

非球対称星の進化計算を目指した 平衡形状の研究

Chiba **I**nstitute of **T**echnology
Nobutoshi Yasutake

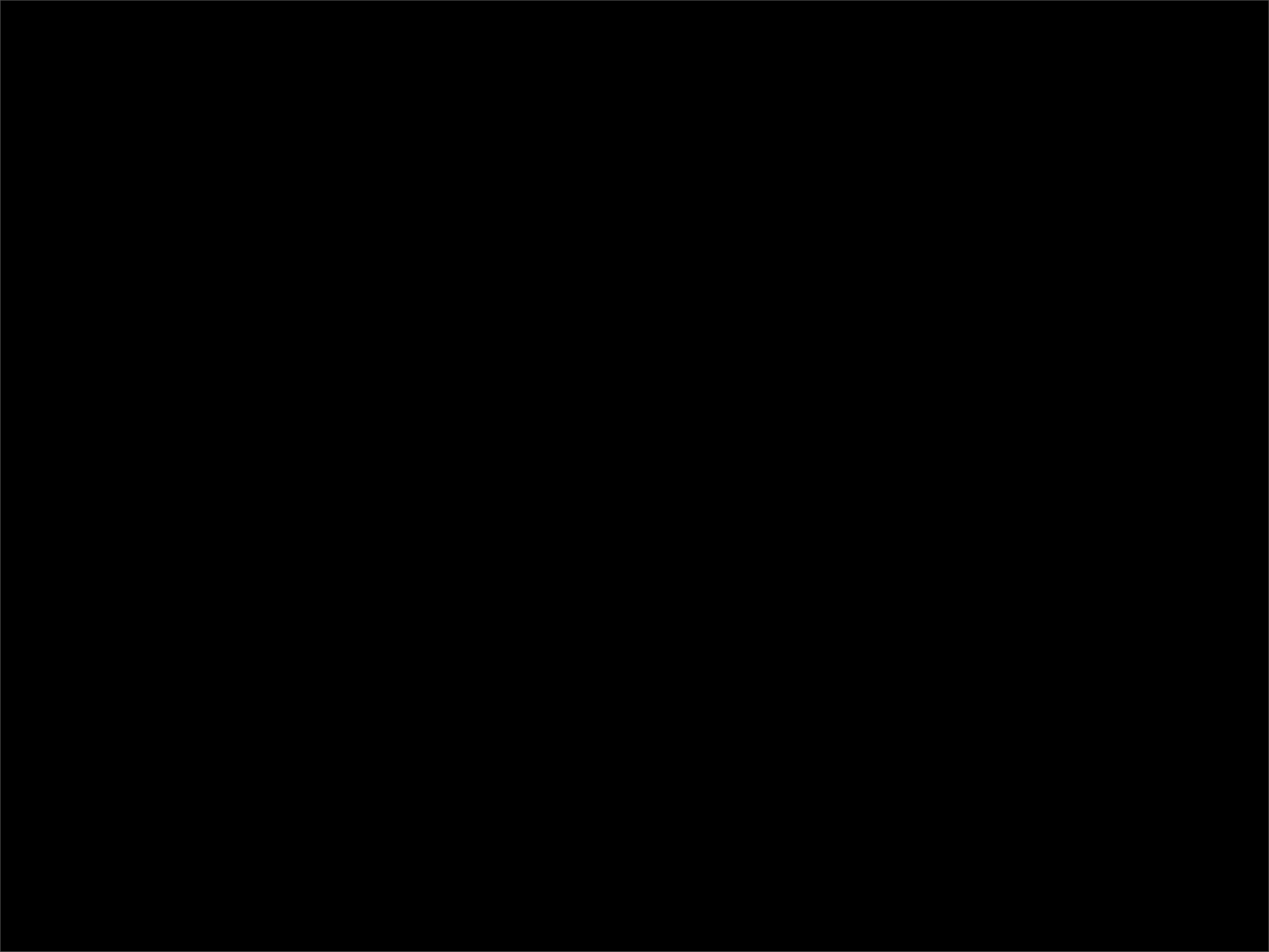
① PASJ(2014) 60, 50

M. Kutsuna, T.Shigeyama(Univ. of Tokyo), **K. Kotake**(Fukuoka univ.),

② MNRAS Letter (2015) 446, 56

K.Fujisawa, S.Yamada(Waseda univ.)

公募研究



脱球対称

“磁場を伴った中性子星の熱的進化”

PASJ(2014) 60, 50

NY, M. Kutsuna, T.Shigeyama(Univ. of Tokyo), **K. Kotake**(Fukuoka univ.),

Temperature distribution

① NY, Kotake, Kutsuna, Shigeyama (2014) PASJ

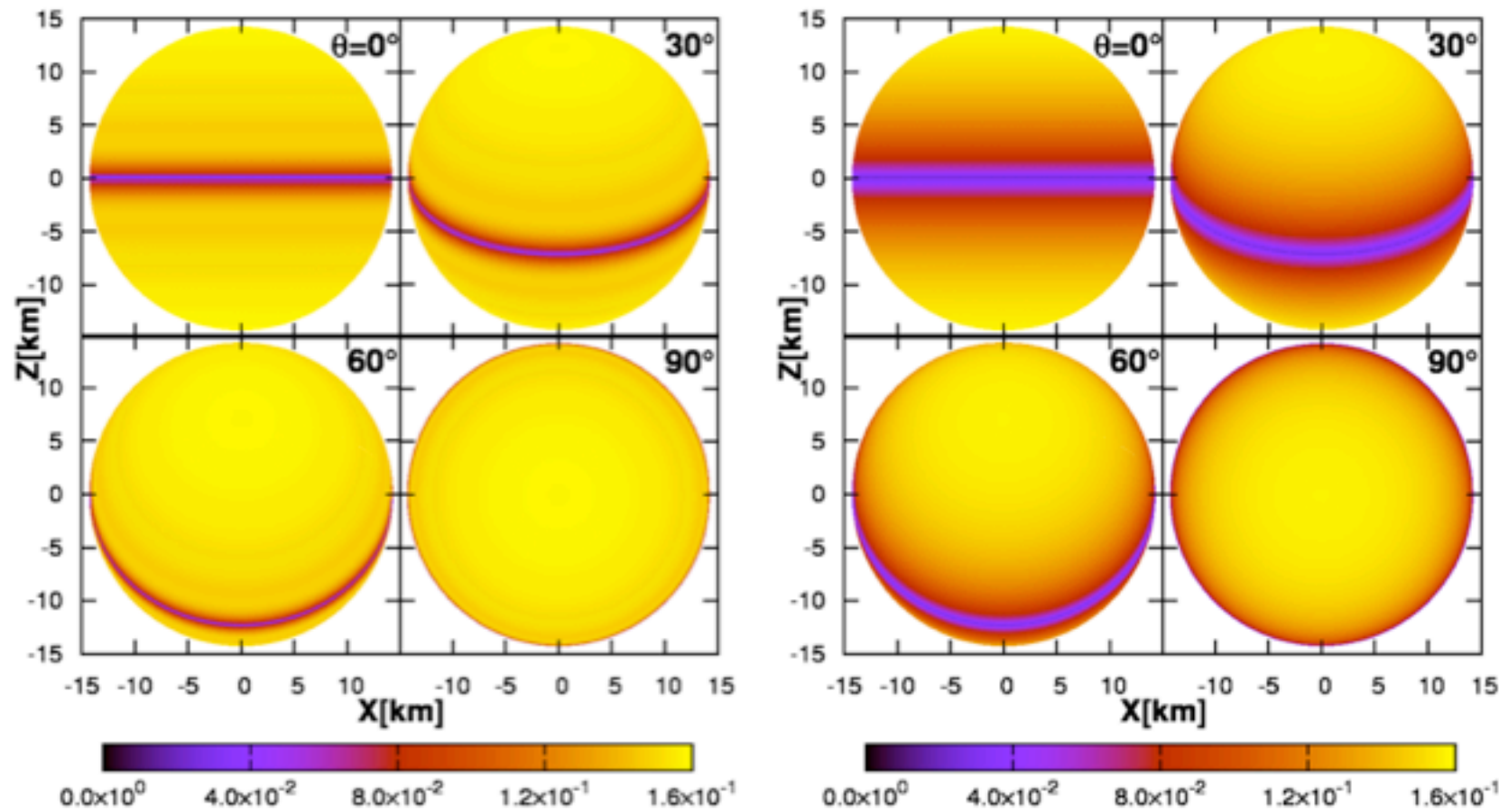
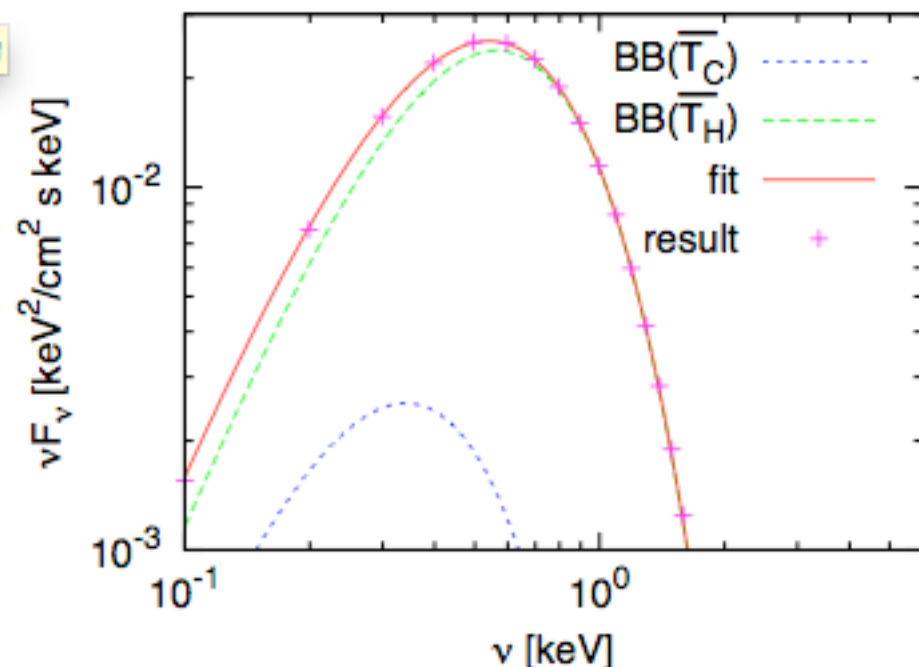


FIG. 7: (Color online) Temperature distribution for model "mSUK" after 10^4 years depended on the inclination angle θ . The unit of color contour is [keV].

移動



Our results

NY, Kotake, Kutsuna, Shigeyama
(2014) PASJ

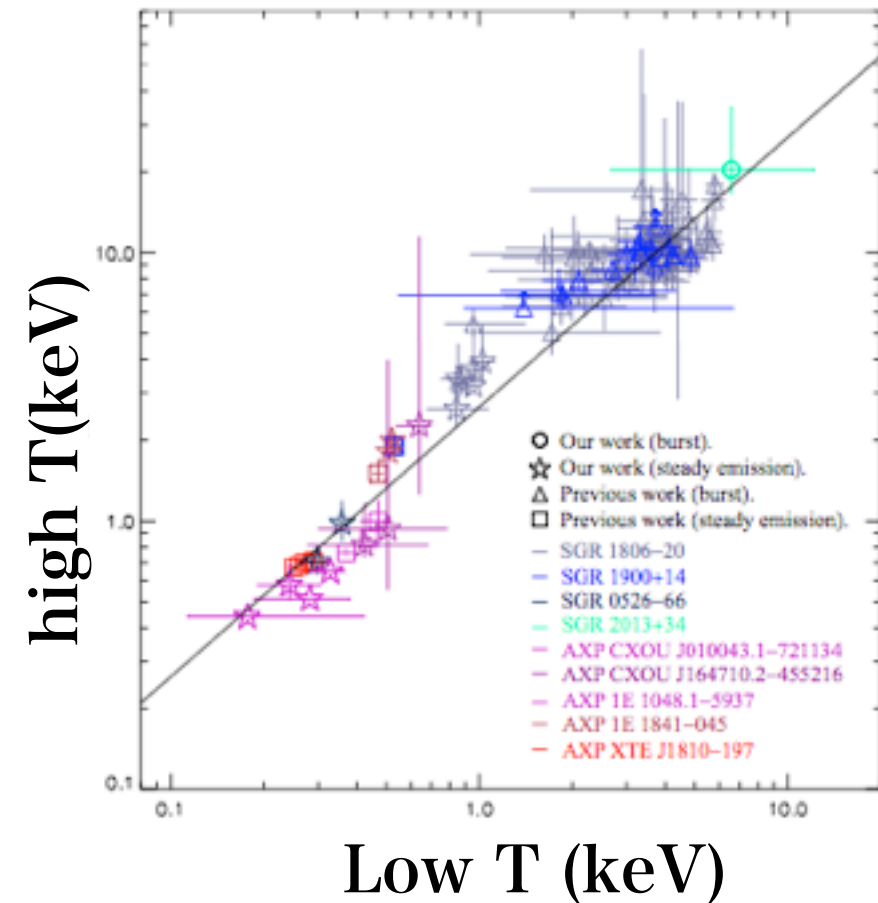


Fig. 3. Relationship between the 2BB temperatures kT_{LT} and kT_{HT} . The triangles and squares denote the previous work on the bursts (Feroci et al. 2004; Olive et al. 2004; Götz et al. 2006a; Nakagawa et al. 2007) and the quiescent emission (Morii et al. 2003; Gotthelf et al. 2004; Gotthelf & Halpern 2005; Tiengo et al. 2005; Mereghetti et al. 2006a), respectively. The circles and stars denote our work on the bursts and the quiescent emission, respectively. The line represents the best-fit power law model.

Observation, Yujin, et al. (2009) PASJ

How to calculate the thermal evolution of compact stars ?

EOS

Quark, hyperon, normal matter,
pion-condensation, kaon-condensation, etc.
(P)NJL, (D)BHF, RMF, variational principle etc.
Landau effects magnetization etc.

structure

w/wo rotation, w/wo magnetic field, axisymmetric etc.

cooling

URCA, MURCA, HURCA, quark beta decay, superfluid
etc.

heating

Ohmic decay, Hall effect, ambipolar diffusion etc.
vortex etc.

evolution

thermal conduction in strong magnetic field
etc.

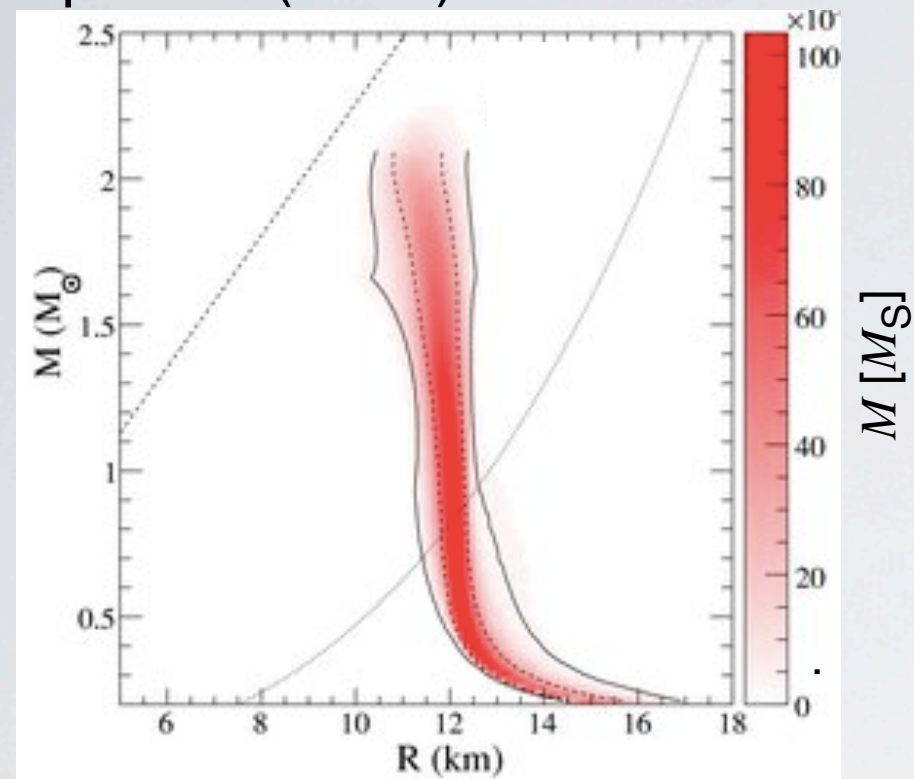
atmosphere
& envelope

Fe including effects strong magnetic field

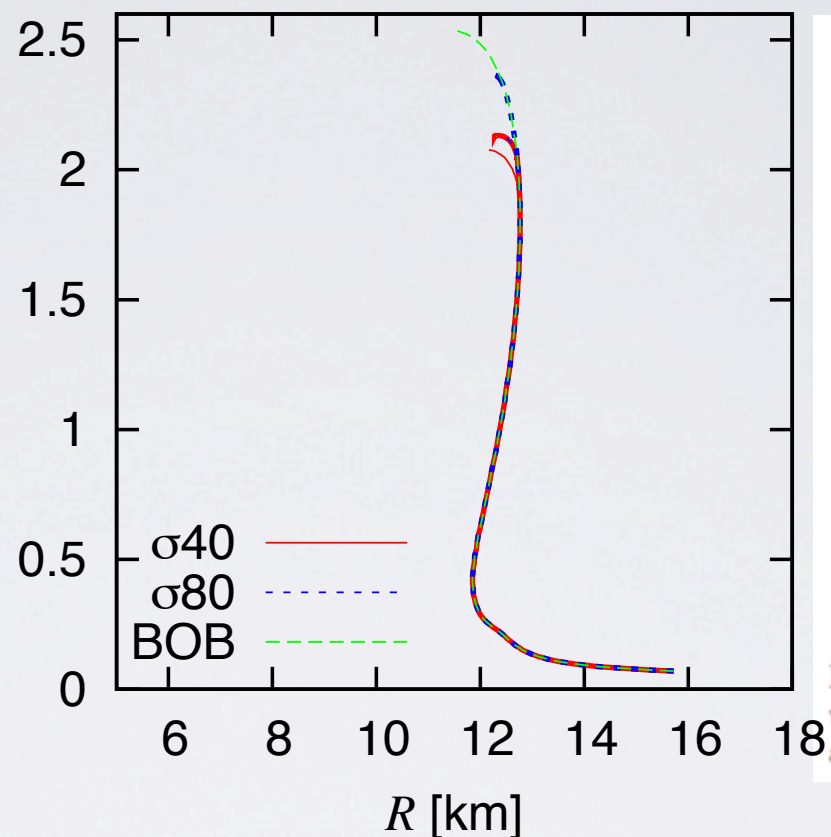
!!
Comparison with observations

EOS & Mass-Radius relation DS+BHF(BOB+TBF)

