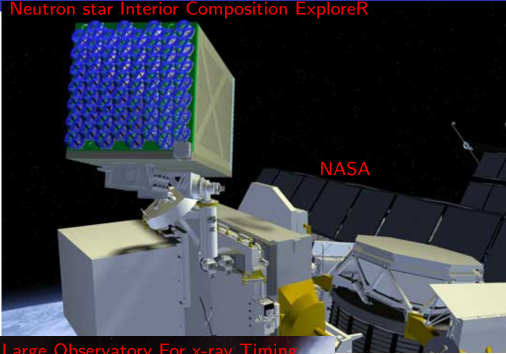


Lopes & Menezes (2013)



Additional Proposed Radius and Mass Constraints

- Pulse profiles
  Hot or cold regions on rotating neutron stars alter pulse shapes: NICER and LOFT will enable timing and spectroscopy of thermal and non-thermal emissions. Light curve modeling → M/R; phase-resolved spectroscopy → R.
- Moment of inertia
  Spin-orbit coupling of ultra-relativistic binary pulsars (e.g., PSR 0737+3039) vary i and contribute to $\dot{\omega}$: $I \propto MR^2$.
- Supernova neutrinos
  Millions of neutrinos detected from a Galactic supernova will measure BE$= m_B N - M$, $< E_\nu >$, τ_ν.
- QPOs from accreting sources
  ISCO and crustal oscillations



Constraints from Observations of Gravitational Radiation

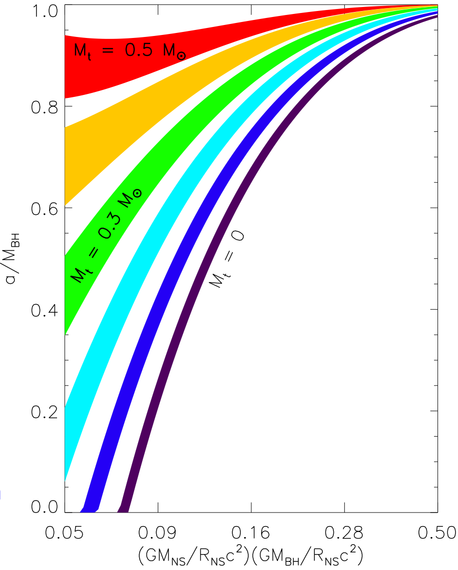
Mergers:

Chirp mass $\mathcal{M} = (M_1 M_2)^{3/5} M^{-1/5}$ and tidal deformability $\lambda \propto R^5$ (Love number) are potentially measurable during inspiral.

$\bar{\lambda} \equiv \lambda M^{-5}$ is related to $\bar{I} \equiv I M^{-3}$ by an EOS-independent relation (Yagi & Yunes 2013). Both $\bar{\lambda}$ and $\bar{I}$ are also related to M/R in a relatively EOS-independent way (Lattimer & Lim 2013).

- Neutron star - neutron star: $M_{\rm crit}$ for prompt black hole formation, $f_{\rm peak}$ depends on R.
- Black hole - neutron star: $f_{\rm tidal\ disruption}$ depends on $R, a, M_{\rm BH}$. Disc mass depends on $a/M_{\rm BH}$ and on $M_{\rm NS} M_{\rm BH} R^{-2}$.

Rotating neutron stars: r-modes



Urca Processes

Gamow & Schönberg proposed the direct Urca process: nucleons at the top of the Fermi sea beta decay.

$n \to p + e^- + \nu_e$,
$p \to n + e^+ + \bar{\nu}_e$

Energy conservation guaranteed by beta equilibrium

$\mu_n - \mu_p = \mu_e$

Momentum conservation requires $|k_{Fn}| \le |k_{Fp}| + |k_{Fe}|$.

Charge neutrality requires $k_{Fp} = k_{Fe}$, therefore $|k_{Fp}| \ge 2|k_{Fn}|$.

Degeneracy implies $n_i \propto k_{Fi}^3$, thus $x \ge x_{DU} = 1/9$.

With muons $(n > 2n_s), x_{DU} = \frac{2}{2+(1+2^{1/3})^3} \simeq 0.148$

If $x < x_{DU}$, bystander nucleons needed: modified Urca process.

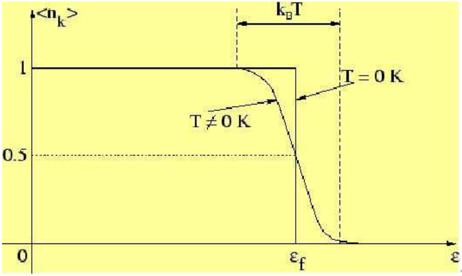
$(n,p) + n \to (n,p) + p + e^- + \nu_e$,
$(n,p) + p \to (n,p) + n + e^+ + \bar{\nu}_e$

Neutrino emissivities:

