重イオン衝突における陽子・中性子・クラスターのダイナミクス

小野　章

東北大理

新学術領域研究会「中性子星核物質」，2015 年 3 月 12 日〜14 日

共同研究者：池野なつ美（東北大理，京大基研）

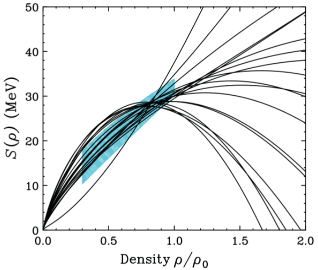
Symmetry energy from many approaches

Nuclear EOS (at $T = 0$)

$$(E/A)(\rho_p, \rho_n) = (E/A)_0(\rho) + S(\rho)\delta^2 + \cdots$$

$$\rho = \rho_p + \rho_n, \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

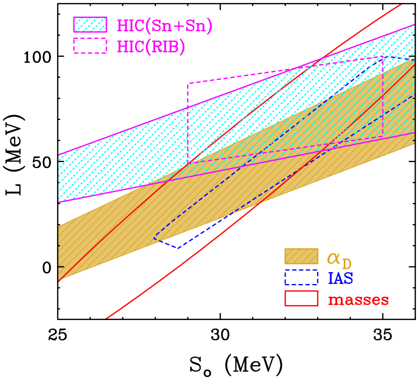
- $S_0 = S(\rho_0)$ at the saturation density
- $L = 3\rho_0 (dS/d\rho)_{\rho=\rho_0}$



$S(\rho)$ for Skyrme interactions

Constrains on $S(\rho)$

Horowitz et al., J. Phys. G: Nucl. Part. Phys. 41 (2014) 093001.



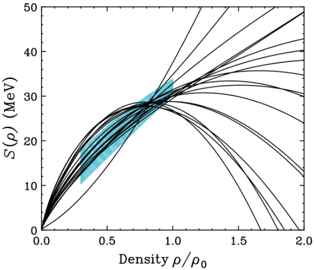
Symmetry energy from many approaches

Nuclear EOS (at $T=0$)

$$(E/A)(\rho_p,\rho_n) = (E/A)_0(\rho) + S(\rho)\delta^2 + \cdots$$

$$\rho = \rho_p + \rho_n, \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

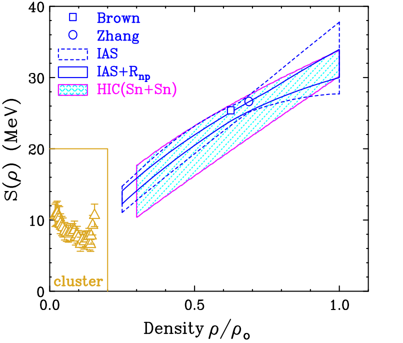
- $S_0 = S(\rho_0)$ at the saturation density
- $L = 3\rho_0 (dS/d\rho)_{\rho=\rho_0}$



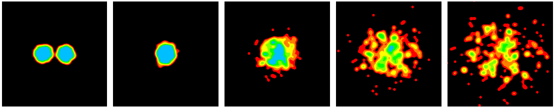
$S(\rho)$ for Skyrme interactions

Constrains on $S(\rho)$

Horowitz et al., J. Phys. G: Nucl. Part. Phys. 41 (2014) 093001.

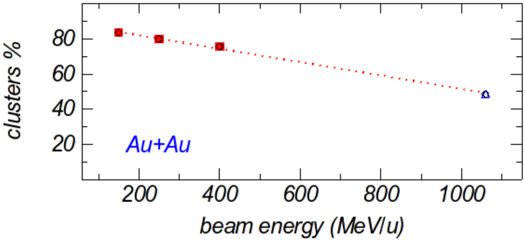


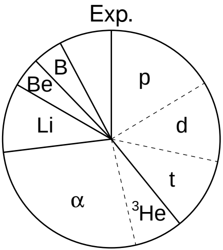
Large fraction of clusters at higher energies



$^{197}Au + ^{197}Au$ at 150 MeV/u

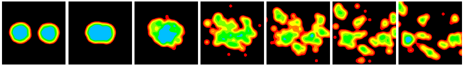
Reisdorf.et al., NPA612(1997)493.