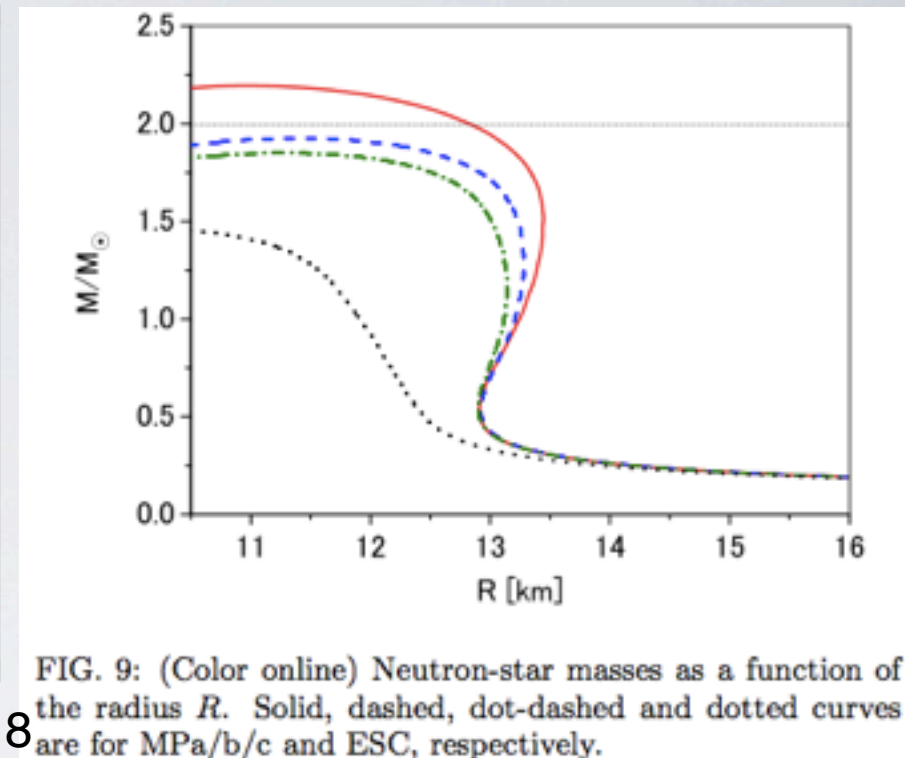
A.W.Steiner, J.M.Lattimer, E.F.Brown
ApJ **722** (2010) 33



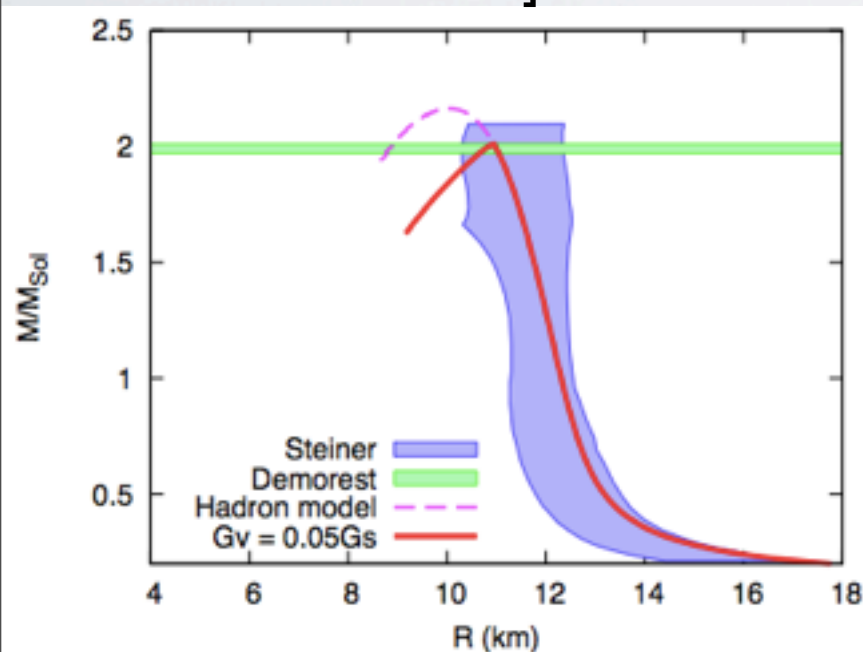
with pasta
[NY et al. in prep.]



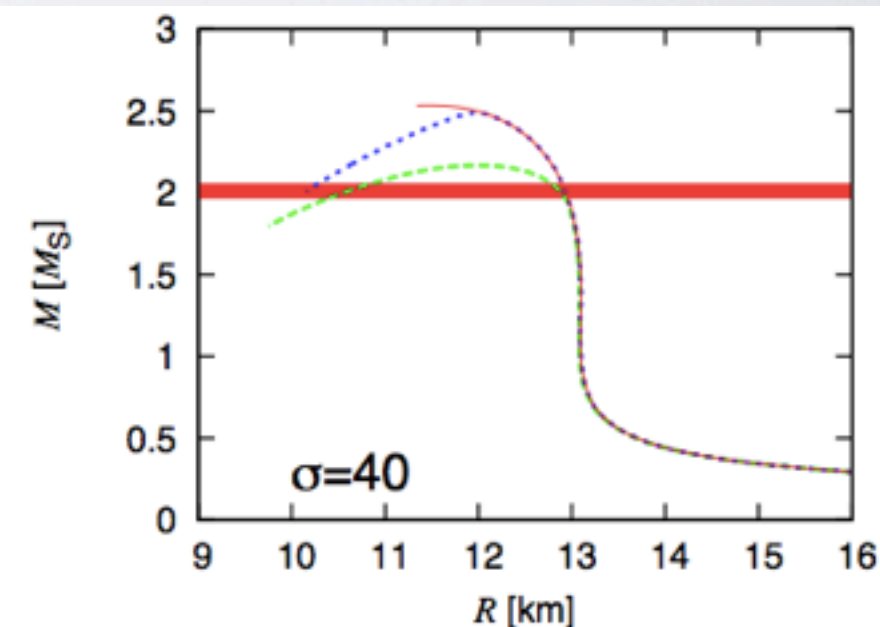
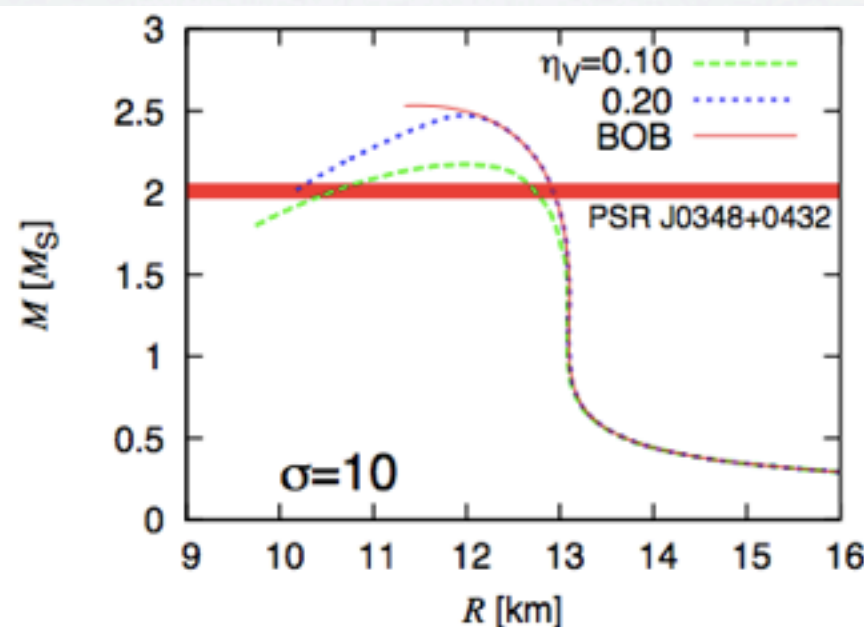
BHF(UTBF by Pomeron without quarks)
[Yamamoto, Furumoto, NY, Rijken, PRC(2014)]



EPNJL+ χ PT
[Sasaki, NY, Khono, Kouno, Yahiro
arXiv:1307.0681]



nINJL+BHF(BOB+TBF) with pasta
[NY et al. PRC2014]



COOLING OF NEUTRON STARS

Cooling...

→ other physical properties

$$c_v e^\Phi \frac{\partial T}{\partial t} + \nabla \cdot (e^{2\Phi} \mathbf{F}) = e^{2\Phi} Q$$

thermal diffusion eq.

heat
capacity

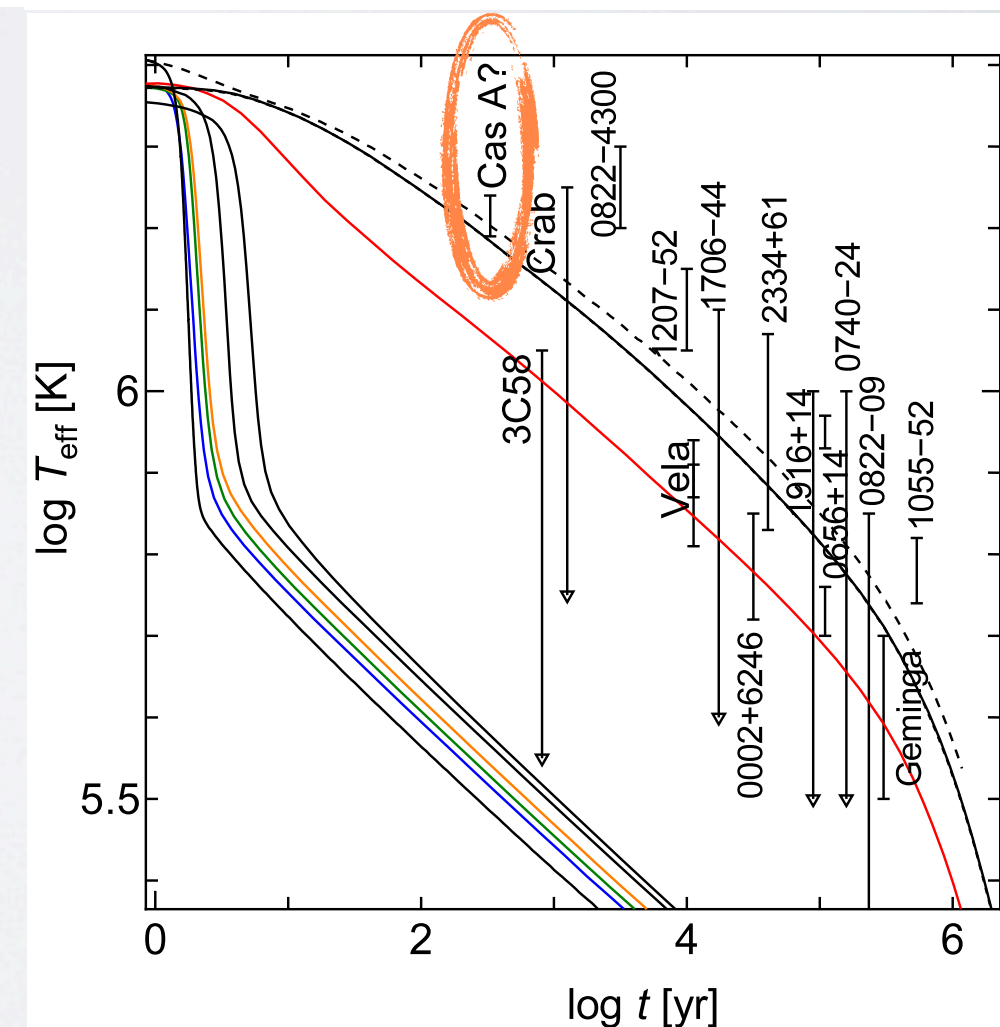
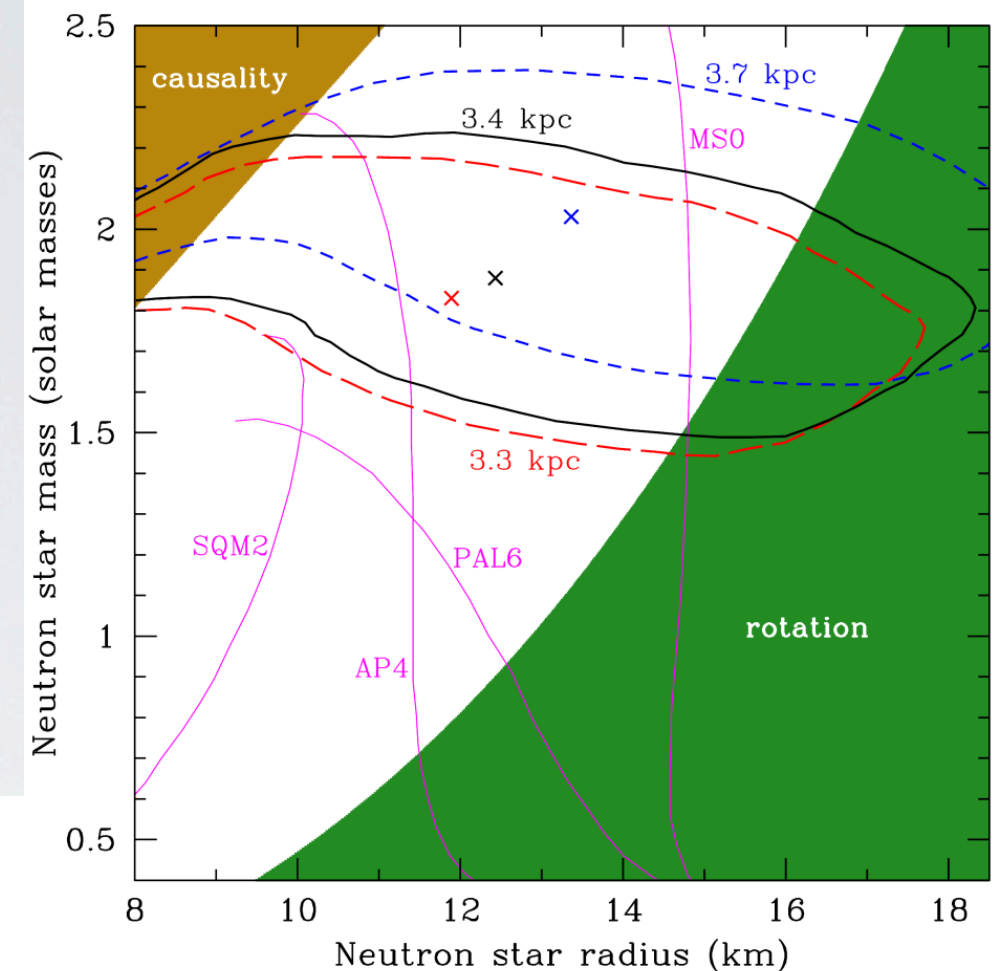
flux
(thermal conductivity)

cooling rate (neutrino)
+
heating rate (magnetic field)

1D: 300×300の行列反転



2D: 10⁵×10⁵の行列反転



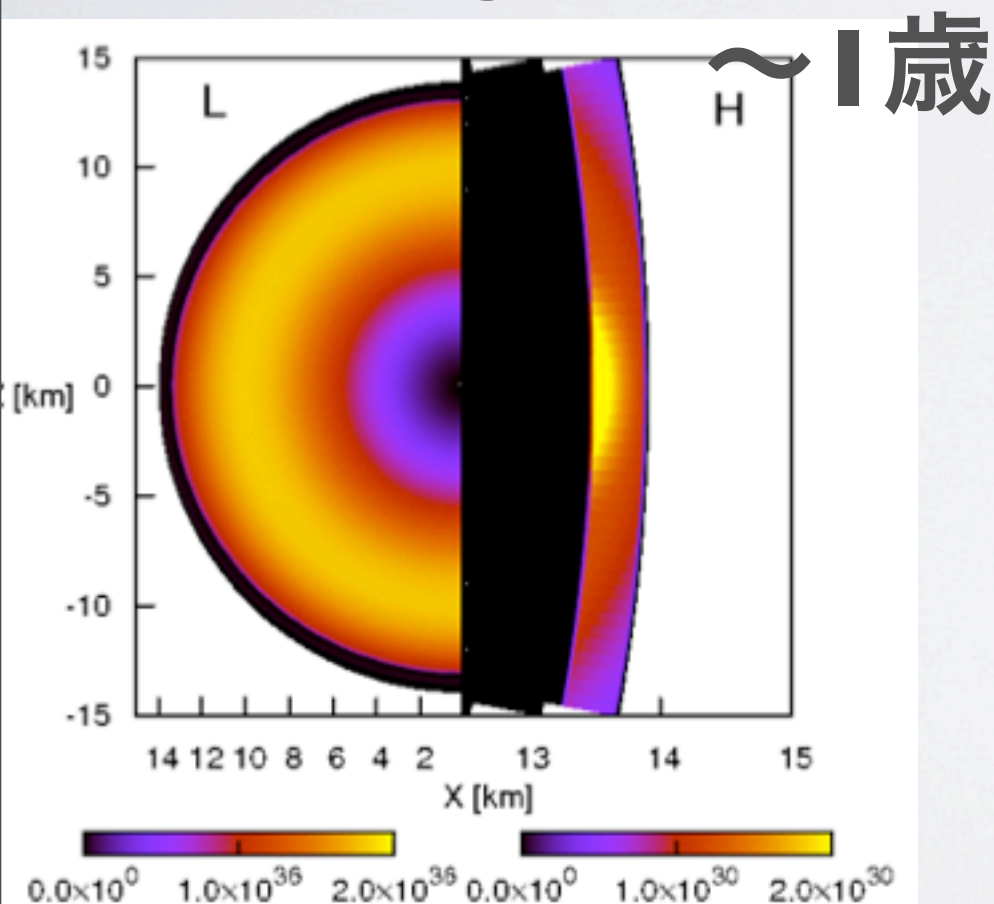
Thermal conduction in strong magnetic field

$$c_v e^\Phi \frac{\partial T}{\partial t} + \nabla \cdot (e^{2\Phi} \mathbf{F}) = e^{2\Phi} Q$$

$$\mathbf{F}_e = -e^\Phi \kappa_e^\perp \left[\nabla \tilde{T} + (\omega_B \tau)^2 (\mathbf{b} \cdot \nabla \tilde{T}) \cdot \mathbf{b} + \omega_B \tau (\mathbf{b} \times \nabla \tilde{T}) \right]$$

- implicit scheme
- operator splitting

cooling rate(L)
& heating rate(H)



Geppert et al.2004

the thermal conductivity

$$\kappa = \begin{pmatrix} \kappa_\perp & \kappa_\wedge & 0 \\ -\kappa_\wedge & \kappa_\perp & 0 \\ 0 & 0 & \kappa_\parallel \end{pmatrix}$$

here

$$\begin{cases} \kappa_0 = \frac{1}{3} c_v \bar{v}^2 \tau = \frac{\pi^2 k_B^2 T n_e}{3 m_e^*} \tau \\ \kappa_\parallel = \kappa_0 \\ \kappa_\perp = \frac{\kappa_0}{1 + (\omega_B \tau)^2} \\ \kappa_\wedge = \frac{\kappa_0 \omega_B \tau}{1 + (\omega_B \tau)^2} \end{cases}$$

TABLE III: Cooling ratio in the cores and crusts we adopt. The details are shown in the references.

process	ratio	reference
Core		
“Modified URCA processes (n-branch)”		
$nn \rightarrow nn\nu\bar{\nu}$		
$pne \rightarrow nn\nu\bar{e}$	$8 \times 10^{21} (n_p)^{1/3} T_9^8$	[31]
“Modified URCA processes (p-branch)”		
$nn \rightarrow nn\nu\bar{\nu}$	$7 \times 10^{19} (n_n)^{1/3} T_9^8$	[31]
$np \rightarrow np\nu\bar{\nu}$	$1 \times 10^{20} (n_p)^{1/3} T_9^8$	[31]
$pp \rightarrow pp\nu\bar{\nu}$	$7 \times 10^{19} (n_p)^{1/3} T_9^8$	[31]
“N – N Bremsstrahlung”		
$nn \rightarrow nn\nu\bar{\nu}$	$7 \times 10^{19} Z n_e^{1/3} T_9^8$	[31]
$np \rightarrow np\nu\bar{\nu}$	$1 \times 10^{20} Z n_e^{1/3} T_9^8$	[31]
$pp \rightarrow pp\nu\bar{\nu}$	$7 \times 10^{19} Z n_e^{1/3} T_9^8$	[31]
Crust		
“e – A Bremsstrahlung”		
$e(A, Z) \rightarrow e(A, Z)\nu\bar{\nu}$	$3 \times 10^{12} Z n_e T_9^8$	[32]
“N – N Bremsstrahlung”		
$nn \rightarrow nn\nu\bar{\nu}$	$7 \times 10^{19} Z n_e^{1/3} T_9^8$	[32]

