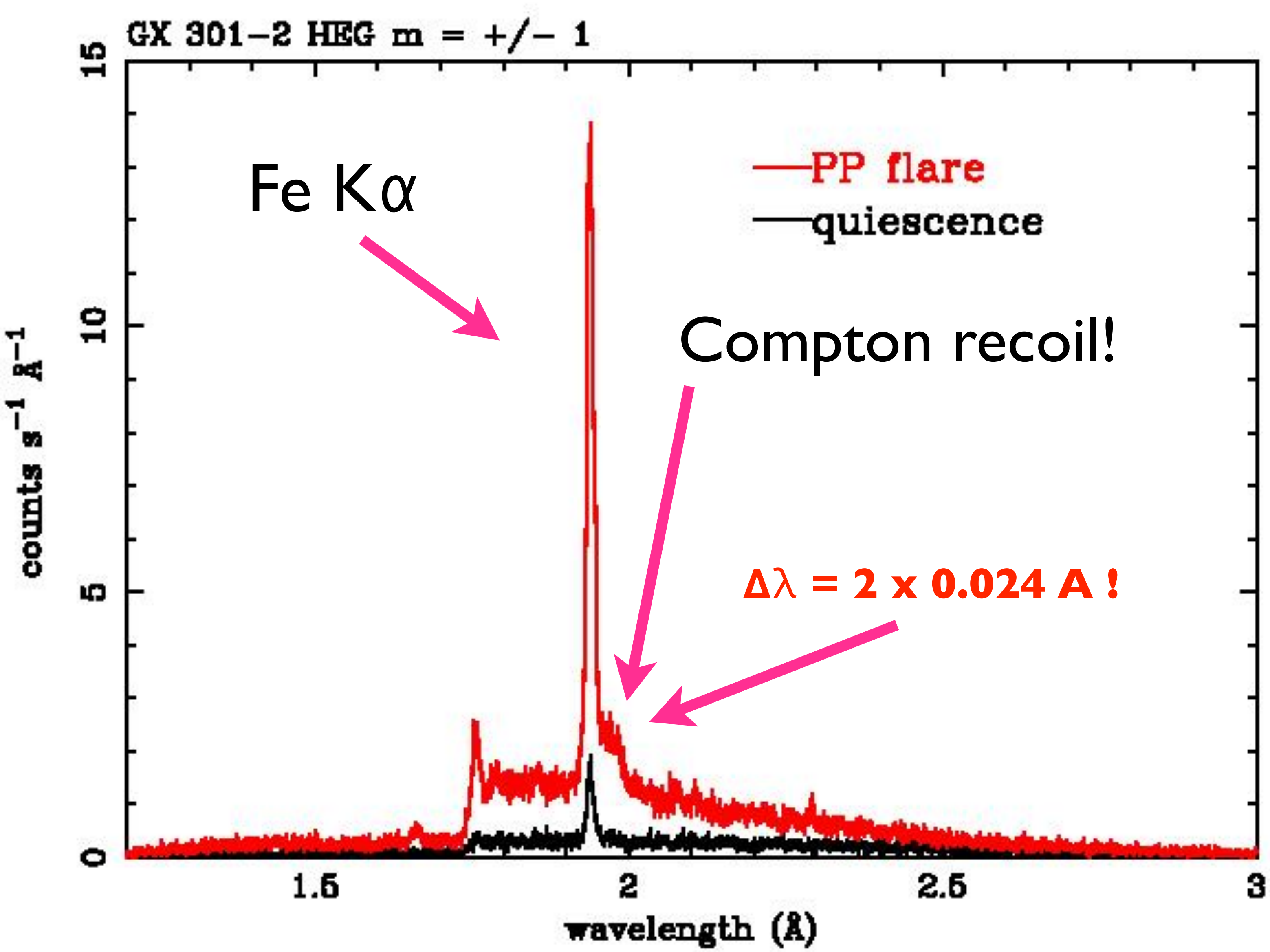
2.1
interaction of X-rays with matter

continuum processes

Compton scattering
incoherent! important in NS atmospheres

GX301-2 Chandra HETGS



with Watanabe-san et al.

bremsstrahlung emission and absorption (*a.k.a.* 'free-free' processes)

thermal bremsstrahlung emissivity, per unit frequency or energy, is ~ flat, up to $E \sim kT$;
therefore,
(inverse) bremsstrahlung absorption cross section is strongly energy-dependent ($\sim E^{-3}$)
(apply Kirchhoff's Law in thermodynamic equilibrium)

note: *this means that there is always a strongly frequency-dependent opacity in dense gas, even if there is no photoelectric absorption!*

Note: cross sections change in magnetized plasmas (will see examples later on)

photoelectric absorption and radiative recombination emission

above threshold energy:

$$\omega_n = \alpha^2 mc^2 Z^2/(2\hbar n^2)$$

photoelectric cross section:

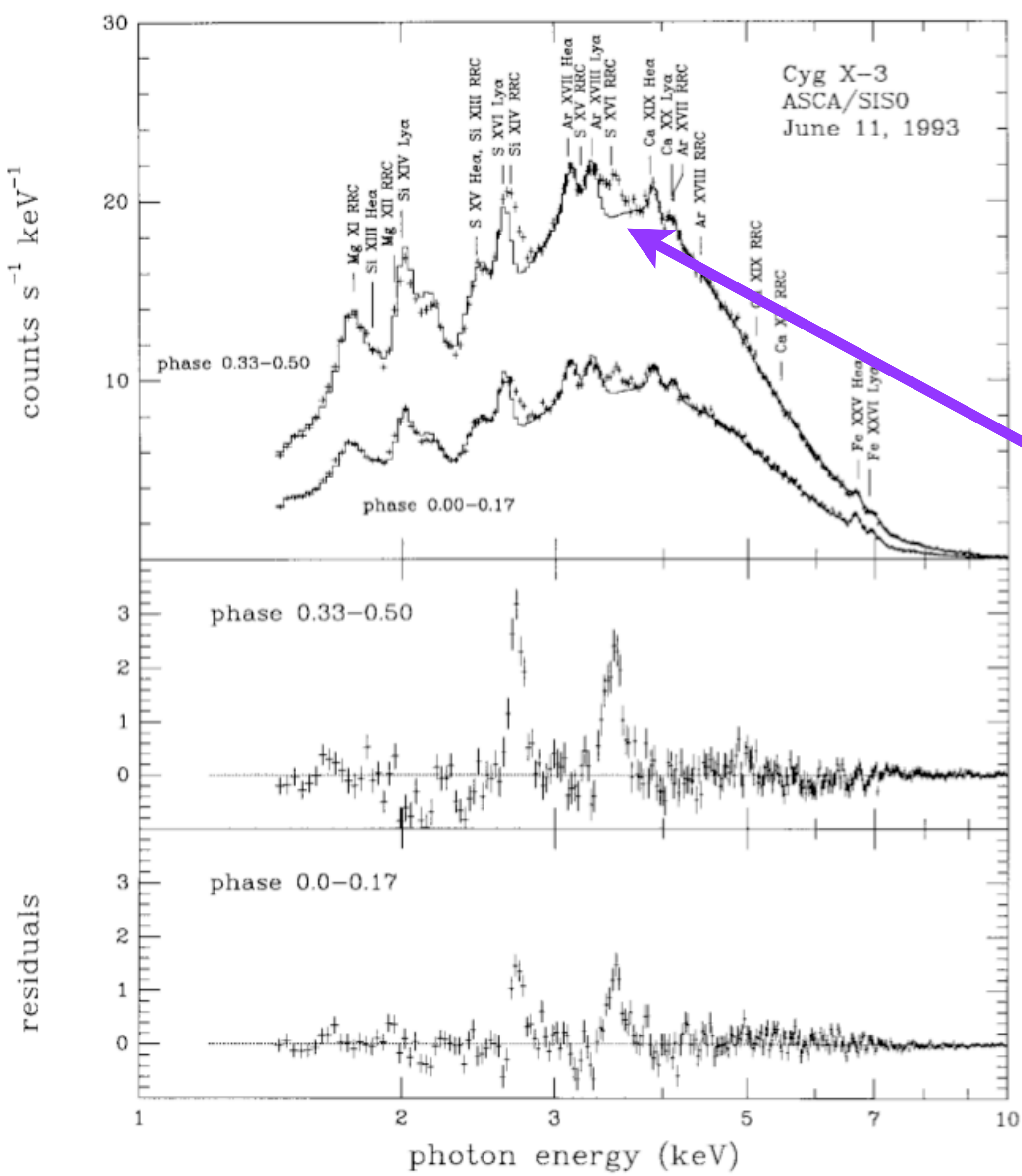
$$\sigma_{\rm bf} = \left(\frac{64\pi n}{3\sqrt{3}Z^2}\right)\alpha a_0^2(\omega_n/\omega)^3 g(\omega, n, l, Z)$$

note: again E^{-3}; also:
proportional to Z^4!
so even trace elements are important

Again: cross sections change in magnetized plasmas
(will see examples later on)

photoelectric absorption
and radiative recombination emission

the inverse process, radiative recombination, can be important, too



Cygnus X-3 again

radiative recombination continuum

Discrete Transitions

in high-density plasmas, will only see E-dipole transitions (no 'forbidden' lines)

classical harmonic oscillator provides simple conceptual framework for *natural* line profile:

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{eE_0}{m} \exp(-i\omega t)$$

leads to expression for absorption cross-section (in cm^2):

$$\sigma_\nu = \frac{\pi e^2}{mc} \frac{\delta/\pi}{(\nu - \nu_0)^2 + \delta^2}; \quad \delta \equiv \gamma/4\pi$$

$$\sigma_\nu = \frac{\pi e^2}{mc} \frac{\delta/\pi}{(\nu-\nu_0)^2+\delta^2}; \quad \delta \equiv \gamma/4\pi$$

quantum mechanics: insert factor *f*: 'oscillator strength',

$$f_{ij} \propto | < j|e\mathbf{r}|i > |^2$$

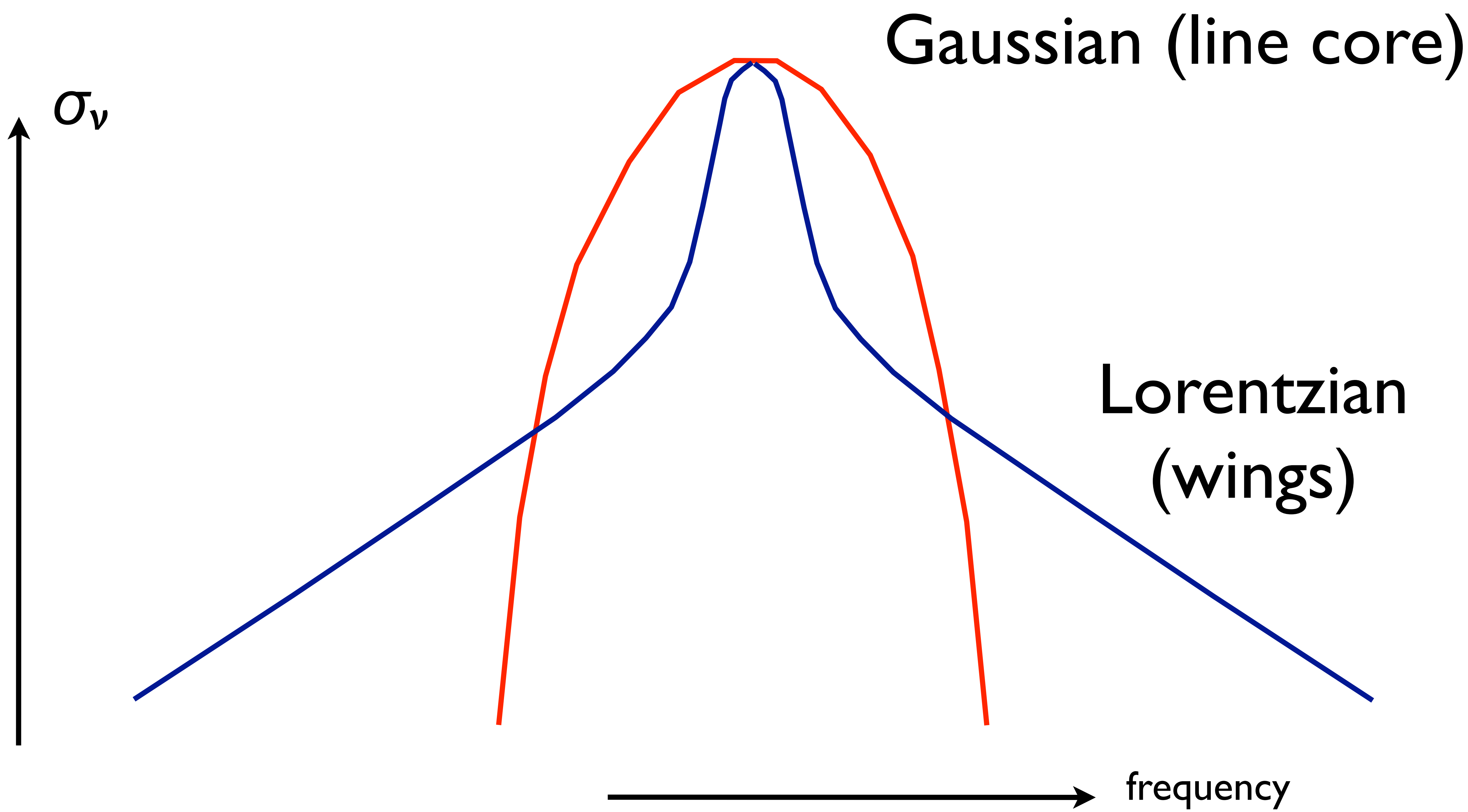
γ is the 'natural width';
to get a rough idea, classical radiation damping gives

$$A = \gamma = \frac{8\pi^2}{3} \frac{e^2}{mc^3} \nu^2 = 2.5 \times 10^{-22} \nu^2 \quad \sec^{-1}$$