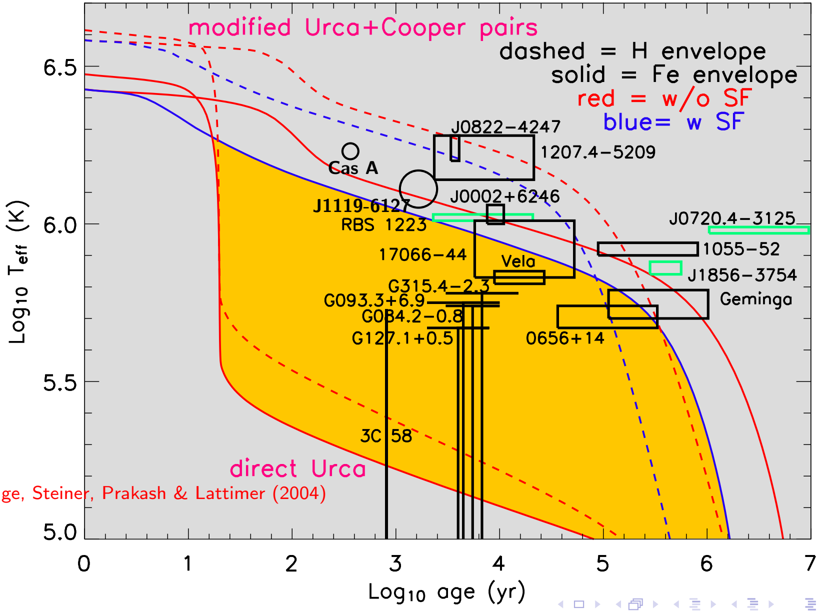
$\dot{\epsilon}_{MU} \simeq (T/\mu_n)^2 \, \dot{\epsilon}_{DU} \sim 10^{-6} \dot{\epsilon}_{DU}$.

Beta equilibrium composition:

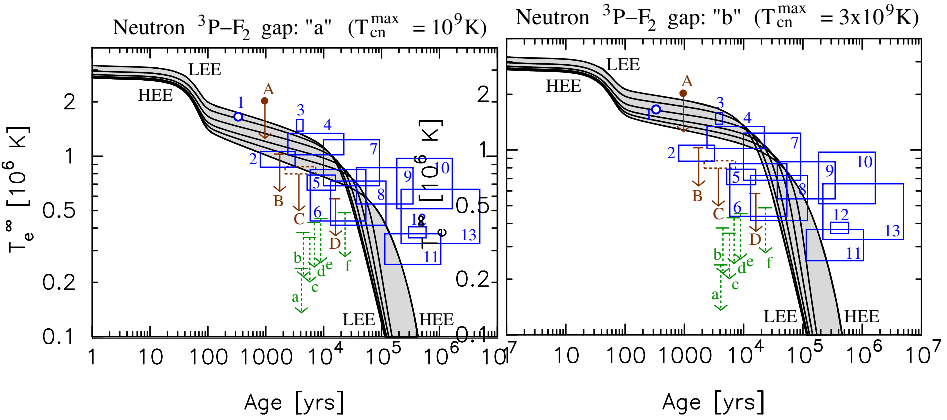
$$x_\beta \simeq (3\pi^2 n)^{-1} (4E_{sym}/\hbar c)^3 \simeq 0.04\,(n/n_s)^{0.5-2} .$$



Neutron Star Cooling



Minimal Cooling



In minimal cooling, it is supposed there is no rapid neutrino cooling due to direct Urca processes among nucleons, hyperons, Bose condensates or quarks.

Envelope and atmospheric variations lie within the shaded regions.

Stellar mass and radii have little effect. Any objects failing to match cooing curves are candidates for rapid cooling (possibly due to high stellar masses).

Transitory Rapid Cooling

MU emissivity: $\dot{\varepsilon}_{MU} \propto T^8$

PBF emissivity ($f \sim 10$):
$\dot{\varepsilon}_{PBF} \propto F(T)\ T^7 \propto T^8 \simeq f\dot{\varepsilon}_{MU}$

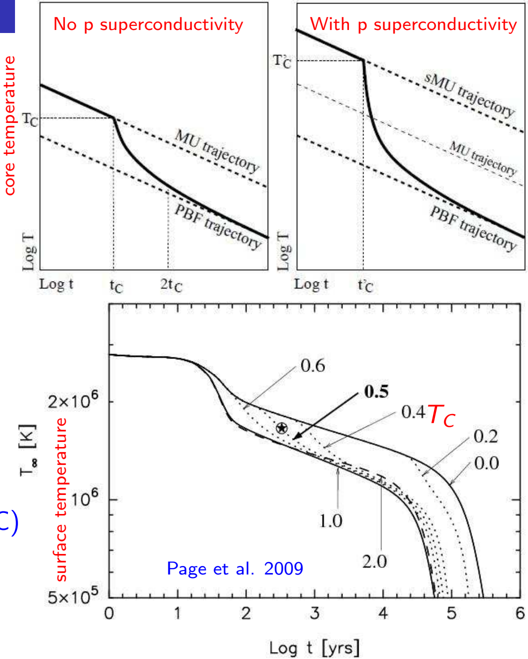
Specific heat: $C_V \propto T$

Neutrino dominated cooling:
$C_V dT/dt = -L_\nu$

$\implies\ T \propto (t/\tau)^{-1/6}$
$\tau_{PBF} = \tau_{MU}/f$

$(d \ln T/d \ln t)_{transitory}$
$\simeq (1-10)(d \ln T/d \ln t)_{MU}$
$\simeq (1-25)(d \ln T/d \ln t)_{MU}$ (p SC)