EOS

+

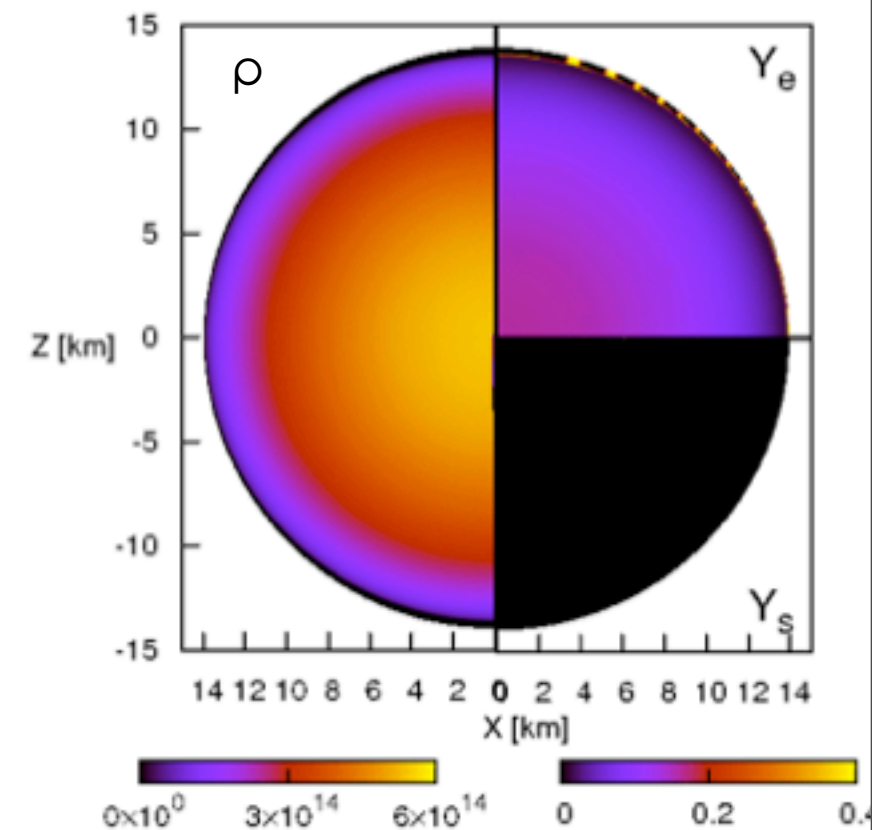
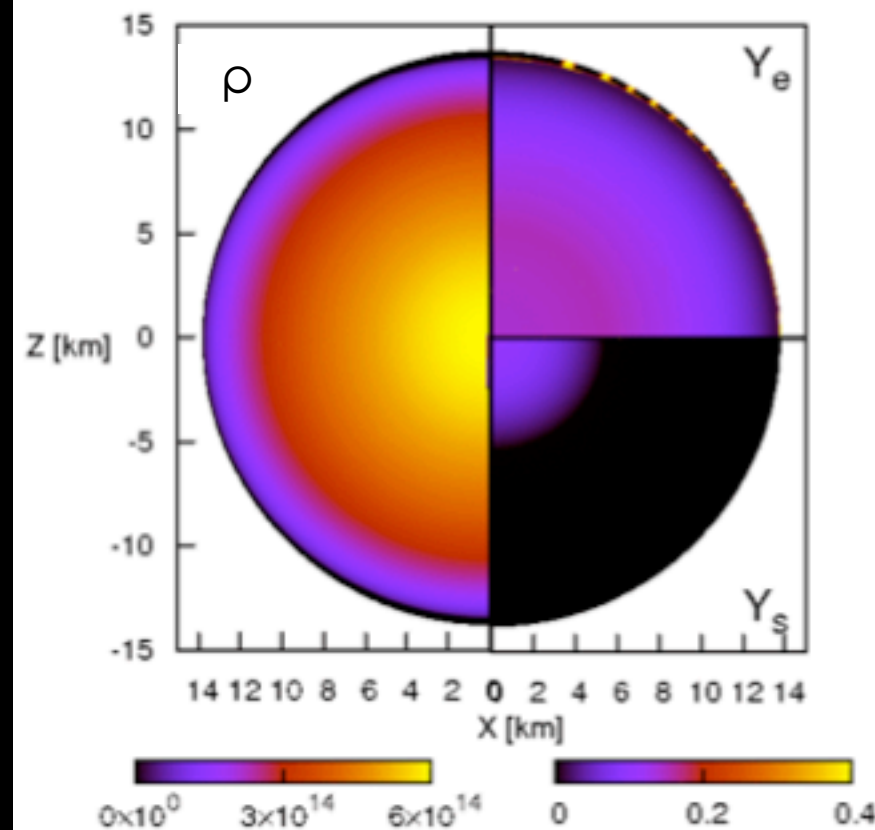
Tomimura & Eriguchi 2005

- (1) axi symmetric
- (2) equatorial symmetric
- (3) no convection
- etc.

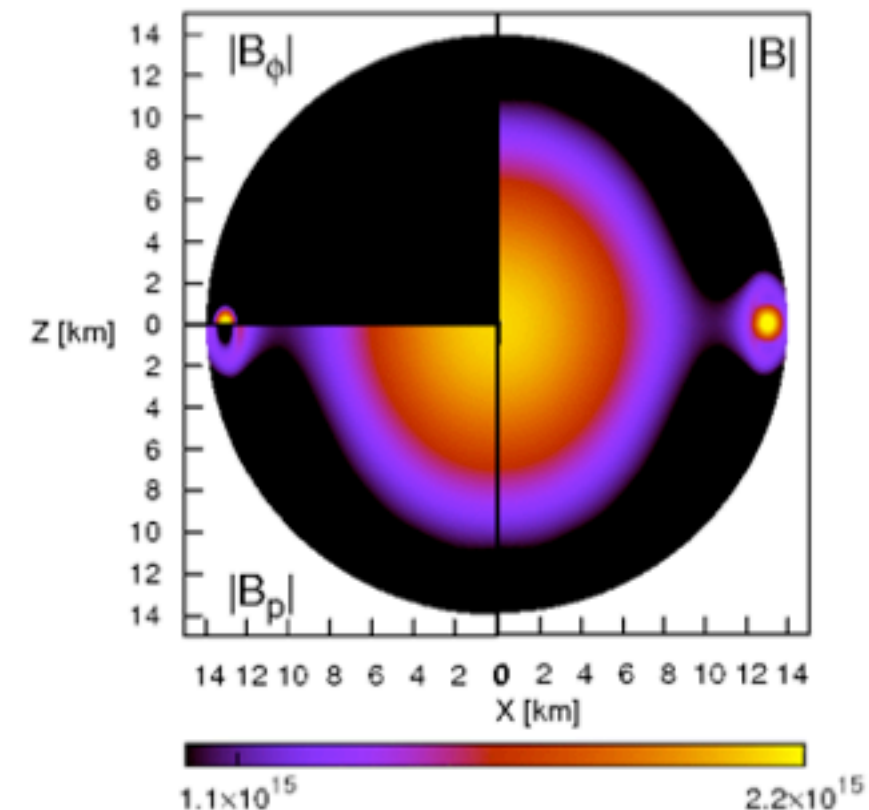
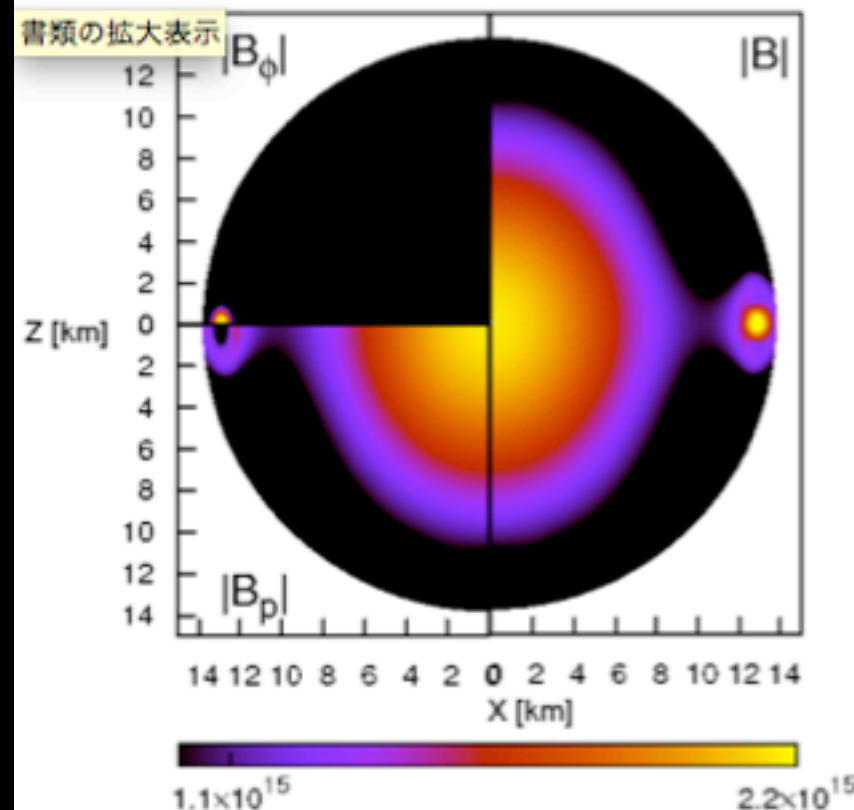
+

GR correction
on the gravitational
potential
ref) Mareck et al. 2006

大きな仮定



Structures with rotation and
magnetic field



EOS

+

Tomimura & Eriguchi 2005

- (1) axi symmetric
- (2) equatorial symmetric
- (3) no convection
- etc.

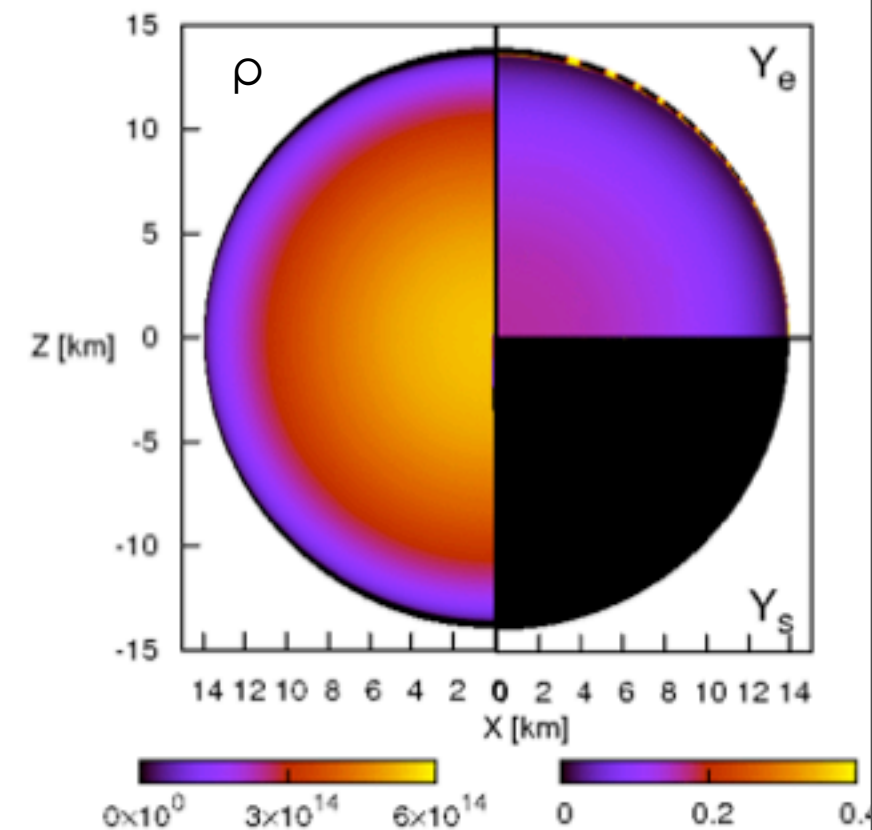
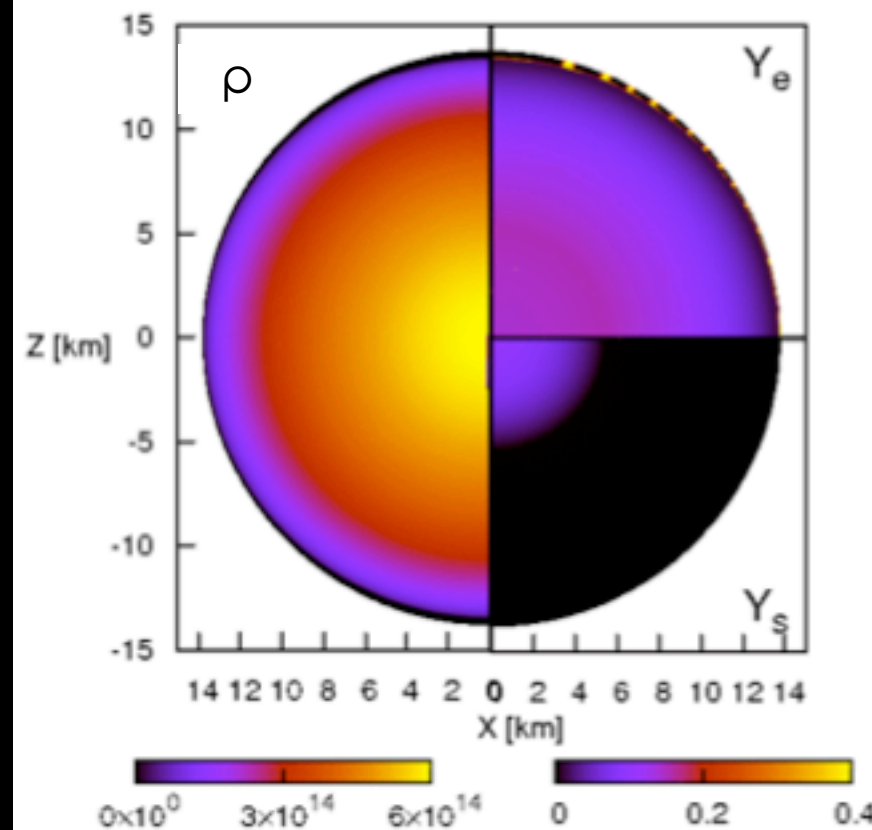
+

GR correction
on the gravitational
potential
ref) Mareck et al. 2006

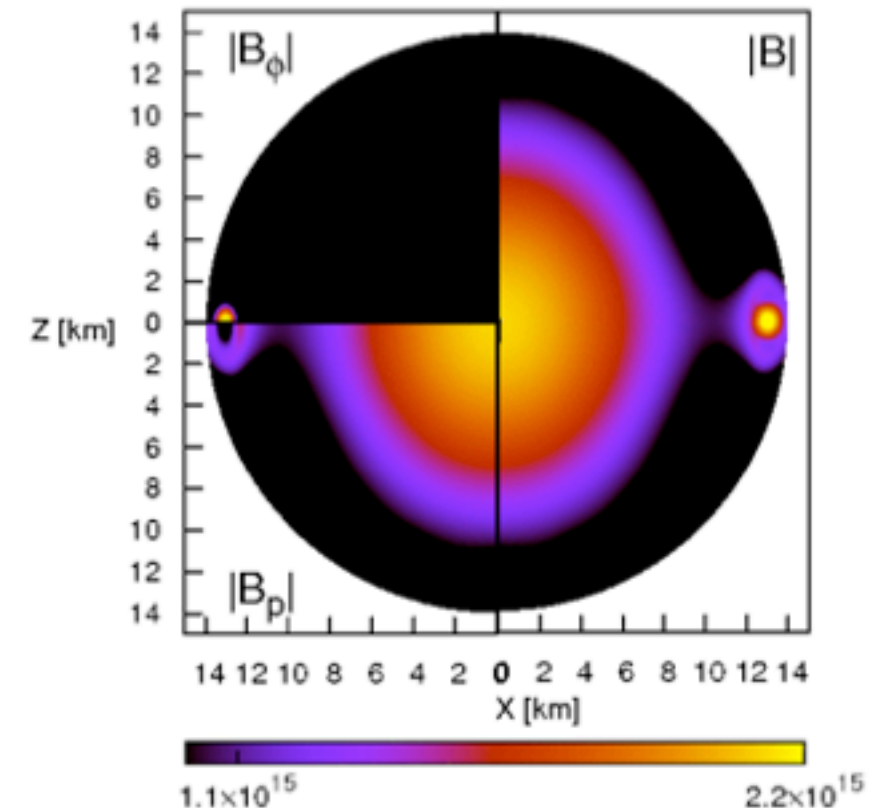
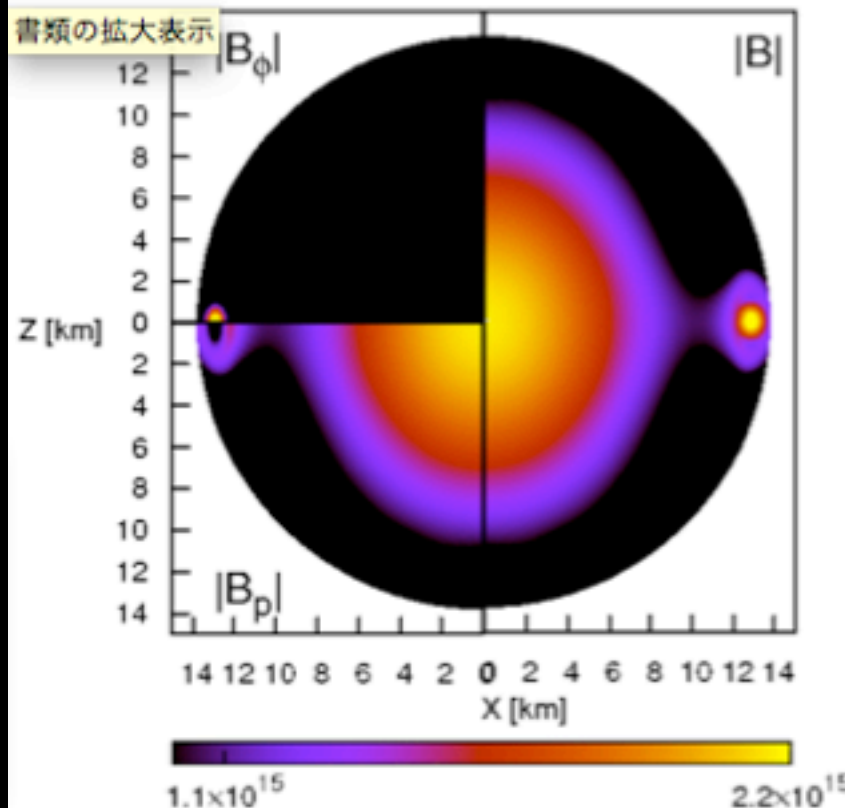
大きな仮定