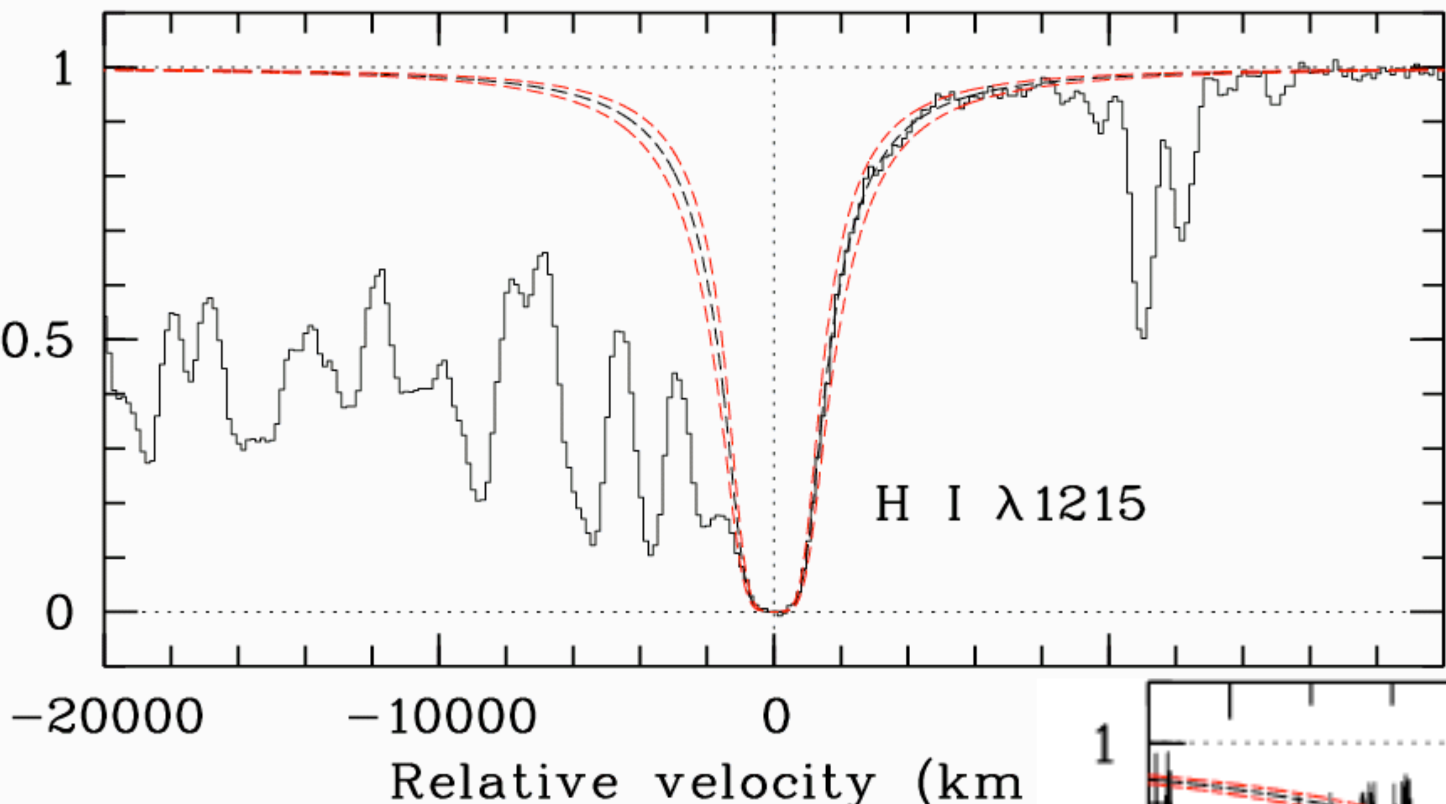
Note: *A* is very large in X-ray band:
E = 1 keV: *A* ~ $10^{13\text{-}14}$ sec^{-1}! (will be important later)

thermal broadening

Convolve the natural profile with a thermal Doppler shift distribution; result is a so-called Voigt profile:

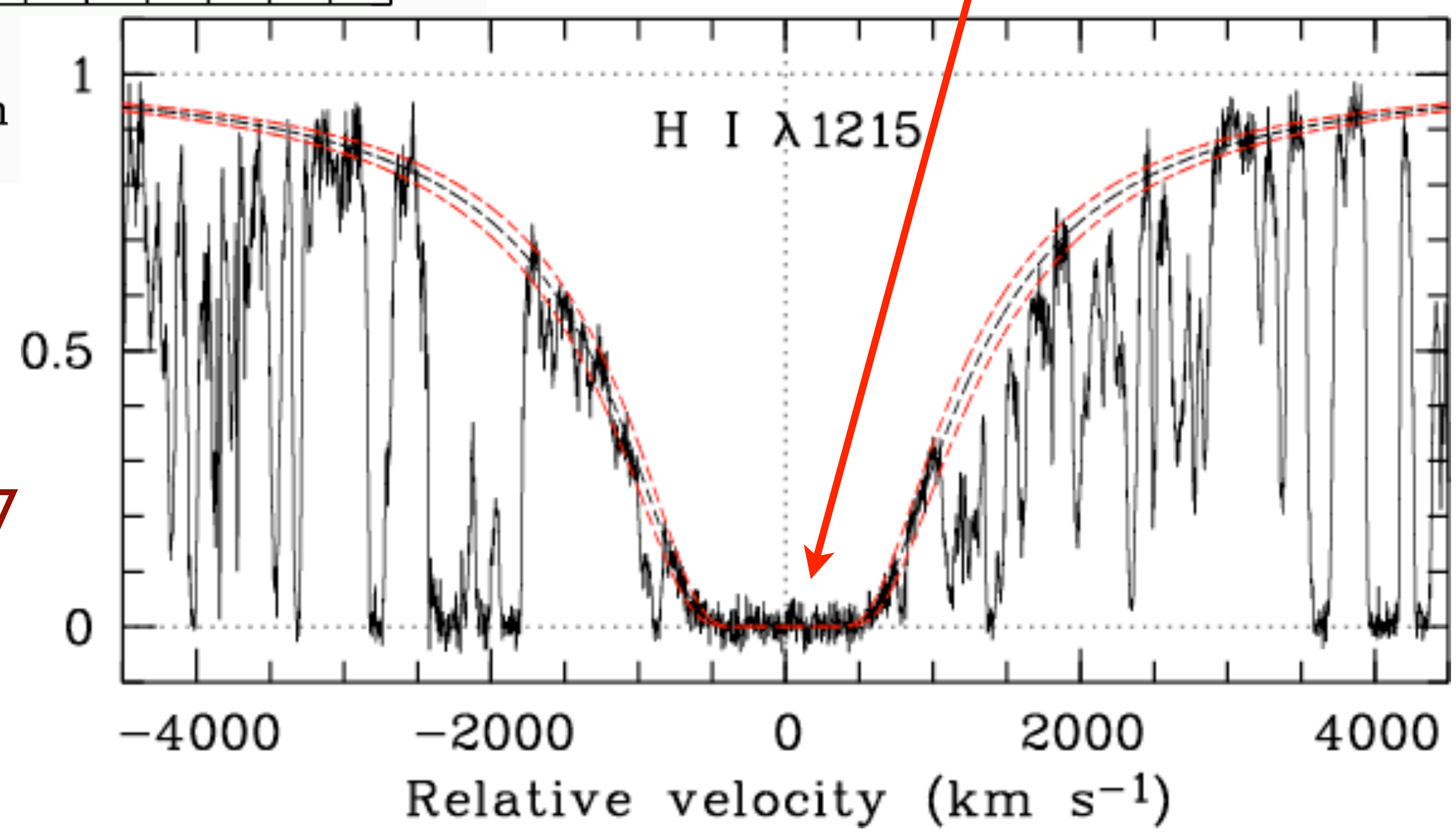


example: absorption by neutral H in the Intergalactic Medium



Small column density (atoms cm^{-2}):
Absorption in Lyα in afterglow of
GRB 060206 (z = 4.408; Fynbo et al. 2006)

note 'saturation'!



High column density:
Absorption in quasar HE2318-1107
at z = 1.989
(Noterdaeme et al. 2007)

other broadening mechanisms: Zeeman

Zeeman $$\Delta E \sim \frac{e\hbar}{2m_e c} B = \mu_{\rm B} B$$

thermal Doppler $$\frac{\Delta E}{E} \sim \left(\frac{kT}{Mc^2}\right)^{1/2}$$

so Zeeman visible for

$$B > 2 \times 10^7 (M/56 m_p)^{-1/2} (T/10^7~{\rm K})^{1/2} E_{\rm keV}~~{\rm Gauss}$$

and once $\mu_B B$ >> atomic binding: Landau quantization

Stark, or pressure broadening:

effect on atomic line profile due to interaction
with other charges in the plasma

very simple 'impact model' ("collision broadening)":
harmonic oscillator phase 'reset' by random collisions:
Lorentzian profile, width ~ collision frequency:

$$\nu_{\rm collision} = 1/\tau_{\rm collision} \sim n_e \sigma v$$

cross section ~ $r^2_{\text{closest approach}}$; from $1/2 m_e v^2 = Ze^2/r$:

$$\nu_{\rm collision} = \frac{4\pi n_e Z^2 e^4}{m_e^{1/2} (3kT)^{3/2}}$$

$$\Rightarrow \Delta E \sim 0.02 (Z/26)(n/10^{23})(T/10^7)^{-3/2} \;\; \text{keV}$$

note the steep T-dependence!

$$\Rightarrow \Delta E \sim 0.02(Z/26)(n/10^{23})(T/10^7)^{-3/2} \ \ \mathrm{keV}$$

compare to thermal width:

$$\Delta E = \left(\frac{kT}{M_i c^2}\right)^{1/2} E =$$

$$= 1.3 \times 10^{-4}(M/56)^{-1/2}(T/10^7)^{1/2} E_{\mathrm{keV}} \ \ \mathrm{keV}$$

pressure broadening dominates line profile
in typical neutron star atmosphere!!

other limit: effect of stationary ions: Stark effect

external electric field ***E*** on bound electron,
(orbital) dipole moment ***p***:

$$\Delta E_{\rm Stark} \sim -\mathbf{p} \cdot \mathbf{E}$$

For ***E***, use "nearest neighbor field":

$$\Delta E_{\rm Stark} = 6 \left(\frac{4\pi}{3} \right)^{2/3} \frac{a_0 e^2 z}{Z} n^{2/3}$$
$$= 0.9 (Z/26)^{-1} (n/10^{23})^{2/3} \ \ {\rm eV}$$

so line core likely dominated by ion broadening,
wings by electron collision broadening

relation to pressure ionization

been measured up to fields of about 10^6 Volt/cm and the presence of order term in (52.5) has been verified at the higher field strengt

