Specific heat in the BCS-BEC crossover regime of an ultracold Femi gas

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Collaborators:
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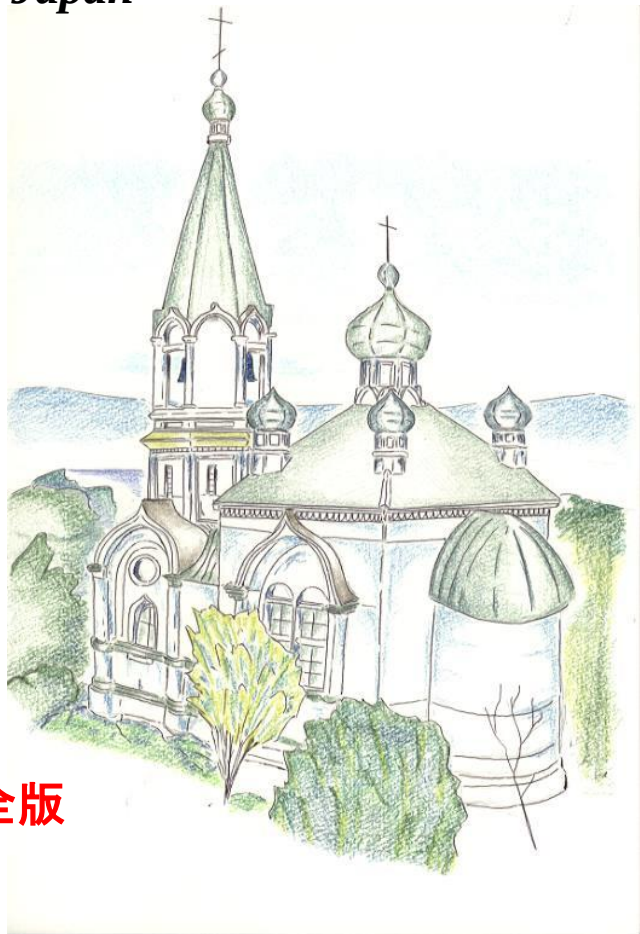
- **Introduction: cold Fermi gas and BCS-BEC crossover**
- **Formulation: Gaussian fluctuation (NSR) theory**
- **Specific heat**
  Determination of "pairing-fluctuation" regime
- **Summary**



これは当日使ったもののダイジェスト版です。図も含めた完全版をご希望の方は大橋までご連絡ください

キックオフ(理研）
2012/10/26-27

理論班研究会
(西浦）2013/3/3

NO PHOTO (?)

公募キックオフ
(東北大）2013/9/12-13

第2回ウインタースクール＆
研究会(理研）2013/12/25-28

B03(?) 国際workshop
（ポハン）2014/5/12-13

第3回研究会（熱川）
2014/9/23-25

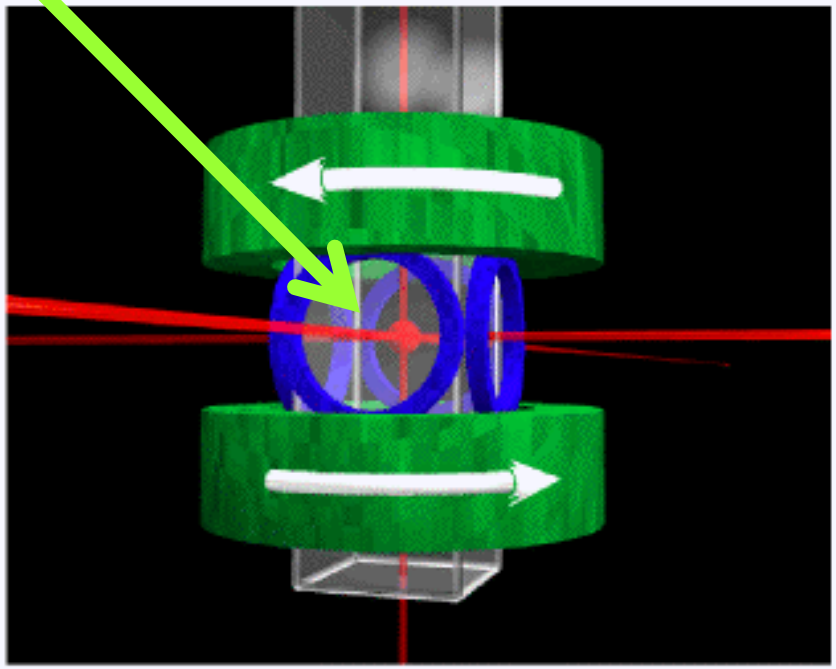
第3回ウインタースクール
（阪大）2014/12/22-23

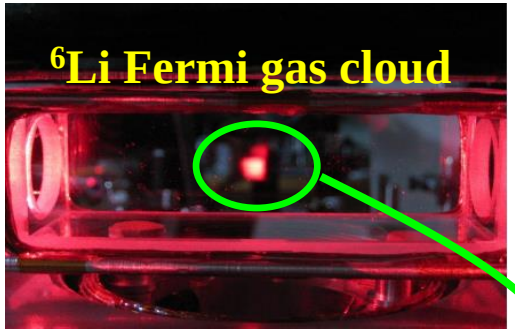
掲載予定（？）

（京大）2015/3/12-14

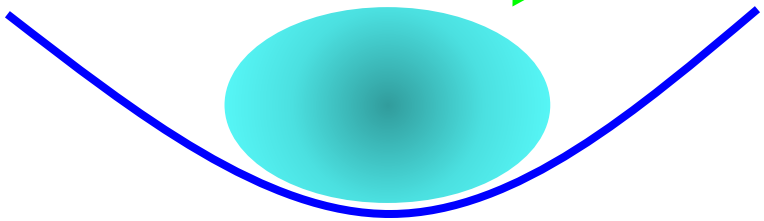
Introduction: Ultracold Fermi gas physics

Fermi atoms (^{6}Li, ^{40}K) are trapped in a magnetic/optical potential, and are cooled down to <O(μK), where various quantum phenomena can be observed, such as superfluidity.



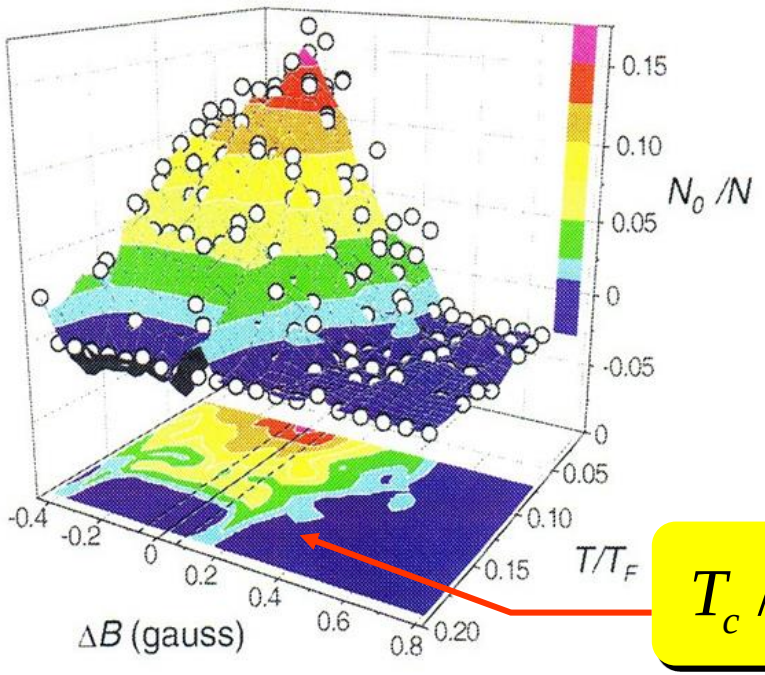


Homepage of Gonokami Lab, University of Tokyo



trap potential

Introduction: superfluid ^{40}K Fermi gas (2004)