① 回転則

② $P=P(\rho)$



Structures with rotation and
magnetic field



EOS

+

Tomimura & Eriguchi 2005

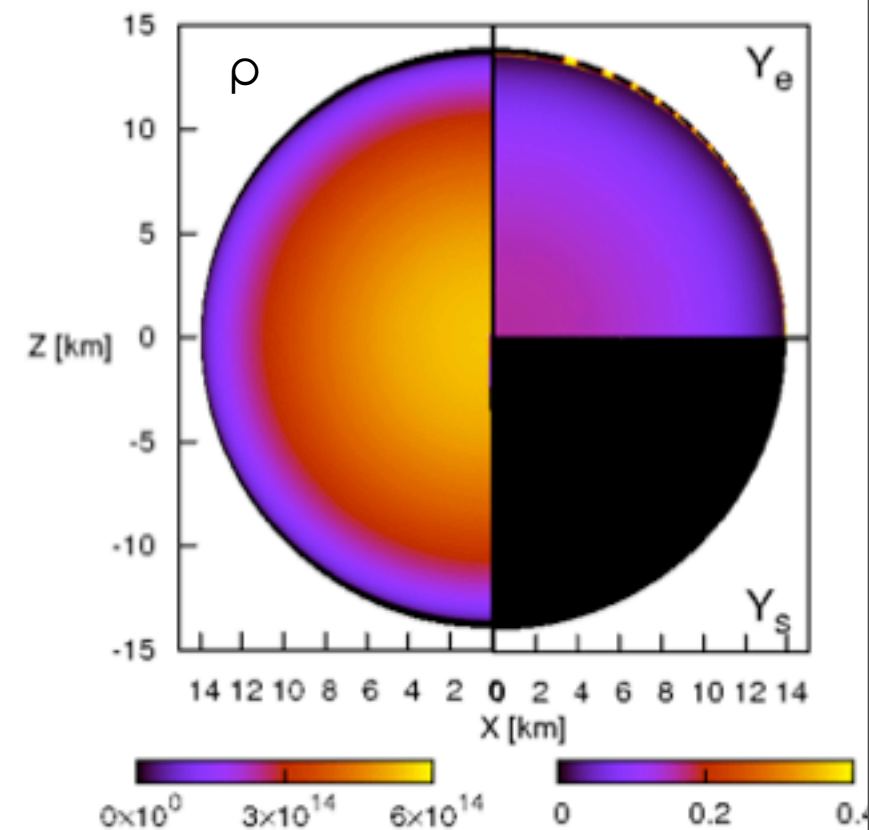
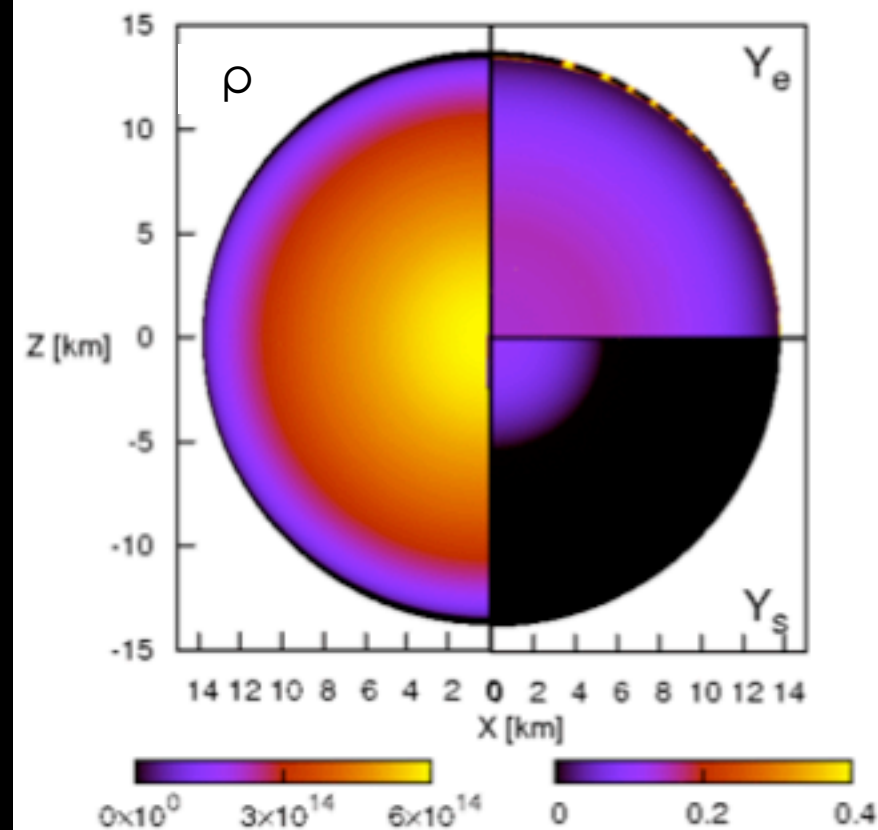
- (1) axi symmetric
- (2) equatorial symmetric
- (3) no convection
- etc.

+

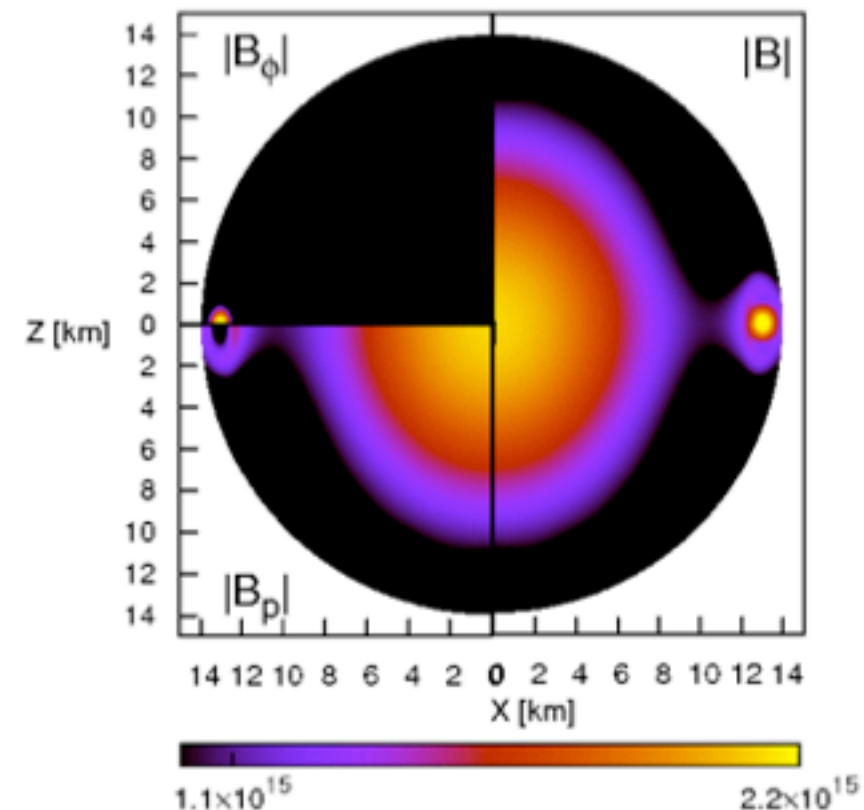
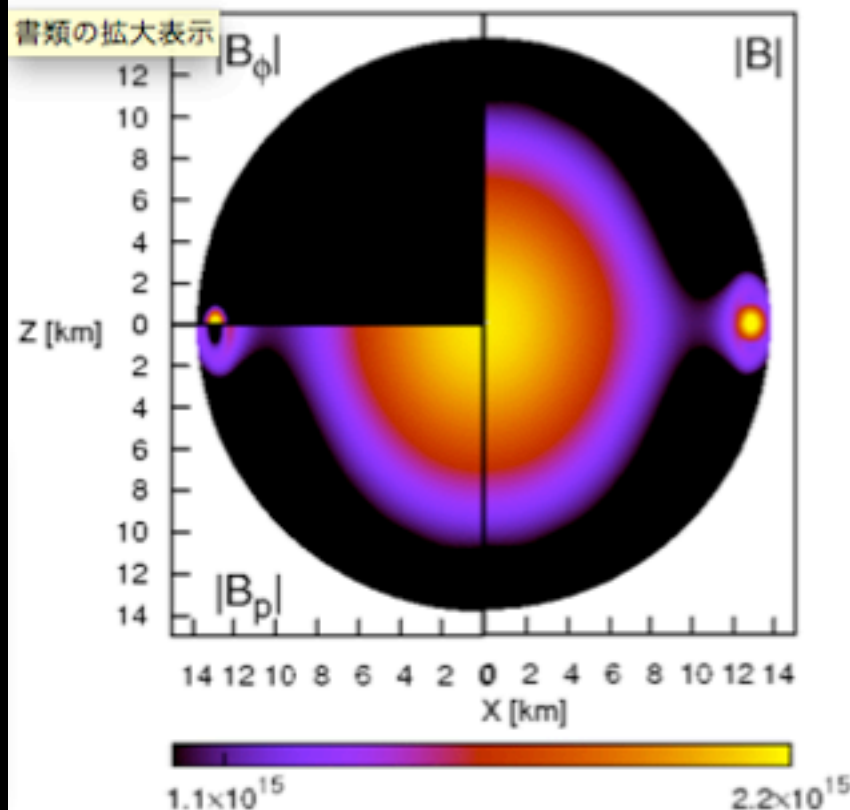
GR correction
on the gravitational
potential
ref) Mareck et al. 2006

大きな仮定

① 回転則
外したい
② $P=P(\rho)$



Structures with rotation and
magnetic field

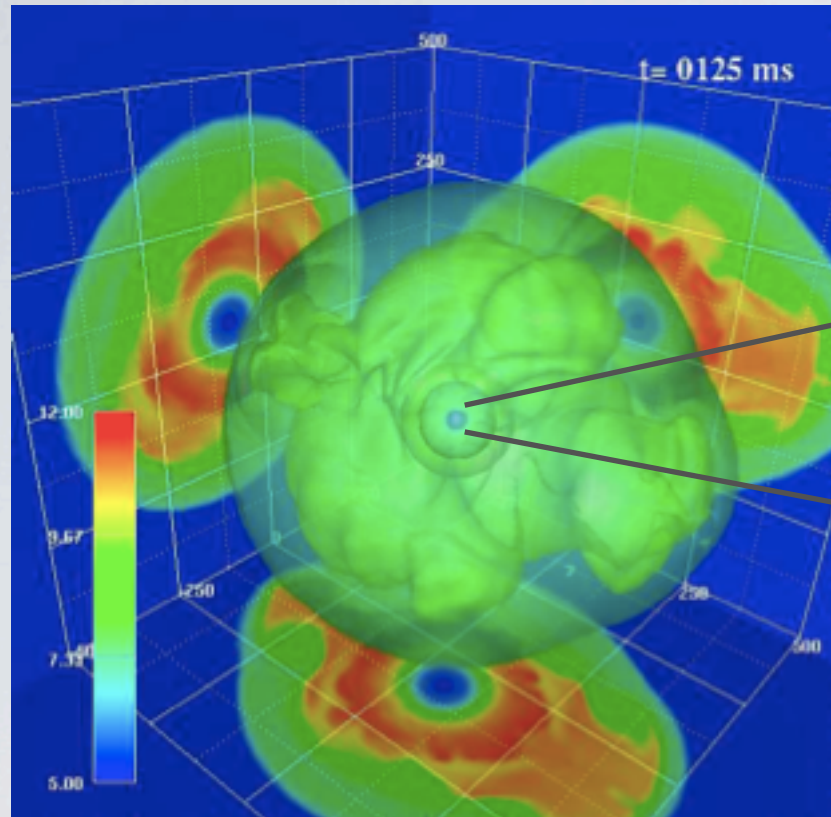


“星の多次元進化計算を目指した 平衡形状の研究”

MNRAS Letter (2015) 446, 56
NY, K.Fujisawa, S.Yamada(Waseda univ.)

Study on (P)NS-structures

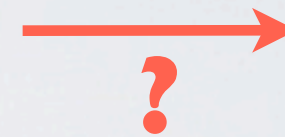
multi-dimensional evolution



PNSs($R \sim 50\text{km}$)

NSs($R \sim 10\text{km}$)

$T \sim 30\text{MeV}$



K.Kotake, K.Sumiyoshi, S.Yamada,
T.Takiwaki, T.Kuroda, Y.Suwa,
H.Nagakura (2012) PTEP



c.f. See the reference;(2010) Physics Today

**Chandrasekhar's role
in 20th-century
science**

Freeman Dyson

Others >> C. Jacobi, R. Dedekind, P.L. Dirichlet, B. Riemann...

cf.) one-dimensional evolution

Heyney method (1964)

星の進化計算(球対称)

cf. Nomoto & Hashimoto(1988) Phys. Rep.

$\xrightarrow{(2.64)}$
 <<静水圧平衡>>
 星が動いては成り立つ。

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2} \qquad \frac{dP}{dm} = -\frac{Gm}{4\pi r^4} \qquad (5.1)$$

<<連続の式>>

$$\frac{dm}{dr} = 4\pi r^2 \rho \qquad \frac{dr}{dm} = \frac{1}{4\pi r^2 \rho} \qquad (5.2)$$

$\xrightarrow{(3.69) \rightarrow (3.70)}$
 <<放射
 物が動いてない。

$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa \rho}{T^3} \frac{F}{4\pi r^2} \qquad \frac{dT}{dm} = -\frac{3}{4ac} \frac{\kappa}{T^3} \frac{F}{(4\pi r^2)^2} \qquad (5.3)$$

$\xrightarrow{(2.64)}$
 <<熱平衡・エネルギー保存>>

$$\frac{dF}{dr} = 4\pi r^2 \rho q \qquad \frac{dF}{dm} = q, \qquad (5.4)$$

<EOS>

$$P = \underbrace{\frac{p_2}{\mathcal{R}} \rho T}_{(3.20)} + P_e + \underbrace{\frac{1}{3} a T^4}_{(3.40)} \qquad (5.5)$$

これに燃焼や冷却を考慮するが
 基本的に速度項は考えなくて良い。➡(準)平衡

星の進化計算(球対称)

cf. Nomoto & Hashimoto(1988) Phys. Rep.

《静圧平衡》

星が動いては成り立つ。

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2}$$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

(5.1)

《連続の式》

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

(5.2)

《放射

物が動いてない。

(3.69) → (3.70)

$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa \rho}{T^3} \frac{F}{4\pi r^2}$$

$$\frac{dT}{dm} = -\frac{3}{4ac} \frac{\kappa}{T^3} \frac{F}{(4\pi r^2)^2}$$

(5.3)

《熱平衡・エネルギー保存》

(3.68) →

$$\frac{dF}{dr} = 4\pi r^2 \rho q$$

$$\frac{dF}{dm} = q,$$

(5.4)

<EOS>

$$P = \frac{\frac{P_2}{\mathcal{R}}}{\frac{\mu_1}{(3.20)}} \rho T + P_e + \frac{\frac{P_{rad}}{3}}{(3.40)} a T^4$$

(5.5)

これに燃焼や冷却を考慮するが
基本的に速度項は考えなくて良い。➡(準)平衡

星の進化計算(球対称)

cf. Nomoto & Hashimoto(1988) Phys. Rep.

《静圧平衡》

足が動かない

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2}$$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

(5.1)

《連続の式》

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

(5.2)

《放射

物が動かない

(3.69) (3.70)

$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa \rho}{T^3} \frac{F}{4\pi r^2}$$

$$\frac{dT}{dm} = -\frac{3}{4ac} \frac{\kappa}{T^3} \frac{F}{(4\pi r^2)^2}$$

(5.3)

《熱平衡・エネルギー保存》

(3.68)

$$\frac{dF}{dr} = 4\pi r^2 \rho q$$

$$\frac{dF}{dm} = q,$$

(5.4)

<EOS>

$$P = \frac{\frac{P_2}{\mathcal{R}}}{\frac{\mu_1}{(3.20)}} \rho T + P_e + \frac{\frac{P_{rad}}{3}}{(3.40)} a T^4$$

(5.5)

これに燃焼や冷却を考慮するが

基本的に速度項は考えなくて良い。➡(準)平衡

進化計算に使える多次元平衡形状がない

星の進化計算(球対称)

cf. Nomoto & Hashimoto(1988) Phys. Rep.

《静圧平衡》

星が動いては成り立つ。

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2}$$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

(5.1)

《連続の式》

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

(5.2)

《放射

物が動いてない。

(3.69) → (3.70)

$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa \rho}{T^3} \frac{F}{4\pi r^2}$$

$$\frac{dT}{dm} = -\frac{3}{4ac} \frac{\kappa}{T^3} \frac{F}{(4\pi r^2)^2}$$

(5.3)

《熱平衡・エネルギー保存》

(3.68) →

$$\frac{dF}{dr} = 4\pi r^2 \rho q$$

$$\frac{dF}{dm} = q,$$

(5.4)

質量座標系が良しとされる

<EOS>

$$P = \frac{\frac{P_2}{\mathcal{R}}}{\frac{\mu_1}{(3.20)}} \rho T + P_e + \frac{\frac{P_{rad}}{3}}{(3.40)} a T^4$$

(5.5)

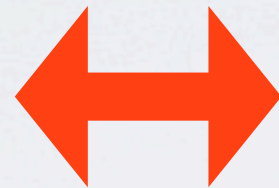
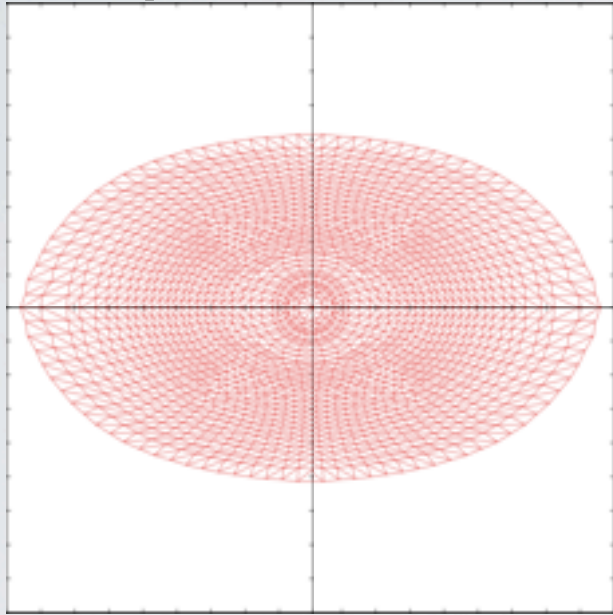
これに燃焼や冷却を考慮するが

基本的に速度項は考えなくて良い。→(準)平衡

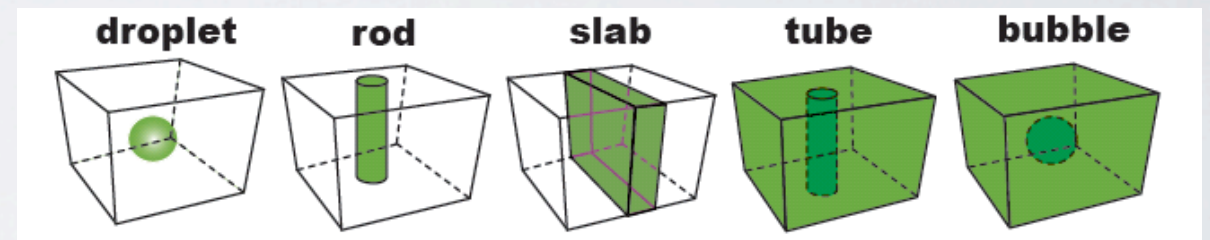
進化計算に使える多次元平衡形状がない

“DEFORMED STAR/PASTA DUALITY”

Deformed stars
(non-spherical stars)

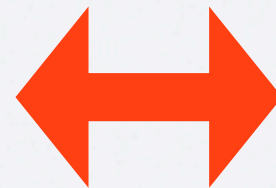


Pasta structures which are non-uniform structures in phase transitions; neutron drip, quark-hadron phase transition, etc.)