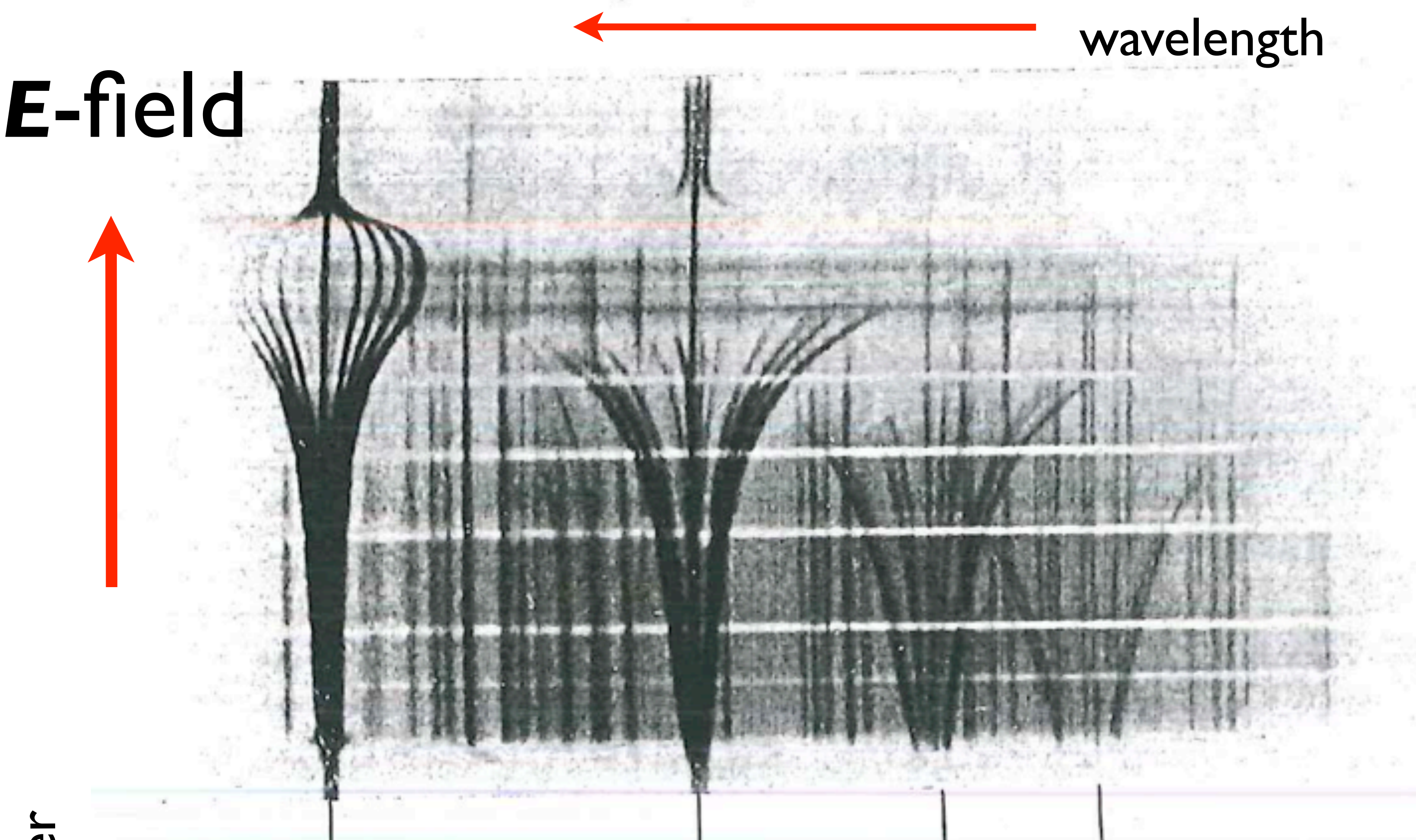


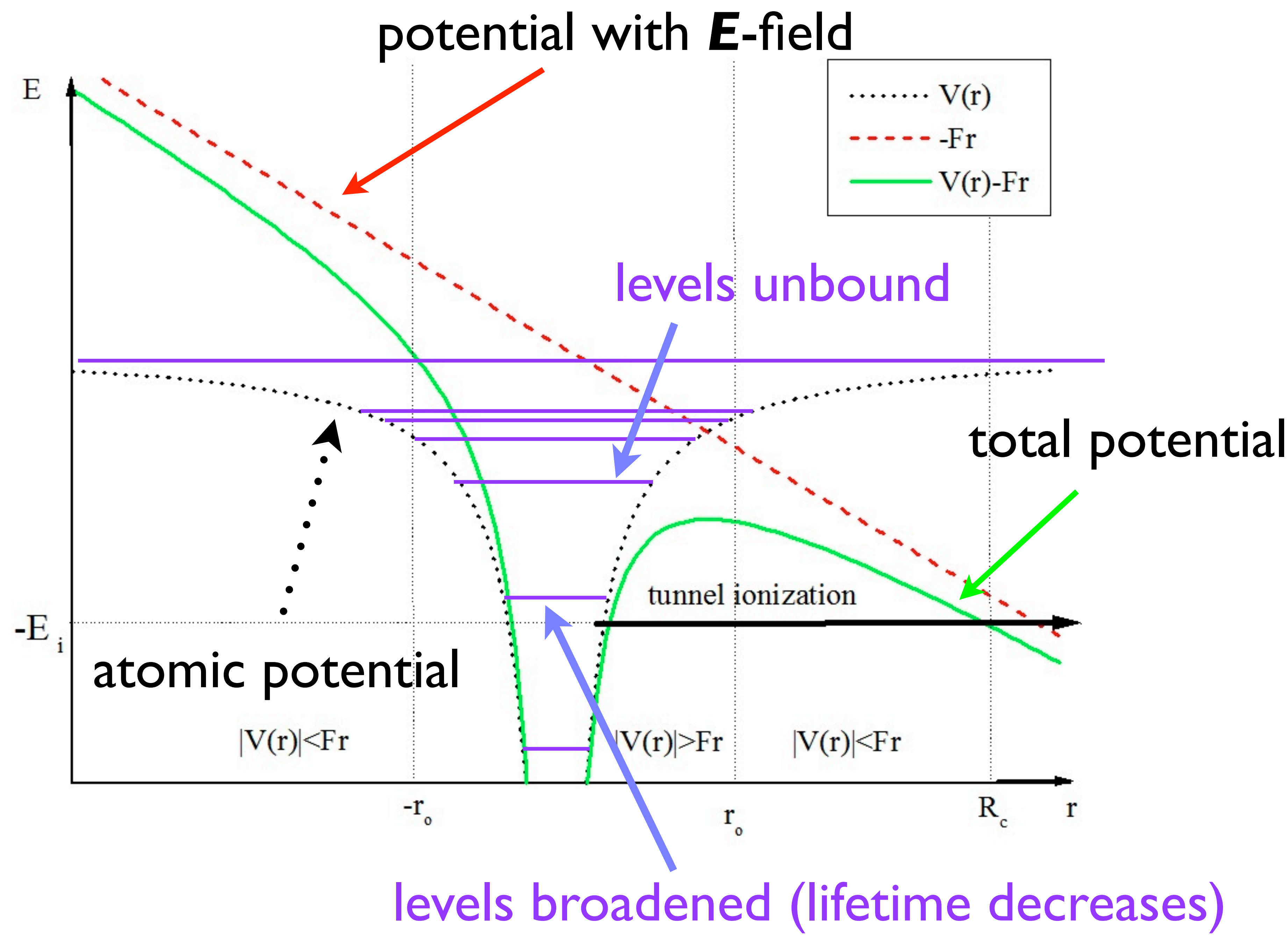
Fig. 26. STARK effect on some of the BALMER lines (experiments by RAUSCH v. TRAUBENBERG). The electric field strength increases from the bottom of the picture upwards, the maximum value (a little below the top of the picture) is 1.14 million Volt/cm, the horizontal white lines are lines of constant field strength.

of the STARK effect in the BALMER se in Fig. 26. The fiel creases from the b picture towards the mum field is 1.1 × Note that the comp red side (to the left free (vertical) lines more than the viol (quadratic STARK also that each line above a critical fiel This quenching of discussed in Sect. 5

Bethe and Salpeter

so p-ionization also has a spectroscopic signature

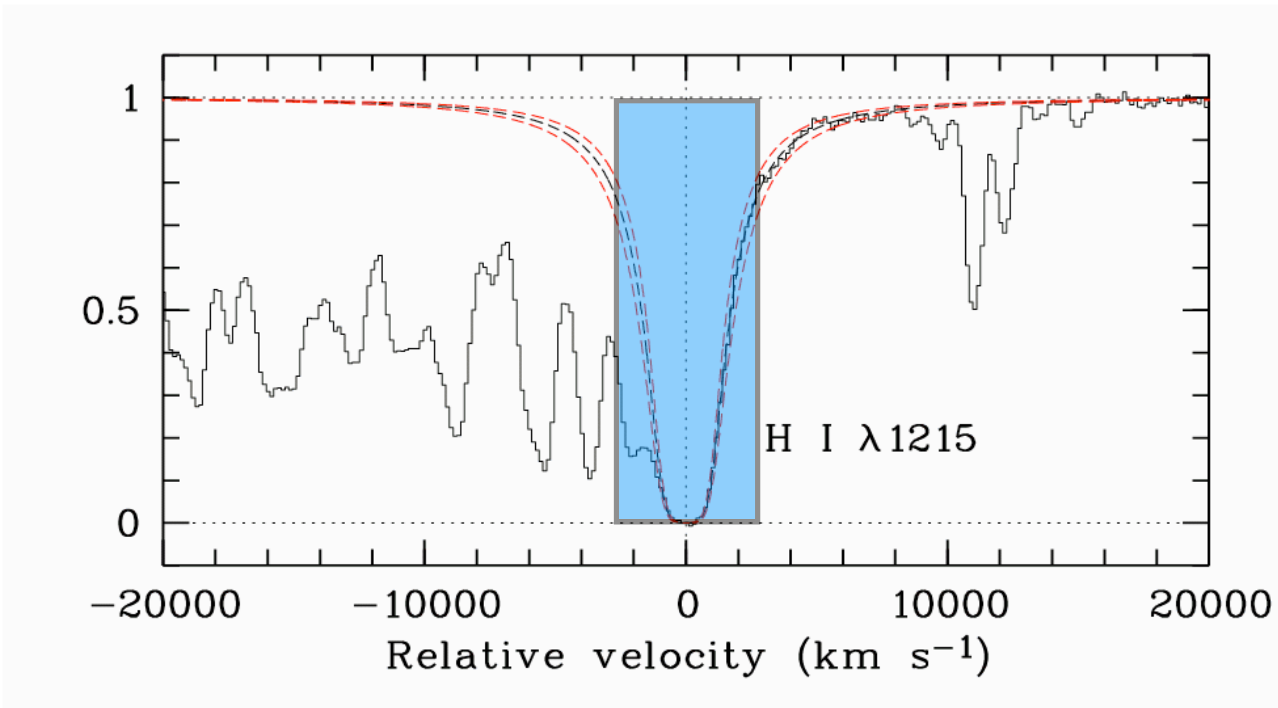
relation to pressure ionization



p-ionization also has a spectroscopic signature

equivalent width and 'curve of growth'

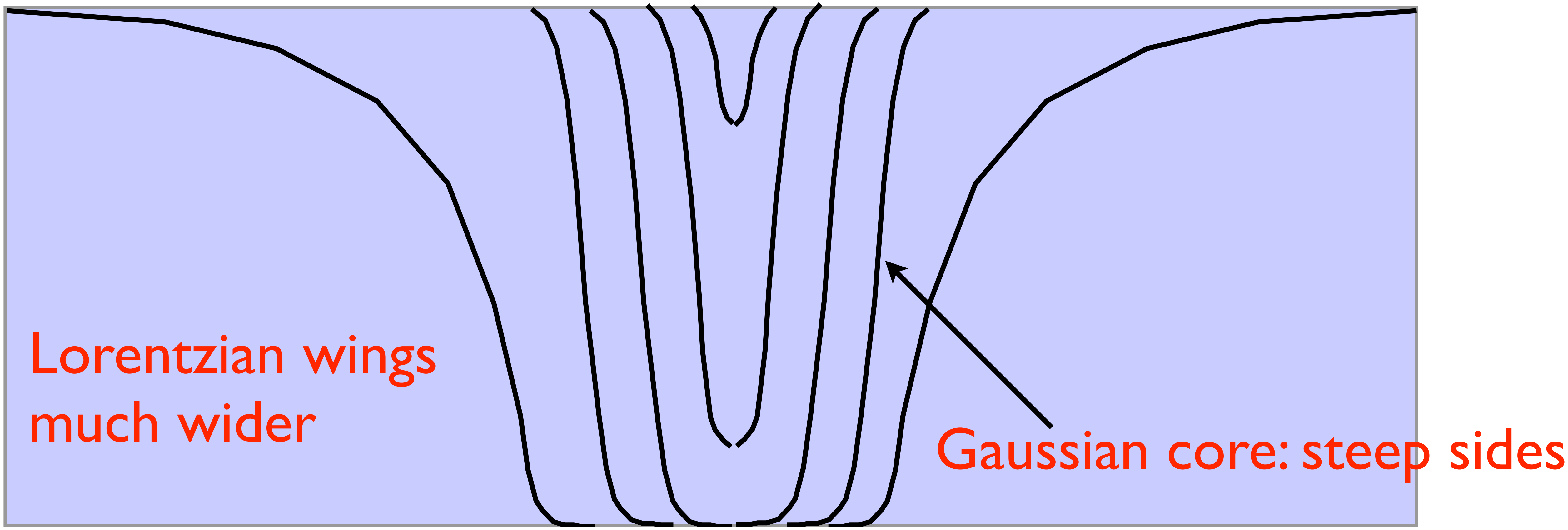
we discussed line*width*, but what about '*contrast*'?

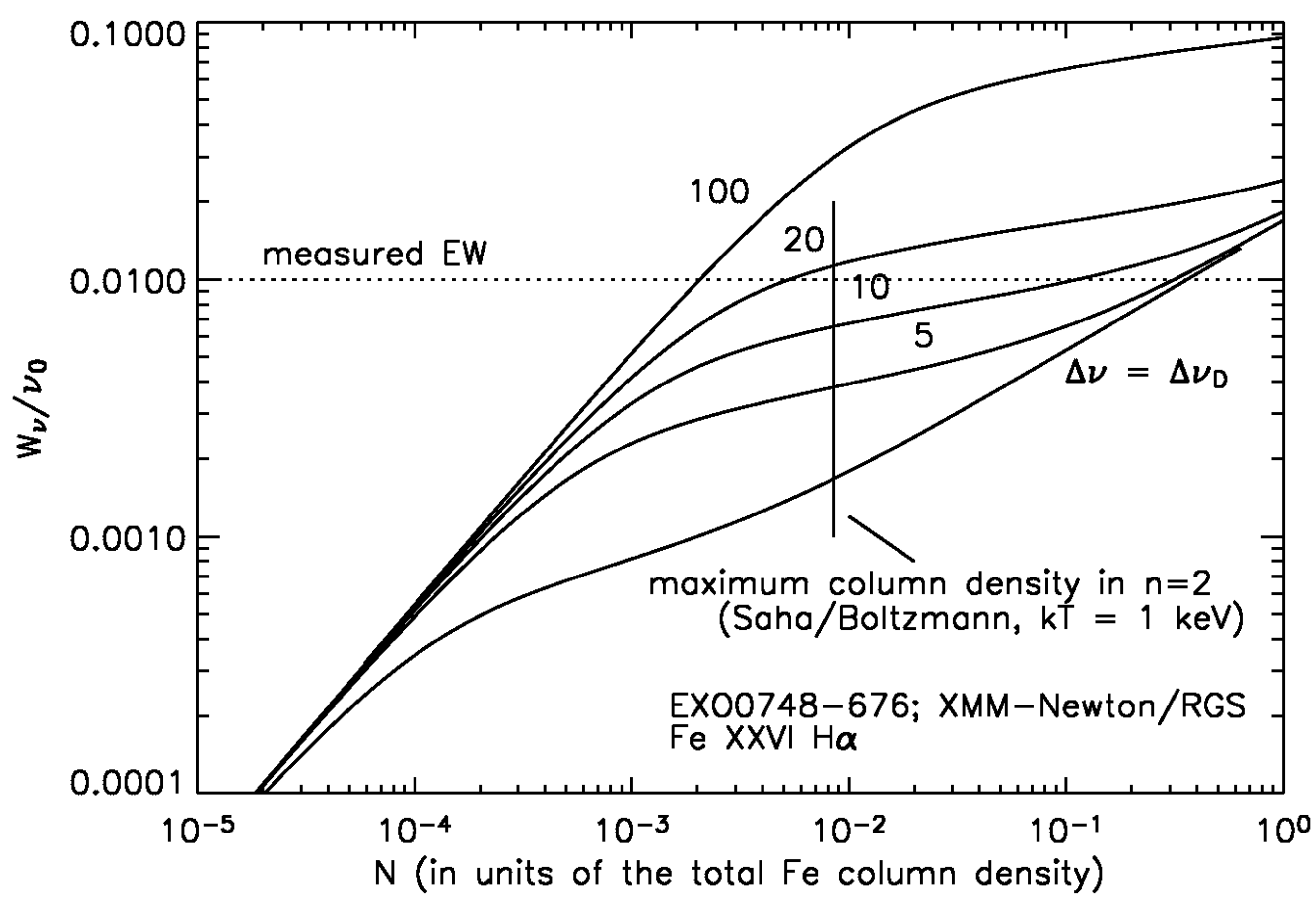


width of the blue rectangle: 'equivalent width' (EW); measured in wavelength, frequency or velocity units

very useful: EW invariant under instrument resolution

what happens when lines get darker ('more absorbers') ?