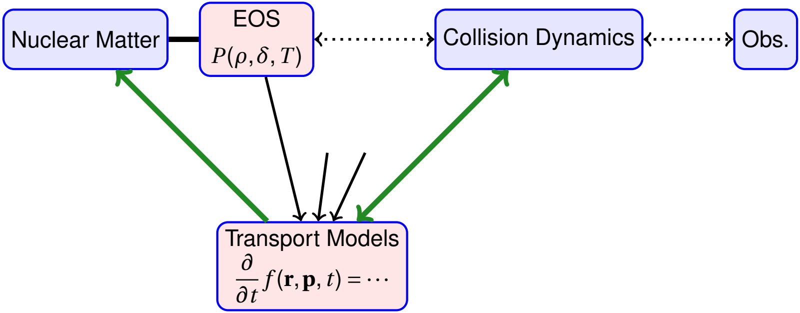




- Clusters (and fragments) are always the important part of the system.

Linking Nuclear Matter and HIC

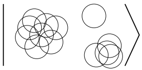




EOS ⇔ 重イオン衝突ダイナミクス ⇔ 観測量

関係はどの程度直接的か？

メカニズムの十分な理解が必要



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = $(2.5\ \text{fm})^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Time-dependent variational principle

$$\delta\int_{t_1}^{t_2}\frac{\langle\Phi(Z)|(i\hbar\frac{d}{dt}-H)|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle}dt = 0, \qquad \delta Z(t_1) = \delta Z(t_2) = 0$$

Equation of motion for the wave packet centroids Z

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathscr{H}\}_{\text{PB}} \qquad \text{or} \qquad i\hbar\sum_{j=1}^{A}\sum_{\tau=x,y,z} C_{i\sigma,j\tau}\frac{dZ_{j\tau}}{dt} = \frac{\partial\mathscr{H}}{\partial Z_{i\sigma}}$$

Motion of wave packets in the mean field (c.f. $C_{i\sigma,j\tau} = \delta_{ij}\delta_{\sigma\tau}$ in QMD)

$$\mathscr{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + (\text{c.m. correction}),$$

H: Effective interaction (e.g. Skyrme force)

Effective Interaction

Skyrme force, in recent calculations.

$$v_{ij} = t_0(1+x_0P_\sigma)\delta(\mathbf{r}) + \frac{1}{2}t_1(1+x_1P_\sigma)[\delta(\mathbf{r})\mathbf{k}^2+\mathbf{k}^2\delta(\mathbf{r})] \qquad \mathbf{r}=\mathbf{r}_i-\mathbf{r}_j$$

$$+t_2(1+x_2P_\sigma)\mathbf{k}\cdot\delta(\mathbf{r})\mathbf{k} + t_3(1+x_3P_\sigma)[\rho(\mathbf{r}_i)]^\alpha\delta(\mathbf{r}) \qquad \mathbf{k}=\frac{1}{2\hbar}(\mathbf{p}_i-\mathbf{p}_j)$$

$$\langle V\rangle = \int \mathcal{V}\Big(\rho(\mathbf{r}),\tau(\mathbf{r}),\triangle\rho(\mathbf{r}),\mathbf{j}(\mathbf{r})\Big)d\mathbf{r} \qquad \sim A^2 \times \text{Volume}$$

$$\rho(\mathbf{r}) = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}}\sum_{i=1}^{A}\sum_{j=1}^{A} e^{-2\nu(\mathbf{r}-\mathbf{R}_{ij})^2}B_{ij}B^{-1}_{ji}, \qquad \mathbf{R}_{ij}=\frac{1}{2\sqrt{\nu}}(\mathbf{Z}_i^*+\mathbf{Z}_j)$$

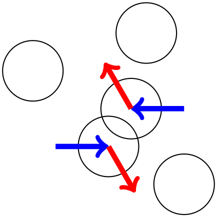
高エネルギー衝突にも使えるように，運動量依存性（t_1 と t_2 の項）は修正．