Hydro-static equilibrium

$$\frac{\delta E[\xi]}{\delta \xi} = \nabla P + \rho \nabla \phi - \frac{\rho j^2}{\varpi^3} \mathbf{e}_\varpi = 0.$$

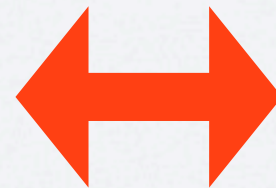


Chemical equilibrium

$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_C^Q, & \mu_d &= \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_C^Q, \\ \mu_n &= \mu_\Lambda = \mu_B, & \mu_p &= \mu_B + \mu_{C,H}, & \mu_{\Sigma^-} + \mu_p &= 2\mu_B, \\ \mu_L^{H(Q)} &= \mu_{\nu_e}^{H(Q)}, & \mu_C^{H(Q)} &= \mu_L^{H(Q)} - \mu_e^{H(Q)}, \end{aligned}$$

ex) quark-hadron phase transition

repulsion = pressure and rotation
attraction = gravitation



repulsion = Coulomb interaction
attraction = surface tension

“NODES & EDGES”



Put some nodes, numbering them randomly.

“NODES & EDGES”

Step 1

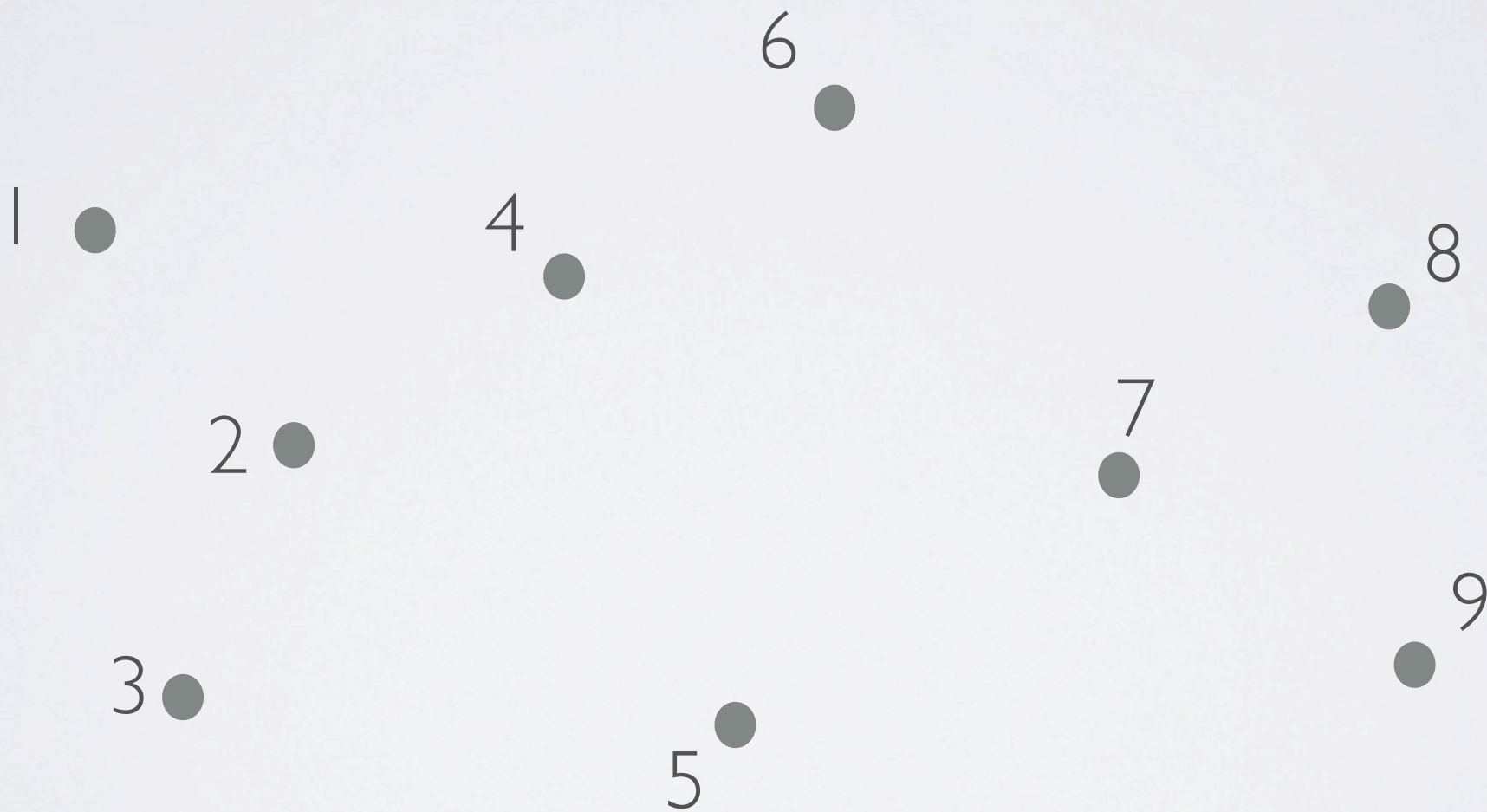
Put some nodes, numbering them randomly.



“NODES & EDGES”

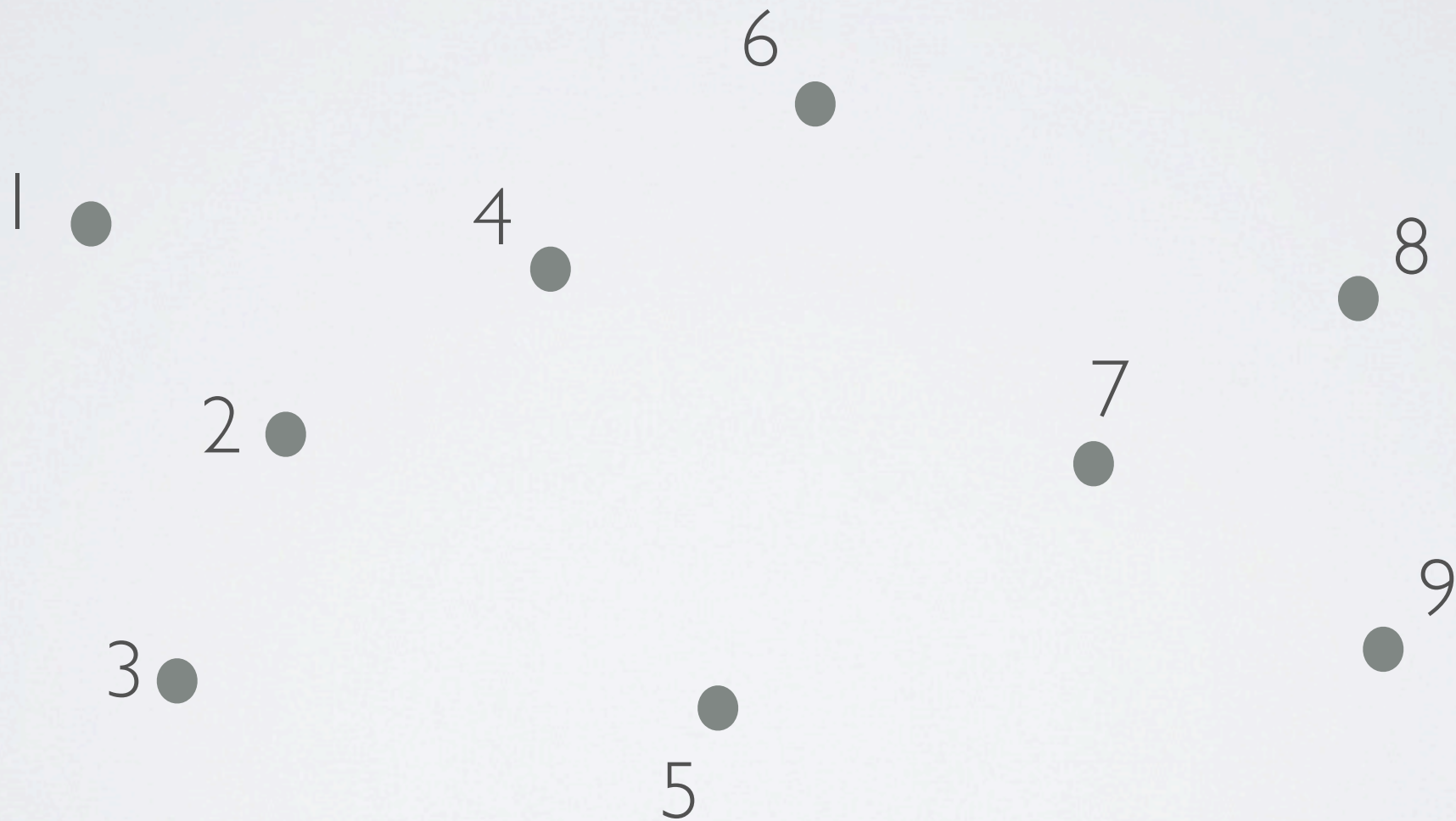
Step 1

Put some nodes, numbering them randomly.



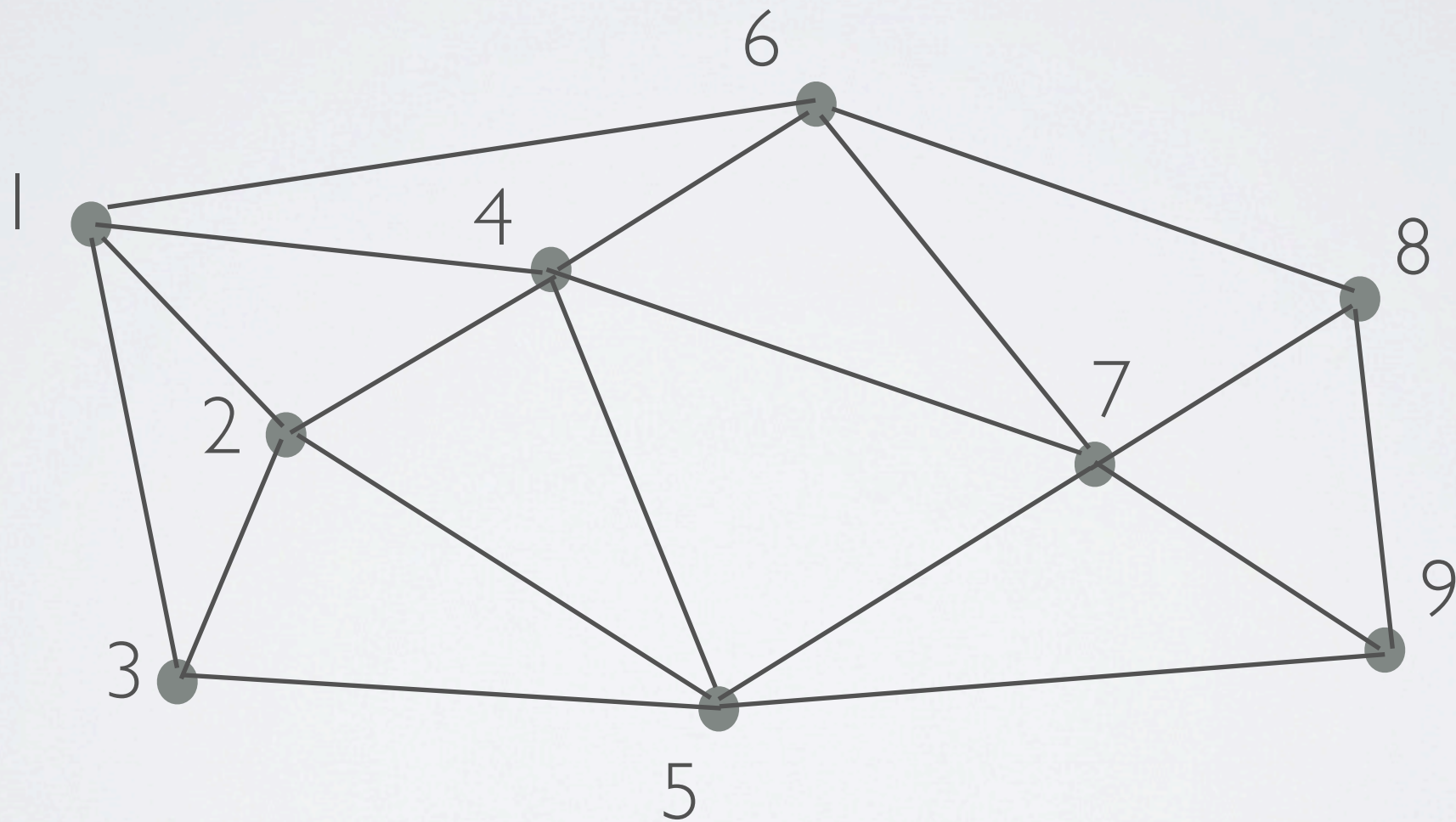
Step II

Connect the nodes with edges making triangles as you want.



Step II

Connect the nodes with edges making triangles as you want.



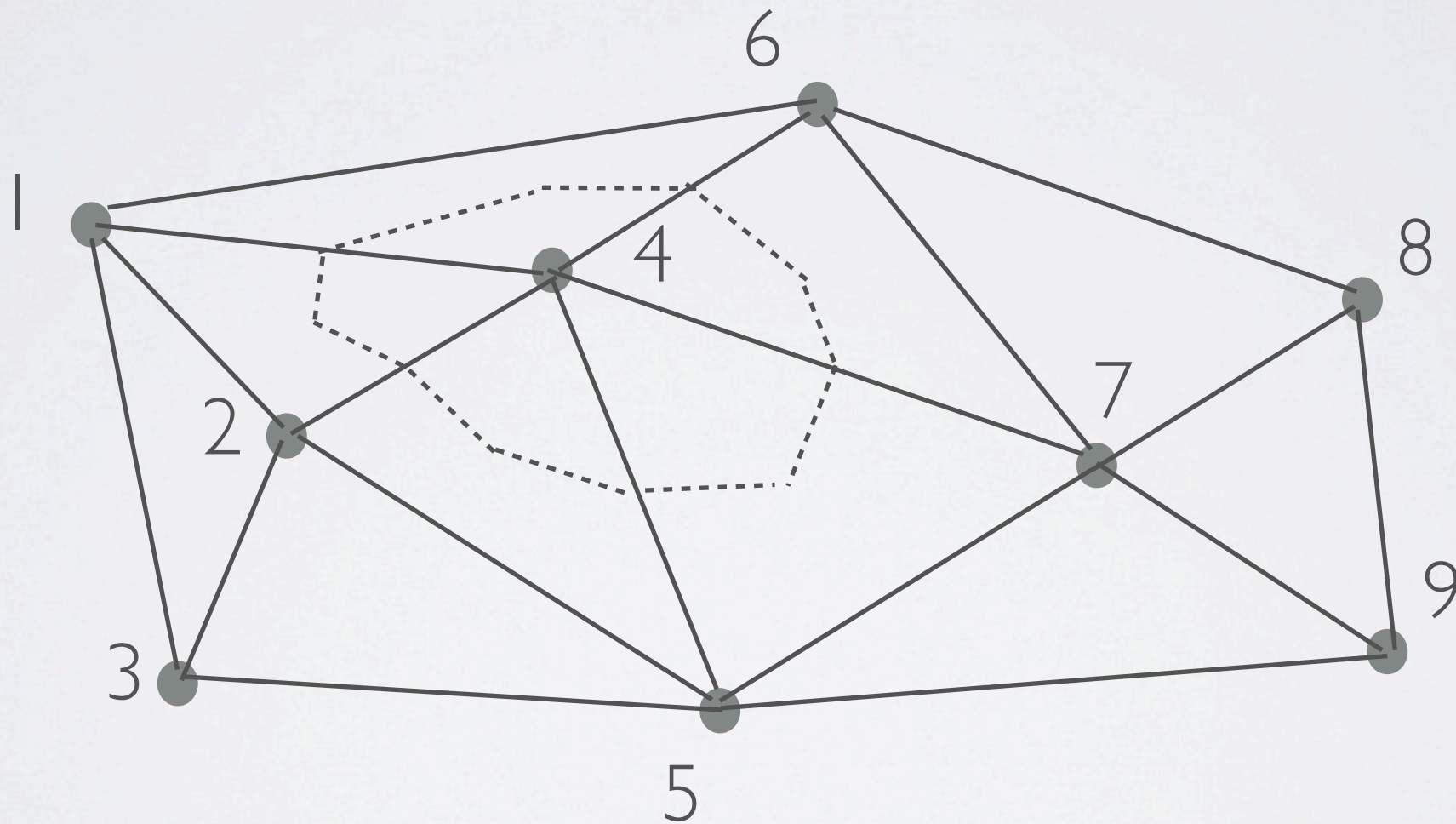
StepIII

Make the matrix which shows the relationship between the nodes; if they have an edge between two nodes, put “1”, otherwise put “0” in the matrix. Then, you will get a beautiful symmetric matrix, which is known as an “adjacency matrix” in *the graph theory*.