$$|9/2,-7/2\rangle + |9/2,-9/2\rangle$$

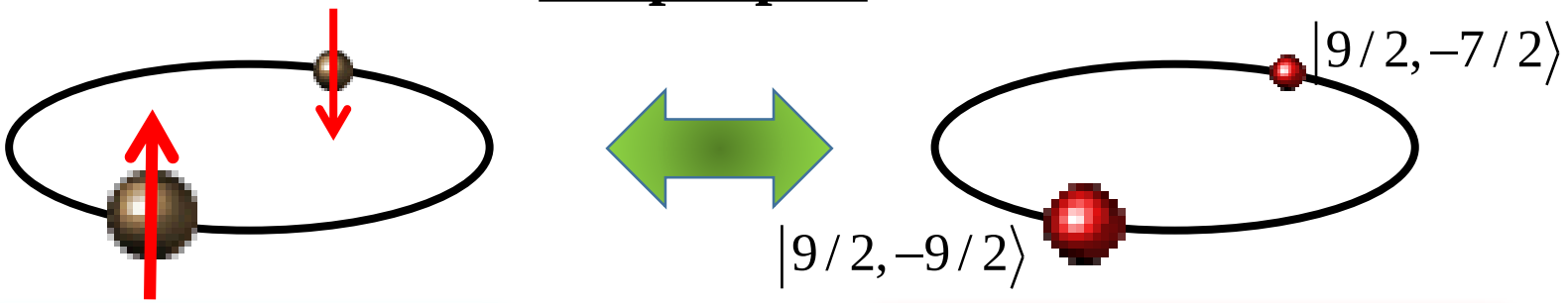
$$T_F = 0.35\mu K$$

$$N = 10^5$$

C. A. Regal, et al. PRL 92 (2004) 040403.

$$T_c/T_F \sim 0.08-0.2 >> 10^{-4}-10^{-2}\,(\text{metal})$$

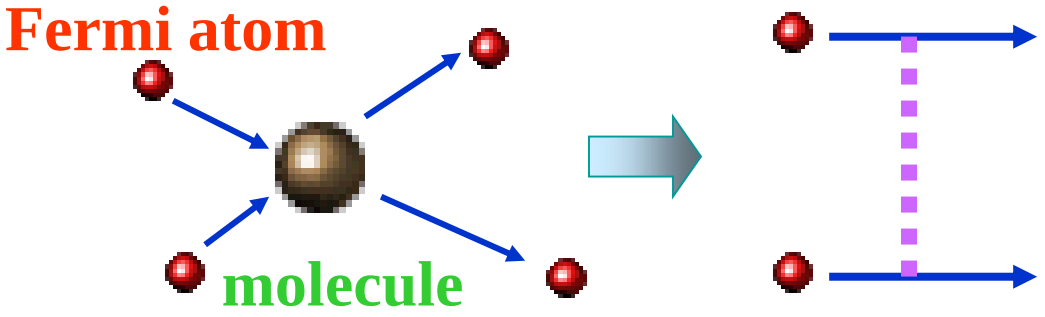
Cooper pair



superconductivity

Fermi gas superfluid

Feshbach resonance: pairing mechanism of superfluid Fermi gas



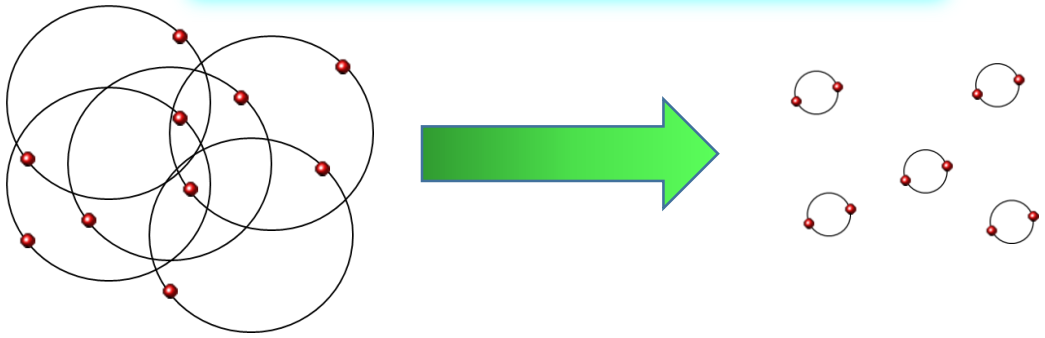
$$V_{eff} = -g^2 \frac{1}{2\nu}$$

tunable by magnetic field

(Timmermans (2001), Holland (2001))

tunable **pairing interaction**

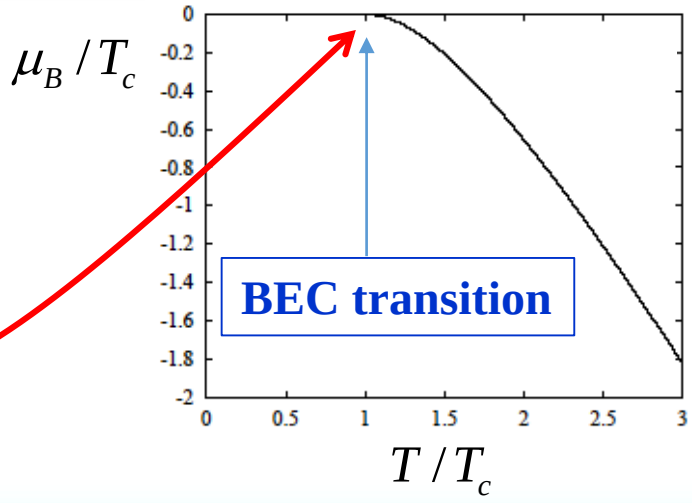
BCS-BEC crossover



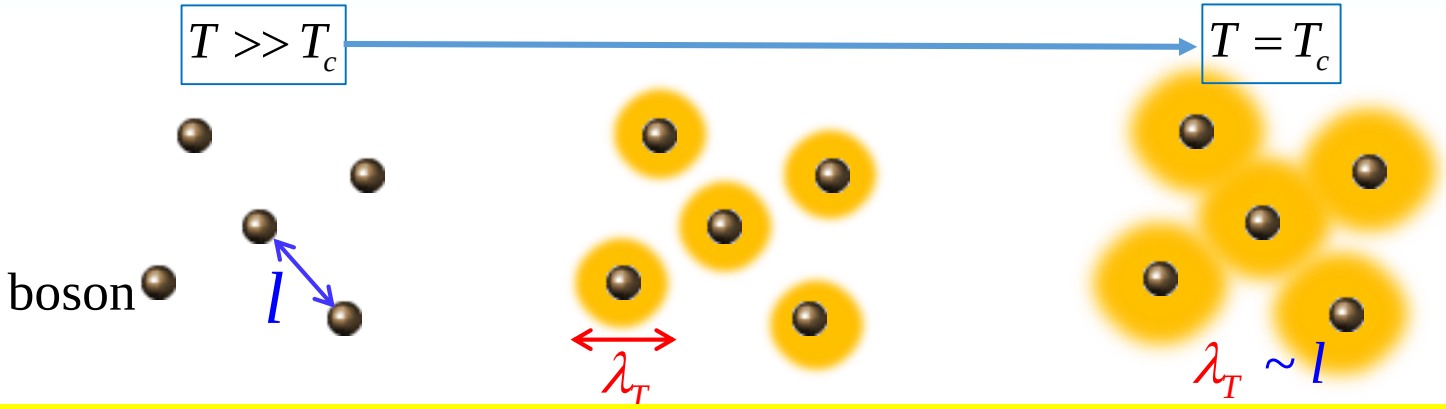
Essence of BCS-BEC crossover

Bose-Einstein condensation (BEC) of an ideal Bose gas

$$N = \sum_{\mathbf{q}} \frac{1}{e^{\beta(\varepsilon_{\mathbf{q}} - \mu_B)} - 1}$$