AMD with Two-Nucleon Collisions (very old version)

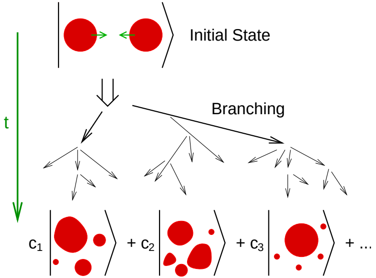
Stochastic two-nucleon collisions

- Cross section $\dfrac{d\sigma_{NN}}{d\Omega}(E,\theta)$
- Pauli blocking for the final state. (Almost automatic in AMD)

Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.



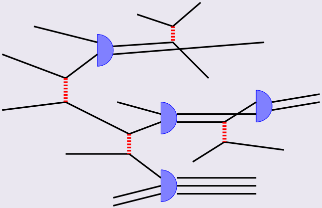
$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



Stochastic equation of motion

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\mathrm{PB}} + \text{(NN collisions)}$$

NN collisions with cluster formation



At each two-nucleon collision, **cluster formation** is considered for the final state.

$N_1 + B_1 \; + \; N_2 + B_2 \;\rightarrow C_1 + C_2$

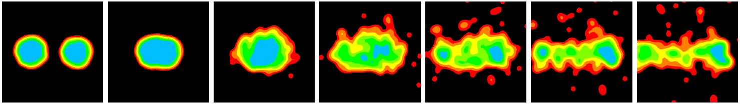
$$v\rho\, d\sigma = \frac{2\pi}{\hbar} |\langle \mathrm{CC}|V_{NN}|\mathrm{NBNB}\rangle|^2 \delta(\mathscr{H} - E)\, p_{\mathrm{rel}}^2 \, d p_{\mathrm{rel}} \, d\Omega$$

AO, J. Phys. Conf. Ser. 420 (2013) 012103

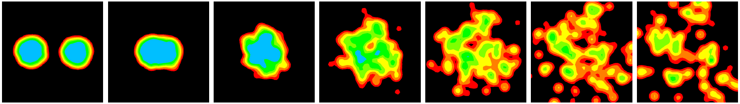
- 終状態のクラスターとしては，単独核子から α クラスターまでを考え，複数のガウス波束を同じ位相空間の点に置くことで表す．
- 二核子の行列要素 $|\langle \mathrm{NN}|V_{NN}|\mathrm{NN}\rangle|^2$ としては，通常の二核子衝突と同じものを用いる ($\Leftarrow \sigma_{NN}$)．
- 結果として，「クラスター + 核子」や「クラスター + クラスター」の弾性散乱や非弾性衝突も含まれる．

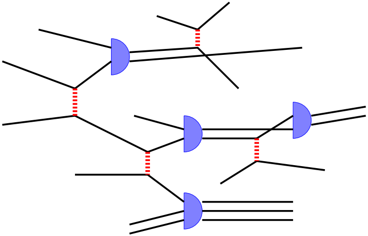
Effect of Clusters on the Density Evolution

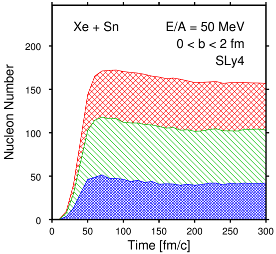
Without cluster correlations (AMD with NN collisions)



With cluster correlations







Non-clustered

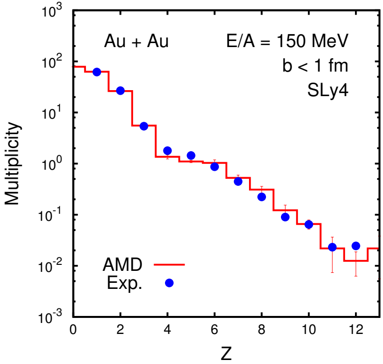
(2N)

(3N)

(4N)