	1	2	3	4	5	6	7	8	9
1	0	1	1	1	0	1	0	0	0
2	1	0	1	1	1	0	0	0	0
3	1	1	0	0	1	0	0	0	0
4	1	1	0	0	1	1	1	0	0
5	0	1	1	1	0	0	1	0	1
6	1	0	0	1	0	0	1	1	0
7	0	0	0	1	1	1	0	1	1
8	0	0	0	0	0	1	1	0	1
9	0	0	0	0	1	0	1	1	0

Step IV

If you give each node the physical quantities; position, and Lagrange values (mass, angular momentum, entropy, fractions ...), you can get Eulerian values (partial volume, density, ...) considering with the relation between neighboring nodes (and/or the adjacency matrix).



each node: $x, y, m, j, s, Y_e, Y_n, Y_p, Y_{He} \dots \rightarrow dv, \boldsymbol{\rho}, P, T, u, \dots$

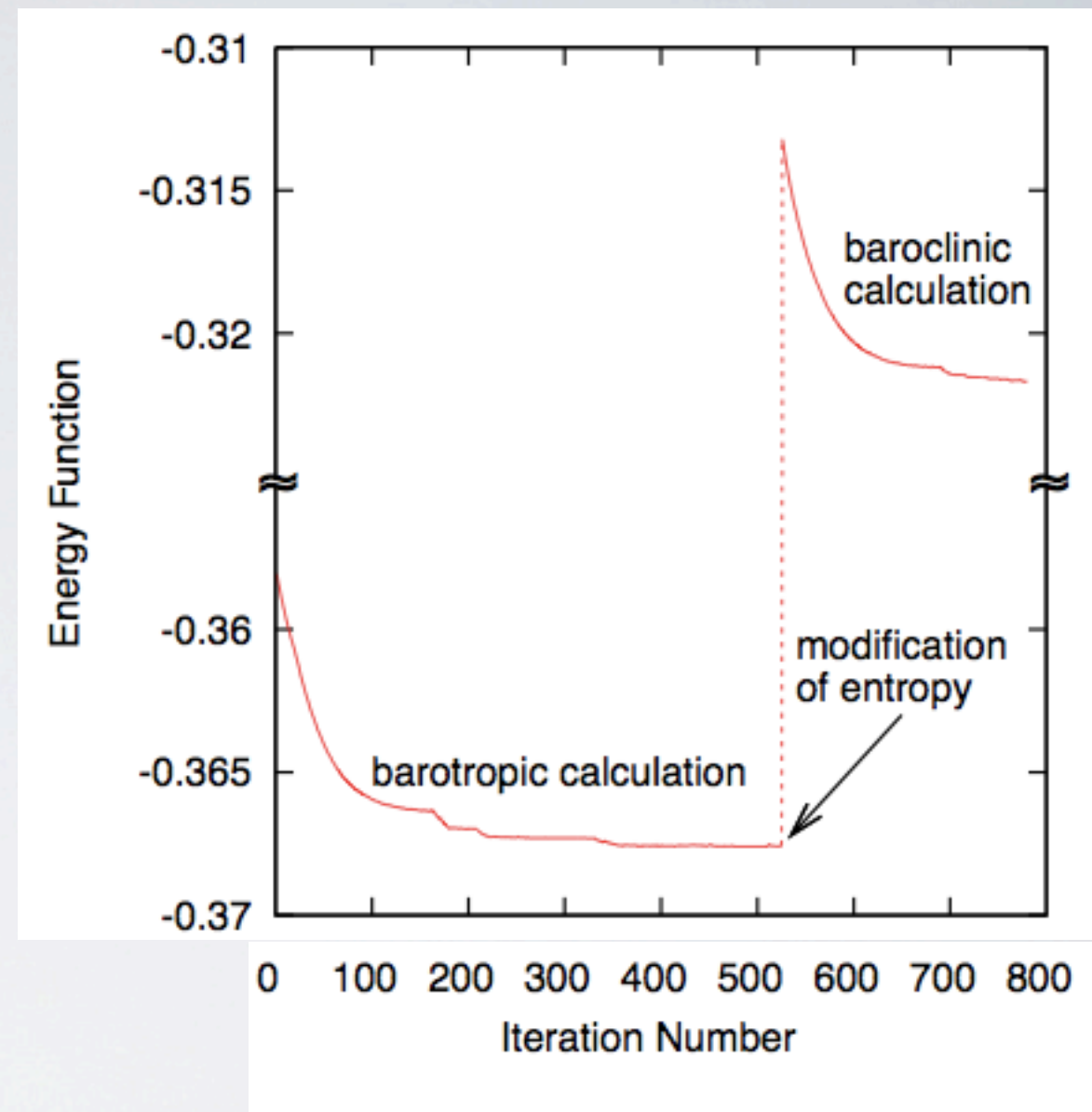
Step V

Find the most optimal arrangement by changing the positions of nodes finding the minimum total energy.

$$E_{\text{FEM}}(\mathbf{r}_i) = \sum_i \varepsilon_i m_i + \frac{1}{2} \sum_i \phi_i m_i + \sum_i \frac{1}{2} \left(\frac{j_i}{\varpi_i} \right)^2 m_i.$$

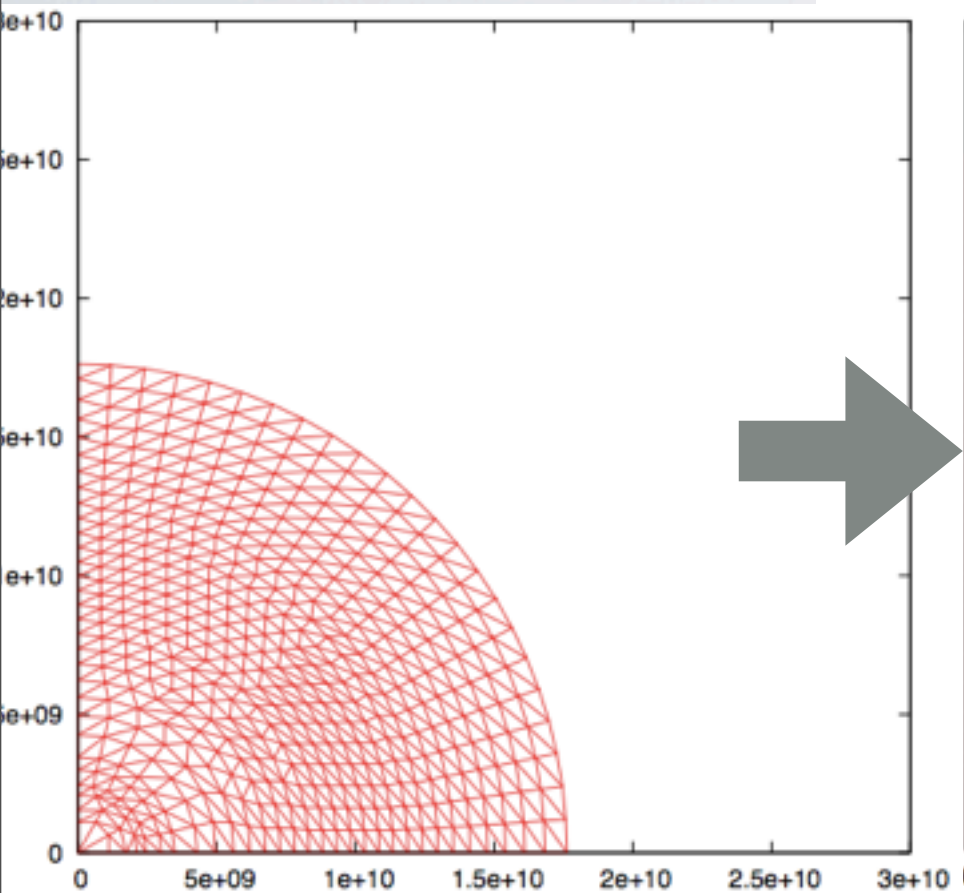
Caution

Now, you may think that this is about two dimension. Of course, if you give (x_i, y_i) as the positions on nodes, it becomes a two-dimensional topic. But, if you give $(x_i, y_i, z_i, \dots)$, it becomes the multi-dimensional.

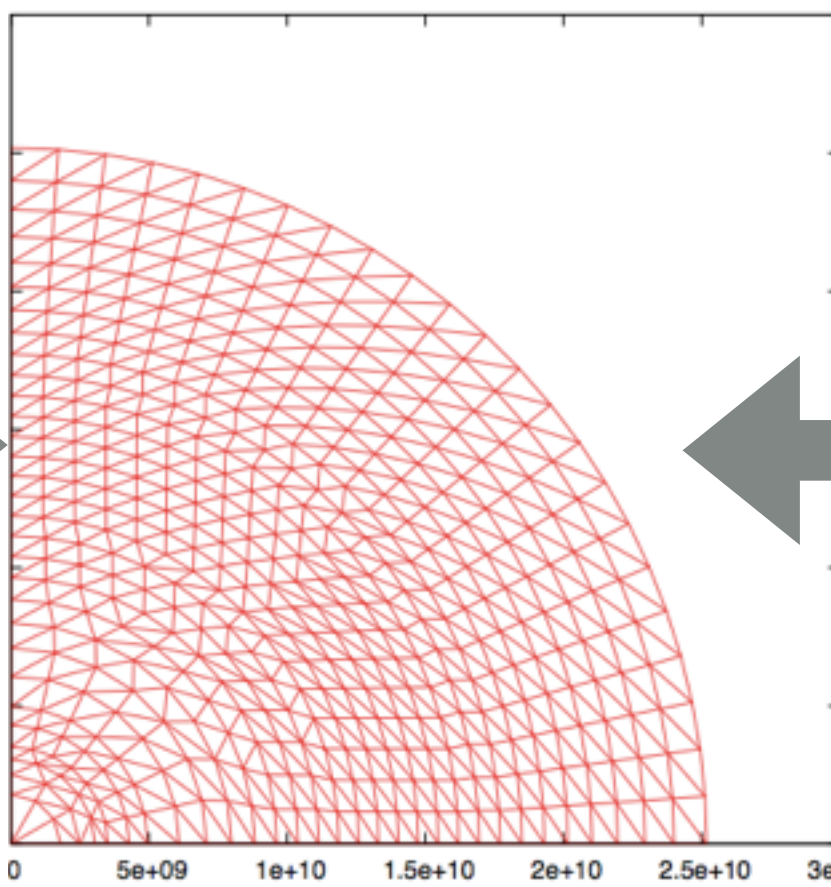


example①

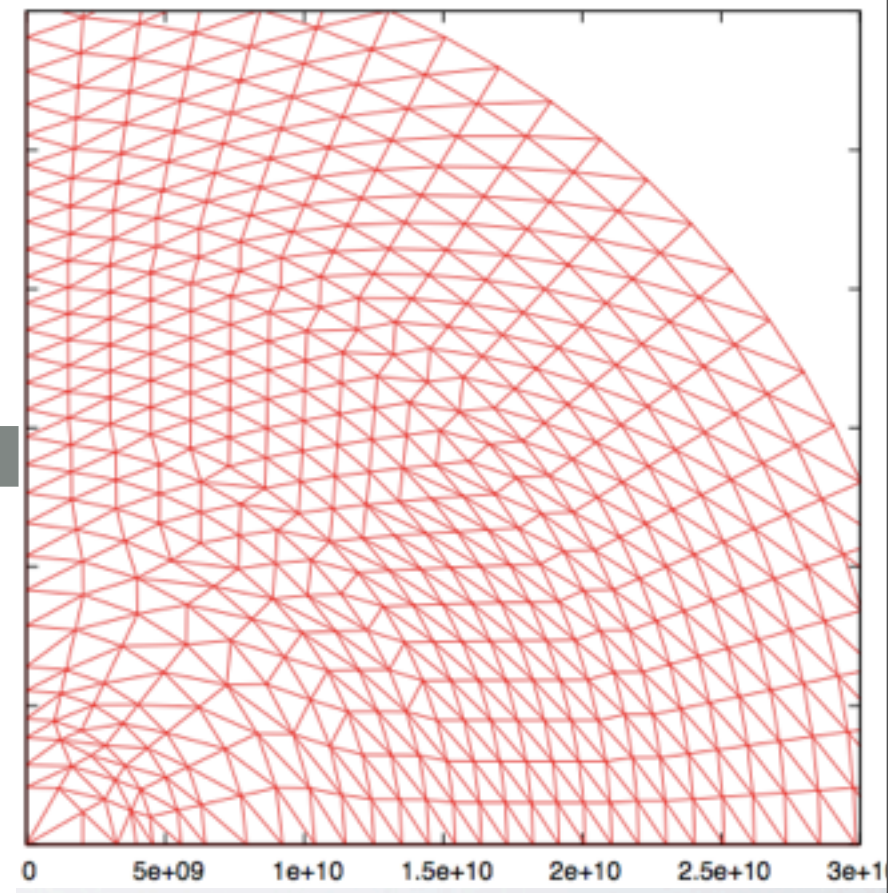
initial guess (i)



exact solution



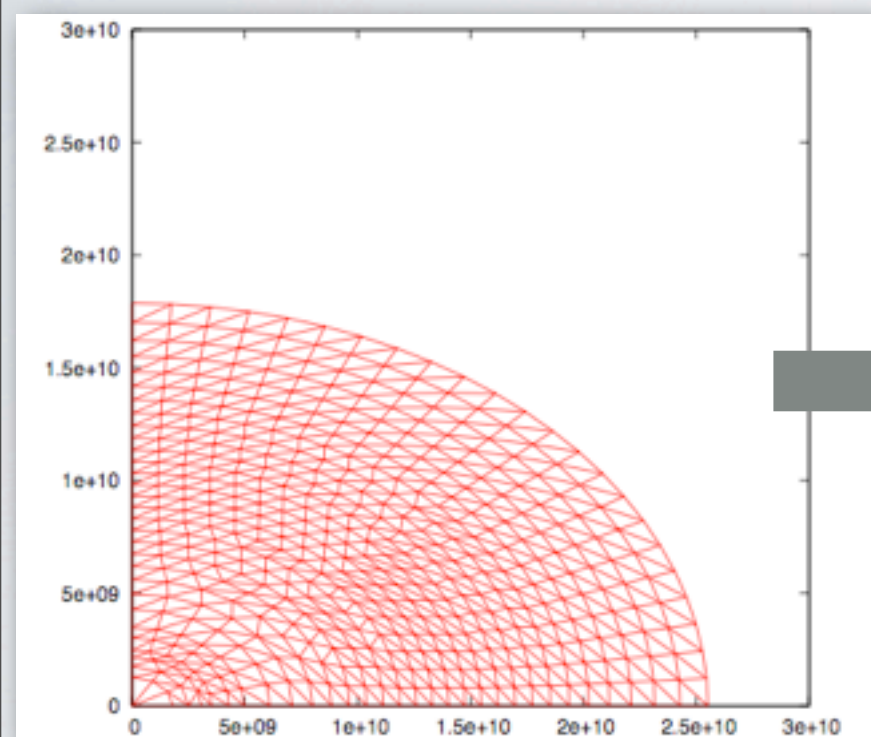
initial guess (ii)



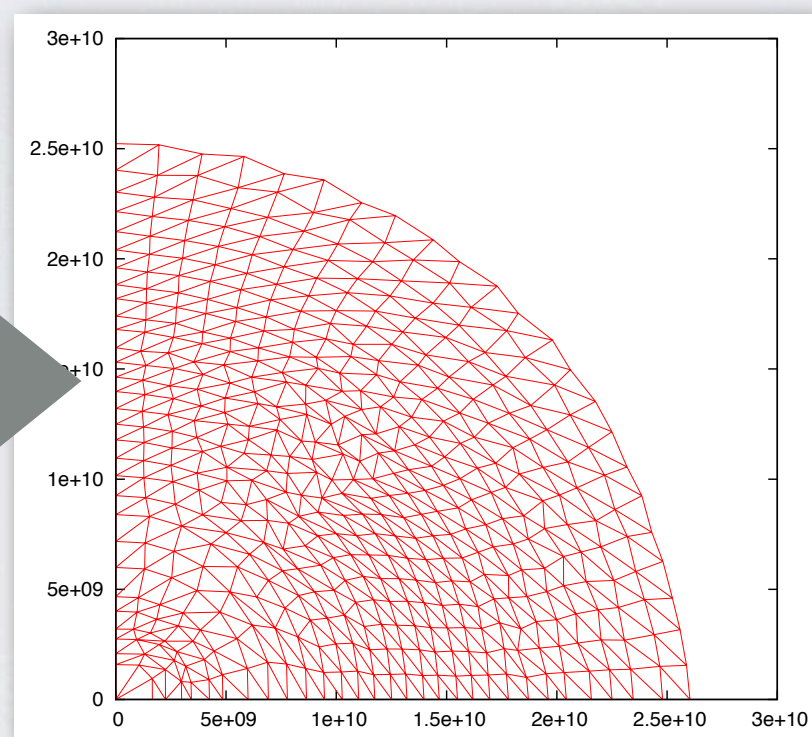
Any initial models(without rotation)
go back to exact solution.

example②

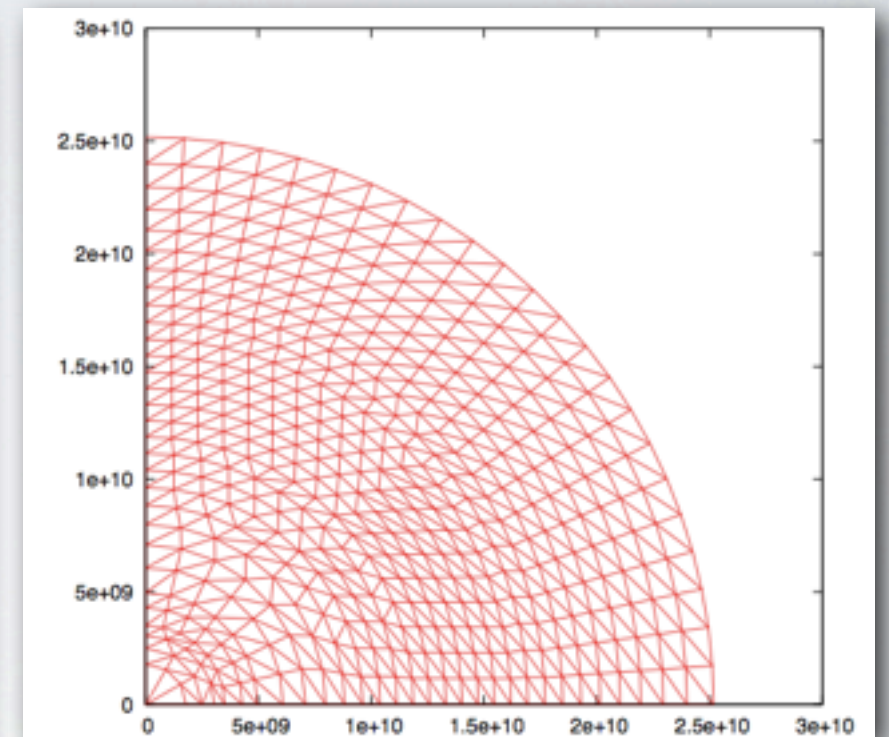
initial guess (iii)



result



exact solution



Any initial models go back to exact solution.

