核媒質中の中性パイ中間子凝縮

--- 玉垣先生から学んだこと ---

国広　悌二（京大理）
T. Kunihiro

新学術領域研究会「中性子星核物質」
基礎物理学研究所
2015年3月14日（土）

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--- 核媒質中の中性パイ中間子凝縮の物理を中心として ---

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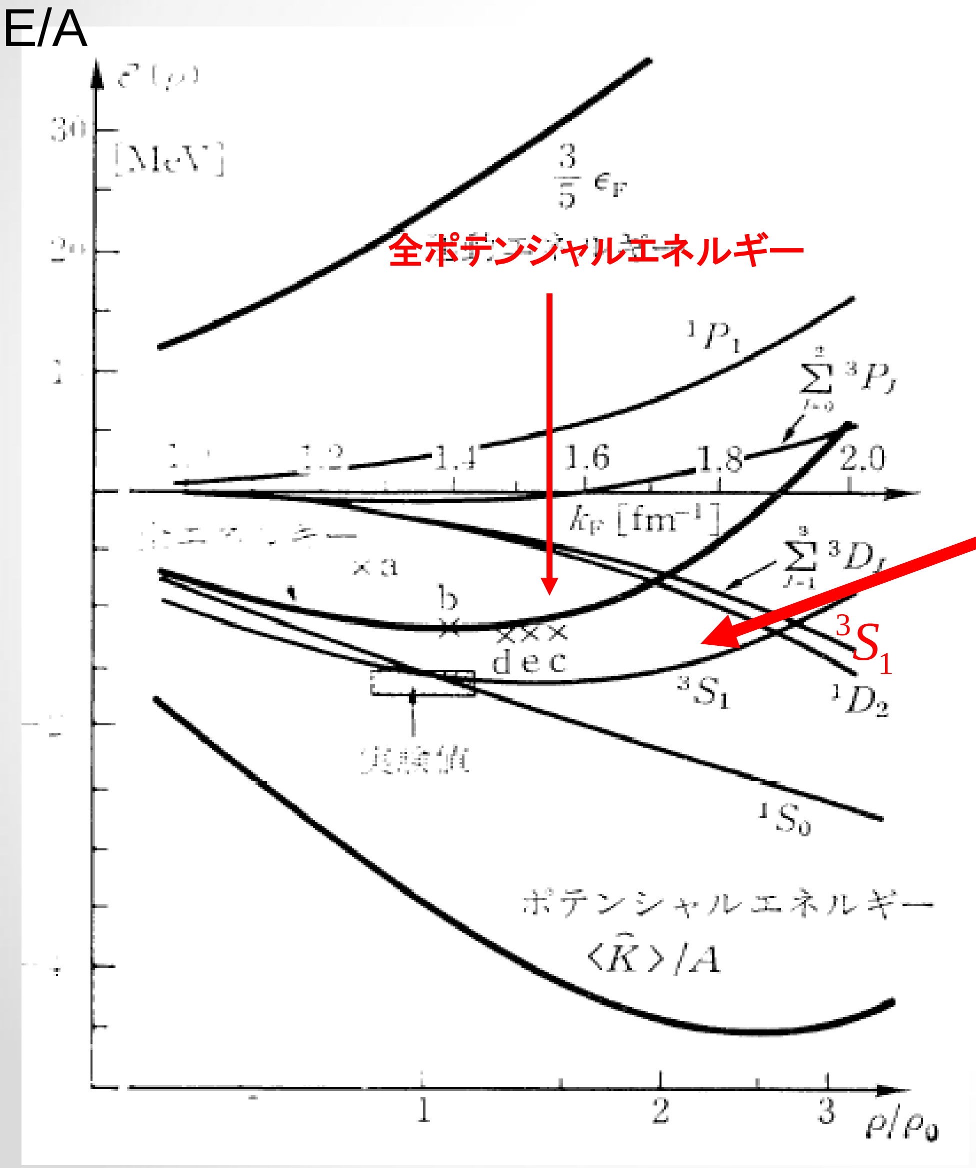
原子核物理の基礎

対称核物質の状態方程式；2核子相互作用の部分波ごとの寄与



ブリュックナー理論（多重散乱理論）による計算

$^3S_1$ ：重陽子のチャンネル

ポテンシャルエネルギーへの
テンソル力の2次の量子効果が
特異な密度依存性を低密度で示す。

⇩

密度の飽和性の第一の起源

玉垣良三
講談社サイエンティフィク
「大学院原子核物理」(1996)

See also,
M.A. Preston and R.K. Bhaduri,
Structure of the Nucleus,
Addison-Wesley, 1975

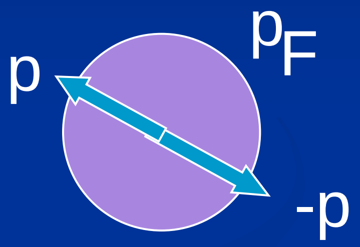
核子（陽子、中性子）；　フェルミオン

パウリの排他律

フェルミ縮退（フェルミ面）

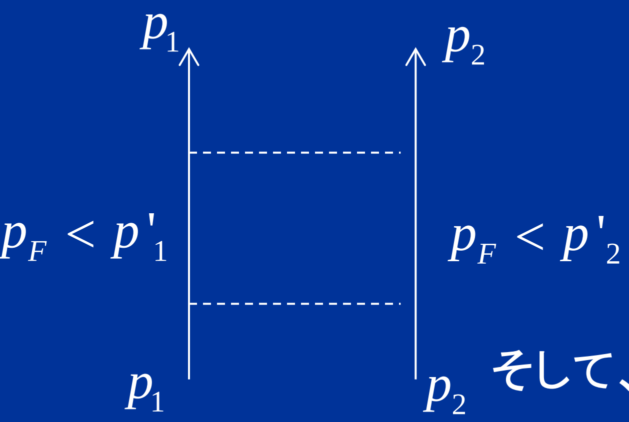
テンソル力

核物質の密度と結合エネルギーの飽和性



粒子間に引力が働けば、
Cooper不安定性により、
そのチャンネルで超伝導（流動）
が起こる。

中重核の表面付近、中性子星の内部の核子超流動

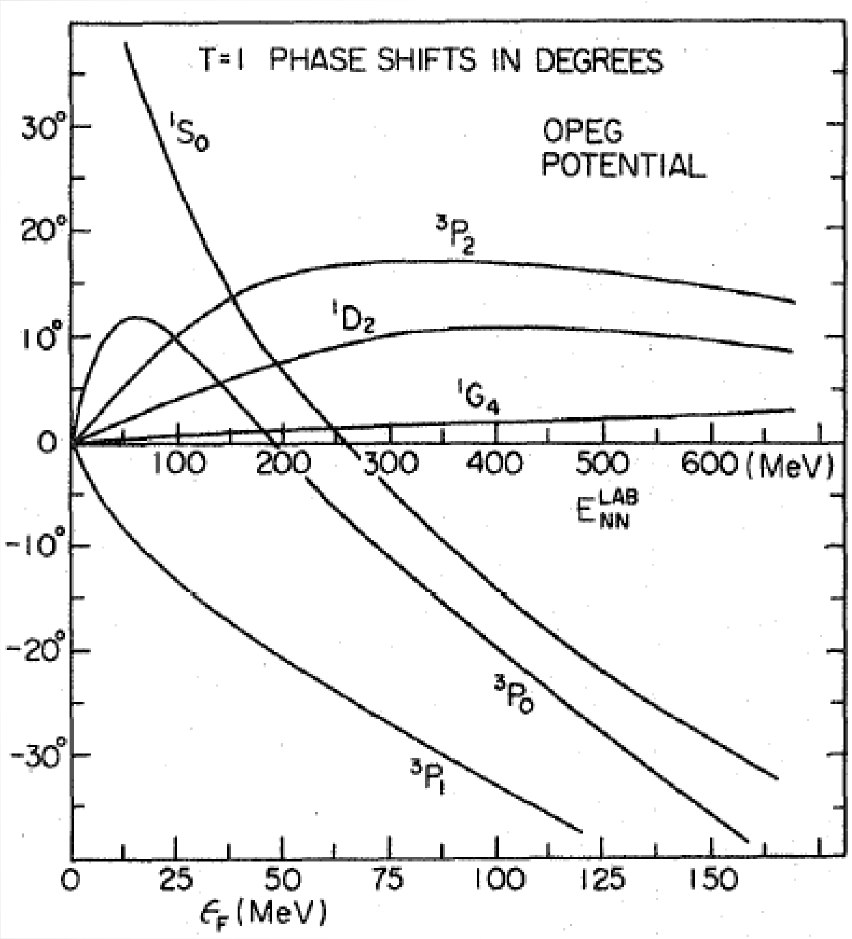


そして、軽い原子核でのアルファクラスター構造

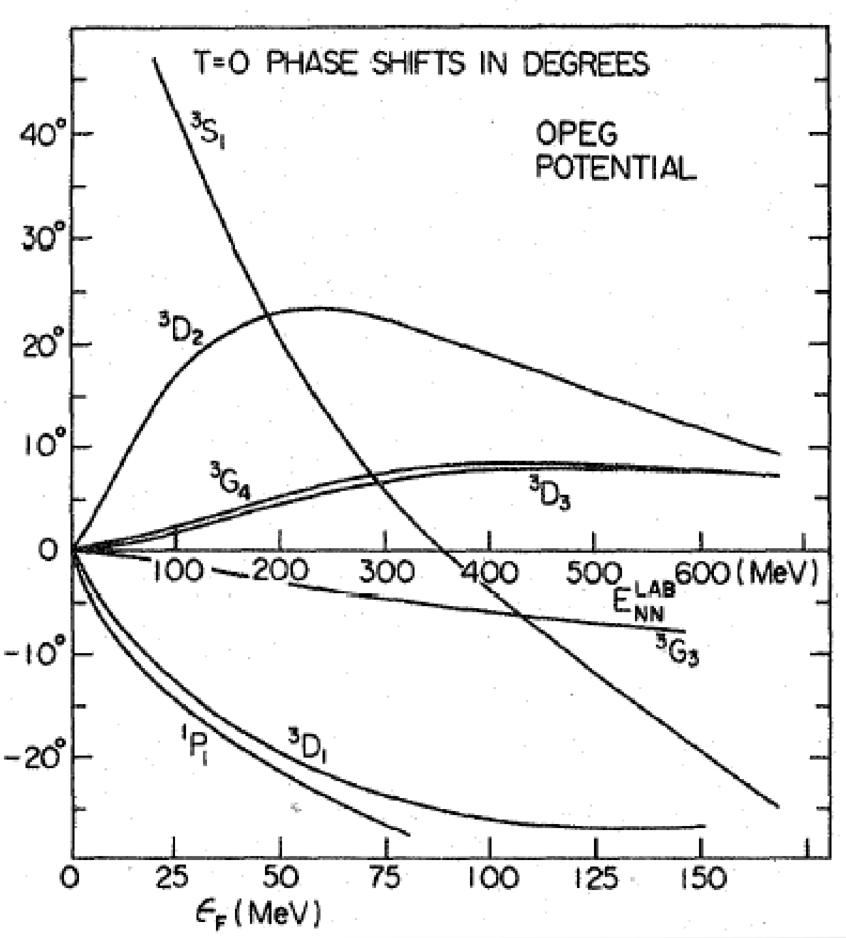
(outer-weak-inner-strong)