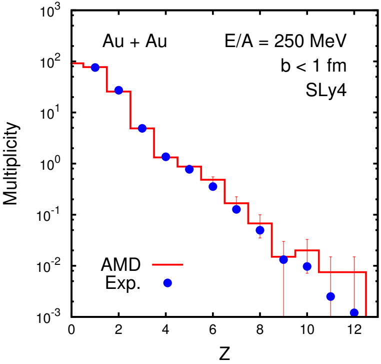
AMD results: Au + Au Central Collisions at 150 and 250 MeV/nucleon

$E/A = 150$ MeV



	with C & C-C	FOPI
$M(p)$	32.8	26.1
$M(\alpha)$	20.1	21.0
$Z_{\text{gas}}/Z_{\text{tot}}$	71%	73%

$E/A = 250$ MeV

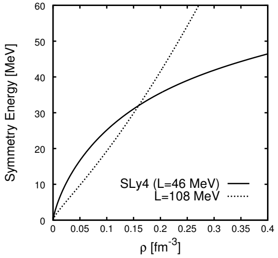


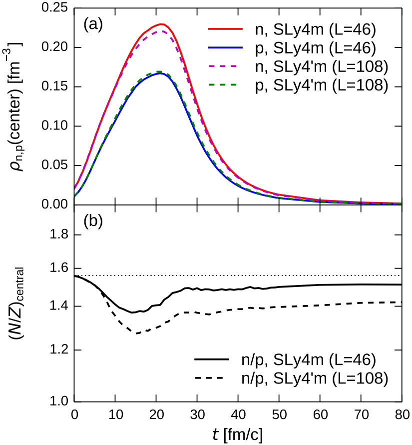
	with C & C-C	FOPI
$M(p)$	42.0	31.9
$M(\alpha)$	19.4	18.2
$Z_{\text{gas}}/Z_{\text{tot}}$	80%	83%

FOPI data: Reisdorf et al., NPA 612 (1997) 493.

$^{132}Sn + ^{124}Sn$, $E/A = 300$ MeV, $b \sim 0$

For the symmetry energy at high densities $\rho \sim 2\rho_0$.



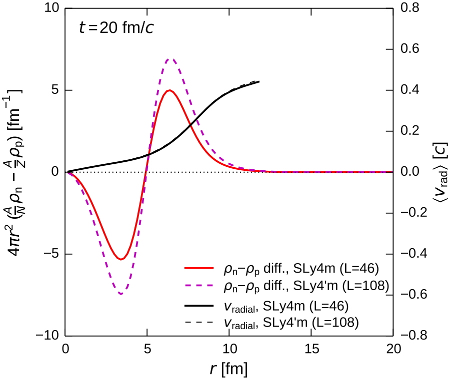


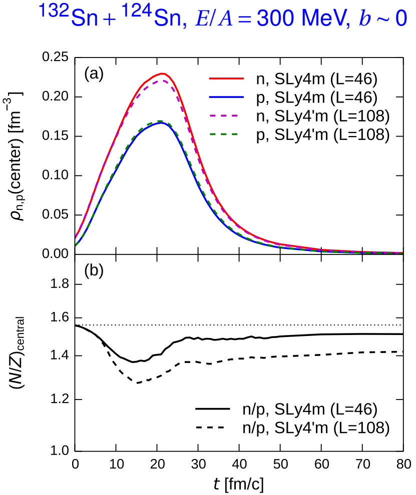
対称エネルギー ⇒ 圧縮時 ⇒ 観測量

- （この発表）核子の観測量
- （次の発表）パイオンの観測量

“central”: within a radius from the center of mass of the system that contains 25 % of the total nucleons.