'curve of growth':
EW vs. absorber column density

other broadening mechanisms: rotation (NS spin)

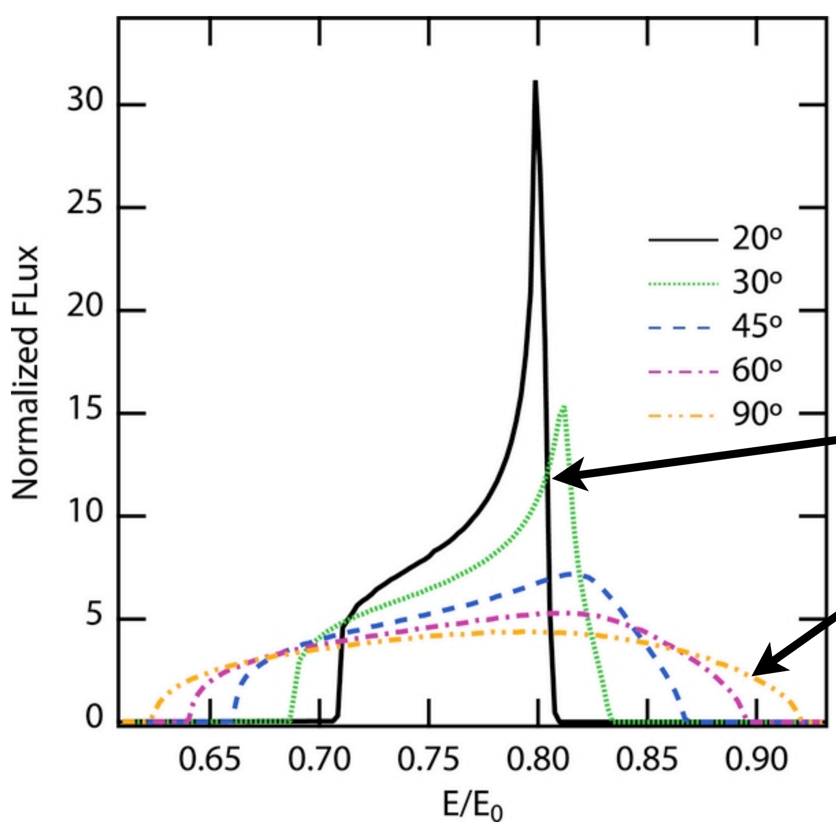
When does spin dominate line width?
Set spin Doppler broadening equal to collision broadening: spin dominates for $\nu_{spin} \gtrsim 100$ Hz

NB: spin broadening does not change the EW (to first order in v/c), so Stark sensitivity preserved.
But a very broad, shallow line is hard to detect...

other broadening mechanisms:
rotation (NS spin)

spin measurement is itself major spectroscopic item:
with ν_{spin} , can get R!!

GR makes this even more interesting!



self-lensing, beaming, distortion of star due to spin (effect on z_{grav})

near 'pole on'

seen in equatorial plane

Bauböck, Psaltis, Özel 2013

will this work??

what do we know about chemical composition?

pure H? (gravitational settling)

radiative levitation of heavy ions?

pure C? (slow downward diffusion & burning of H)

pure Fe?

accreting NS: if dM/dt large enough:

abundance equal to accreting gas

what do we know about chemical composition?

accretion/diffusion equilibrium

$$\left(\frac{Z}{H}\right)^{\rm star} = \left(\frac{Z}{H}\right)^{\rm accretion} \frac{\dot{M}}{\dot{M} + 4\pi R^2 \rho v_{\rm D}}$$

$$\frac{L}{L_{\rm Eddington}} = \frac{\dot{M}\sigma_{\rm T}}{4\pi R m_p c} \approx 1 \times 10^{-2} (\dot{M}/10^{16}\ {\rm g\ s^{-1}})$$

so

$$\dot{M} >> 4\pi R^2 \rho v_{\rm D} = 2 \times 10^{12} (n/10^{23})(v/1\ {\rm cms^{-1}})$$

as long as $L > 0.01\ L_{\rm Eddington}$

2.2
Neutron Star Atmospheres

a few definitions

definition of effective temperature:

$\sigma_{SB} T^4_{eff} = F$ (total flux)

for a blackbody, $T_{eff} = T_{BB}$

$$F \equiv \int_0^\infty F_\nu d\nu$$

“stellar atmosphere”

Self-gravitating gas sphere,
Hydrostatic Equilibrium:

$$\frac{dP(r)}{dr} = -\rho g$$

Near surface, g constant; assume ideal gas:

$$P = nkT = \frac{\rho kT}{\mu m_p} \Rightarrow$$

$$\frac{d\rho}{dr} = -\frac{\rho}{H}; \quad H \equiv \frac{kT}{\mu m_p g}$$

Note: at very high density, phase transition to solid surface may occur!

the ‘scale height’, H:

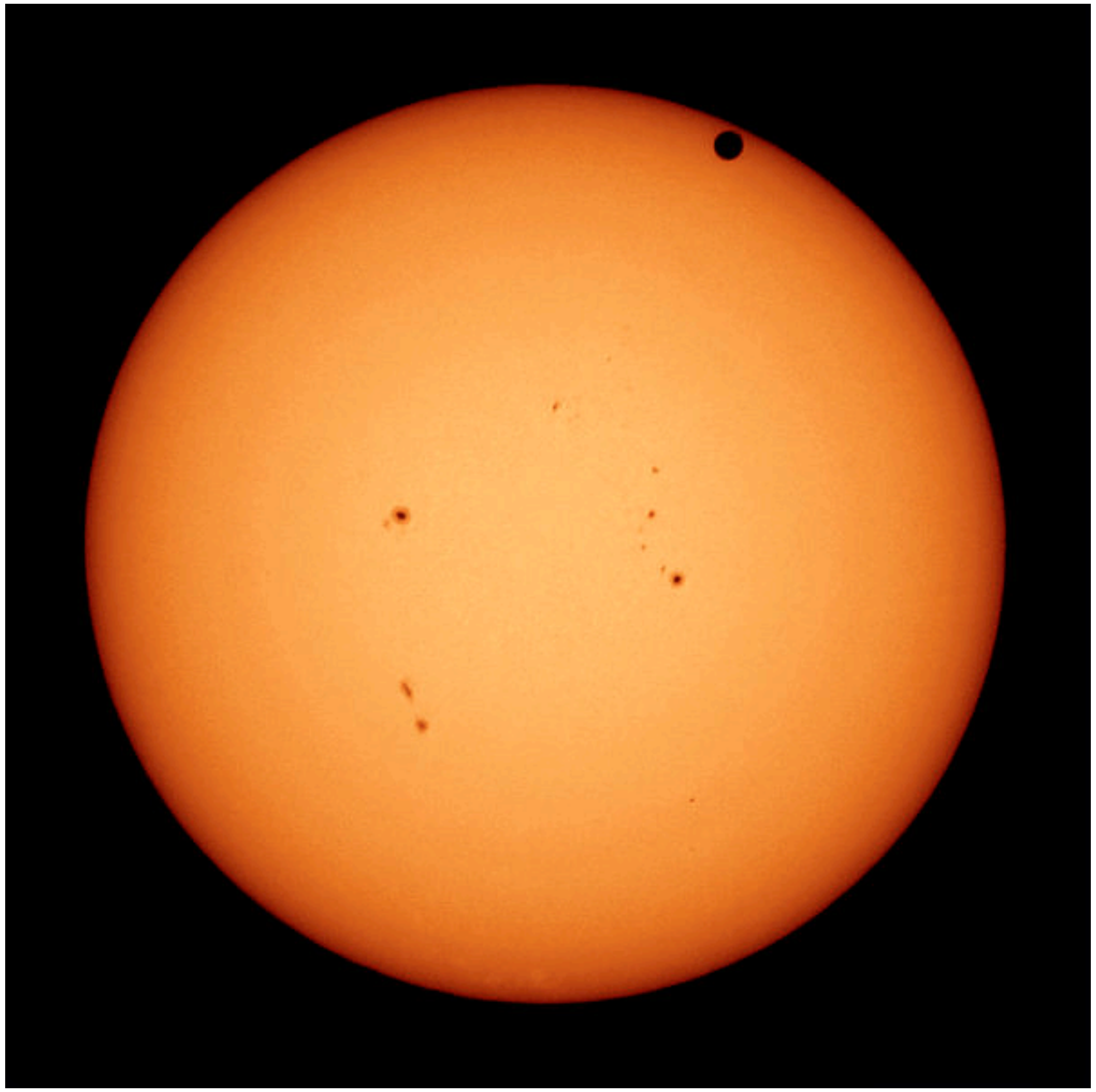
Sun: T = 5800 K, $g = 3.10^4$ cm/s^2:
$H = 1.6\ 10^7$ cm, or $H/R \sim 2\ 10^{-4}$

neutron star: $T = 10^7$ K, $g = 1.10^{14}$ cm/s^2:
H = 8 cm!! or $H/R \sim 10^{-5}$

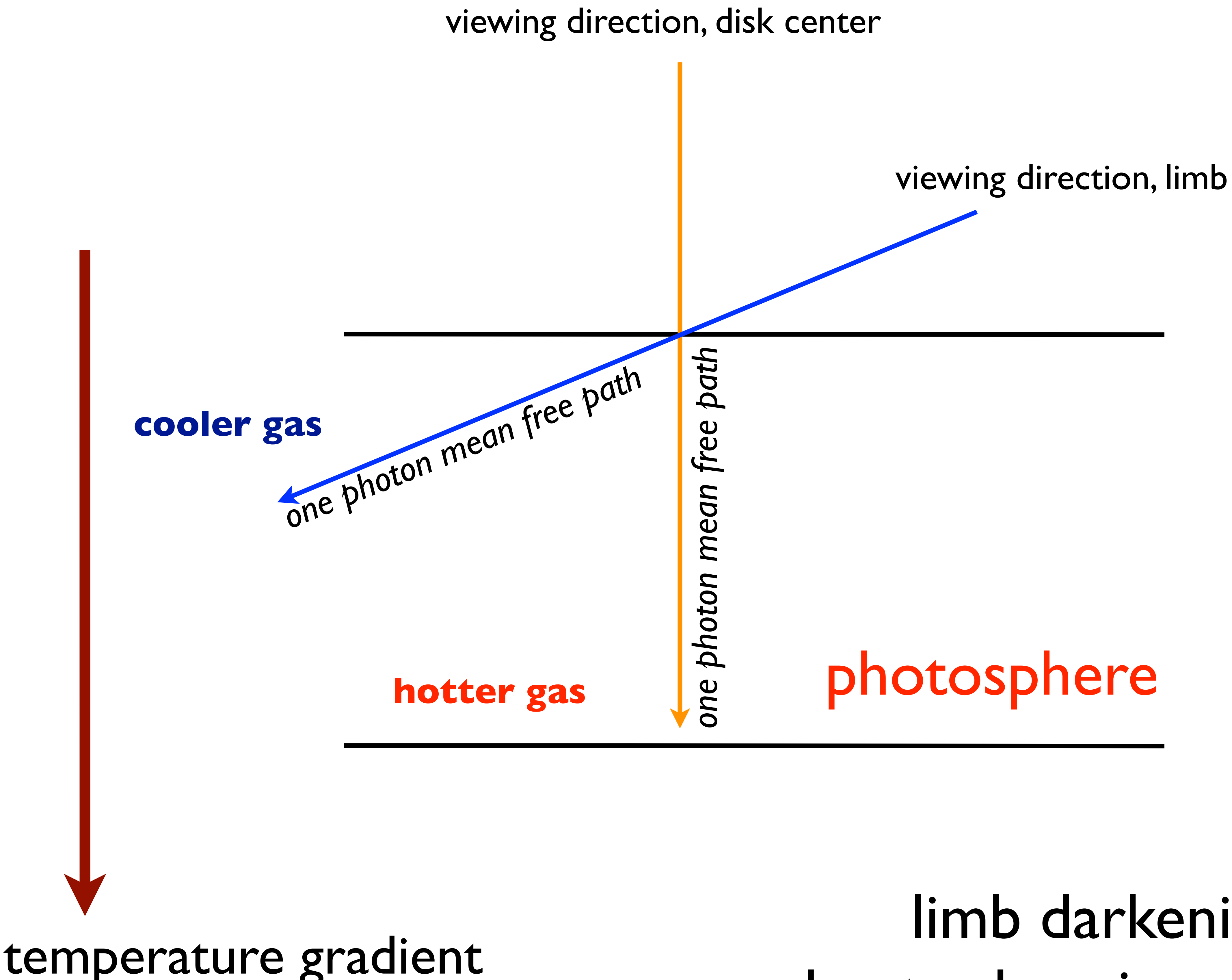
Atmosphere is the thin layer where
the pressure/density ~exponentially fall to zero