核力の状態依存性：核子散乱の位相差のエネルギー依存性

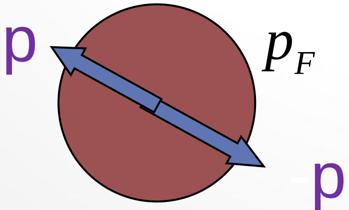
T=1; pp, nn etc



T=0; (pn-np)/



高密度でのBCS
ペアリングは高エネ
ルギー散乱である。



玉垣良三
講談社サイエンティフィク
「大学院原子核物理」(1996)

核子超流動(伝導)ギャップの密度依存性

Tamagaki (’70), Takatsuka-Tamagaki(’71), Takatsuka (’72), Takatsuka-Tamagaki (’94)

c.f. BCS 理論によるギャップ Δ の
結合定数 g依存性

$$\Delta \propto \exp(-c/gN_F)$$

$$N_F = \frac{m^* p_F}{\pi^2}$$:フェルミ面での
単位エネルギーあたりの
状態数）状態密度）

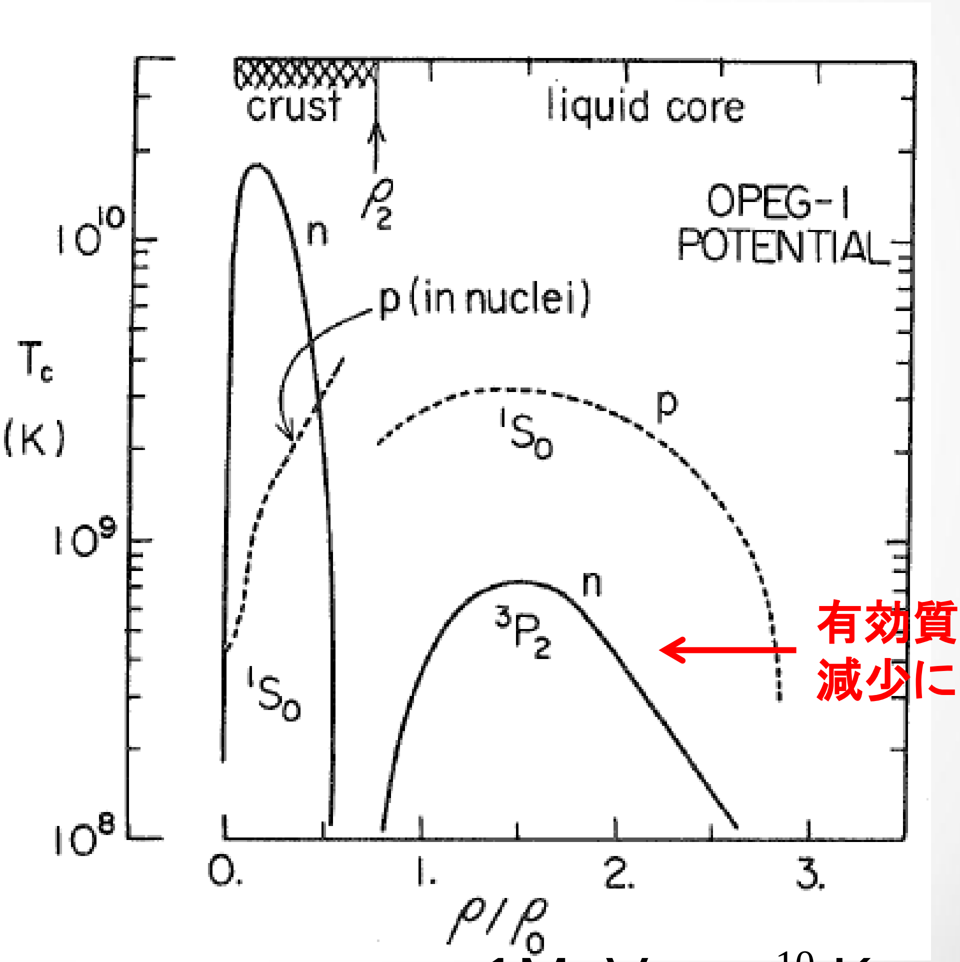
状態密度と有効質量

$$dN = 2\frac{d^3\mathbf{p}}{(2\pi)^3} = \frac{1}{\pi^2}p^2\frac{1}{d\varepsilon/dp}d\varepsilon$$

$$\varepsilon = \varepsilon(p) = \frac{p^2}{2m} + U(p;\rho) \simeq \frac{p^2}{2m^*(\rho)} + U(0)$$

Thus,

$$\left.\frac{dN}{d\varepsilon}\right|_{p=p_F} = \frac{m^* p_F}{\pi^2}$$



有効質量の
減少による

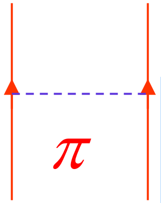
1MeV= 10^{10} K

OPEP(One-Pion-Exchange Potential)

$$V_{\mathrm{OPEP}}(r) = f^2 m_\pi \frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{3}\Big[(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 Y(m_\pi r) + S_{12} Z(m_\pi r)\Big],$$

Central Tensor

$$Y(x) = \exp(-x)/x,\ Z(x) = (1 + 3/x + 3/x^2)Y(x),$$

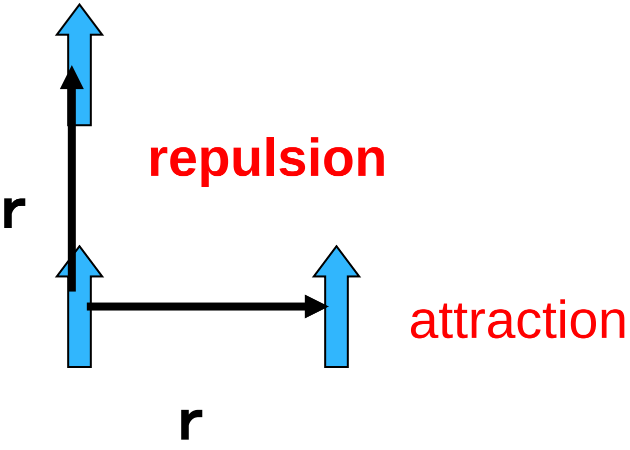


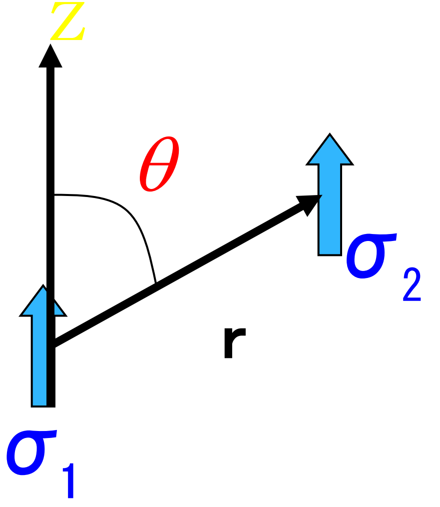
$$S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

Tensor operator $(\hat{\boldsymbol{r}} = \boldsymbol{r}/r).$

$= 3\cos^2\theta - 1$ ↑↑

$= -3\cos^2\theta + 1$ ↑↓





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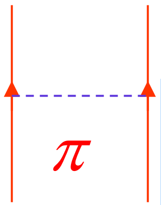
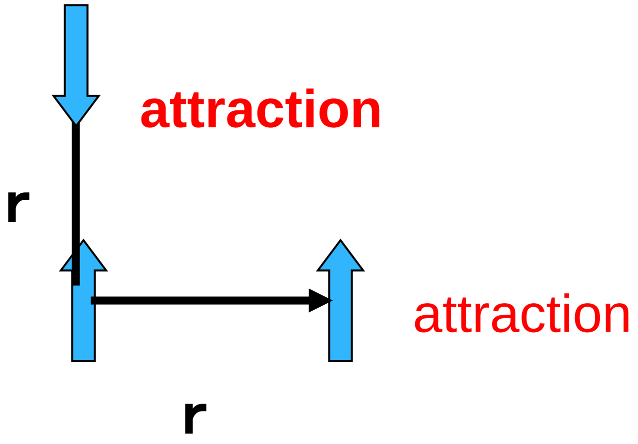
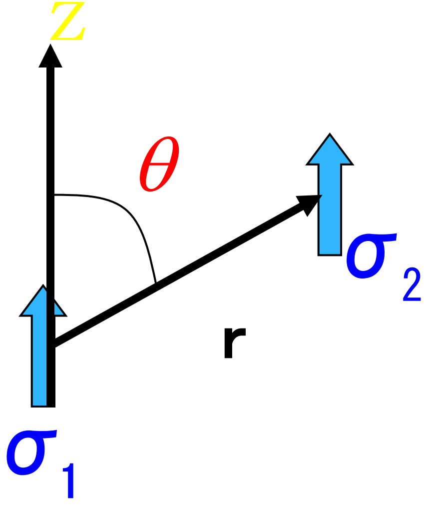


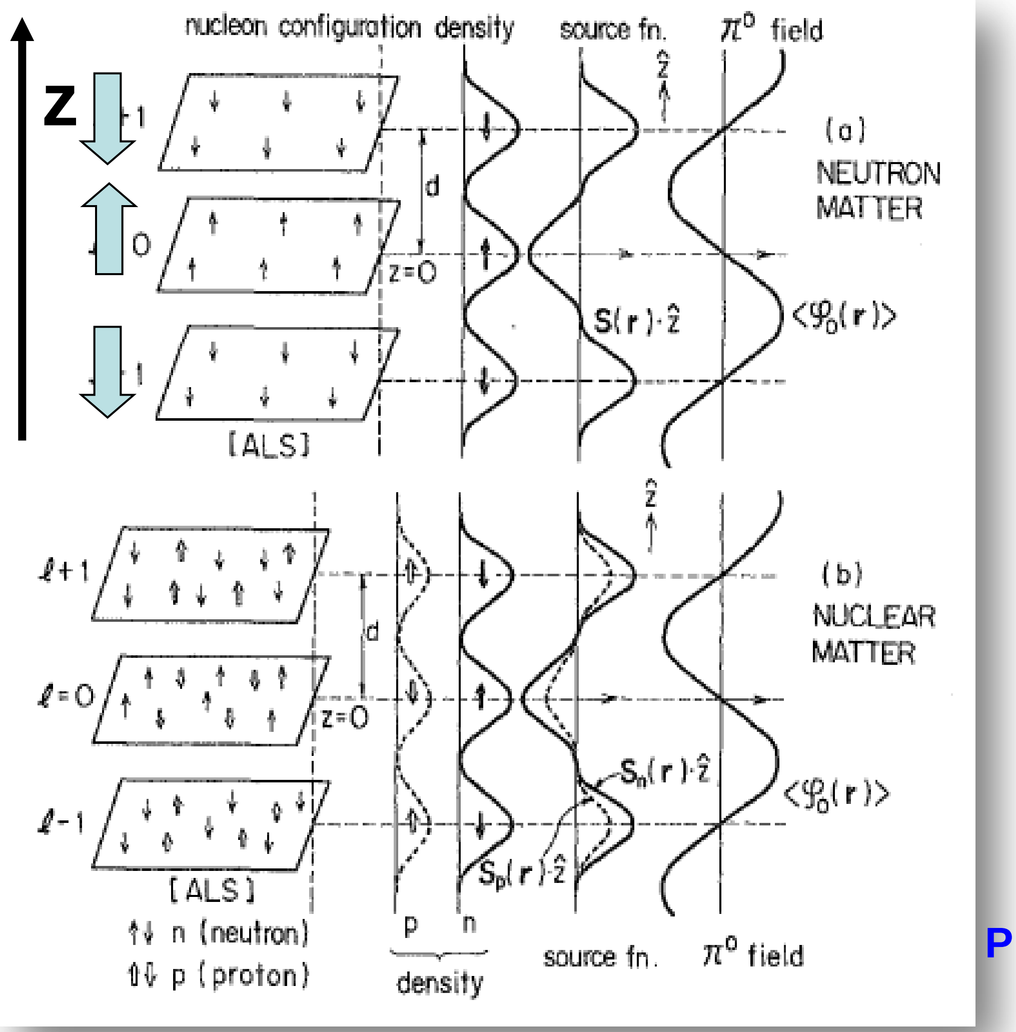
$$S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) = 3\cos^2\theta - 1$$ ↑↑