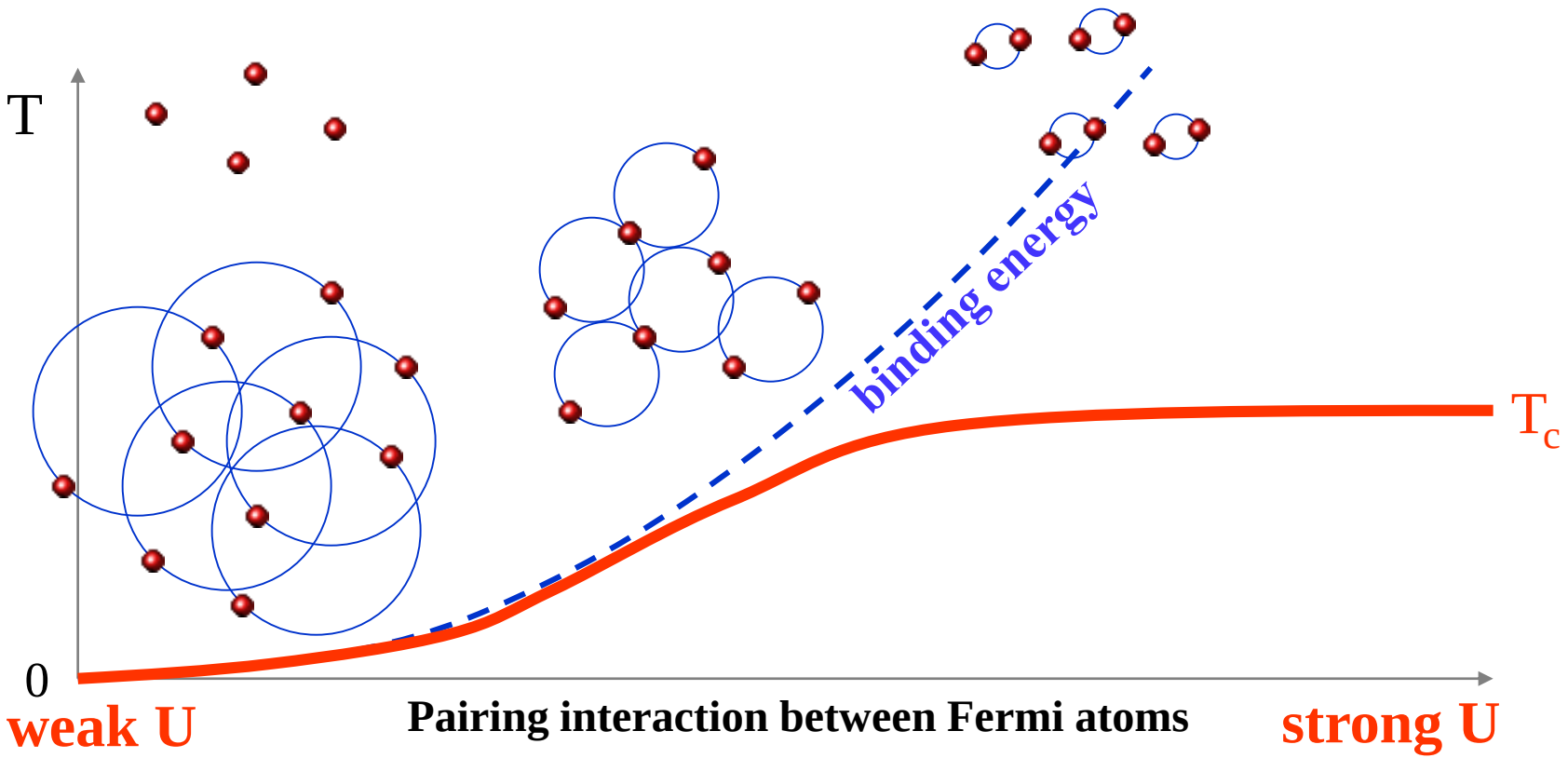


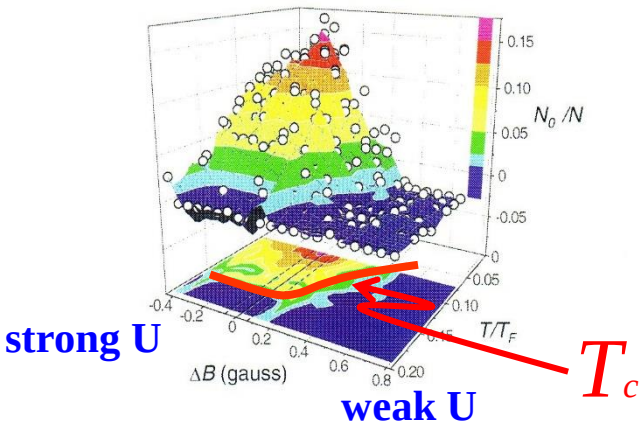
Thermal de Broglie length: $\lambda_T = \dfrac{h}{\sqrt{2\pi m T}}$ ← **quantum (statistical) size of a particle**



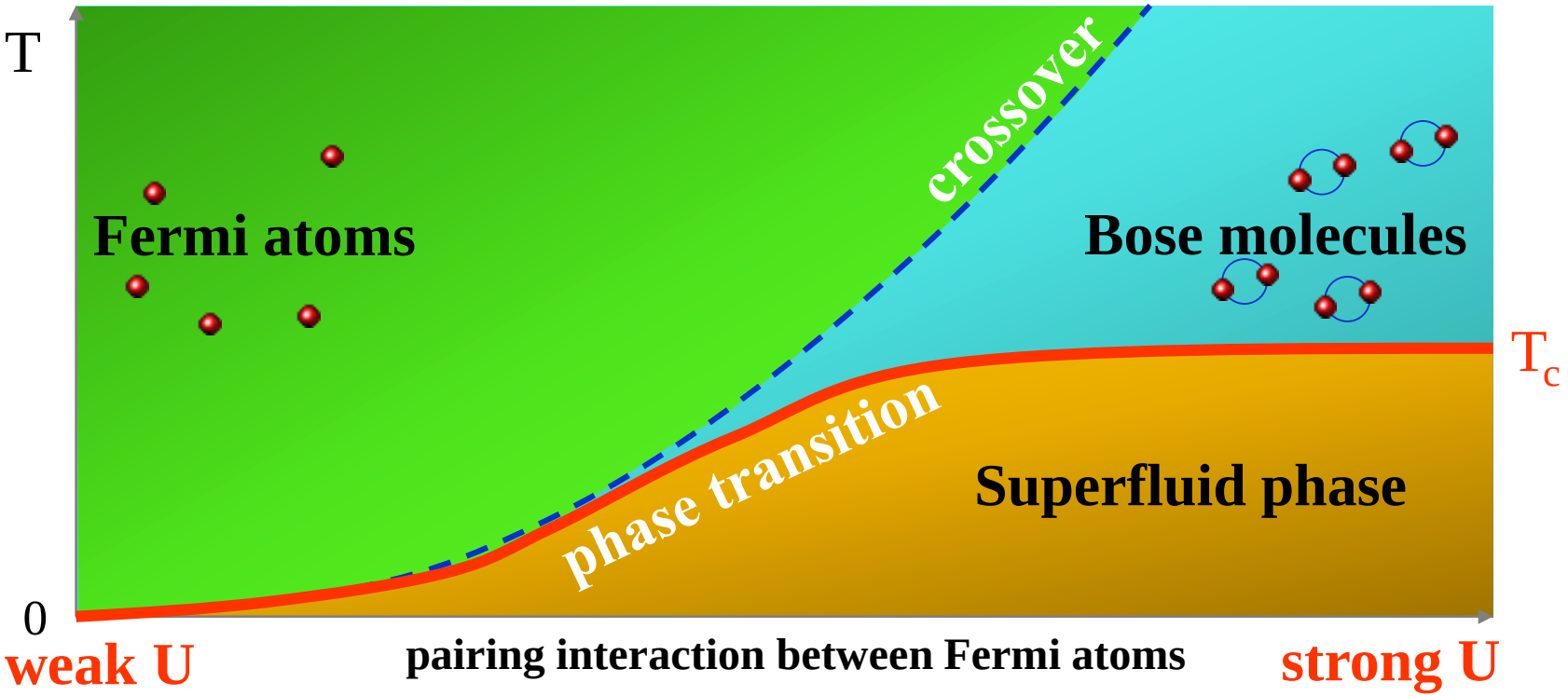
The BEC occurs, when the quantum size of a particle reaches the interparticle distance.