‘photosphere’ is of comparable extent:
layer from which photons can escape

stellar limb is sharply defined

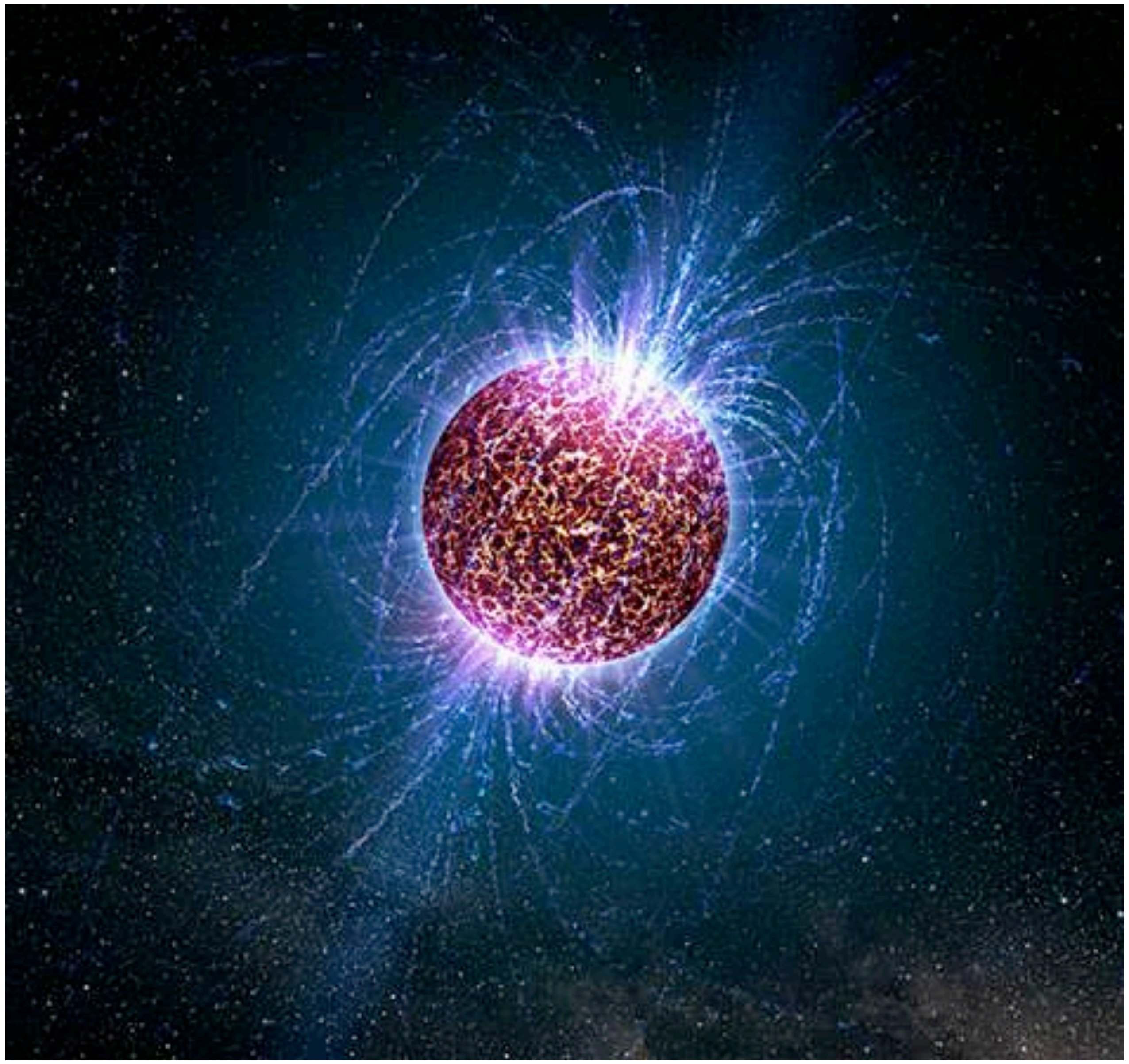


and a star darkens toward the limb:
'limb darkening'; darkening is different in different colors



limb darkening,
due to drop in emissivity
toward top of atmosphere

NB: this is obviously also important
if the viewing angle *changes* with time!!!



neutron star atmosphere:
some more fiducial numbers

characteristic *density*:

$$\frac{dP}{dr} = -\rho g \Rightarrow \frac{1}{\kappa\rho}\frac{dP}{dr} \equiv \frac{dP}{d\tau} = \frac{g}{\kappa}$$

(κ: 'opacity'; $\kappa\rho$ = nσ; d$\boldsymbol{\tau}$ = - $\kappa\rho$ dr; $\boldsymbol{\tau}$: "optical depth")

Integrate from $\boldsymbol{\tau}$ = 0 to $\boldsymbol{\tau}$ = 1, set P($\boldsymbol{\tau}$ =0) = 0;
assume κ constant:

$$P(\tau = 1) = \frac{g}{\kappa} \Rightarrow$$

$$n(\tau = 1) = \frac{g}{\kappa k T} = 2 \times 10^{23}\ (g/10^{14})(\kappa/0.4)^{-1}(T/10^7)^{-1}\ \mathrm{cm}^{-3}$$

!!!

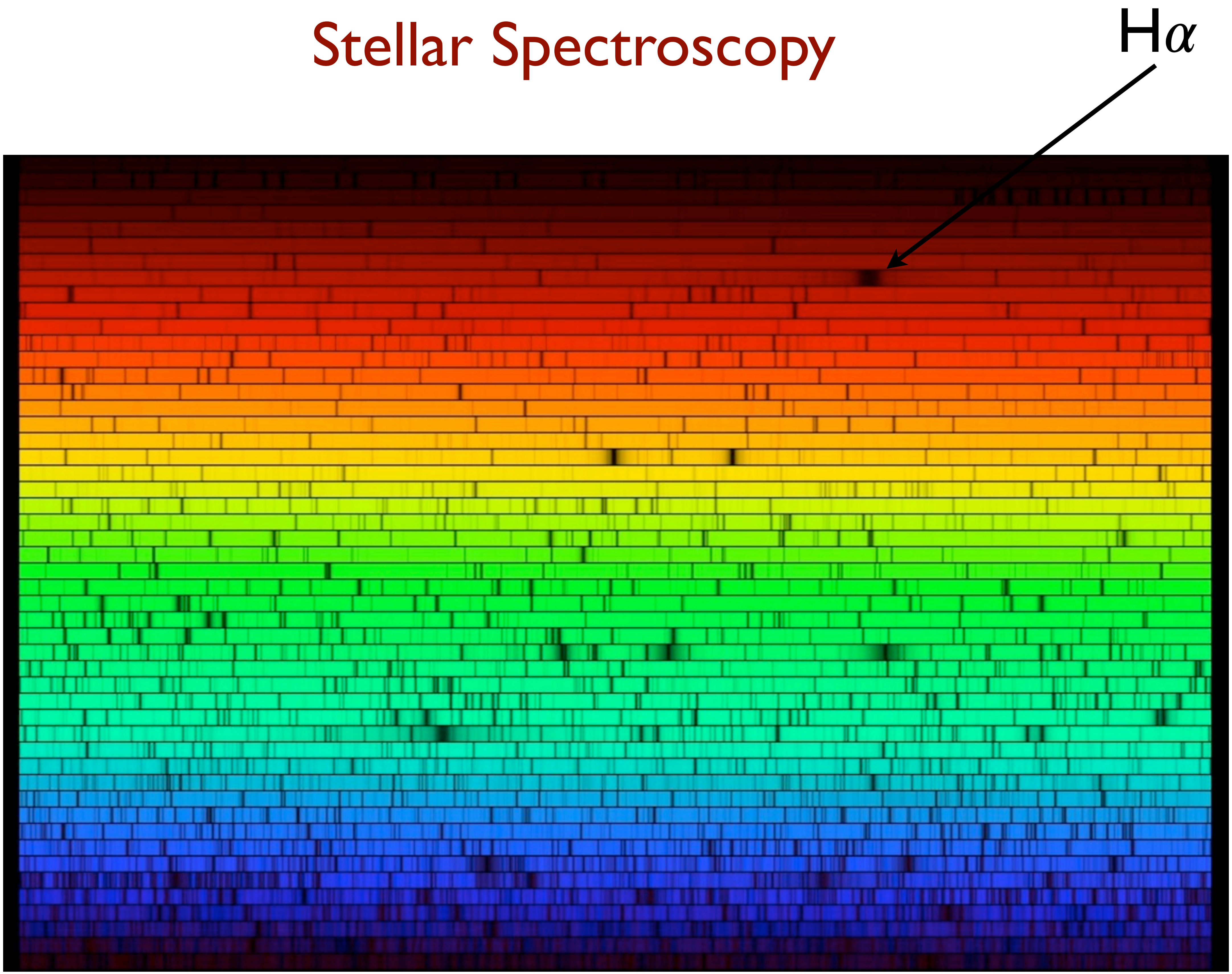
and, not surprisingly, mean free path for photon:

$$\Delta\tau = 1 \Rightarrow \kappa\rho l = 1 \Rightarrow l = \frac{1}{\kappa\rho} \sim 8 \text{ cm}$$

since this is the same expression as for H, but in different form.

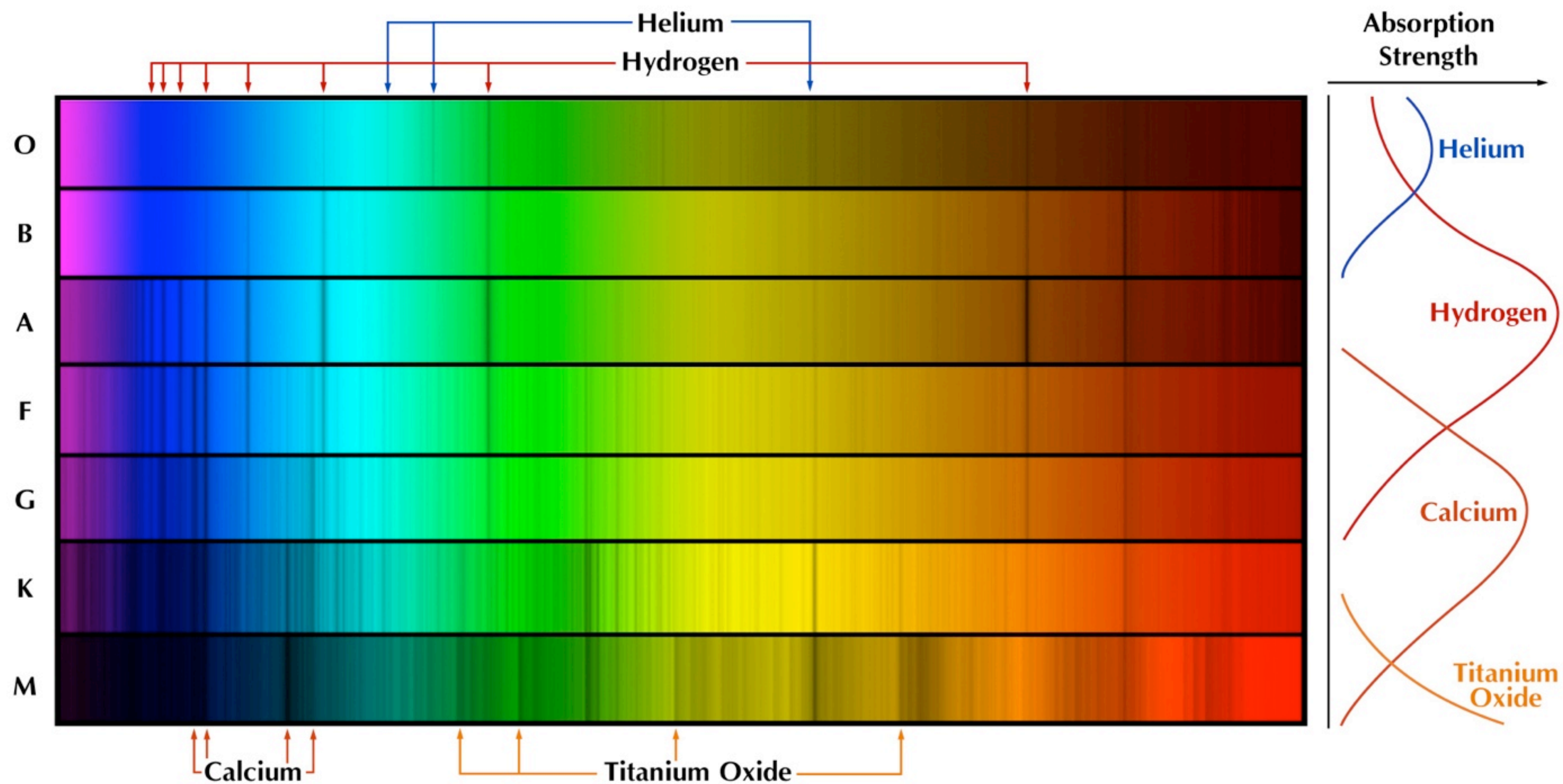
Other than neutrino's, gravitational waves (?),
or seismological data:
the only information on a star we have is
in the radiation that passed through its atmosphere

Stellar Spectroscopy



the optical Solar spectrum

credit: NOAO/AURA



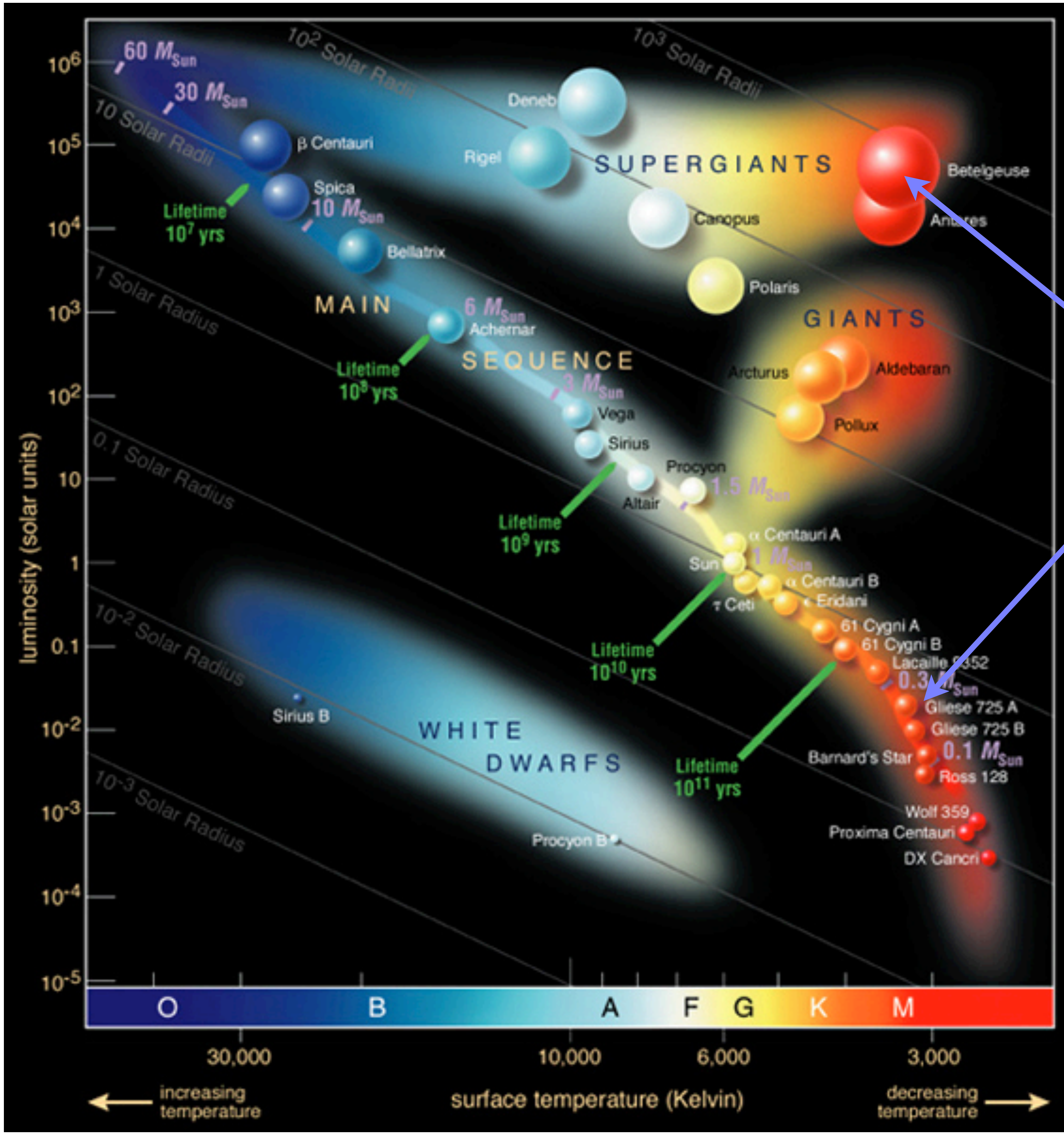
example stellar spectra ranked by color (blue-red),
i.e. effective temperature, i.e. "spectral type" (OBAFGKM)

don’t need to observe all frequencies

careful study gives

correct luminosity (if distance known) from $T_{\rm eff}$ and R
chemical abundances
rotation rate (from absorption line Doppler broadening)
surface gravity