Tensor operator $(\hat{\boldsymbol{r}} = \boldsymbol{r}/r).$ $= -3\cos^2\theta + 1$ ↑↓





p-wave Neutral Pion-condensed Baryonic Matter;
pion-induced tensor-force dominating phase



PTP suppl.112(1993)

A.B. Migdal, Sawyer-Scalapino ('72)

Pion condensed phase

=Alternating-Layer Spin (ALS)

structure of the nucleon System

(R.Tamagaki et al (1976~))

$$(\nabla^2 - m_\pi{}^2)\langle\varphi_0(\boldsymbol{r})\rangle = \tilde{f}\,\nabla\cdot\boldsymbol{S}(\boldsymbol{r})$$

$$\boldsymbol{S} = \langle\Phi_{\rm N}|\psi^\dagger(\xi,t)\tau_3\boldsymbol{\sigma}\psi(\xi,t)|\Phi_{\rm N}\rangle$$

$$\begin{tikzcd}
|\Phi_{\rm ALS}\rangle \arrow[rr, leftrightarrow] \arrow[dr, leftrightarrow] & & \nabla\cdot\boldsymbol{S}(\boldsymbol{r})\neq 0 \arrow[dl, leftrightarrow] \\
& \langle\varphi_0(\boldsymbol{r})\rangle\neq 0\,. &
\end{tikzcd}$$

Pi : longitudinal spin-isospin density wave

⇕

c.f. ρ meson condensation : transverse spin-isospin density wave

T.K., PTP 60 (1978), 1229

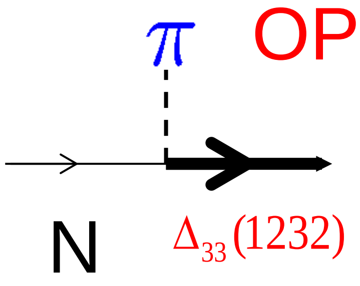
Realistic treatment of π con. with the isobar Δ and Short-range int. and correlations

EOS for pion-condensed N=Z Matter

Effective Force (G0-force)* with $\Delta_{33}(1232)$

the resonance

R.Tamagaki and T.K. PTP. 61 ('79)1107



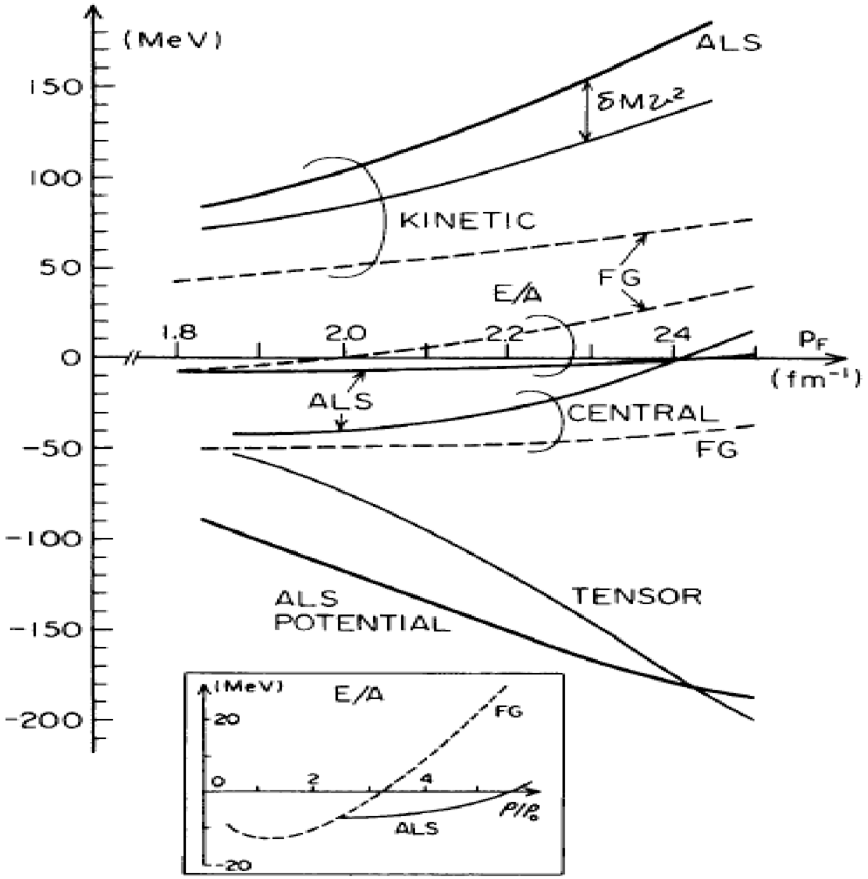
OPEP+g'

$$|\tilde{\Phi}_{ALS}\rangle \equiv \prod_{\alpha}^{(occ)} \tilde{N}_{\alpha}{}^{\dagger}|0\rangle$$

$$\tilde{N}_{\alpha} = u_{\alpha}N_{\alpha} - v_{\alpha}\Delta_{\alpha}$$

$$\tilde{\Delta}_{\alpha} = v_{\alpha}N_{\alpha} + u_{\alpha}\Delta_{\alpha}$$

$$\binom{u^2}{v^2} = \frac{1}{2}\{1 \pm (\epsilon_{\Delta} - \epsilon_{N})/\sqrt{(\epsilon_{\Delta} - \epsilon_{N})^2 + 4CV^2}\}$$



* D.W.L.Sprung and P.K. Banerjee,NPA168('71); D.W.L. Sprung,NPA182('72),97. (「有効相互作用」との出会い)

有限温度の効果と重イオン衝突によるπ凝縮相生成の可能性
--- 相対論的流体方程式による評価 ---

相対論的流体のショック(Taub 断熱曲線.)

T.Takatsuka, R. Tamagaki and T.K.
PTP 79 (1988) 120

$$\frac{\rho_2}{\rho_1}=\left[\frac{e_2+P_2}{e_1+P_1}\cdot\frac{e_2+P_1}{e_1+P_2}\right]^{1/2}$$

$$\gamma=1/\sqrt{1-v_{\mathrm{CM}}^2}$$

$$=\left[\frac{e_1+P_2}{e_1+P_1}\cdot\frac{e_2+P_1}{e_2+P_2}\right]^{1/2}$$

$e_1=(m_N+E_N)\rho_1,\quad P_1=0$: normal state

$$E^{(\mathrm{CM})}=(\gamma-1)\cdot(m_N-B)$$

混合相:

$$\rho_2=(1-x)\rho_{c1}(T_2)+x\rho_{c2}(T_2)$$

$$e_2=(1-x)e_{\mathrm{FG}}(\rho_{c1},T_2)+xe_{\mathrm{ALS}}(\rho_{c2},T_2)$$

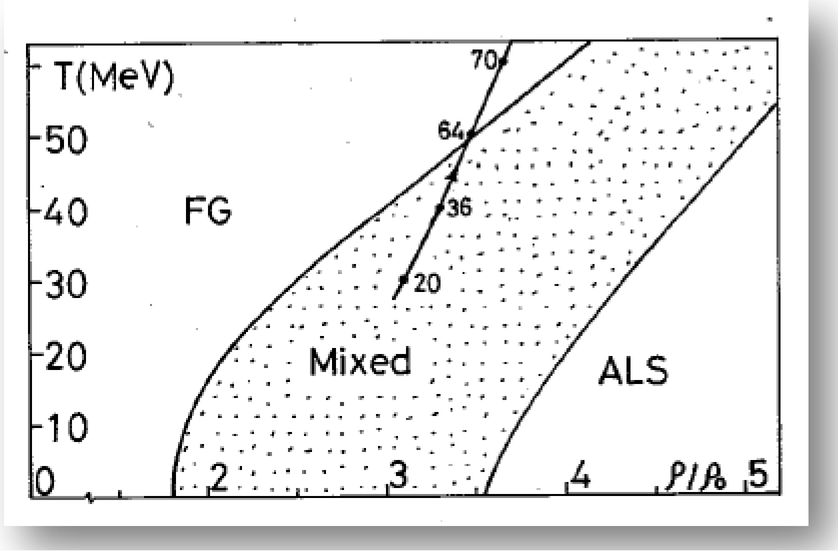
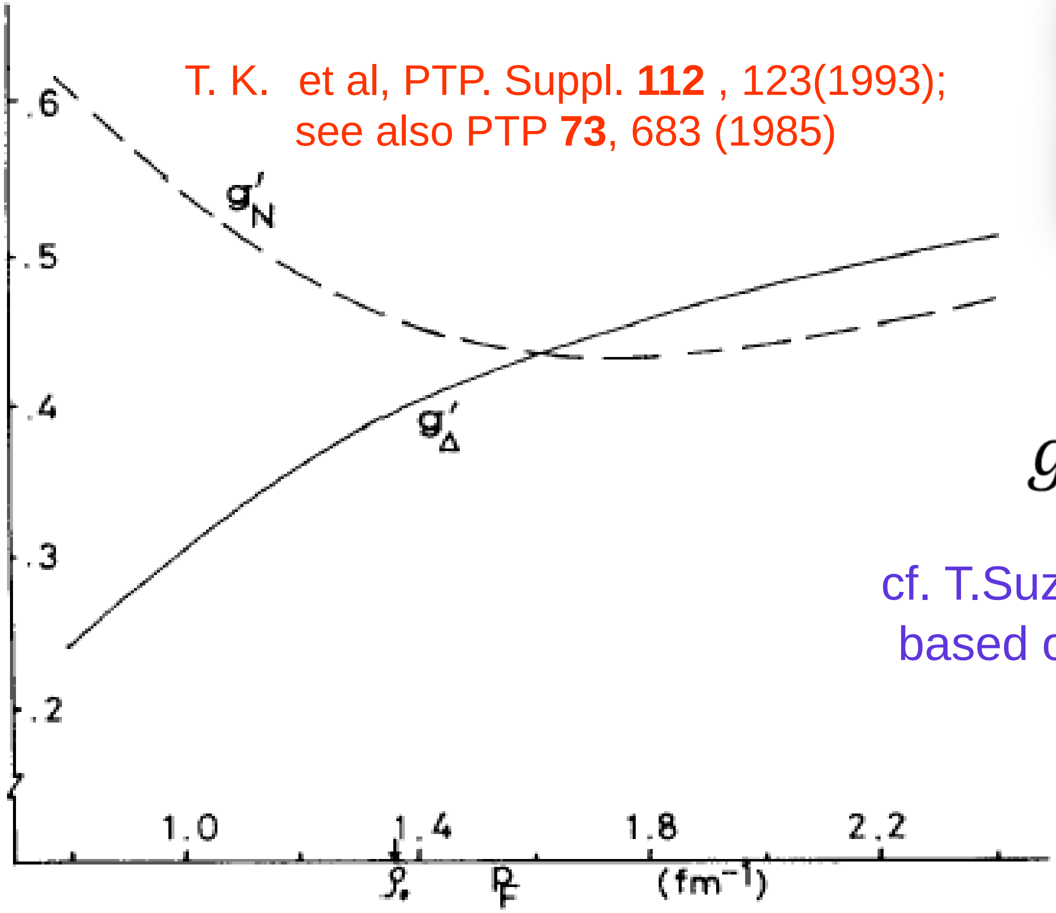


Fig. 8. The phase diagram of the symmetric nuclear matter. The dots indicate the coexistent region of the π°-condensed and normal phase. The curved arrow is the shock adiabat; the attached numbers are the corresponding C.M. energies per particle of HEHIC in a fluid dynamical model.