Very sensitive to n $^1S_0$ critical temperature (T_C) and existence of proton superconductivity



Cas A

Remnant of Type IIb (gravitational collapse, no H envelope) SN in 1680 (Flamsteed).

3.4 kpc distance

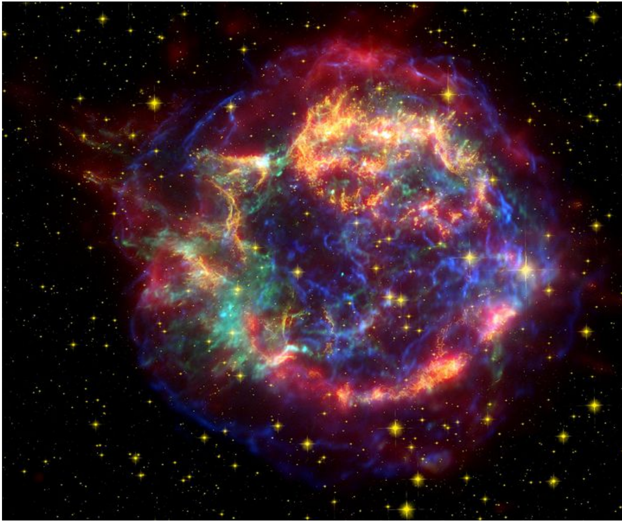
3.1 pc diameter

Strongest radio source outside solar system, discovered in 1947.

X-ray source detected (Aerobee flight, 1965)

X-ray point source detected (Chandra, 1999)

1 of 2 known CO-rich SNR (massive progenitor and neutron star?)



Spitzer, Hubble, Chandra

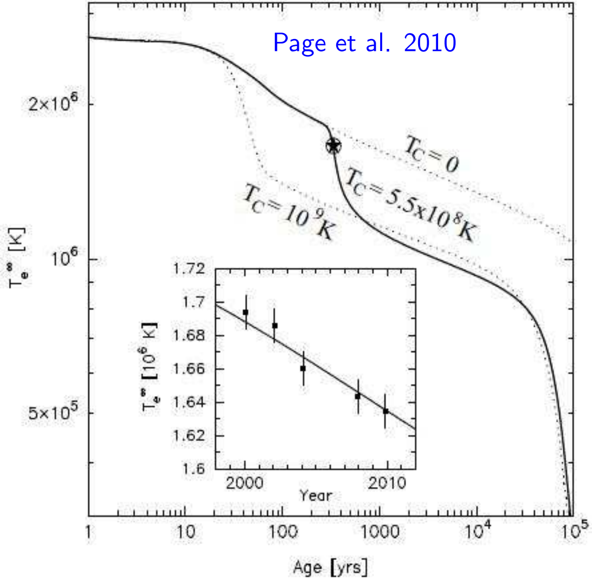
Cas A Superfluidity

X-ray spectrum indicates thin C atmosphere,
$T_e \sim 1.7 \times 10^8$ K
(Ho & Heinke 2009)

10 years of X-ray data show cooling at the rate
$\frac{d \ln T_e}{d \ln t} = -1.23 \pm 0.14$
(Heinke & Ho 2010)

Modified Urca:
$\left(\frac{d \ln T_e}{d \ln t}\right)_{MU} \simeq -0.08$

We infer that
$T_C \simeq 5 \pm 1 \times 10^8$ K
$T_C \propto (t_C L/C_V)^{-1/6}$



Conclusions

- Nuclear experiments set reasonably tight constraints on symmetry energy parameters and the symmetry energy behavior near the nuclear saturation density.
- Theoretical calculations of pure neutron matter predict very similar symmetry constraints.
- These constraints predict neutron star radii $R_{1.4}$ in the range 12.0 ± 1.4 km.
- Combined astronomical observations of photospheric radius expansion X-ray bursts and quiescent sources in globular clusters suggest $R_{1.4} \sim 12.1 \pm 0.6$ km.
- The nearby isolated neutron star RX J1856-3754 appears to have a radius near 12 km, assuming a solid surface with thin H atmosphere (Ho et al. 2007).
- The observation of a 1.97 $M_{\odot}$ neutron star, together with the radius constraints, implies the EOS above the saturation density is relatively stiff; abundance of hyperons or any phase transition must be small.

What Was Not Included

- Neutron Star Mergers
- Gravitational Radiation from Mergers
- Nucleosynthesis from Mergers
- Quadrupole Polarizability