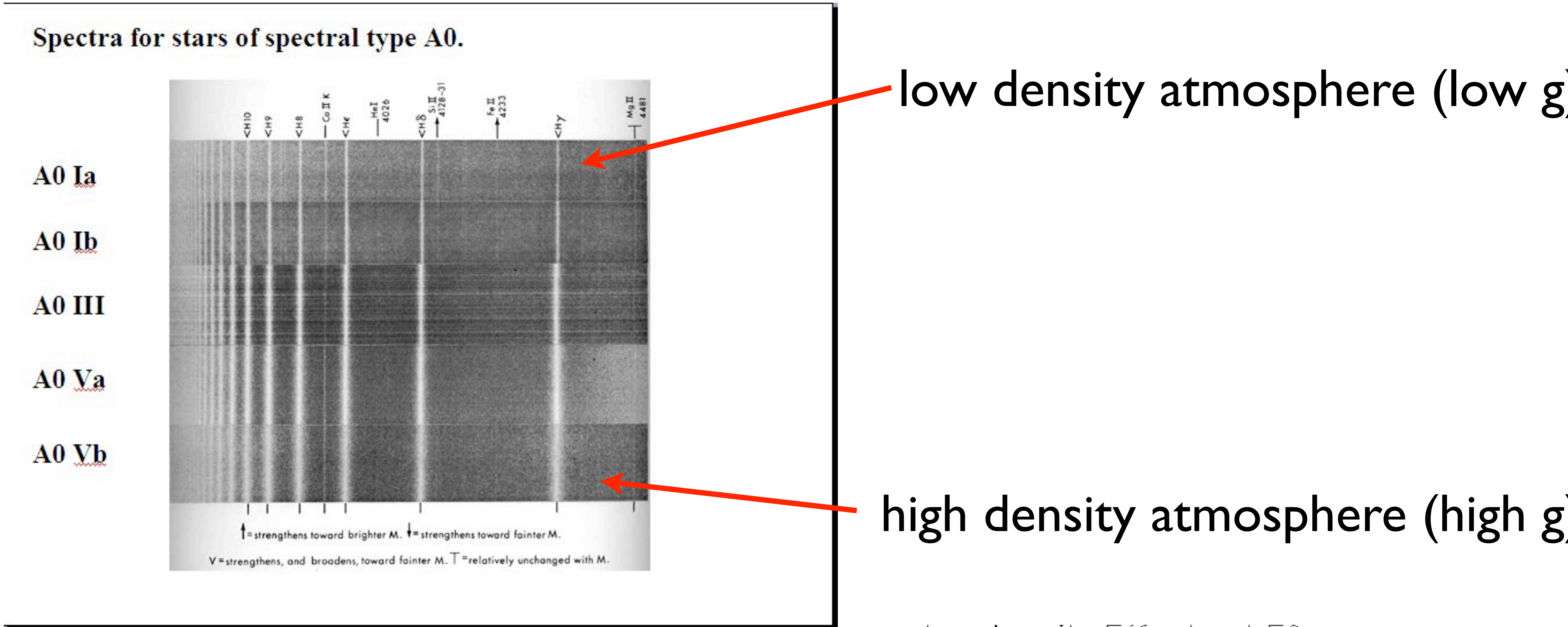
but masses come from binary orbital dynamics only...
(not true for neutron stars, as we will see)



these stars have very similar spectra (same T_{eff}); how do we know whether they are dwarfs or giants?

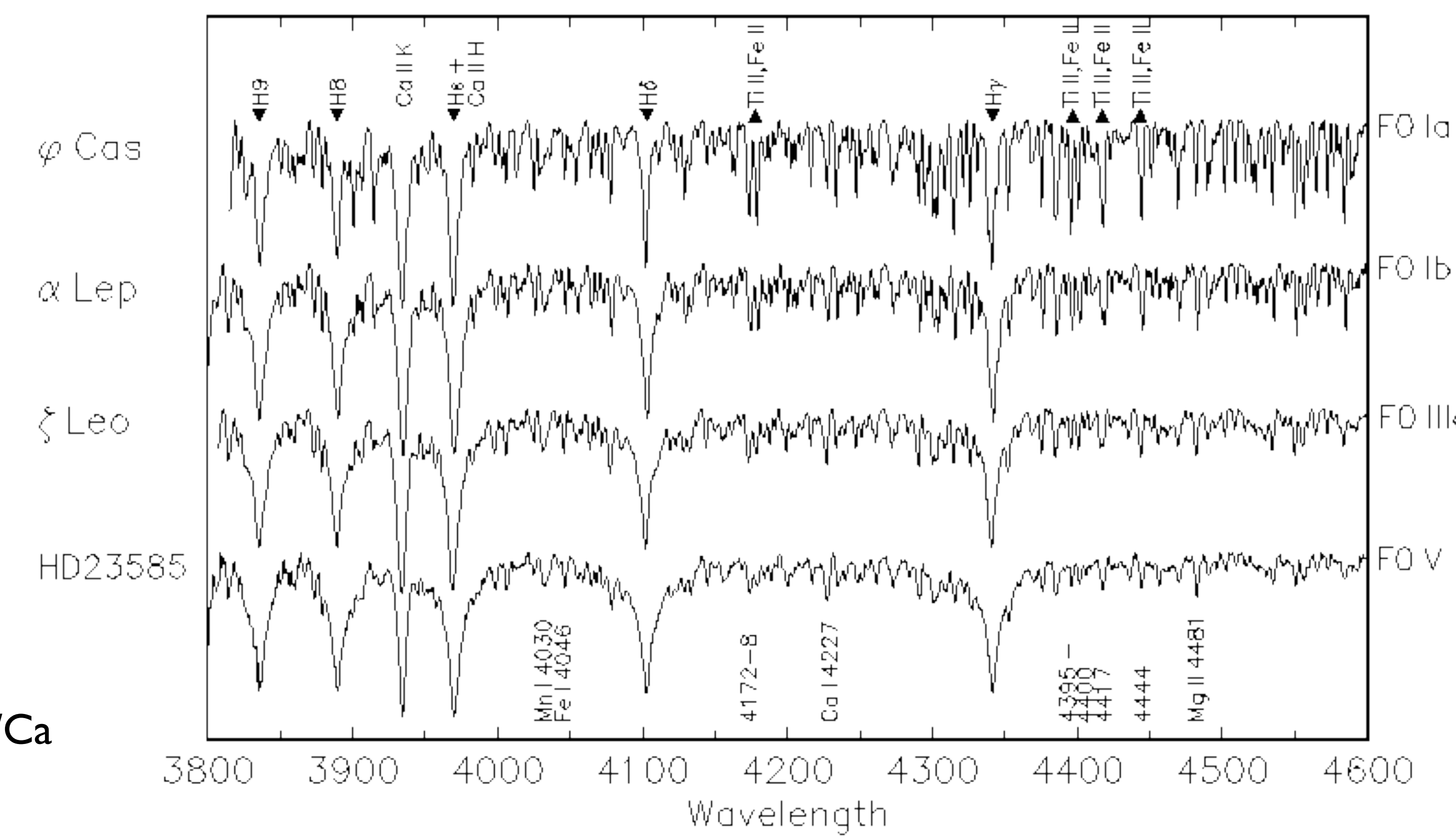
example of the power of stellar spectroscopy

the Hertzsprung-Russell Diagram

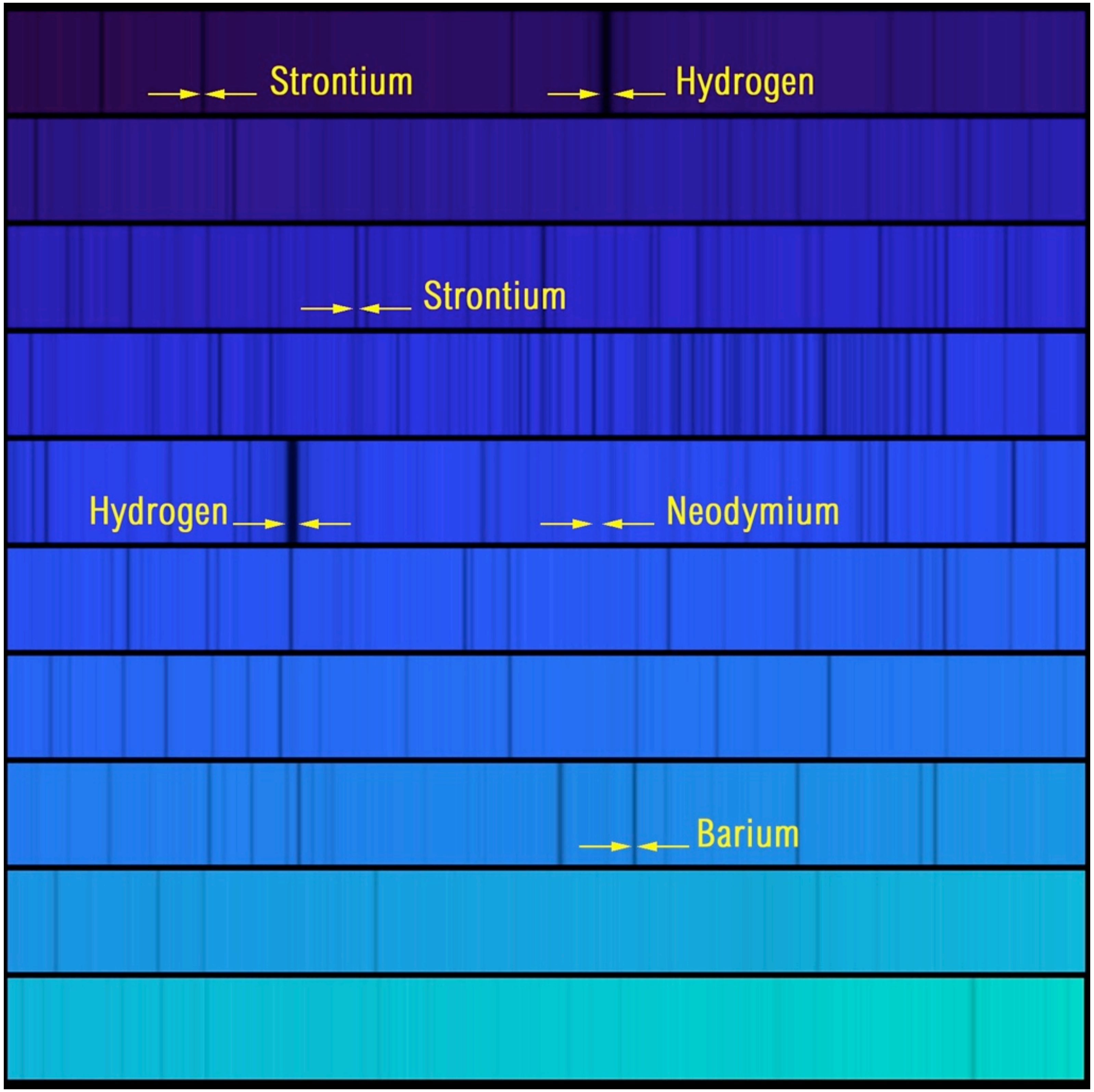


Luminosity Effects at F0

pressure broadening and log g in classical stellar spectroscopy



credit SMU/Ca



spectrum of HD126587, 4066-4710 Å

credit: NOAO/AURA

structure of a stellar atmosphere, and the emergent intensity spectrum

Simplest case:

mechanical equilibrium
radiative equilibrium (radiation flux conserved)
(plane parallel geometry):
g, T_{eff}, composition specify unique model

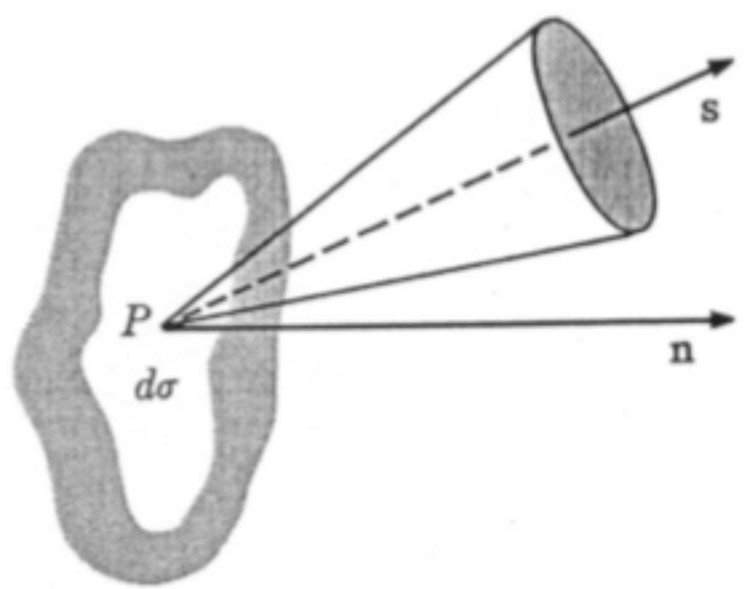
generally considered exotic (but relevant to our case!):
inhomogeneous composition
General Relativistic effects (changing z?)
....

definition of effective temperature: $\sigma_{SB} T^4_{eff} = F$ (total flux)

Equation of Radiative Transfer

ignoring polarization, the radiation field is completely specified by the monochromatic intensity, $I_\nu(\mathbf{n})$