The density dependence of the Landau-Migdal parameteters in the N-N and Δ -N channels from G-0 force



T. K. et al, PTP. Suppl. **112** , 123(1993);
see also PTP **73**, 683 (1985)

$$G_\Delta'(\boldsymbol{p}_1, \boldsymbol{p}_2)\boldsymbol{\sigma}_1 \cdot \boldsymbol{S}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{T}_2$$

$$G_\Delta'(\boldsymbol{p}_1, \boldsymbol{p}_2) = \sum_{l=0}^{\infty} G'_{\Delta l} P_l(\cos\theta).$$

$$g_\Delta'(ff^*/m_\pi^2) = G'_{\Delta l=0}$$

$$g'_N \sim .6 \qquad g'_\Delta \sim .2$$

cf. T.Suzuki and H.Sakai, PLB455('99),25
based on T. Wakasa et al, PRC, ('97), 2909

Gamow-Teller giant resonance;
K.Ikeda, S.Fujii and J.I. Fujita, PL .3('63),271

スピン-アイソスピンモードのソフト化とパイ中間子凝縮臨界条件

Restoring force for the (longitudinal) spin-isospin density wave

$$\varepsilon_{\sigma\tau}(q) = \frac{\varepsilon_F}{3} \cdot \frac{1}{\Phi(q/2p_F)} + \frac{\rho}{2} \cdot (v_\pi(q) + g')$$

(T.K. PTP **65** (1981), 1098)

Lindhard fun.

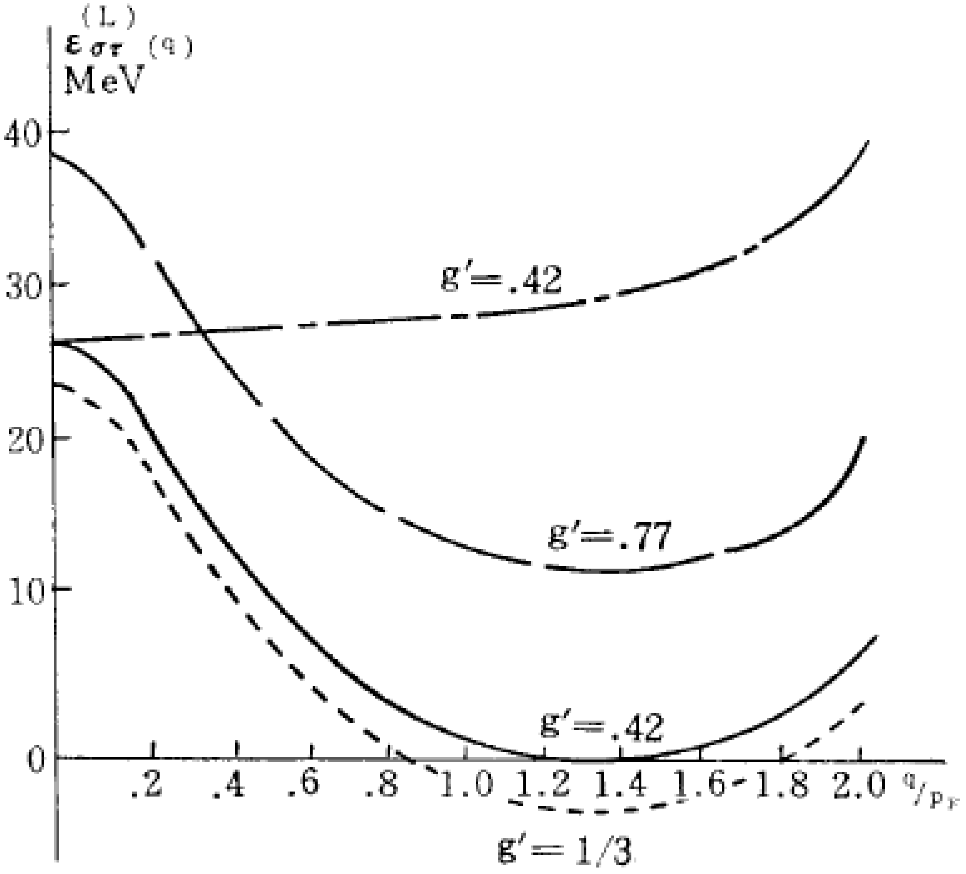
In the Steinwedel-Jensen model:

$$\omega_n^{(L)} = q_n^{(L)} \sqrt{2\varepsilon_{\sigma\tau}(q_n^{(L)})/M} \equiv \omega(q_n^{(L)})$$

$z_n^{(L)}$ ：n-th zero of the derivative of the spherical Bessel function

$$q_n^{(L)} = z_n^{(L)}/R$$

$v_\pi(q)$: OPEPのフーリエ成分



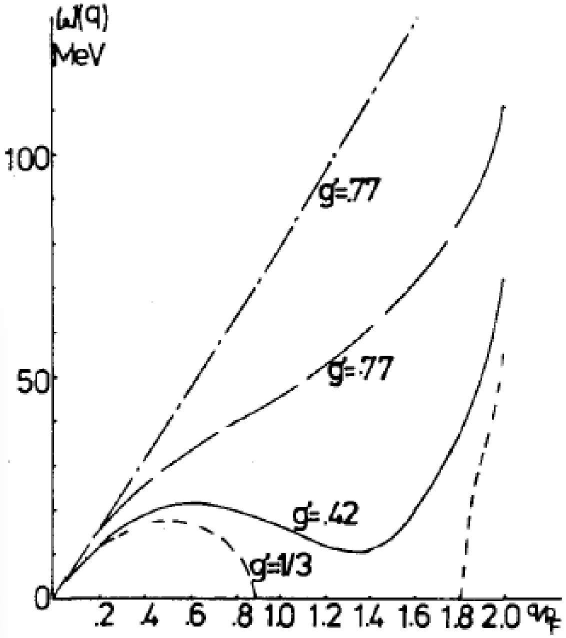
Dispersion relation of the spin-isospin excitation

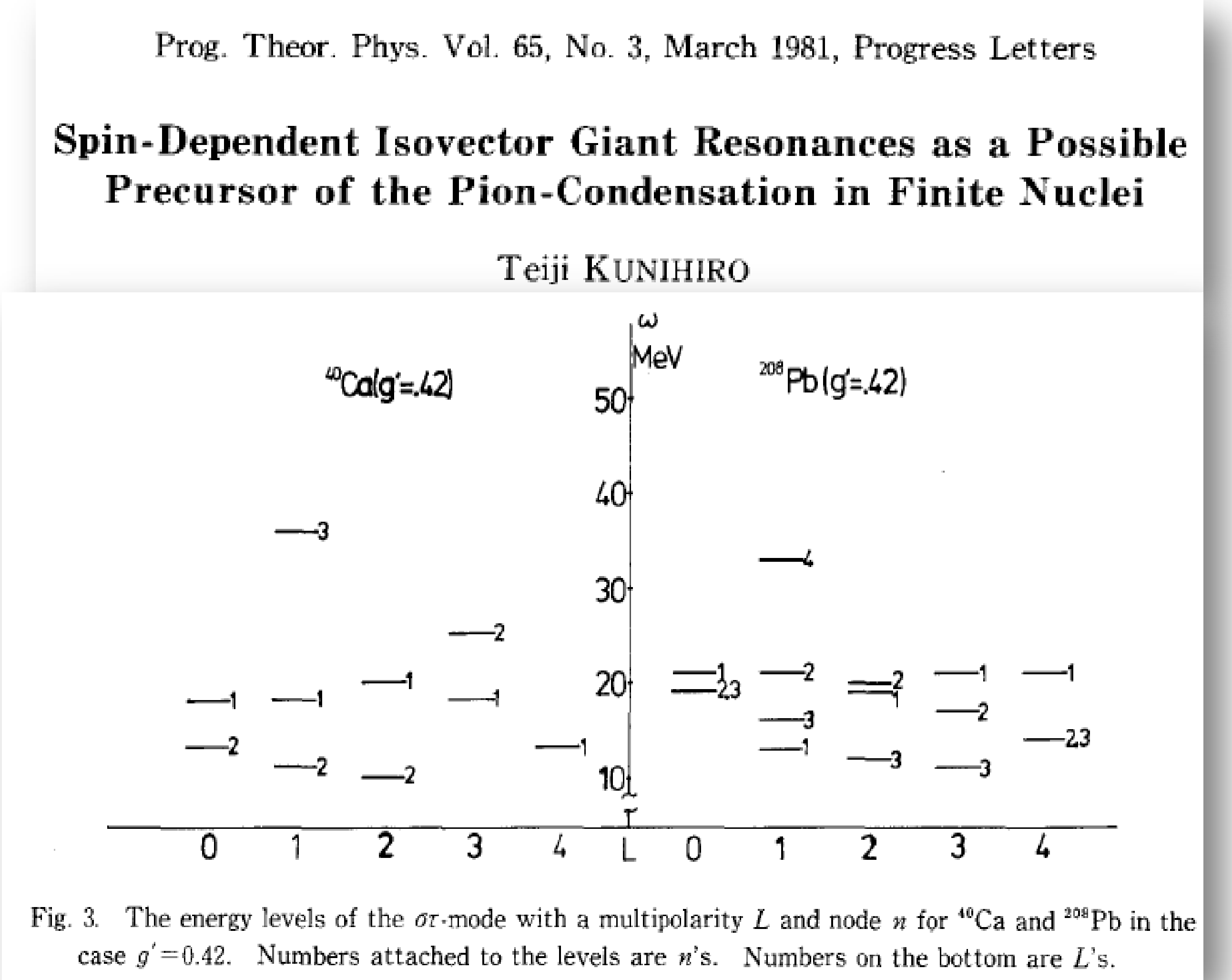
Softening at finite q !

and hence larger L
In finite nuclei.