Stellar structures without Høiland criteria

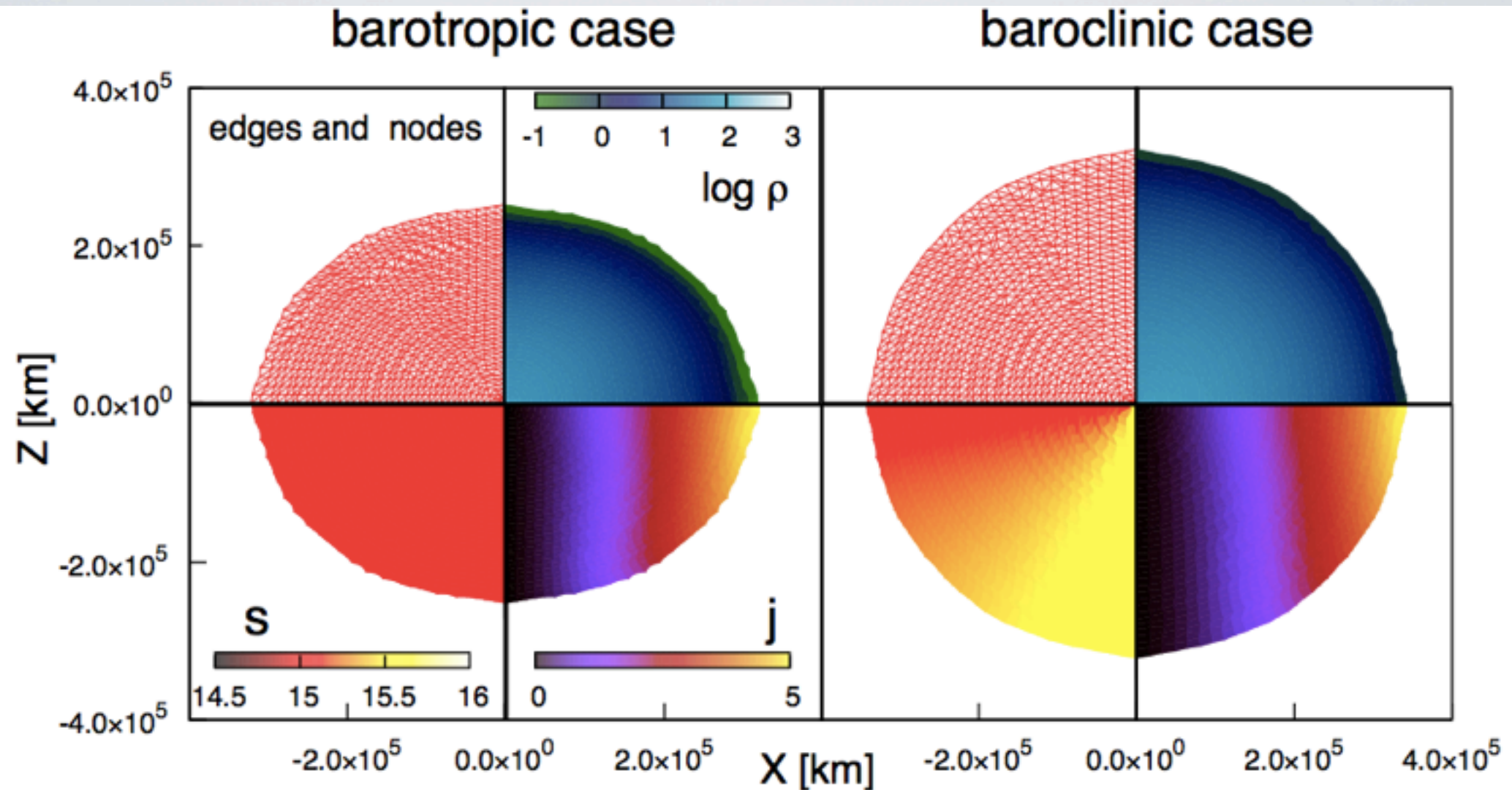
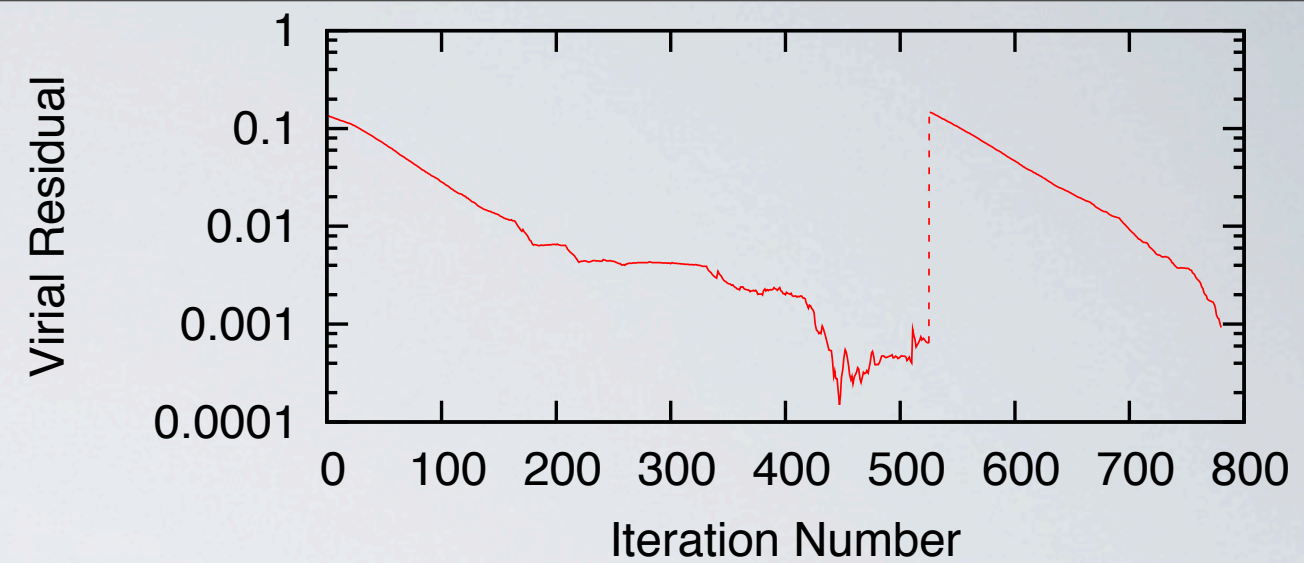
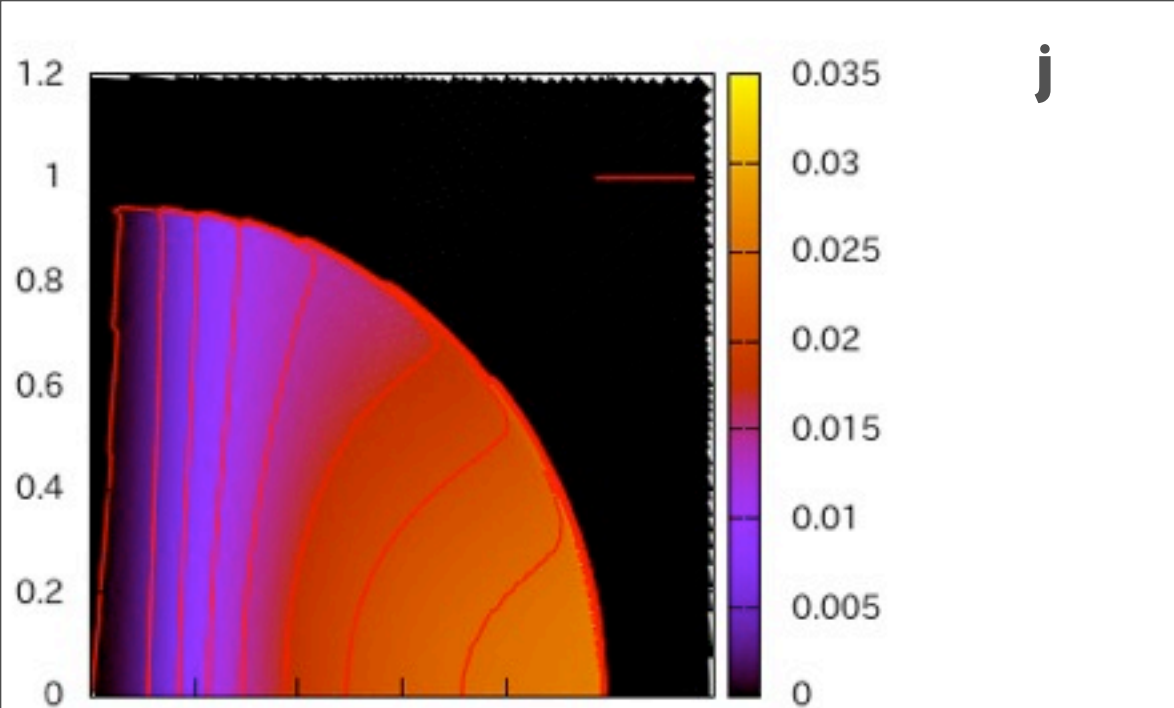


Figure 1. (color on line). Structures of a star in rotational equilibria for barotropic (left four panels) and baroclinic (right four panels) EOS's. The upper left quadrants show the nodes and edges in the triangulated mesh. The other panels display clockwise the color contours of logarithmic density in g/cm^3 , specific entropy in k_B and specific angular momentum in $10^{18}\text{cm}^2/\text{s}$. The color scales are identical for both cases.

We succeeded to get **hydro-static equilibriums for stars with realistic (baroclinic) EOS in Lagrange coordinate**. The distribution of specific angular momentum j is not cylindrical as shown in the right panel.

→ **Bjerkness theorem.**

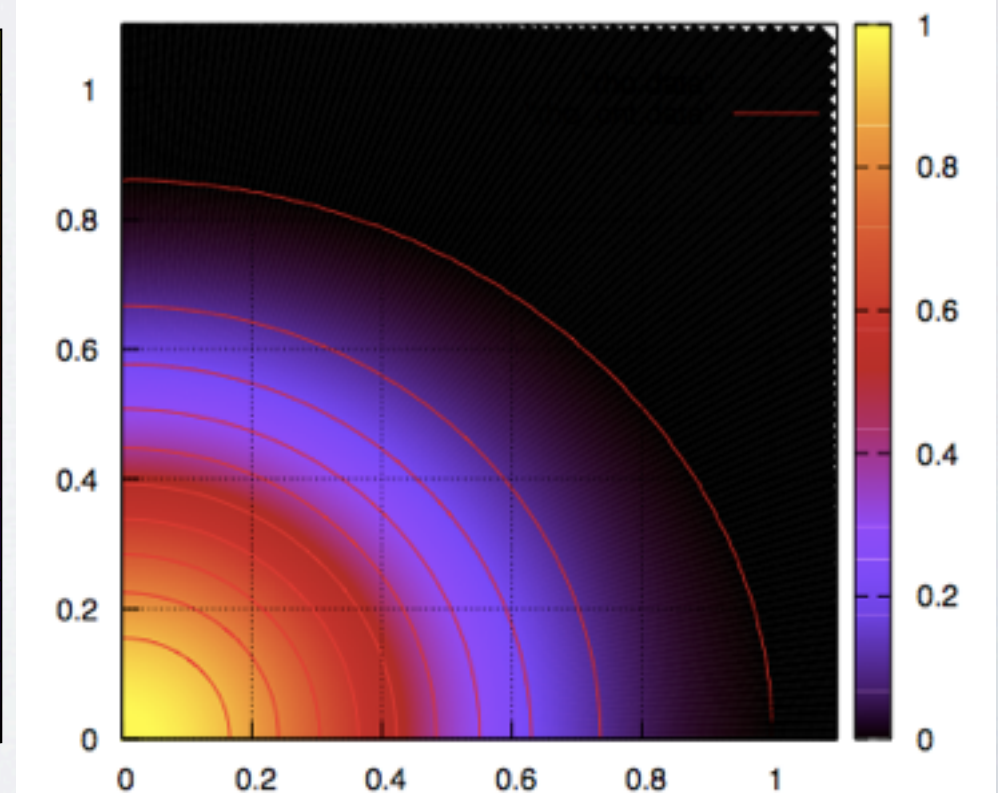
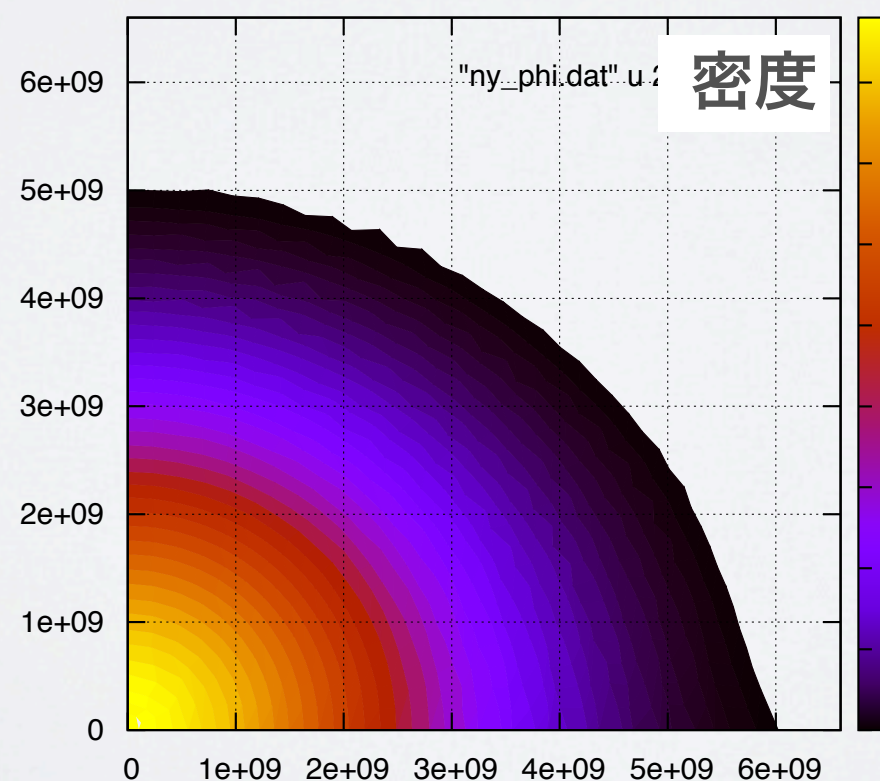


$$V_C = \left| \frac{2T + W + 3 \int P dV}{W} \right|, \sim O(10^{-4})$$

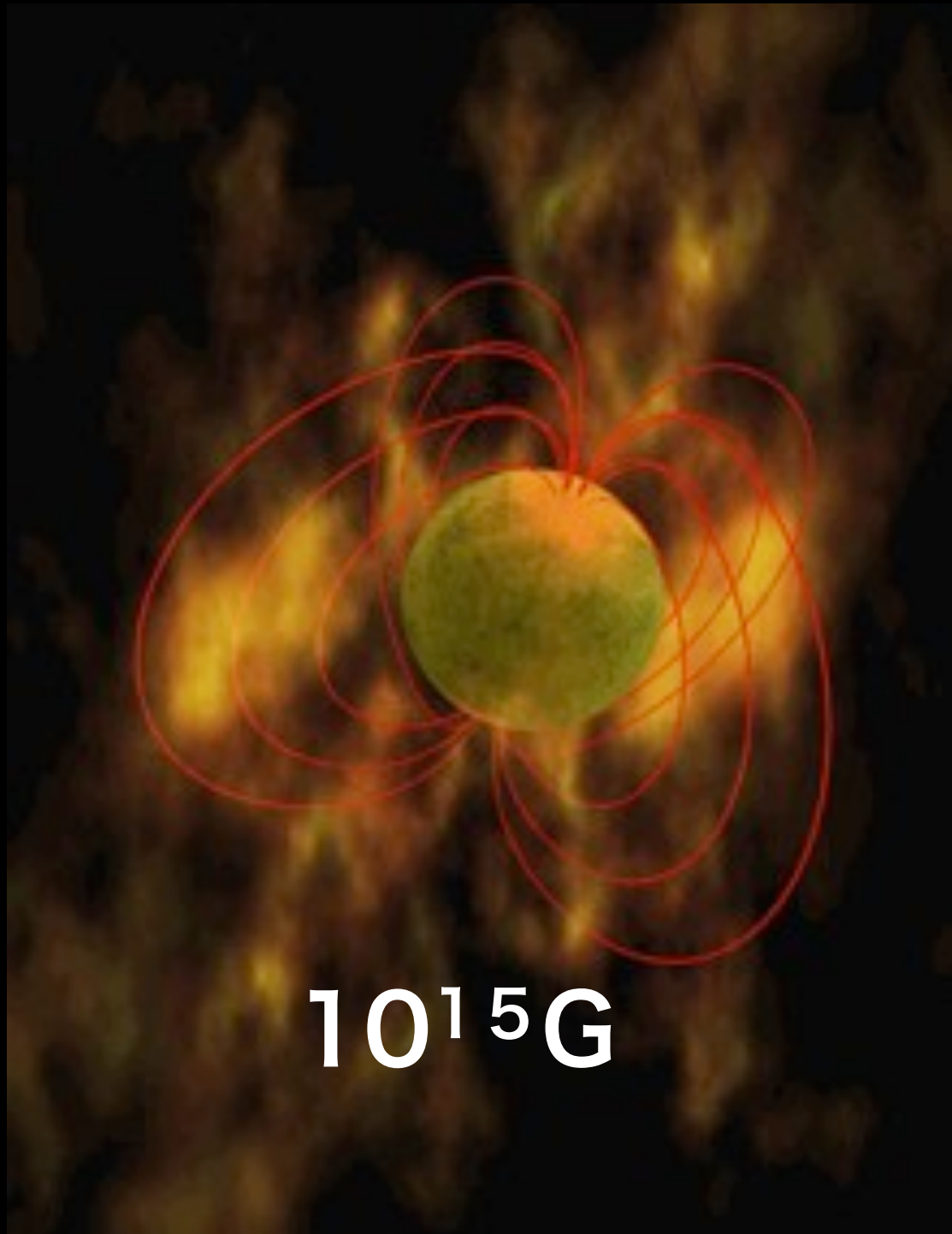
Virial constant → OK

**Now checking
Uryu&Eriguchi method,
Fujisawa method (new!)**

Consistent with Hachisu method



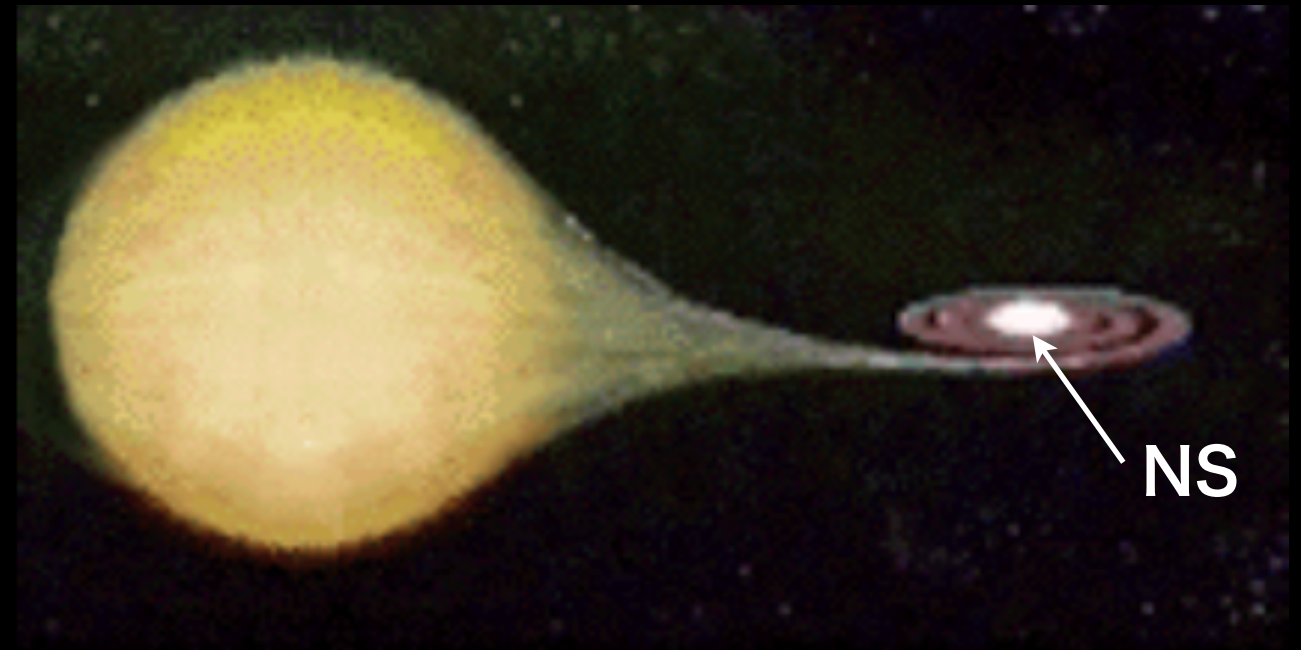
“ON GOING PROJECT”



1979~

observation of magnetars

→ **What is the mechanism?**
What does exist inside?



2007~

Some X-ray transits have
 strong cooling mechanism.