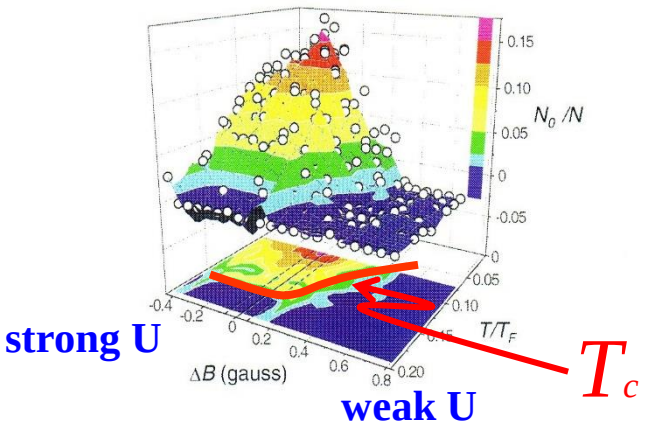
Essence of BCS-BEC crossover phenomenon





Phase diagram of ultracold Fermi gas





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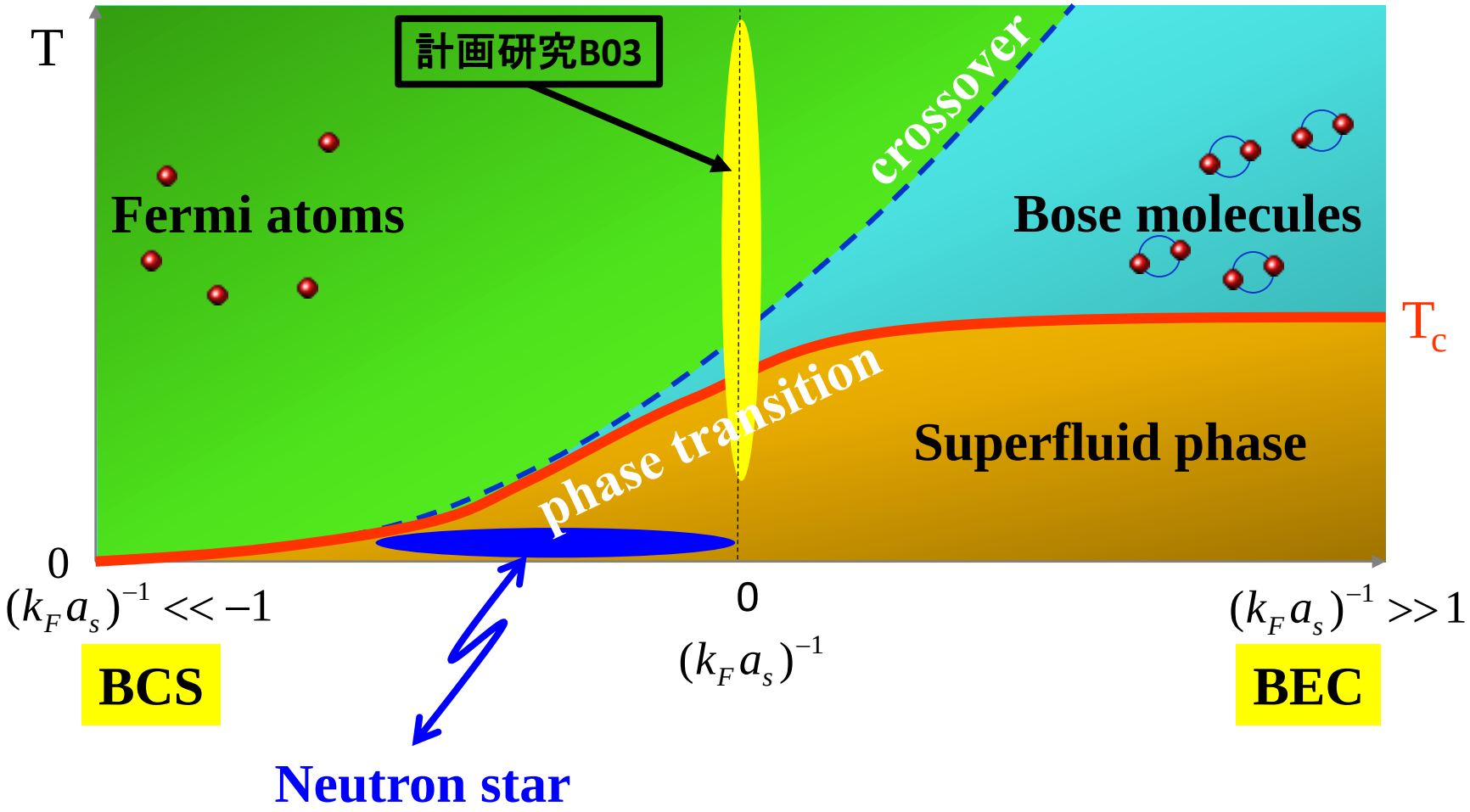