T.K. PTP **65** (1981), 1098

自然だ。（玉垣さん）





Prog. Theor. Phys. Vol. 65, No. 3, March 1981, Progress Letters

Spin-Dependent Isovector Giant Resonances as a Possible Precursor of the Pion-Condensation in Finite Nuclei

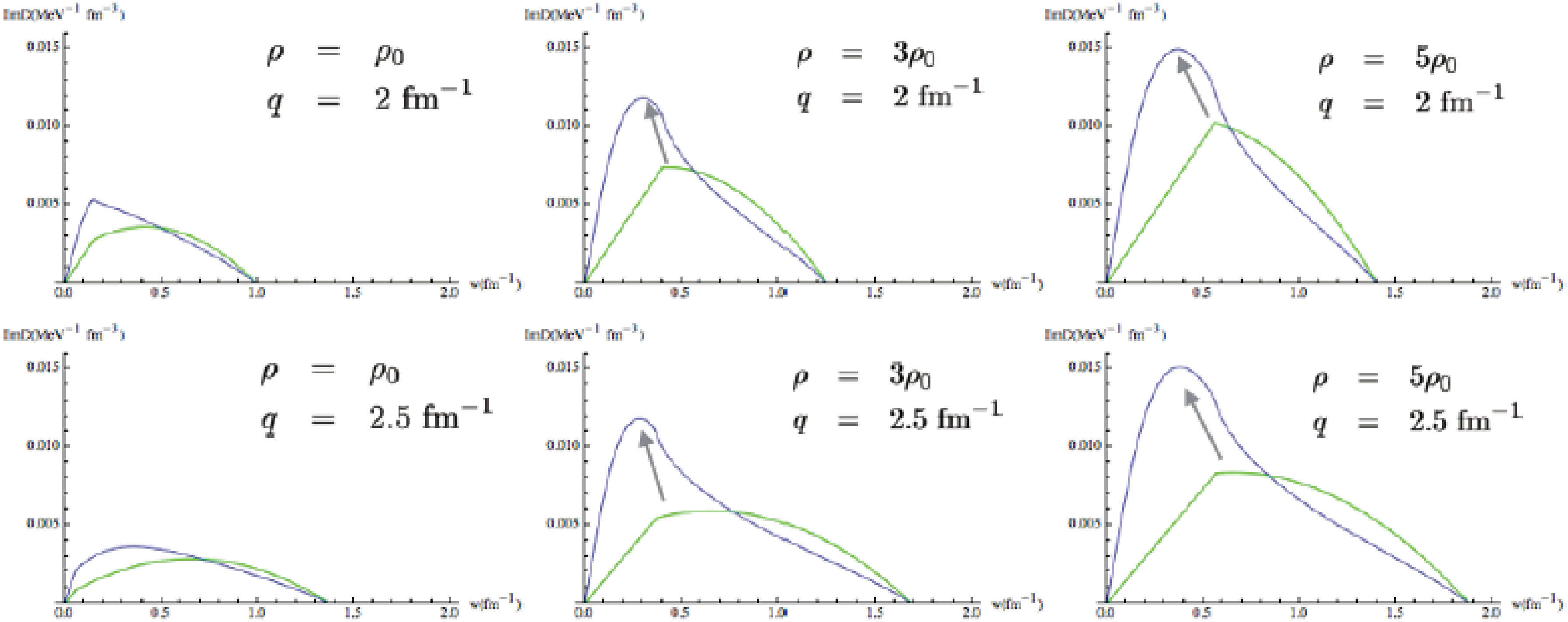
Teiji KUNIHIRO

Fig. 3. The energy levels of the $\sigma\tau$-mode with a multipolarity L and node n for ^{40}Ca and ^{208}Pb in the case $g'=0.42$. Numbers attached to the levels are n's. Numbers on the bottom are L's.

G-0 力を用いたパイ中間子チャンネル（σ-τモード）のスペクトル関数

(Y. Kikuchi , T.K. (‘14))

(— : $\mathrm{Im}D_{\mathrm{ph}}$(free fermion), — : $\mathrm{Im}D_{\mathrm{RPA}}$)



- 核子間相互作用によりパイ中間子モードがソフト化している。
- 密度が高いほど、ソフトモードの強度が大きくなっている。
- パイ中間子凝縮はみられない。

⇒ Δ-空孔の効果の導入の効果は？

核力とシグマ中間子

- **Responsible for the intermediate range attraction in the nuclear force**.

「核力のシグマは``σ”:粒子として存在しない。」（玉垣さん）

カイラル対称性とシグマ中間子

カイラル対称性の自発的破れ in NJL model

$$-(2g)^{-1}M = 2N_c \int_{|\mathbf{p}|<\Lambda} \frac{d\mathbf{p}}{(2\pi)^3} \frac{-M}{\sqrt{M^2+\mathbf{p}^2}}$$

Gap equation

The dispersion relations of the pion and sigma meson;

=0

$$-D_\pi^{-1}(q^2)/m_0^2 = \boxed{1-\frac{2gN_c}{\pi^2}\int_0^\Lambda \frac{p^2dp}{E_p}[1}-\frac{q^2}{q^2-4E_p^2}]$$

Gap eq.!

$$-D_\sigma^{-1}(q^2)/m_0^2 = \boxed{1-\frac{2gN_c}{\pi^2}\int_0^\Lambda \frac{p^2dp}{E_p}[1}-\frac{q^2-4M^2}{q^2-4E_p^2}]$$

=0

Thus,

$$D_\pi^{-1}(q^2=0)=0 \qquad\qquad D_\sigma^{-1}(q^2=(2M)^2)=0$$

$$\Updownarrow \qquad\qquad\qquad\qquad \Updownarrow$$

$$m_\pi = 0 \qquad\qquad\qquad\qquad m_\sigma = 2M$$

Nambu-Goldstone boson!

QCDのHiggs としてのσ中間子の存在はNGボソンとしてのパイオンの存在と同等に基本的。

c.f.「核力のシグマは``σ”:粒子として存在しない。」（玉垣さん）

しかも、カイラル対称性の回復(M→減少）とともに質量が減少する！
ハドロンの性質が環境によって変化する！
So do the Higgs in principle.

逆に、
重大性
を示している！

The significance of the σ meson in low energy hadron physics and QCD

1. The pole in this mass range observed in the pi-pi S-matrix.

   As a compilation of the pole positions of the σ obatined in the modern analyses: Significance of respecting chiral symmetry,unitarity and crossing symmetry to reproduce the phase shifts both in the σ (s)- and ρ, (t)-channels with a low mass σ pole;(Igi and Hikasa(1999),I.)). Caprini, G. Colangero and H. Leutwyler, PRL(2006

2. Seen in decay processes from heavy particles; $D^+ \to \pi^-\pi^+\pi^+$

   E. M. Aitala et al, Phys. Rev. Lett. (86), 770 (2001)

3. Accounts for Δ I=1/2 enhancement in K → 2π compared with $K^+ \to \pi^+\pi^0$.

   E.P. Shabalin (1988); T. Morozumi, C.S. Lim and I. Sanda (1990).

4. π-N sigma term 40-60 MeV (naively ~ 15 MeV) enhanced by the collectiveness of the σ (.T.Hatsuda and T.K.(1990)).

5. The σ :

   the quantum fluctuation of the chiral order parameter

   ⟷ The Higgs particle in the WSG model

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the quantum fluctuation of the chiral order parameter

⟺ The Higgs particle in the WSG model

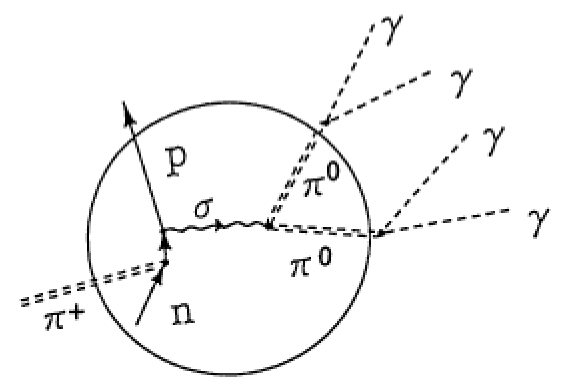
σ on the Lattice

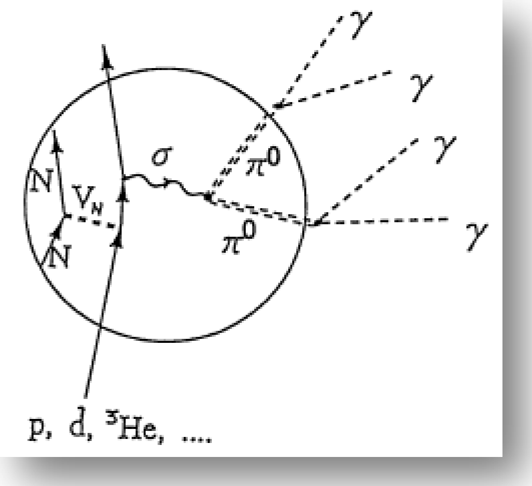
SCALAR collaboration:
Muroya, Nakamura, Nonaka, Sekiguchi, Wada, T.K. PRD 70 (2004) 034504

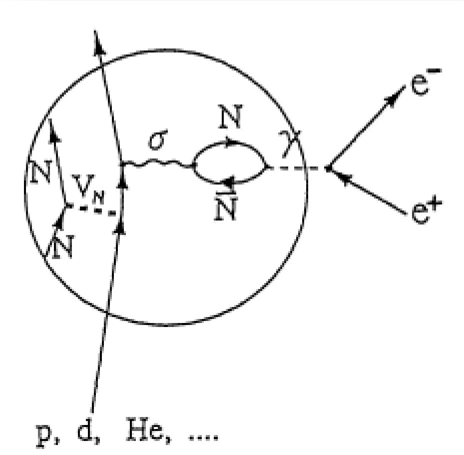
T. Hatsuda, H.Shimizu, T.K.,
``Precursor of chiral symmetry restoration in the nuclear medium,'‘ PRL 82 (1999),2840

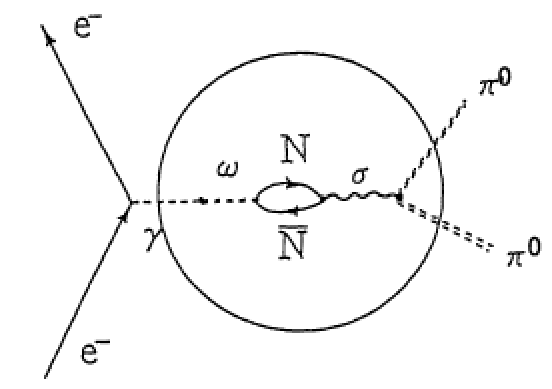
核媒質中でのカイラル対称性の部分的回復の検証
––– 原子核中にσを作る –––

PTP Suppl.120 (1995) 75







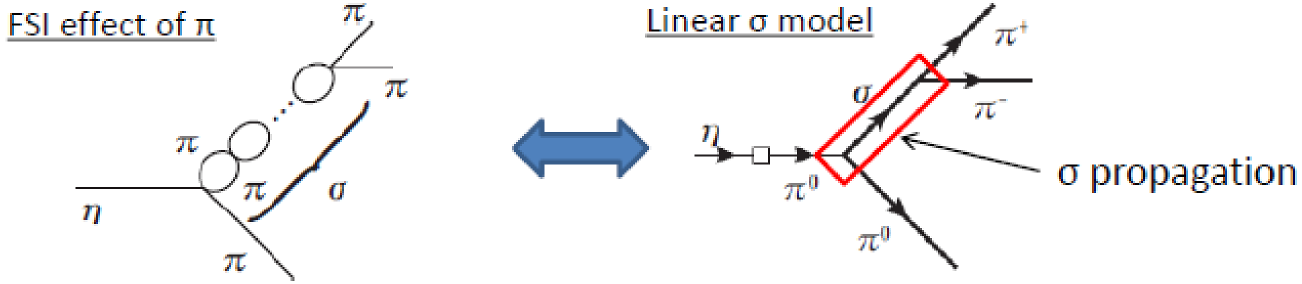


The η decay into 3π in asymmetric nuclear medium and possible relevance of partial restoration of chiral symmetry in nuclear medium

S. Sakai and T.K., PTEP ('15) 013D03

Significance of the final-state int.

Analysis with explicit σ degree of freedom may be interesting.