($\boldsymbol{\nu}$ is the radiation frequency, **n** a direction vector)



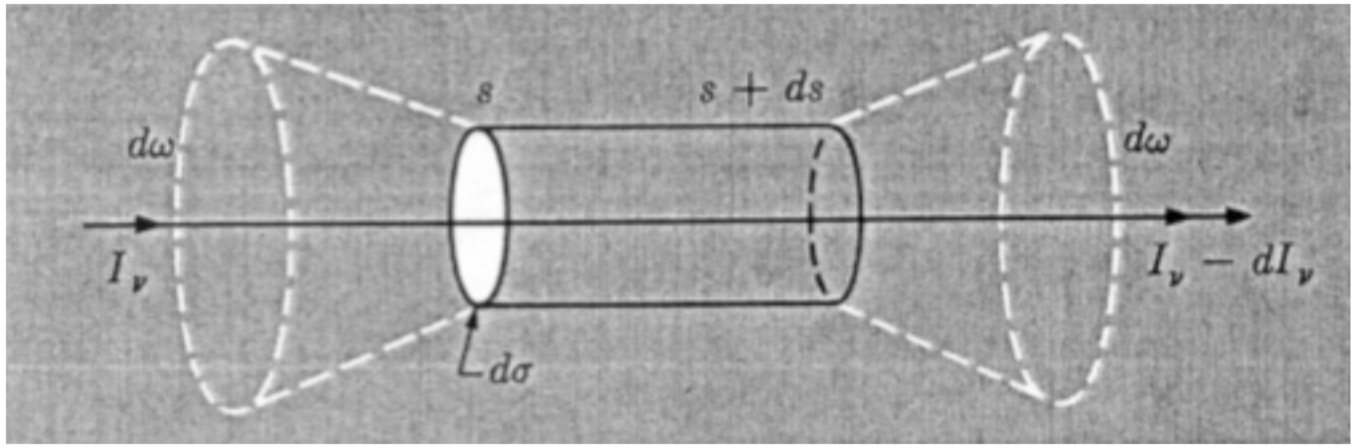
$I_\nu(\mathbf{n})$: erg cm^{-2} s^{-1} sr^{-1} Hz^{-1}

flowing along **n**

recommended text: J.T. Jefferies, *Spectral Line Formation*

default text: D. Mihalas, *Stellar Atmospheres*

Equation of Radiative Transfer



$$\frac{dI_\nu}{ds} = -\kappa_\nu \rho I_\nu + \rho j_\nu$$ (*ds* along **n**)

κ_ν: cm^2 gr^{-1}

j_ν : erg gr^{-1} s^{-1} sr^{-1} Hz^{-1}

energy lost from the beam

energy added to the beam
(may also be radiation scattered into the beam)

(it's just the Boltzmann Equation for photons!)

Equation of Radiative Transfer

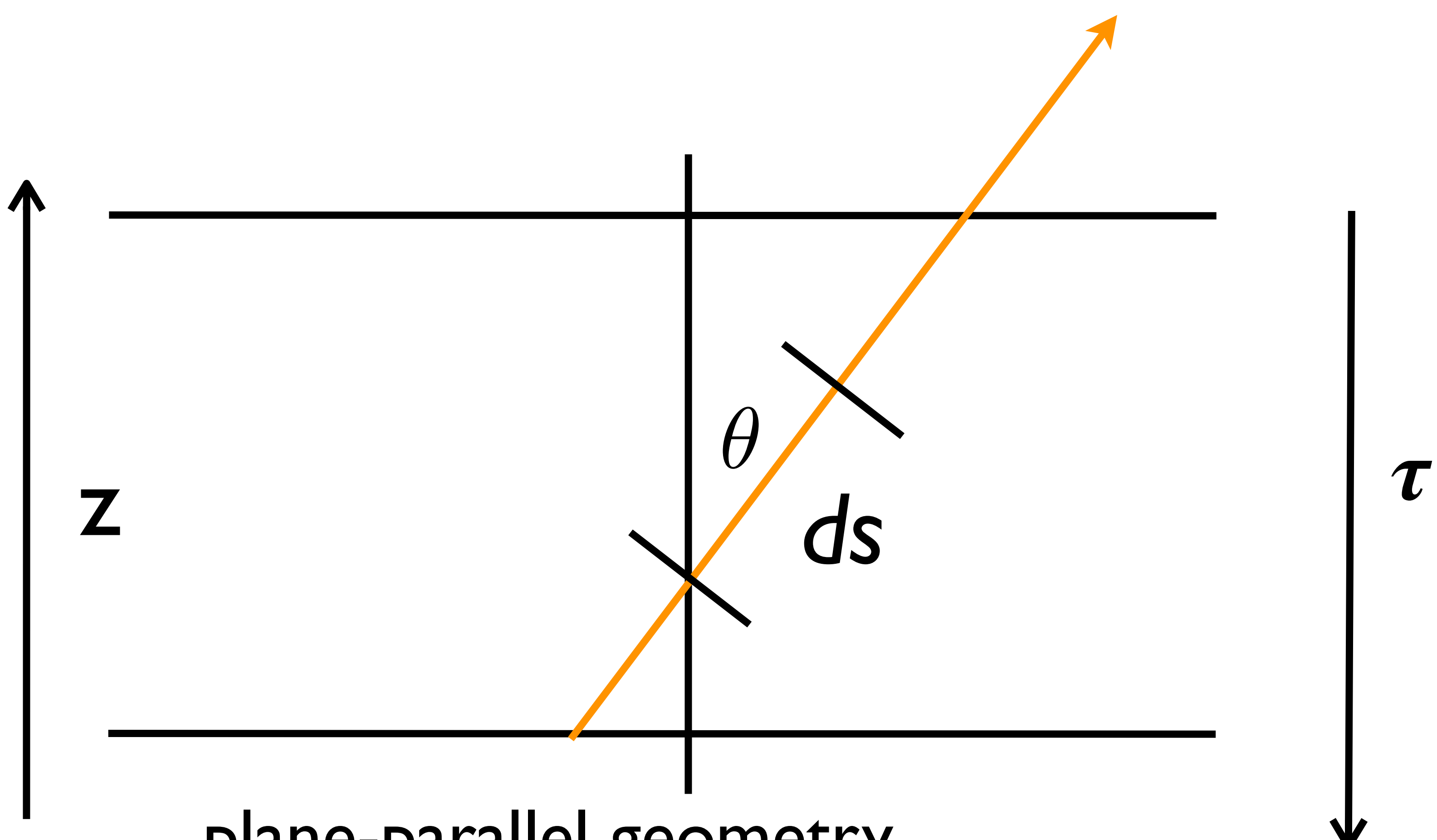
$$\kappa_\nu \rho \; ds \equiv -d\tau_\nu / \mu \quad \Rightarrow$$

note the minus sign!

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \frac{j_\nu}{\kappa_\nu} \equiv I_\nu - S_\nu$$

S_ν is the 'source function'

radiative transfer problems often difficult because S depends on I



plane-parallel geometry

$$\mu = \cos\theta$$

Illustrative analytical example:

set S_ν = a + bτ_ν

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu = I_\nu - (a + b\tau_\nu)$$

Use integrating factor exp(τ_ν/μ) and integrate between 0 and ∞:

$$I_\nu(\tau_\nu = 0) = a + b\mu = S_\nu(\tau_\nu = \mu)$$

so, e.g., at normal incidence (μ=1):

emergent intensity equals S_ν at τ_ν = 1 !

Also note how this elegantly explains limb darkening!

the source function and 'LTE'

in Thermodynamic Equilibrium, at temperature T:

$$\frac{j_\nu}{\kappa_\nu} = B_\nu(T)$$

with B_ν the Planck function (Kirchhoff's Law).

In presence of a mild *T*-gradient, as long as

$T\,(dT/dr)^{-1} \ll$ *photon mean free path*

and

all emission thermal *(no scattering)*

the emissivity depends only on local conditions:

'Local Thermodynamic Equilibrium', LTE.

Then, $S_\nu = B_\nu(T(\mathbf{r}))$; and Saha, Boltzmann eq. apply

to BB or not to BB?

at any frequency, see radiation field roughly corresponding to $\tau_\nu \sim 1$;

corresponds to different physical depths for different ν, (*)

so even if $S_\nu = B_\nu(T(\mathbf{r}))$,

the emergent intensity will not be BB!

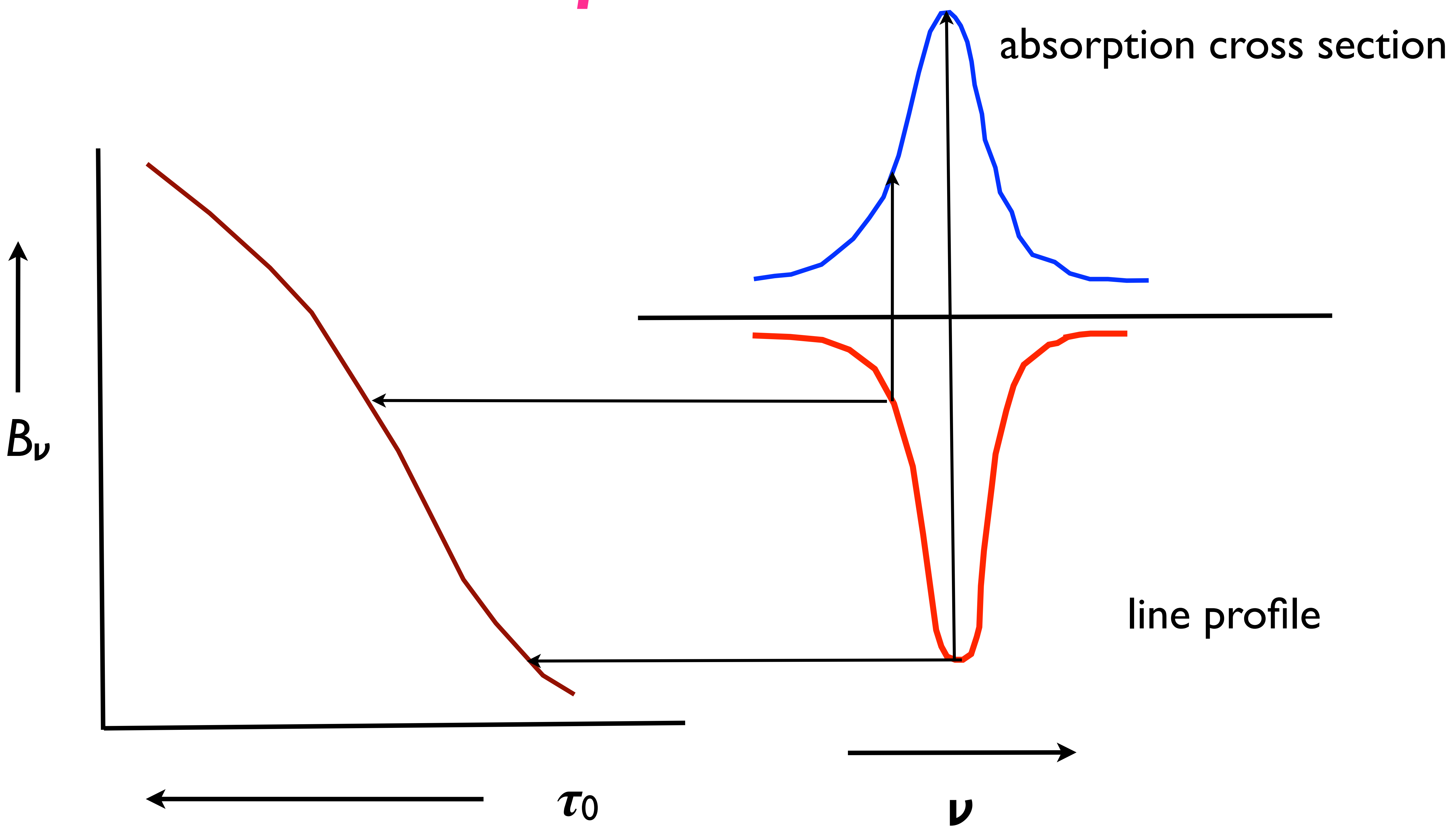
It will be $\sim B_\nu(T(\tau_\nu \sim 1))$*!*

(more generally, $S_\nu(\tau_\nu \sim 1)$)

We should not expect the NS spectrum to be BB!

(*) Recall how ff and bf opacities go as ν^{-3} !

this also ~explains appearance of absorption lines



another classical example: the 'scattering atmosphere'

assume *isothermal* atmosphere, include electron scattering:

$$S_\nu = \frac{\kappa_\nu B_\nu(T) + n_e \sigma_\mathrm{T} J_\nu}{\kappa_\nu + n_e \sigma_\mathrm{T}}; \qquad J_\nu \equiv \frac{1}{2}\int_{-1}^{1} I_\nu d\mu$$

set $\epsilon_\nu \equiv \dfrac{\kappa_\nu}{\kappa_\nu + n_e \sigma_\mathrm{T}}$

then

$$S_\nu = \epsilon_\nu B_\nu(T) + (1 - \epsilon_\nu) J_\nu$$

introduce moments of I_ν; already seen J_ν:

$$H_\nu \equiv \frac{1}{2}\int_{-1}^{1} I_\nu \mu \, d\mu; \quad K_\nu \equiv \frac{1}{2}\int_{-1}^{1} I_\nu \mu^2 d\mu$$

another classical example: the 'scattering atmosphere'

here is the trick: even in a mildly anisotropic radiation field

$$K_\nu = \frac{1}{3} J_\nu \qquad \text{(the 'Eddington approximation')}$$

so then

$$\frac{1}{3}\frac{d^2 J_\nu}{d\tau_\nu^2} = J_\nu - S_\nu = \epsilon_\nu (J_\nu - B_\nu)$$

and since B_ν is constant:

$$\frac{1}{3}\frac{d^2}{d\tau_\nu^2}(J_\nu - B_\nu) = \epsilon_\nu (J_\nu - B_\nu)$$

assume ϵ_ν independent of depth,

another classical example: the ‘scattering atmosphere’

solution:

$$J_\nu - B_\nu = \text{const.} \times \exp(-\sqrt{3\epsilon_\nu}\tau_\nu)$$

fix the constant by using boundary condition at $\boldsymbol{\tau_\nu} = 0$:

no incident radiation. Using a ‘two stream approximation’,

find: at $\boldsymbol{\tau_\nu} = 0$

$$J_\nu = \frac{1}{\sqrt{3}}\frac{dJ_\nu}{d\tau_\nu} \;\Rightarrow\; \text{const.} = -\frac{B_\nu}{1+\sqrt{\epsilon_\nu}}$$

now the emergent flux: from the first moment of the transfer equation, find

$$H_\nu = \frac{dK_\nu}{d\tau_\nu} = \frac{1}{3}\frac{dJ_\nu}{d\tau_\nu}$$

another classical example: the ‘scattering atmosphere’

so that

$$H_\nu(\tau_\nu = 0) = \frac{1}{\sqrt{3}} \, \frac{\sqrt{\epsilon_\nu}}{1 + \sqrt{\epsilon_\nu}} \, B_\nu(T)$$

and for small ϵ_ν:

$$H_\nu(\tau_\nu = 0) \approx \frac{1}{\sqrt{3}} \sqrt{\epsilon_\nu} \, B_\nu(T)$$

For free-free absorption, ϵ_ν is a steep function of frequency!!
So: *even* in an isothermal atmosphere, when scattering is important, emergent spectrum is ***not*** a *constant* x $B_\nu(T)$!!!!