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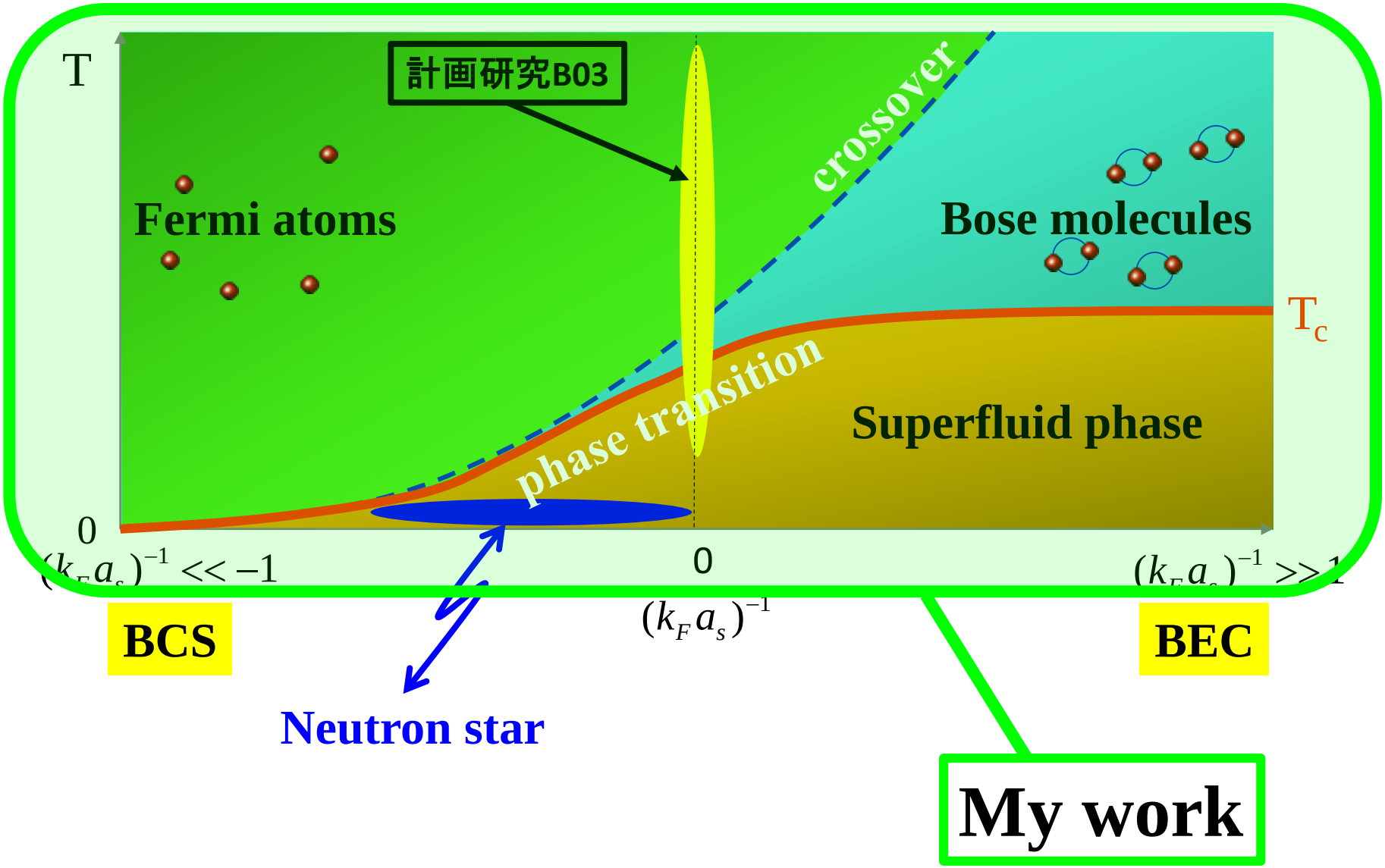
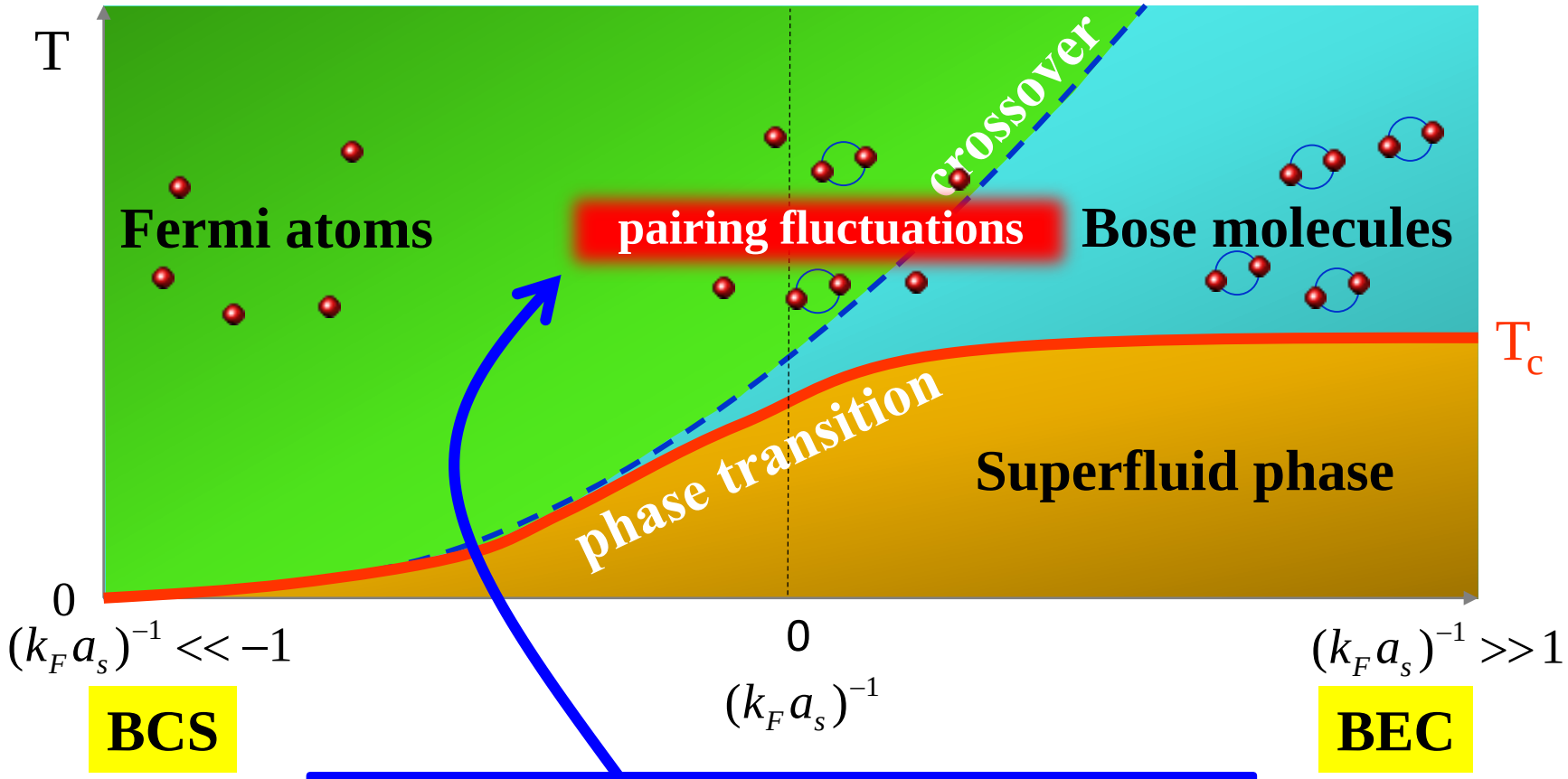