FSI effect of π

Linear σ model

σ propagation

※Relevance of the FSI of π in scalar channel and σ mode

まとめに代えて：玉垣先生から学んだこと

- 湯川秀樹の偉さを学べ、そして、
湯川につながる者としての誇りを持て

- 自分の問題を持ち、その物理を育てること

- 「外国人何する者ぞ」の気概：eg. 核力の短距離斥力の起源とG. Brown

- 知られていることの論理的帰結か本当に新しいかを判断し、
本当に新しいことをするように努めること

- 本質をえぐり出すことばへの執着：

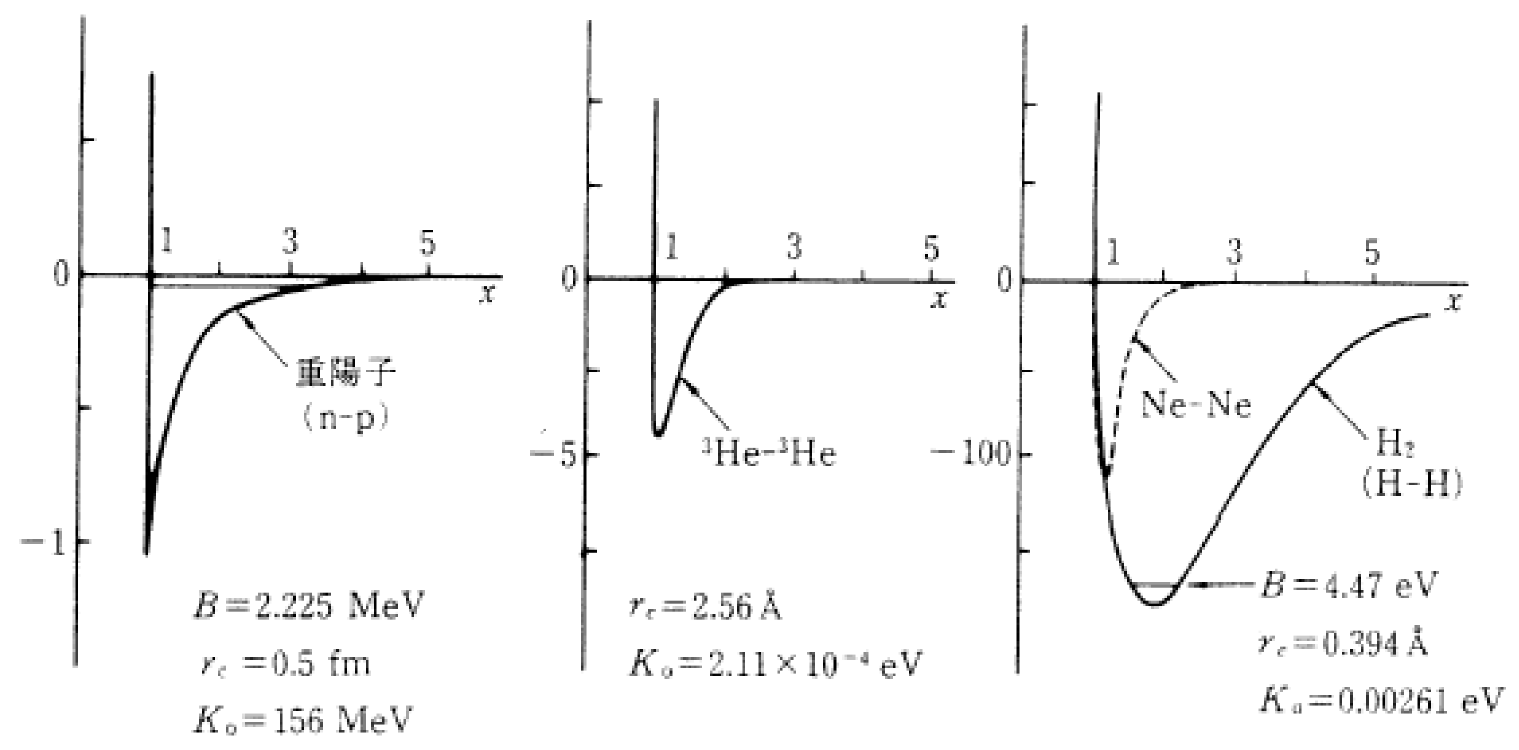
  tensor-force-dominating phase, outer-weak-inner-strong, dual role of Pauli principle, ALS(Alternating-Layer-Spin) model 等々

- 核物理/核物質への基本的視点：
テンソル力とパウリ原理

補足

核子系は量子ゆらぎが大きい！

重陽子　He2体系　水素分子



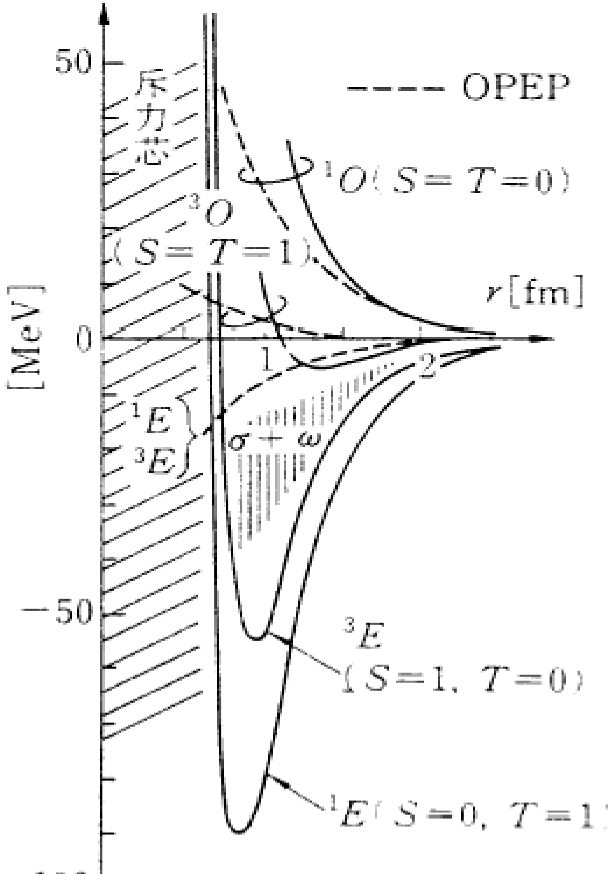
距離の単位= ハードコアの半径rc

エネルギーの単位= rcでのゼロ点エネルギー $K_0\equiv\hbar^2/M_0 r_c^2$

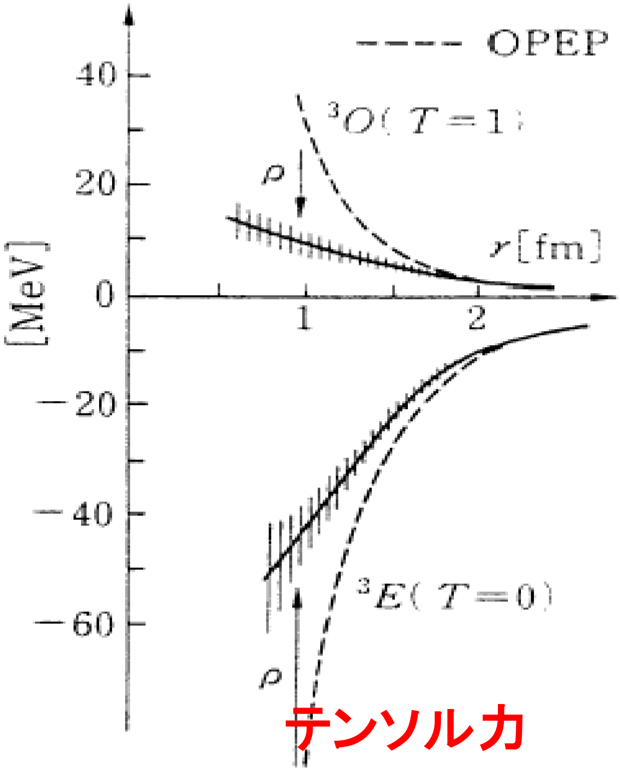
玉垣良三
講談社サイエンティフィク
「大学院原子核物理」(1996)

核力の特異性:テンソル力と核物質の結合

唯一の2核子結合状態
=重陽子　T=0, 3S1-3D1**状態**



(c) 中心力 V_C



テンソル力

(a) テンソル力 V_T

T=1の2核子結合状態は存在しない;
1Eの引力は十分には強くない。
しかし、3Eの引力はもっと小さい！？

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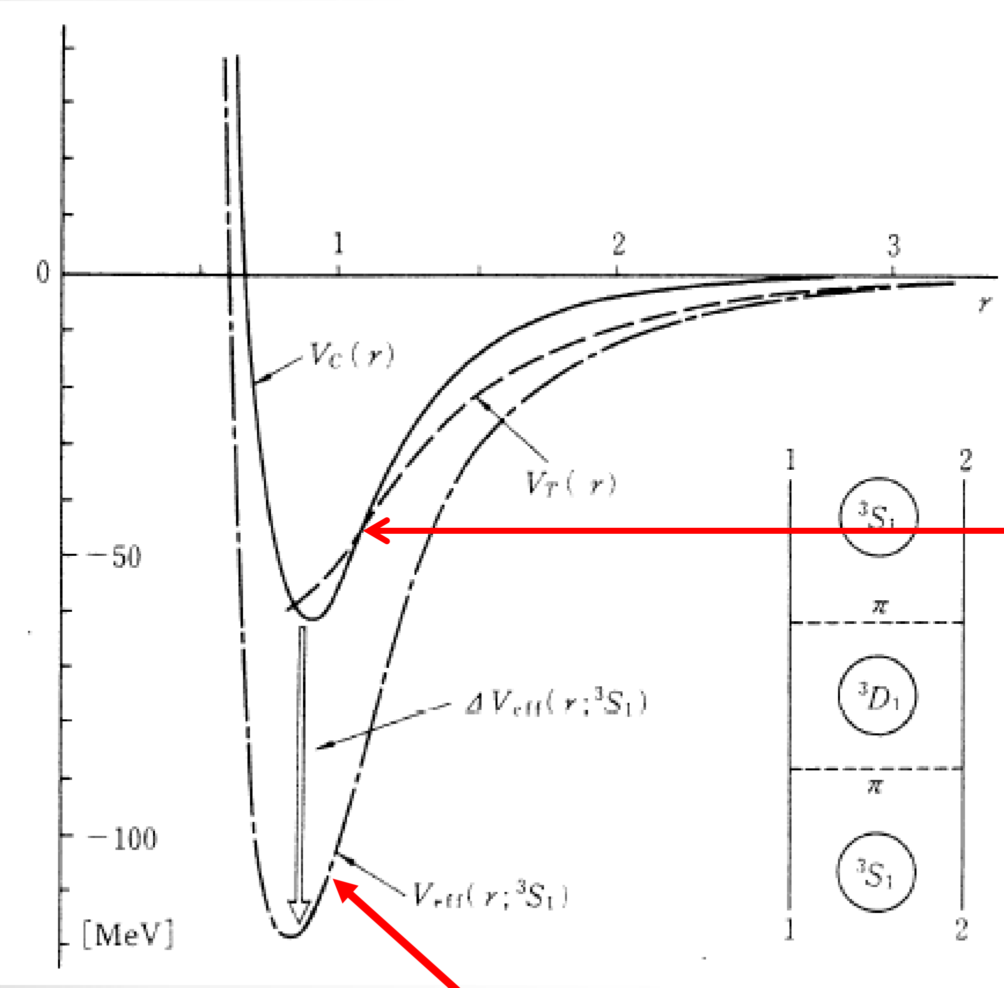
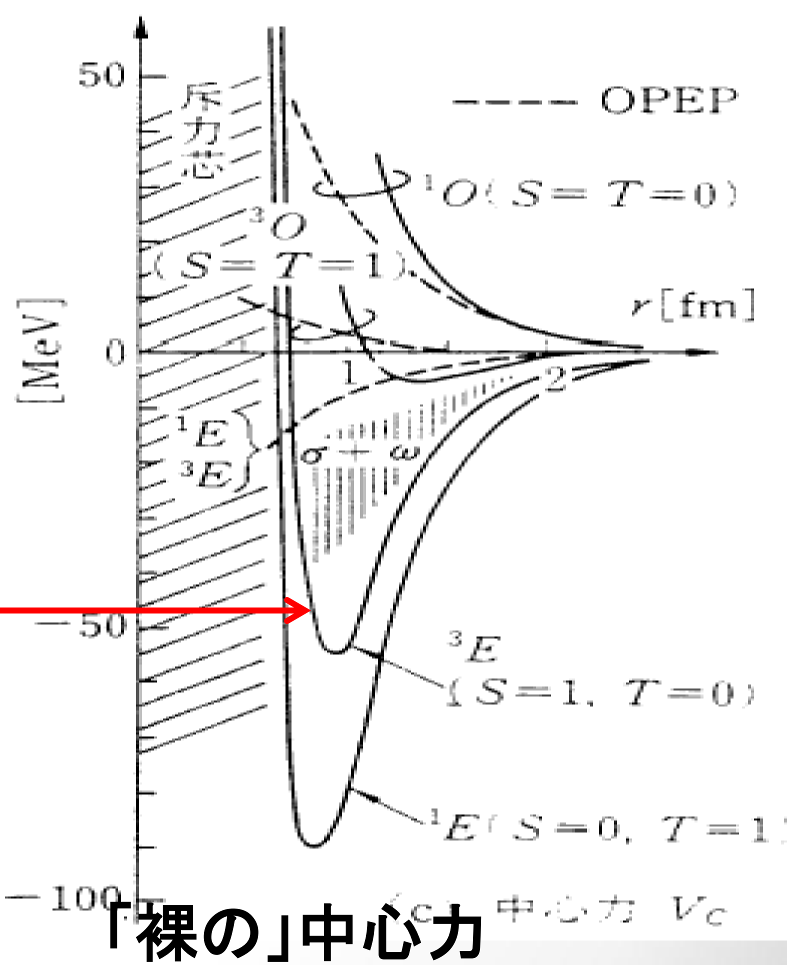
「大学院原子核物理」(1996)