So now we have shown:
spectrum is $B_\nu(T)$ only for the unrealistic case of an isothermal atmosphere without any scattering !

Source function:

depends on the ionization balance
excitation

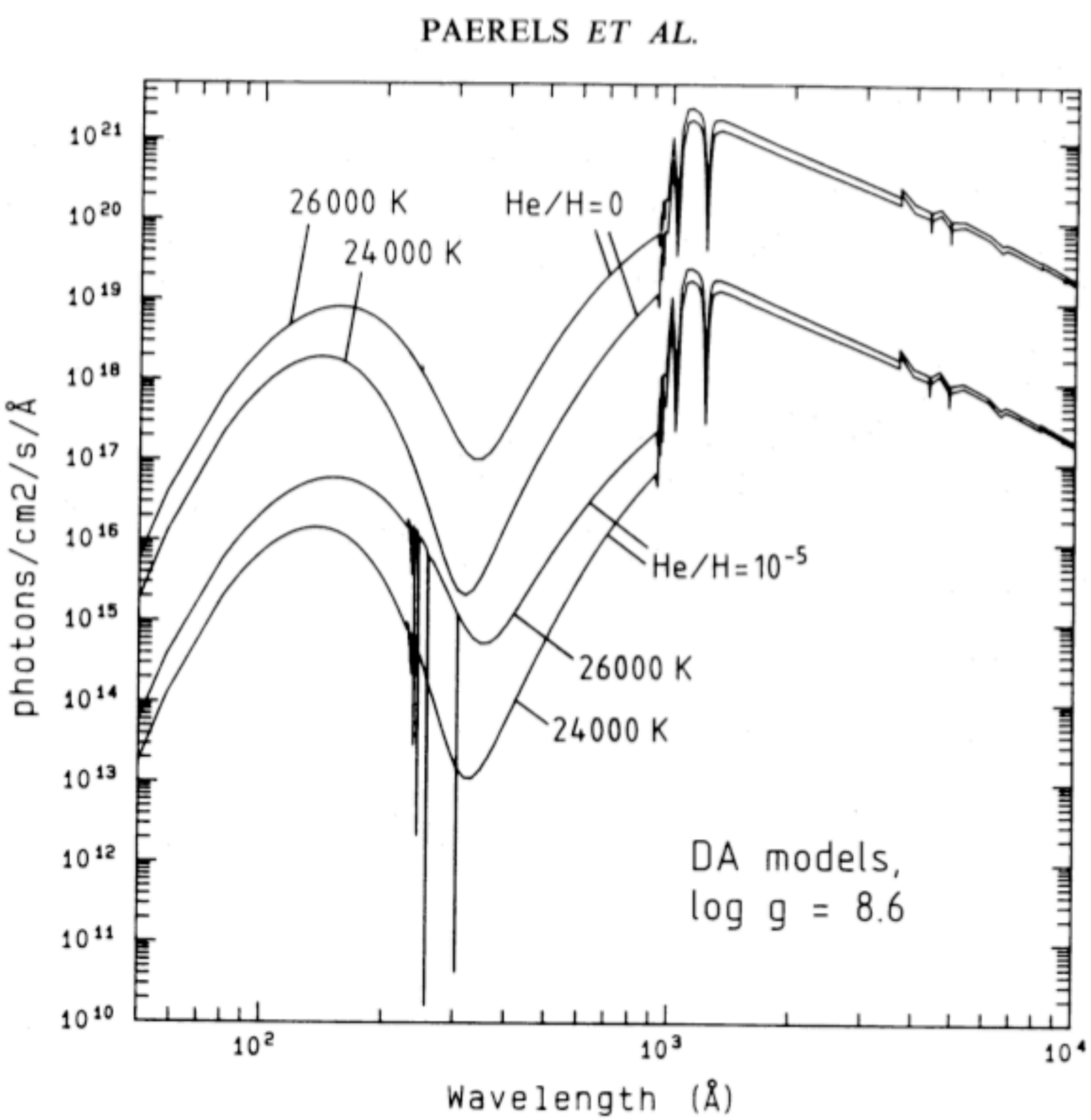
in difficult cases, depends on the distant radiation field (scattering, photoionization, photoexcitation)

When S_ν explicitly depends on the excitation balance and/or I_ν :"**NLTE**"

Have to solve the rate equations together with radiative transfer...
This is a numerical nightmare, but a huge physical bonus!

example:

simple interpretation of the continuum; measurement of T_{eff}

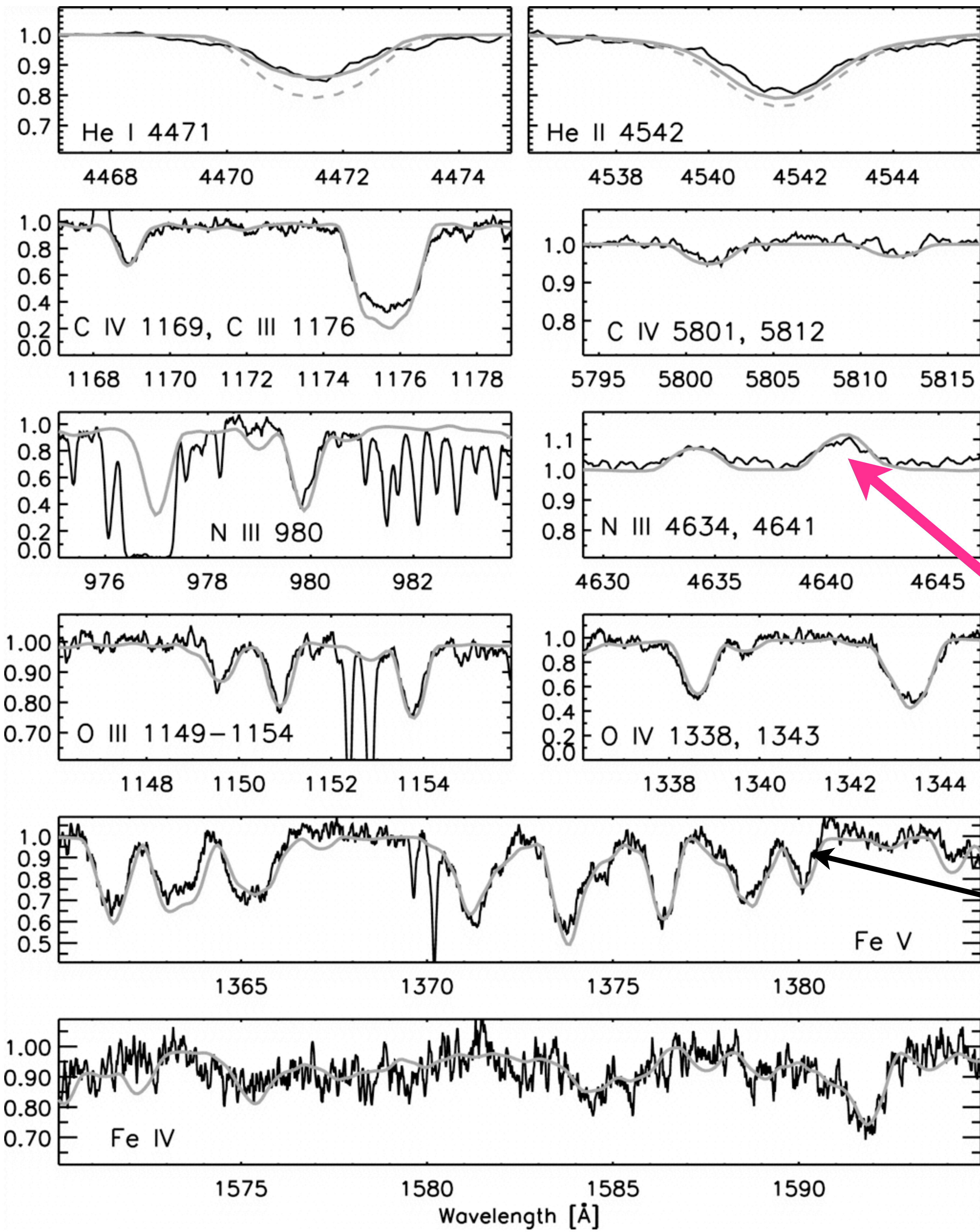


models for hot, pure H white dwarf atmosphere; note the H Lyman lines and edge, and the peculiar shape of the X-ray continuum

Analysis of the soft X-ray spectrum of Sirius B

Paerels et al., 1988

O6.5 II(f) star AV 15



Heap, Lanz, Hubeny (2006)

$T_{\rm eff}$=37,000 K

log g = 3.5

N III appears in emission!

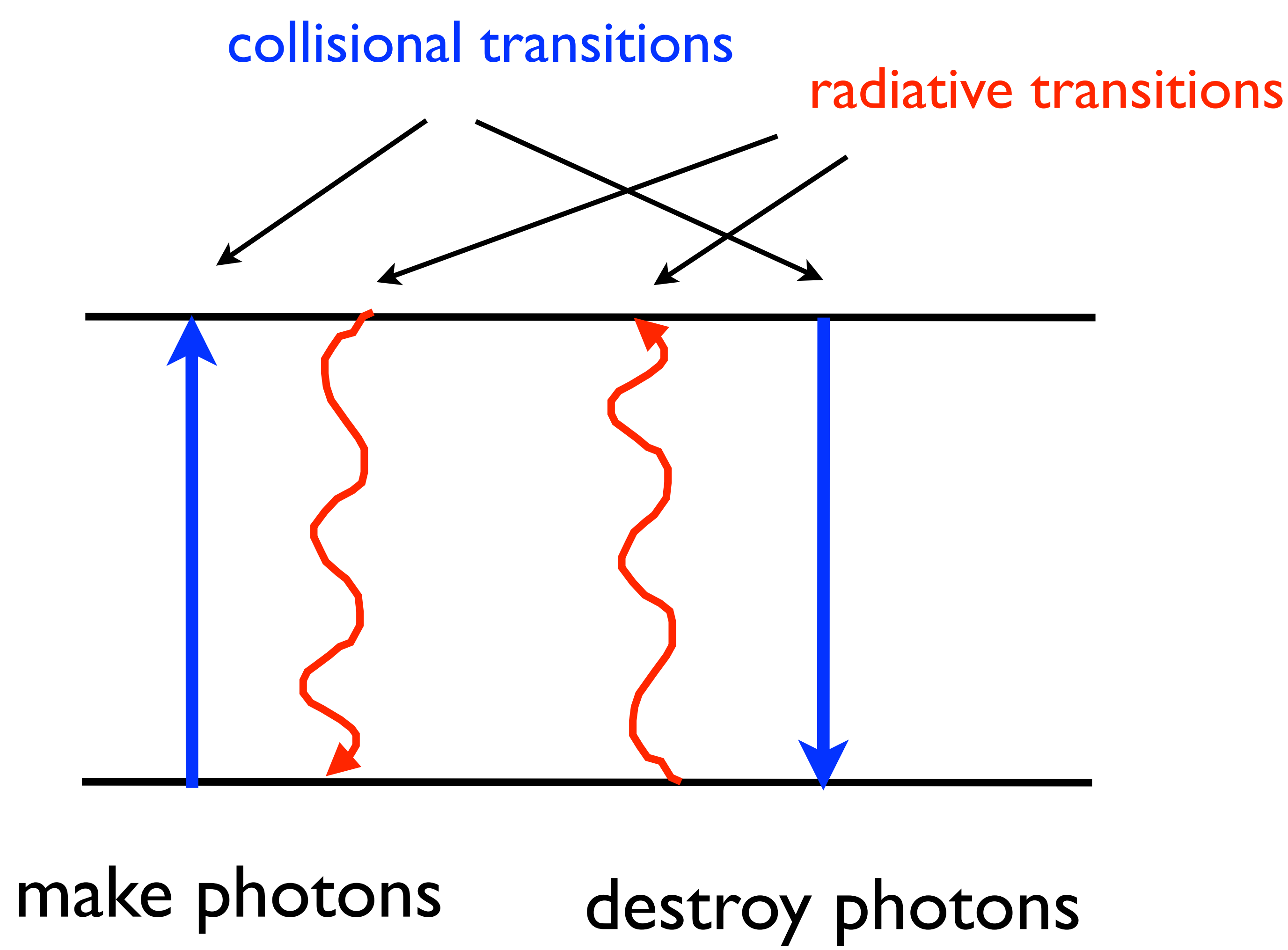
solid grey line: model

modern models match complex spectra in detail when should we expect LTE to break down?

whenever the close interchange between thermal energy and radiative energy is lost:

collision-induced transitions are being overwhelmed

example with the two-level atom



but when $A_{21} > n_e C_{21}$: photons scatter, and ‘travel far’; level populations depart from Boltzmann, now coupled to distant radiation field:

lines can go into emission!

(happens in Fe n=1-2 even at $n_e \sim 10^{23}$ cm^{-3}!! because A_{21} is very large!)

A_{21}: spontaneous radiative decay

C_{21}: collisional deexcitation

so: detailed line spectroscopy:
T_{eff} (ionization/excitation balance),
abundances, ***B*** fields, rotation,
gravitational redshift, and: log g

Applications to Neutron Stars:

tomorrow!