Phase diagram of ultracold Fermi gas

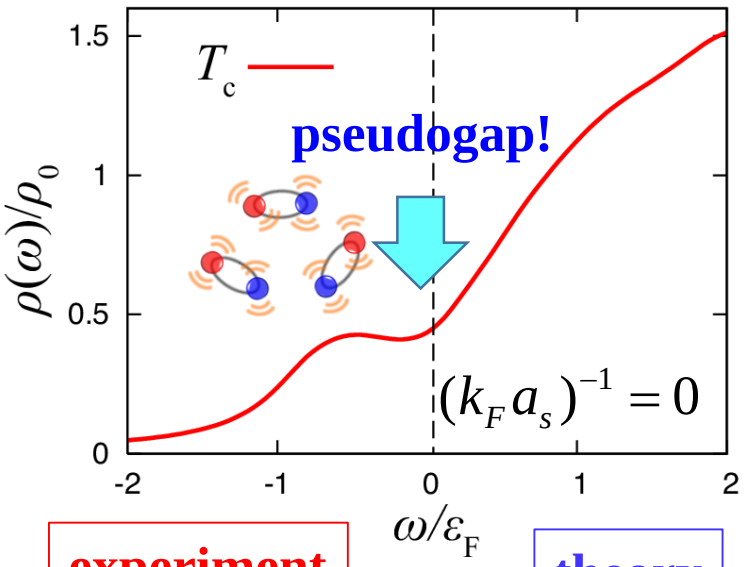


"preformed Cooper-pair" formation

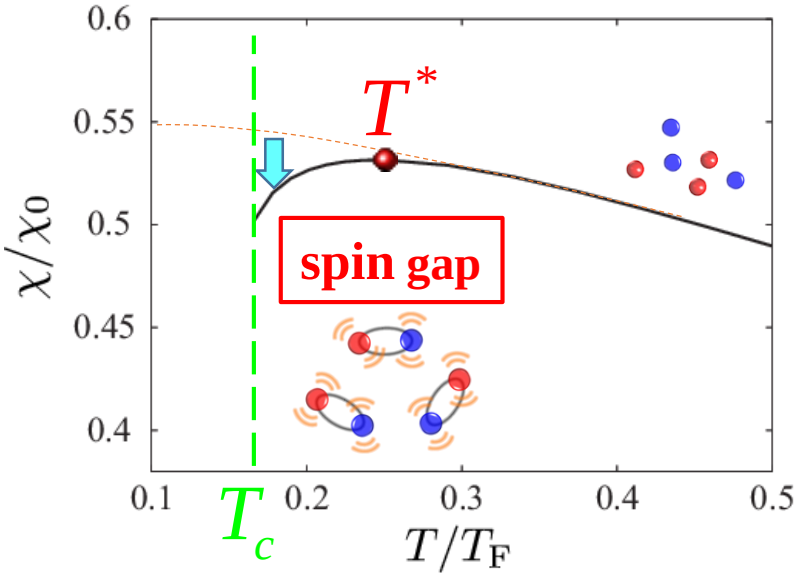
- **BCS-like gap in the density of states (pseudo-gap)**
- **Suppression of spin excitations (spin-gap)**

Determination of "crossover" temperature T*

density of states

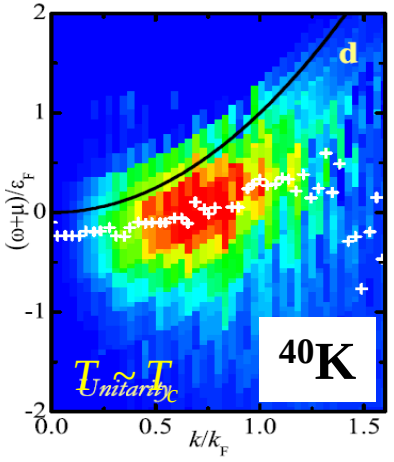


spin susceptibility



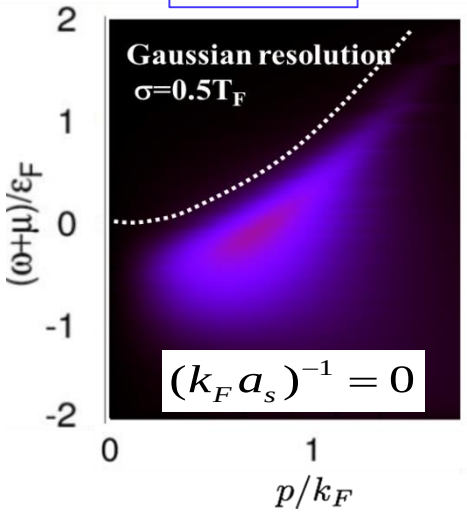
single-particle excitation energy

experiment

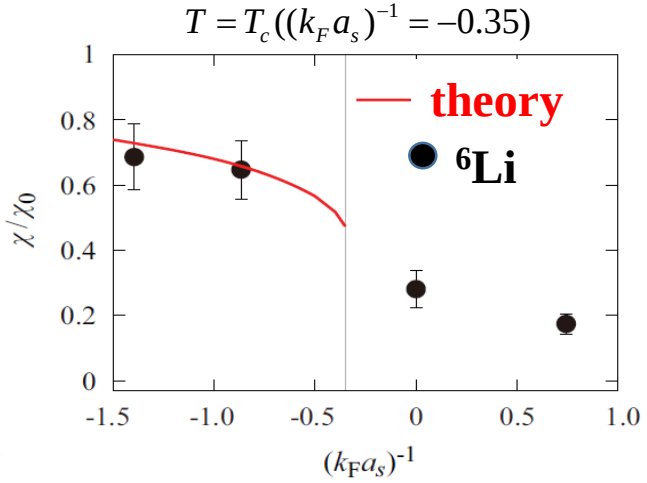


JILA, Nature **454**, 744 (2008).