（重陽子に働く）テンソル力に起因する3S1状態の有効中心力

有効中心力





「裸の」中心力

有効中心力

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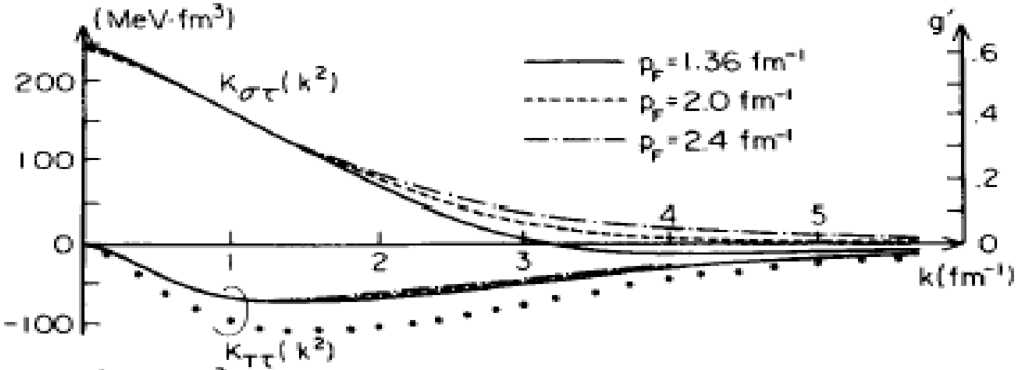
Effective Interaction(G-0 force)* in the $\sigma\tau$ channel

$$V_{\rm ph}(1,2)=\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2[\,V_{\sigma\tau}(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+V_{T\tau}S_{12}(\hat{\boldsymbol{r}})\,]\,,$$

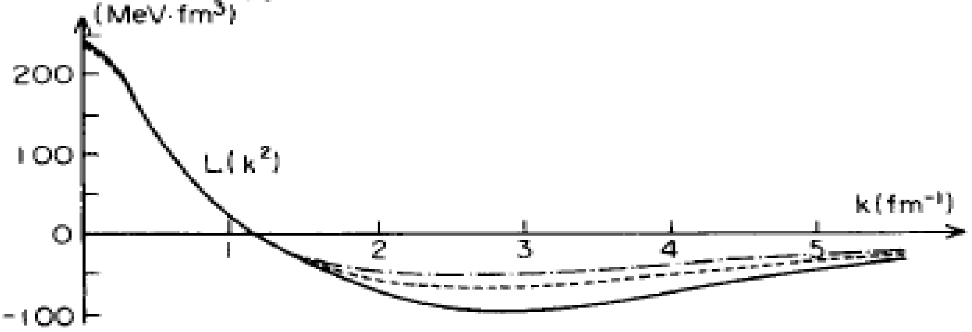
$$=\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\int\frac{d\boldsymbol{k}}{(2\pi)^3}[(\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{k}})(\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{k}})L(k^2)+(\boldsymbol{\sigma}_1\times\hat{\boldsymbol{k}})\cdot(\boldsymbol{\sigma}_2\times\hat{\boldsymbol{k}})T(k^2)]e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$

$$L(k^2)=2K_{T\tau}(k^2)+K_{\sigma\tau}(k^2),\quad T(k^2)=-K_{T\tau}(k^2)+K_{\sigma\tau}(k^2)$$

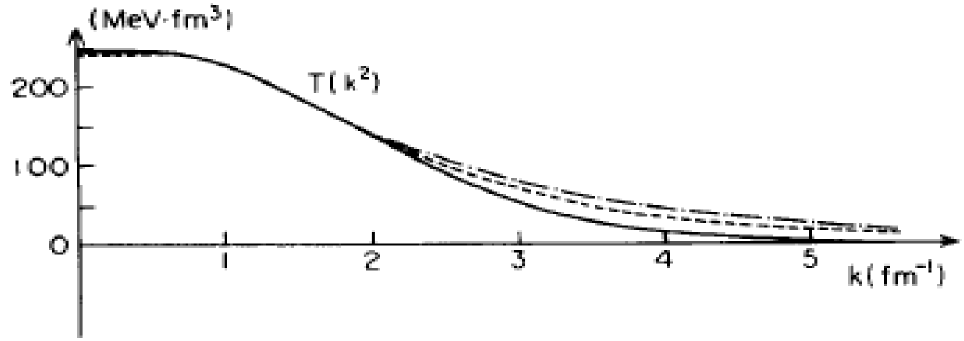
*D.W.L.Sprung and P.K. Banerjee,NPA168('71);
D.W.L. Sprung,NPA182('72),97.



Central and Tensor



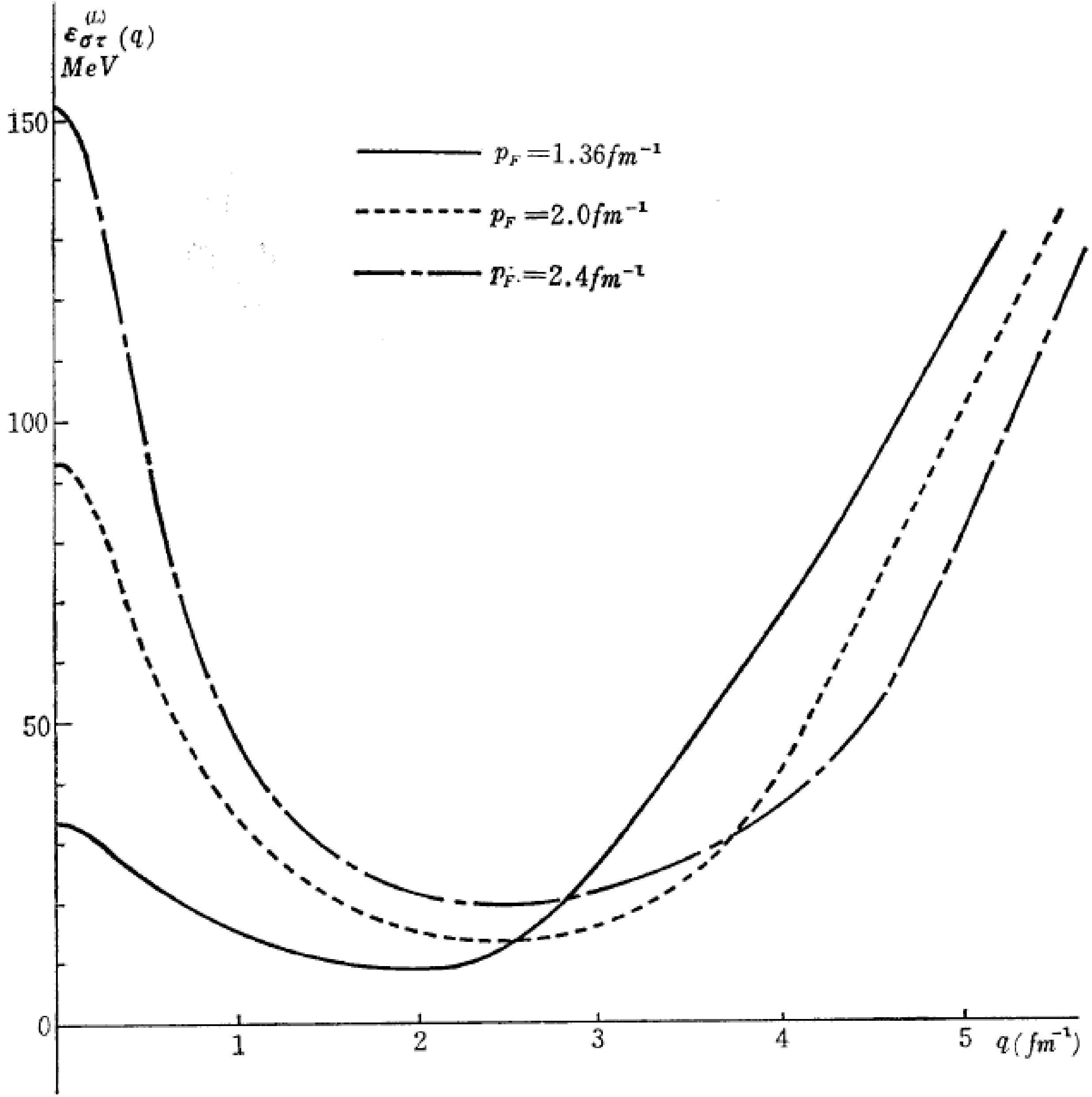
Longitudinal (pion) channel



Transverse(ρ meson) channel

$$\varepsilon^{(L)}_{\sigma\tau}(k) = \epsilon_F/3\Phi(k/2p_F)+\rho L(k^2)/2$$

Spin-isospin
symmetry energy
calculated with
G-0 force
(T.K.('83))



Notice: $\Delta_{33}(1232)$
is not incorporated.