→ **Exotic matter**

Matter inside of NSs

“マグネター進化の研究における情勢”

1 位

	Vigano, Pons et al.	Fujisawa, Pons Kojima etc.	Tsuruta et al. or Yakovlev et al.	共同研究 Wood et al.	NY et al. or (Gregorian et al.) etc.
(熱的)進化	○	×	◎	×	○
誘導方程式による 磁場の時間発展	○	○	×	○	△
次元	2	2	1	3	2
EOSなど諸物理量	◎	○	○	○	◎
観測との比較	○	×	○	×	△

* 磁場、回転、相対論などを考えた平衡形状のみを考えたものは除外。

* 一つでも×や△があると、なかなか観測と比べられない(そういう気にならない)。

“マグネター進化の研究における情勢”

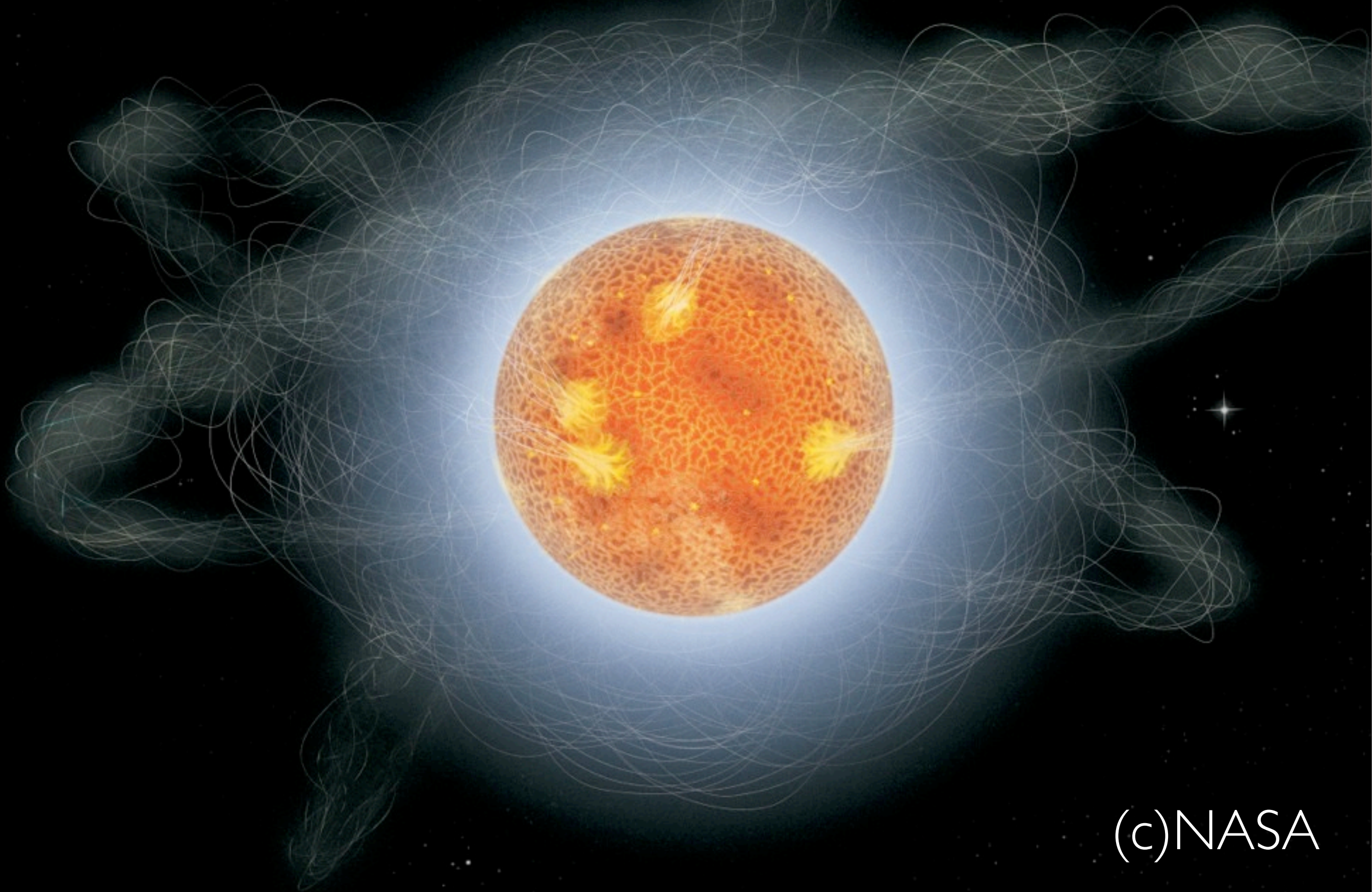
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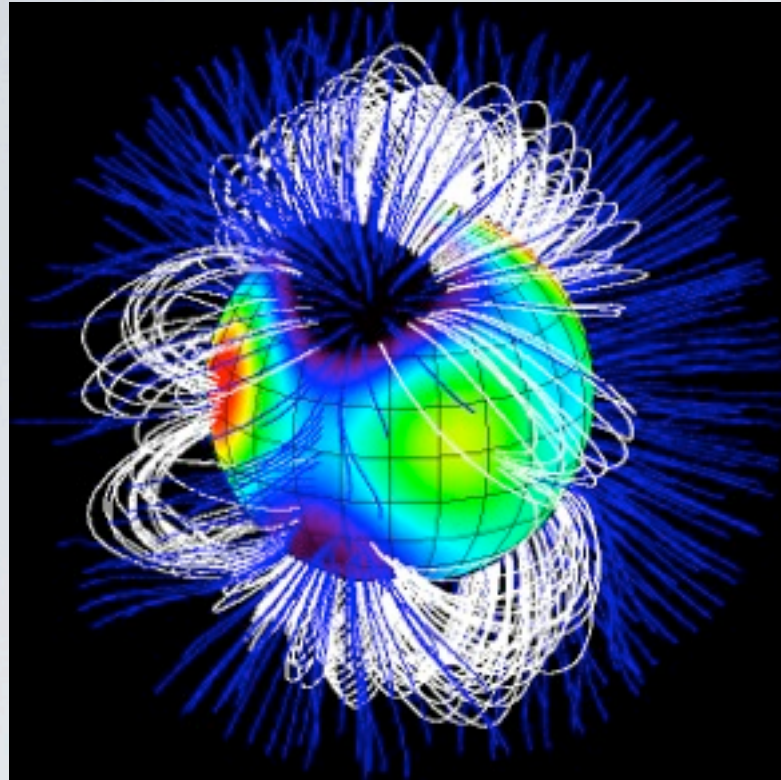
* 一つでも×や△があると、なかなか観測と比べられない(そういう気にならない)。

磁場を考えるならやはり 3Dも必要



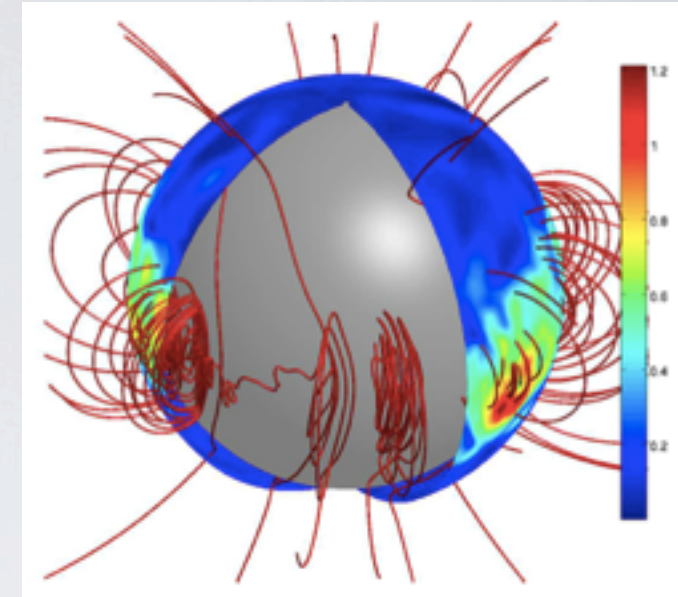
(c)NASA

磁場を伴う星(の進化)と 3 次元



観測

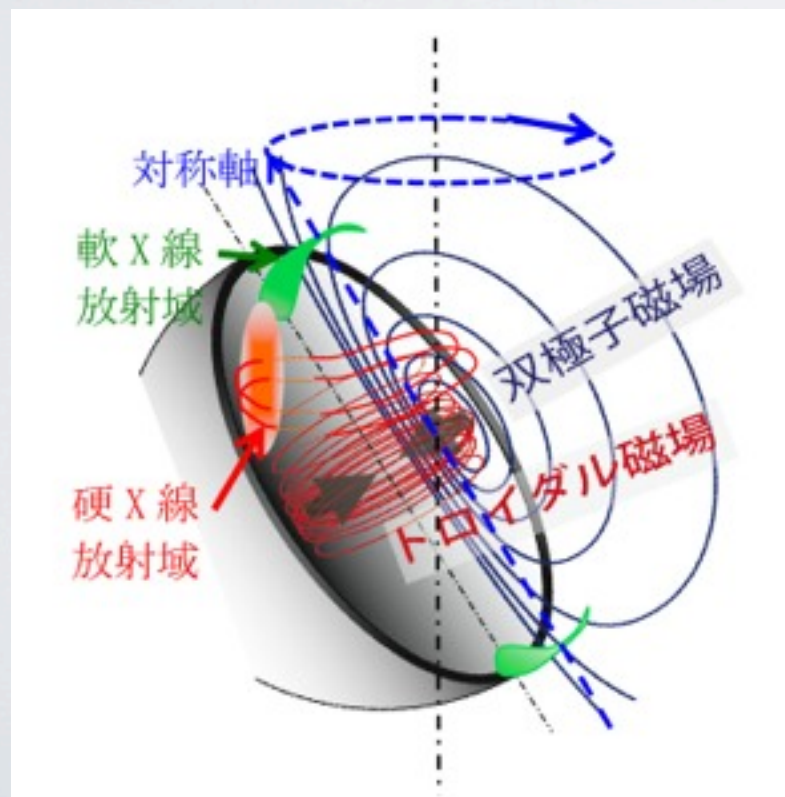
理論



マグネター(クラスト)の磁場進化

Wood & Hollarbach (2015) arXiv 1501.05149

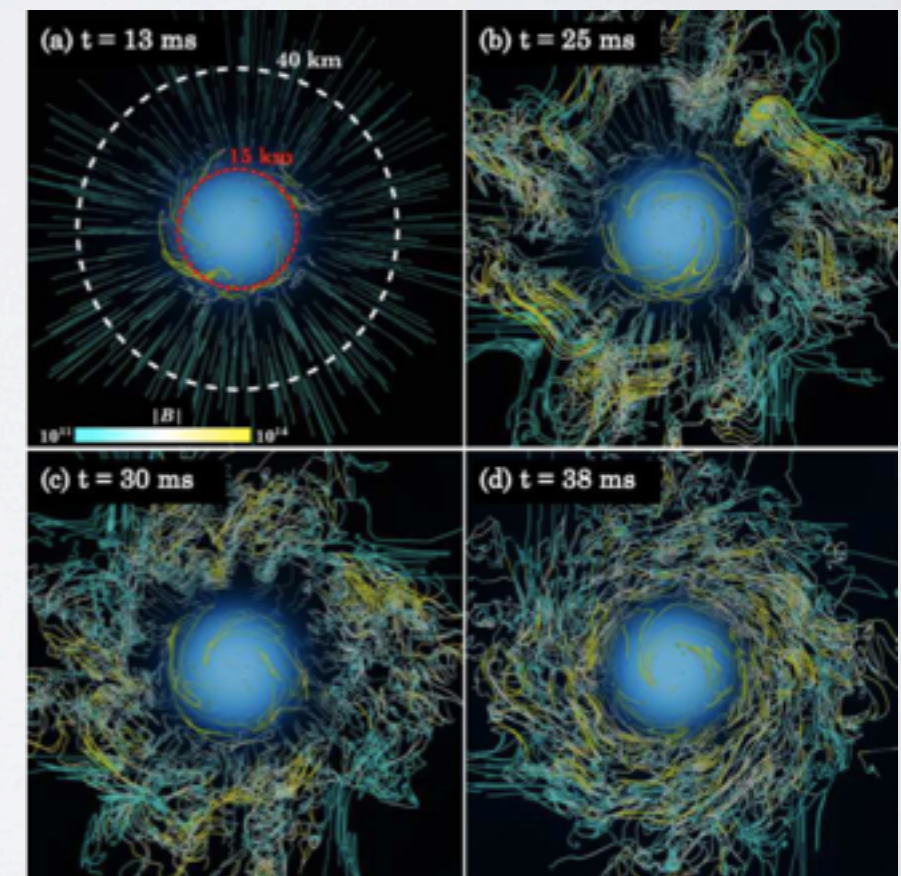
大質量星(OB星)の蠍座タウ星[ただの参考]



東大のWebページ

(c) Nakano

Makishima et al. (2014) PRL 112, 171102



MRIによるNSsの磁場の増幅 (0~38ms)

Masada et al. (2015) ApJ L798.22

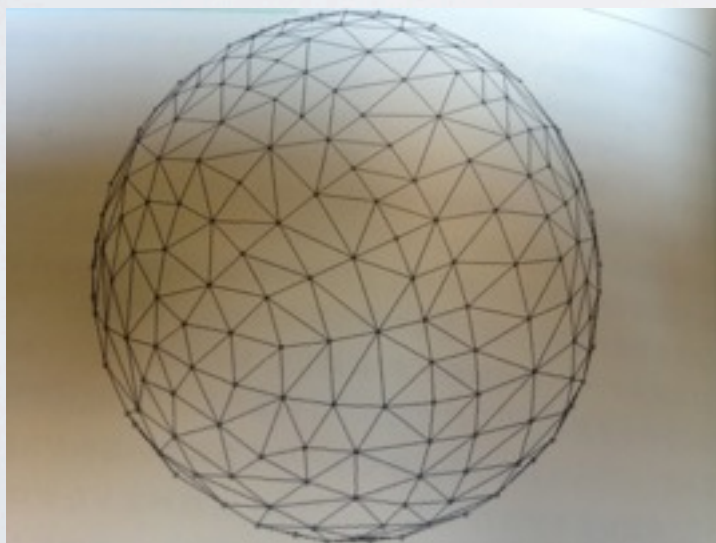
SUMMARY

現実的な状態方程式(baroclinic)で回転則を仮定せずに、
かつ質量座標系のもとでの平衡形状を求める手法を
世界で初めて構築した。

→中性子星に限らず、(原始)恒星、(原始)惑星など
あらゆる星に適用できる！

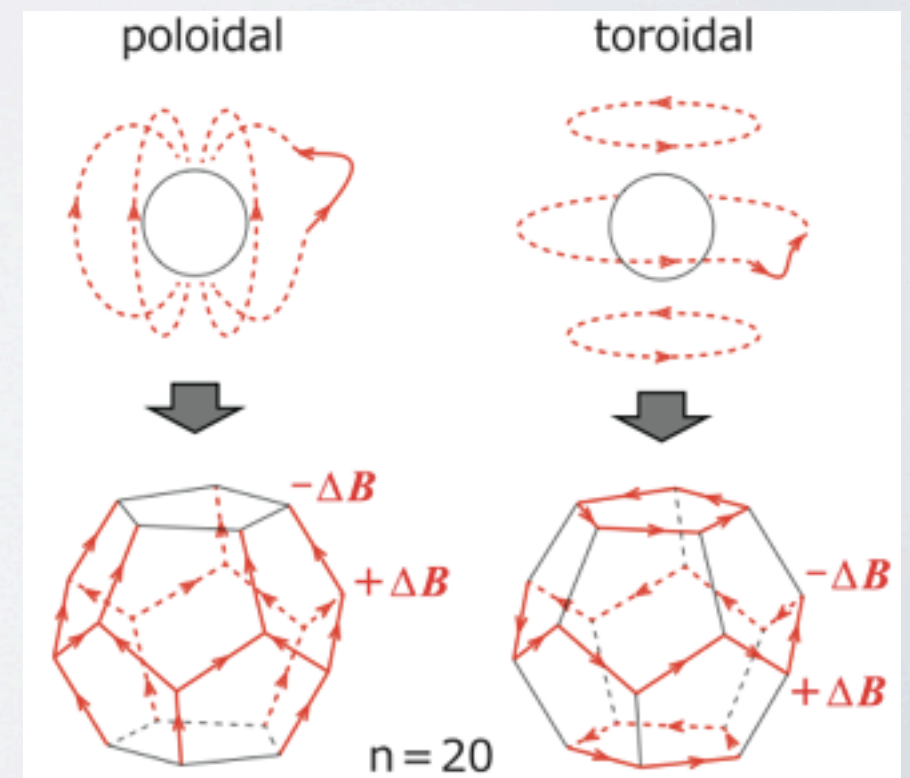
Future works

- ・他の研究者と連携して進化計算を行って観測と比べたい。
 - ・2D軸対称計算でマグネターの進化計算を行えるようになったが3Dに拡張したい
- 2, 3年後？



もし、できたとしても
Cellular Automatonによる

簡単なモデルと似た振る舞いをする？



K.Nakazato 2014 PRD

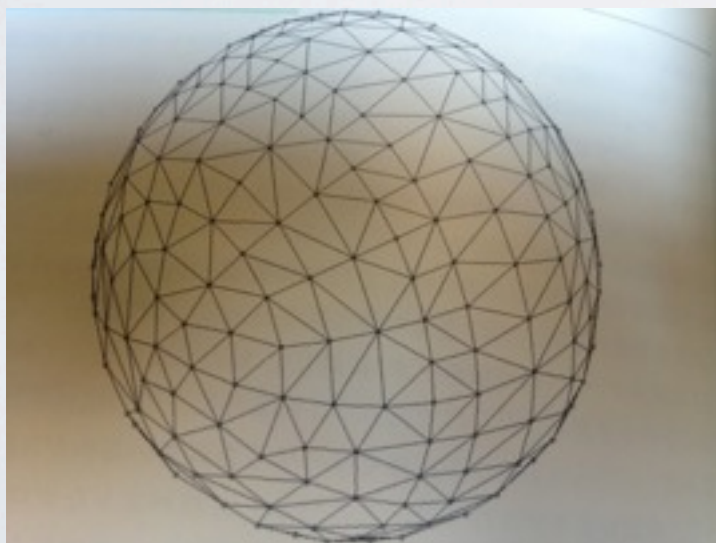
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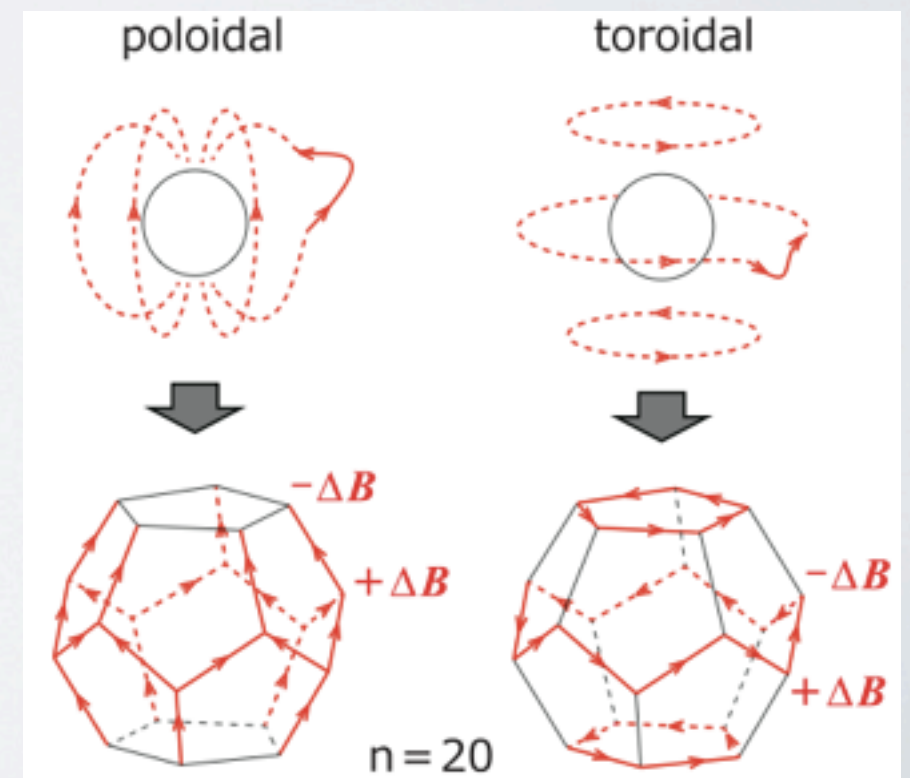
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もし、できたとしても
Cellular Automatonによる

簡単なモデルと似た振る舞いをする？

Fin



K.Nakazato 2014 PRD