theory



Tsuchiya, Watanabe, Ohashi, PRA 82 (2010) 033629

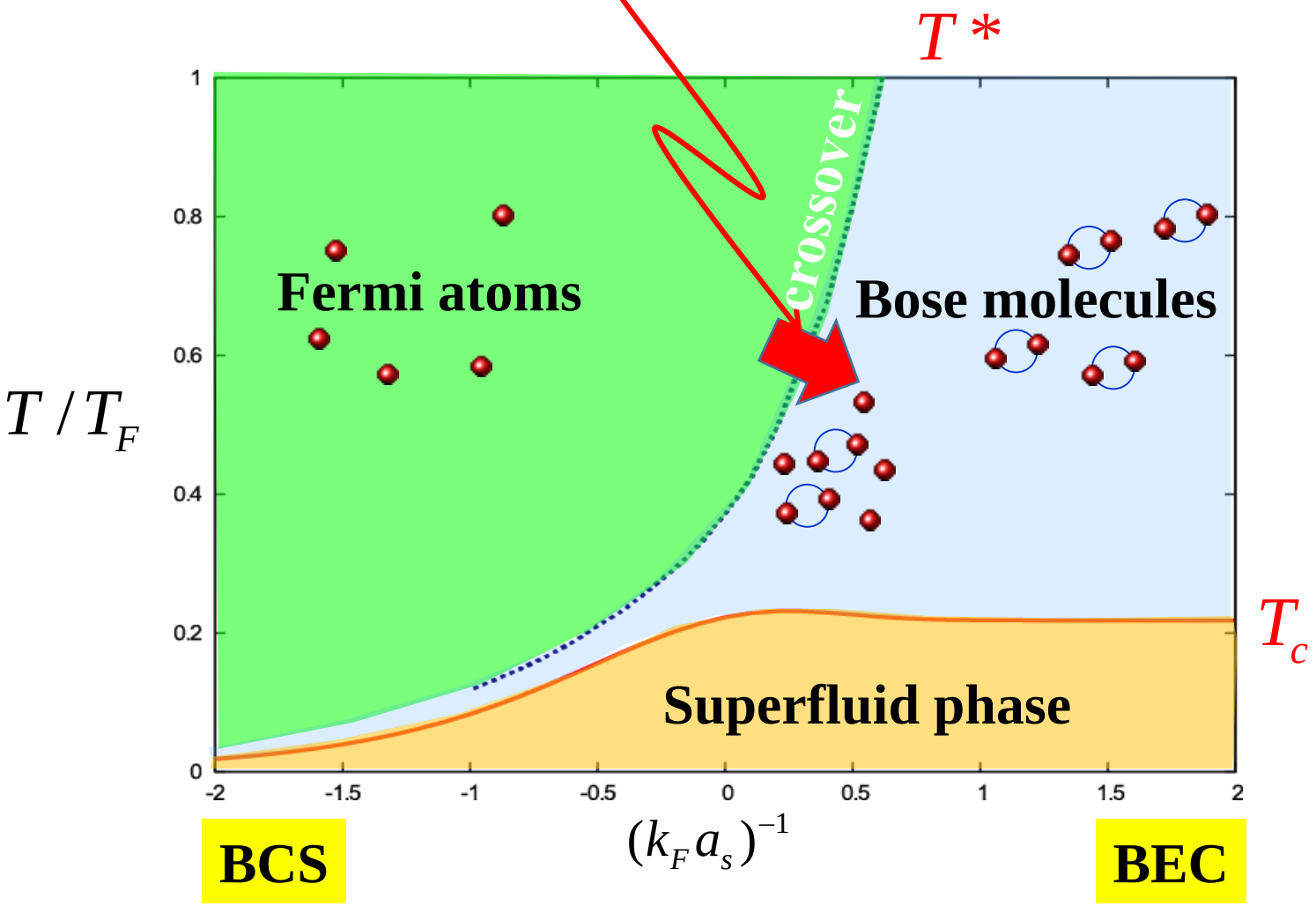


Kashimura, Watanabe, Ohashi, PRA 86 (2012) 043622

Experiemtal data: Sanner et.al, PRL 106 (2011) 010402

Determination of “crossover” temperature T*

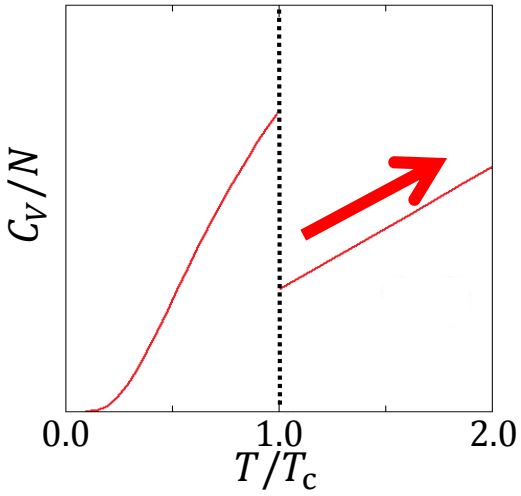
Fermi atoms start to form preformed Cooper pairs (bosons) .



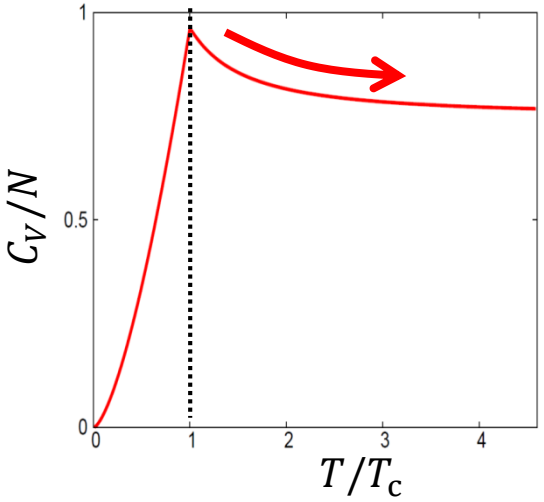
Today's talk

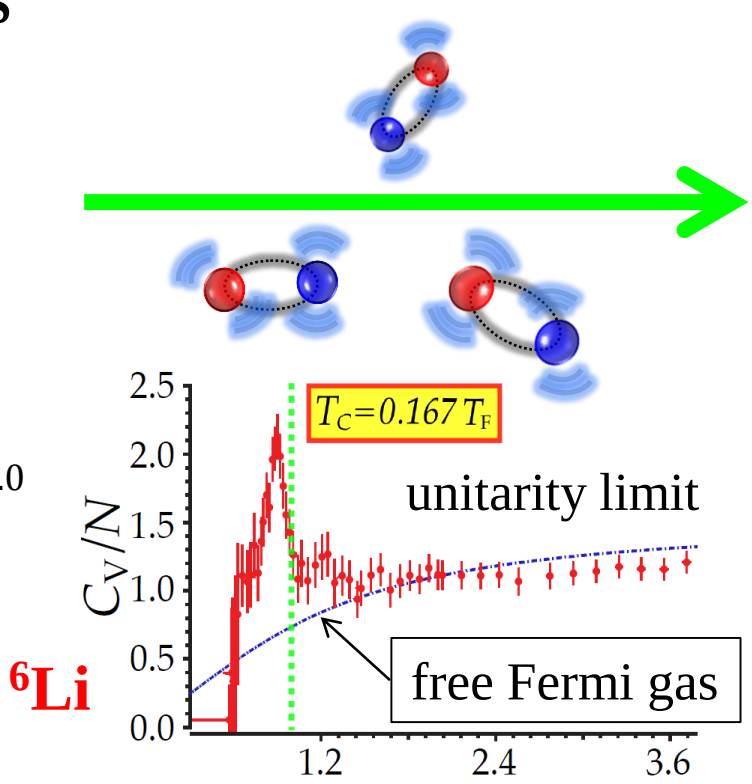
We study specific heat C_v in the BCS-BEC crossover regime of an ultracold Fermi gas. We show that Cv is a useful quantity to determine T which physically distinguish between the Bose gas regime and fluctuation regime.**

weak-coupling BCS



ideal Bose gas





M. Ku *et al.* Science 335, 563 (2012)

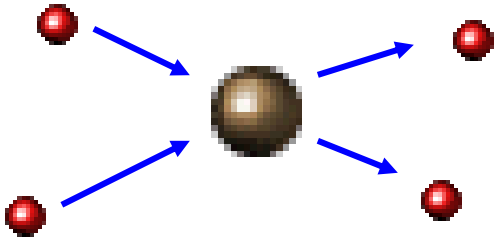
- $C_V \sim \langle E^2 \rangle - \langle E \rangle^2$ ⟷ **Assessment of strong-coupling theory to evaluate EoS**

Formulation: Gaussian fluctuation (NSR) theory

(Minimal theory to describe the whole BCS-BEC crossover region)

$$H=\sum_{p,\sigma}(\varepsilon_p-\mu)c^{\dagger}_{p\sigma}c_{p\sigma}-U\sum_{p,p'q}c^{\dagger}_{p+q/2\uparrow}c^{\dagger}_{-p+q/2\downarrow}c_{-p'+q/2\downarrow}c_{p'\uparrow}$$

- uniform gas is assumed.
- σ : two atomic hyperfine states = pseudospin $\uparrow,\downarrow$
- U : effective pairing interaction associated with the F.R.



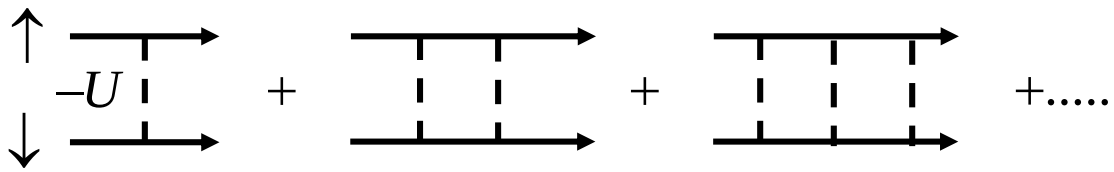
Feshbach resonance

$$\Rightarrow \quad U$$

We treat *U* as a tunable parameter.