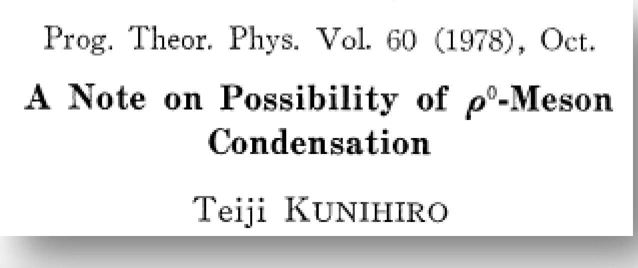
Possible ρ meson condensation and its precursory mode at high density

Pion 凝縮: スピン・アイソスピン密度の縦波成分

$$(-\nabla^2+m_\pi^2)\varphi_c(\mathbf{r})=(f/m_\pi)\boldsymbol{\nabla}\cdot\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$

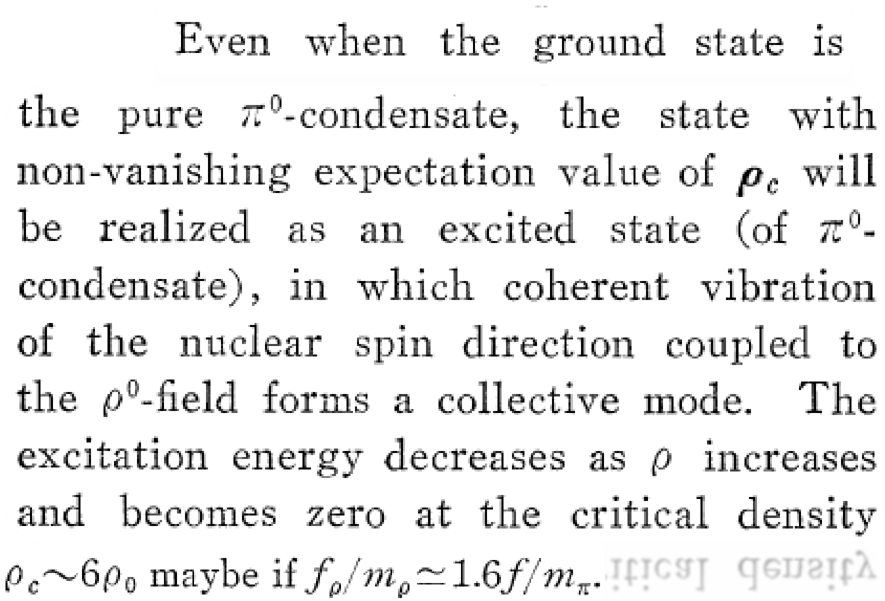
ρ 中間子 凝縮: スピン・アイソスピン密度の横波成分

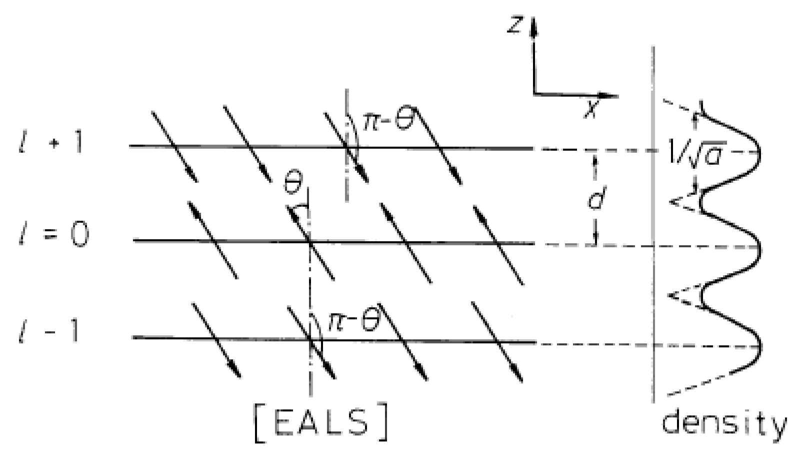
$$(-\nabla^2+m_\rho^2)\boldsymbol{\rho}_c(\mathbf{r})=(f_\rho/m_\rho)\boldsymbol{\nabla}\times\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$


Prog. Theor. Phys. Vol. 60 (1978), Oct.

A Note on Possibility of ρ^0-Meson Condensation

Teiji KUNIHIRO


Even when the ground state is the pure π^0-condensate, the state with non-vanishing expectation value of $\boldsymbol{\rho}_c$ will be realized as an excited state (of π^0-condensate), in which coherent vibration of the nuclear spin direction coupled to the ρ^0-field forms a collective mode. The excitation energy decreases as ρ increases and becomes zero at the critical density $\rho_c\sim6\rho_0$ maybe if $f_\rho/m_\rho\simeq1.6f/m_\pi$.



possible π-ρ coexistence phase

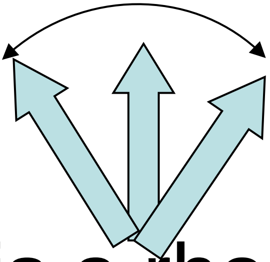
eg. if $f_\rho/m_\rho\simeq1.6f/m_\pi$

pure ρ condensation for $\rho\gtrsim6\rho_0$

Pion to Rho meson condensation

T.K., PTP 60 (1978), 1229;

- The nuclear spin direction oscillates at higher densities,
- And then

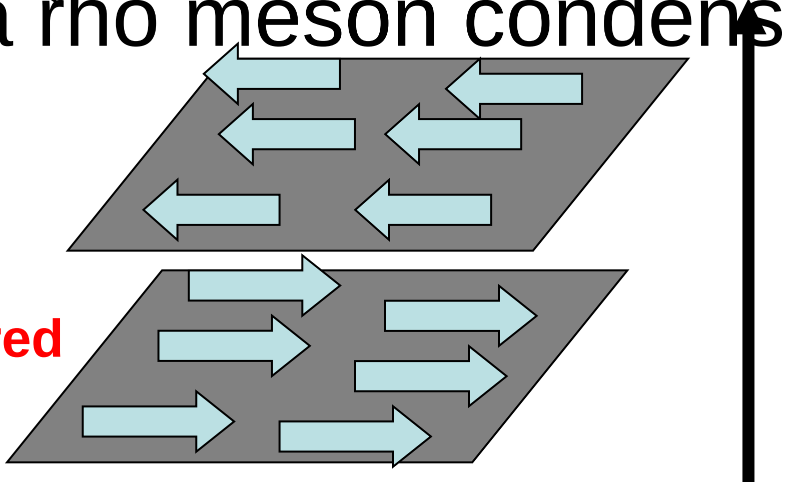


bend down, which is a rho meson condensed state.

Rho meson condensation

↕

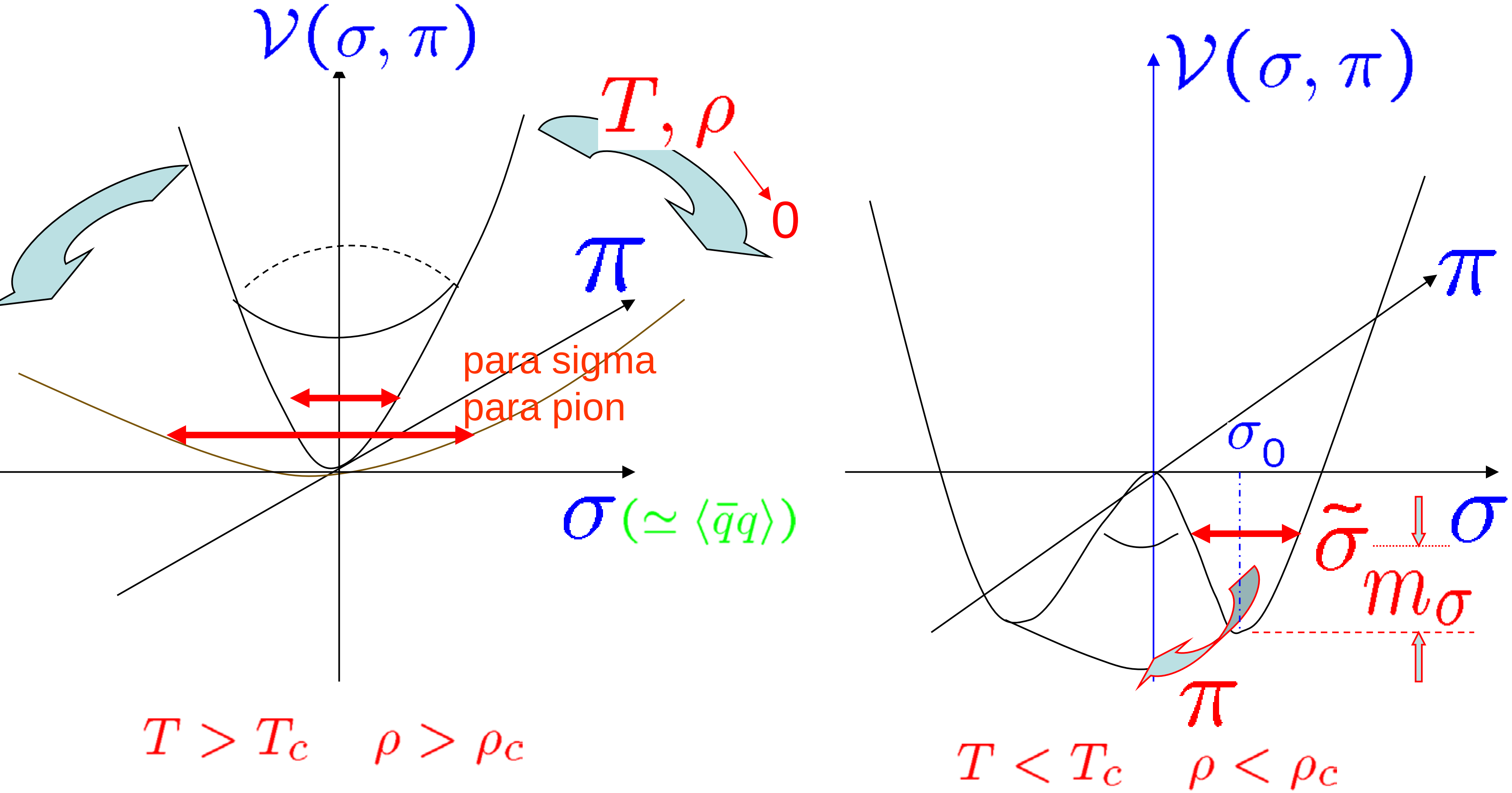
Transverse spin-isospin ordered baryonic matter



Cf. Gluonic phase
in CSC. V.Miransky

*** 3-d crystalline structure can be possible by incorporating other isospin components..**

カイラル相転移と集団モードの変化



$$\sigma = \sigma_0 + \tilde{\sigma}$$

c.f. 標準模型におけるヒッグズ粒子

ϕ ; ヒッグズ場 → $\phi = \langle\phi\rangle + \tilde{\phi}$

ヒッグズ粒子

Role of the vector mesons

- ρ and ω mesons

Tensor coupling v.s. vector coupling

⇑

cf. Chiral bag model(G.E.Brown)

The E.M. formfactor of the nucleon based on
the Vector Meson Dominace

cf. Chiral symmetry and its dynamical breaking
may be responsible for the VMD according to
Hidden Local Symmetry (Bando,Kugo, Yamawaki)

$$\bar{\psi}\sigma_{\mu\nu}\tau^k\psi(\partial^\mu\rho_k{}^\nu - \partial^\nu\rho_k{}^\mu)$$

⇒ $$(\sigma_1\times\mathbf{q})\cdot(\sigma_2\times\mathbf{q}) = -S_{12}(\mathbf{q}) + \frac{2}{3}\sigma_{\mathbf{1}}\cdot\sigma_2 q^2$$

Tensor 力のrange
の決定 ⇐ OPEPと逆
符号

OPEPと同じ符号