Formulation: Tc in the BCS-BEC crossover region

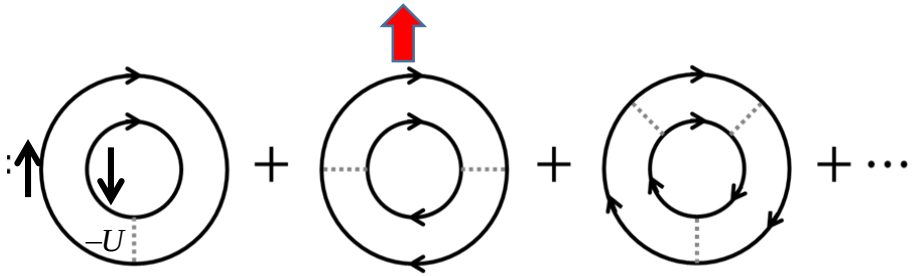
- **Tc: Thouless criterion**

$$\Gamma(\mathbf{q},\omega) =$$ 

pole at $q = \omega = 0$ ⟹

$$1 = U\sum_{\mathbf{p}} \frac{1}{2(\varepsilon_{\mathbf{p}} - \mu)} \tanh\frac{\varepsilon_{\mathbf{p}} - \mu}{2T}$$

- **thermodynamic potential:** $\Omega = \Omega_0 + \Omega_{NSR}$



pairing fluctuations (Gaussian fluctuation level)

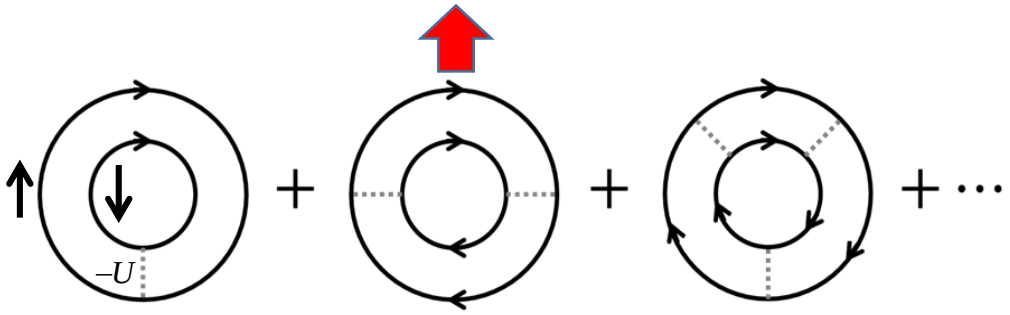
$$N - \frac{\partial\Omega}{\partial\mu} = N_{\text{free}} - T\frac{\partial}{\partial\mu}\sum_{q,\nu_n} e^{i\delta\nu_n}\log\left[1 - U\Pi(\mathbf{q},\nu_n)\right]$$

pair correlation function

Formulation: specific heat in the crossover region (T>Tc)

- **Specific heat:**

$$\Omega = \Omega_0 + \Omega_{NSR}$$



pairing fluctuations

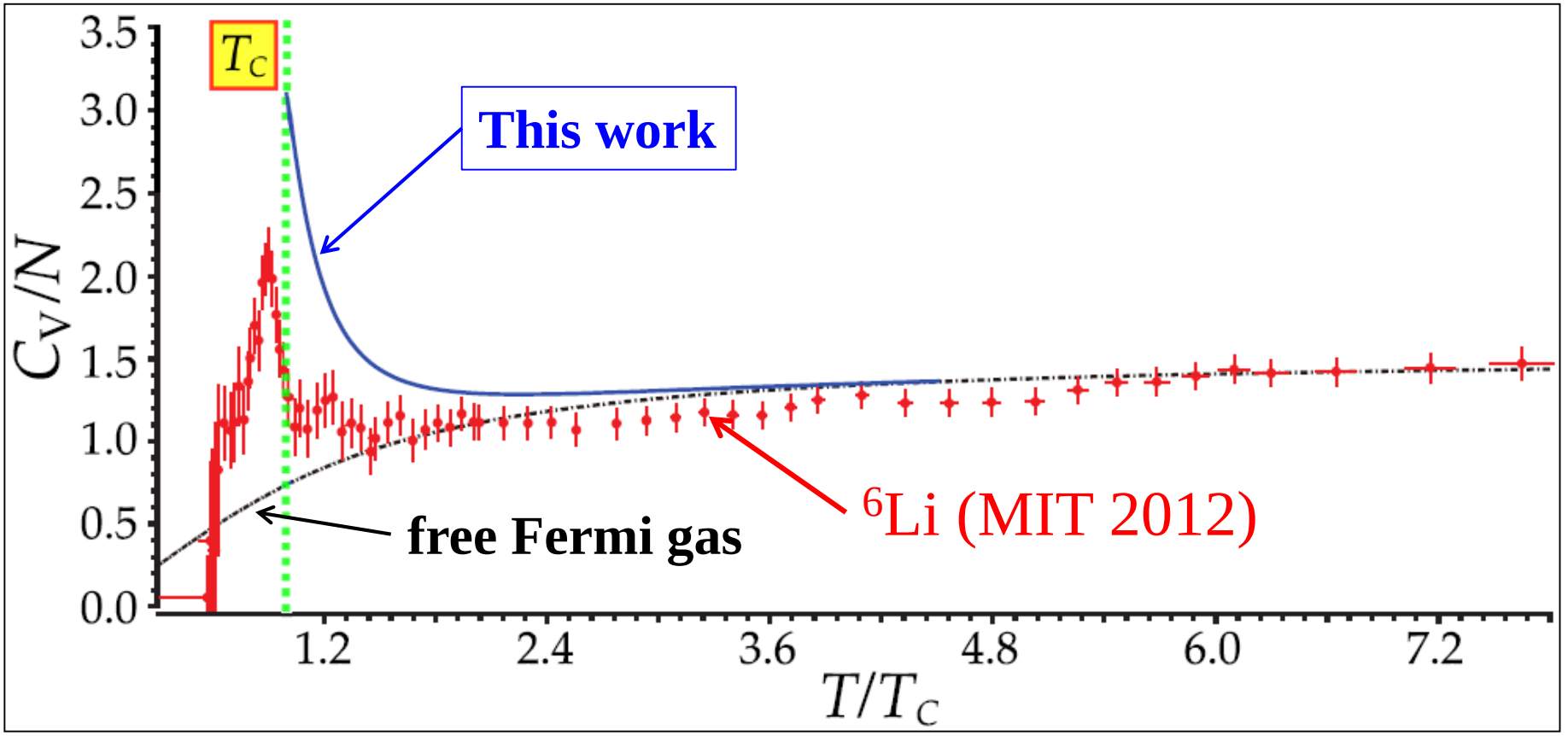
- **Legendre transformation:** $\Omega \rightarrow E$

$$E = E_0 + T\sum_{\boldsymbol{q},i\nu_n} \frac{U}{1 - U\Pi(\boldsymbol{q},i\nu_n)} \left[\mu \frac{\partial \Pi(\boldsymbol{q},i\nu_n)}{\partial \mu} + T\frac{\partial \Pi(\boldsymbol{q},i\nu_n)}{\partial T}\right]$$

$$C_v = \left(\frac{\partial E}{\partial T}\right)_{V,N}$$

pair correlation function

Comparison with recent experiment on a ^{6}Li Fermi gas



The experimental result is affected by finite temperature resolution, which may somehow smear the singularity around Tc.

Summary

Specific heat and effects of pairing fluctuations

We have discussed specific heat in the BCS-BEC crossover regime of an ultra-cold Fermi gas. Within the framework of the NSR-theory, we calculated this thermodynamic quantity as a function of temperature above Tc. From the temperature dependence, we determined the Bose gas regime, as well as the region where the system is dominated by fluctuating preformed pairs, in the phase diagram of an ultra-cold Fermi gas.