Dynamics in Compression and Expansion

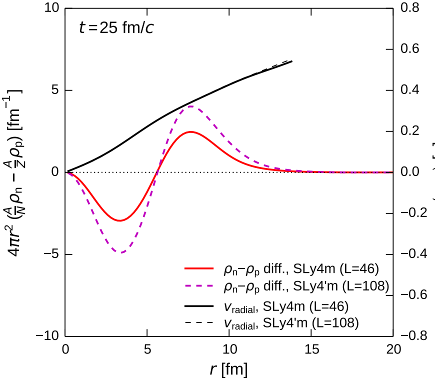




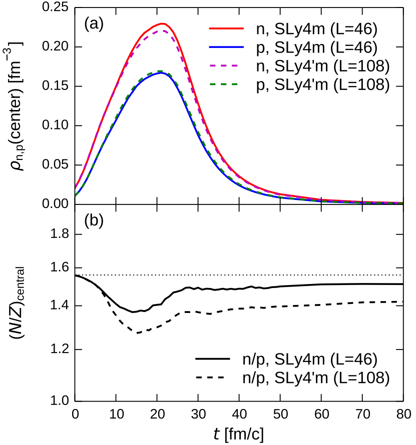
- Difference of angle-averaged densities

$$4\pi r^2\left[\frac{A}{N}\rho_n(r) - \frac{A}{Z}\rho_p(r)\right]$$

- Average radial velocity $v_{\rm rad}(r)$



^{132}Sn + ^{124}Sn, $E/A = 300$ MeV, $b \sim 0$

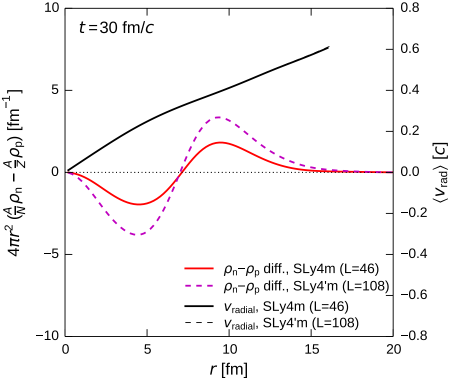


- Difference of angle-averaged densities

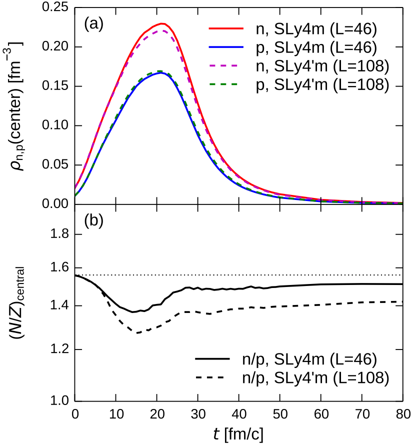
$$4\pi r^2 \left[\frac{A}{N} \rho_n(r) - \frac{A}{Z} \rho_p(r) \right]$$

- Average radial velocity $v_{rad}(r)$

Dynamics in Compression and Expansion



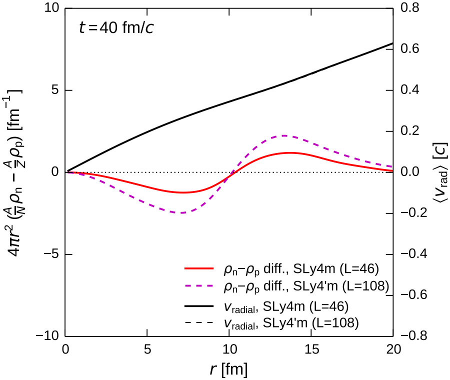
^{132}Sn + ^{124}Sn, $E/A = 300$ MeV, $b \sim 0$

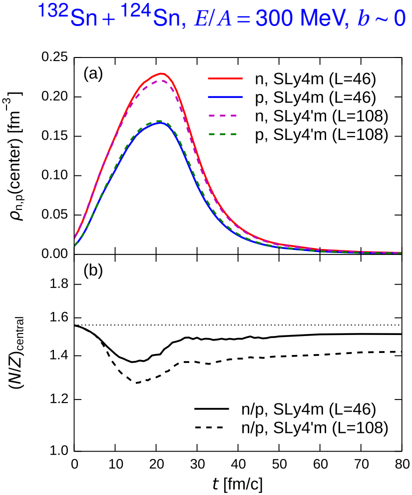


- Difference of angle-averaged densities

$$4\pi r^2 \left[\frac{A}{N} \rho_n(r) - \frac{A}{Z} \rho_p(r) \right]$$

- Average radial velocity $v_{\rm rad}(r)$



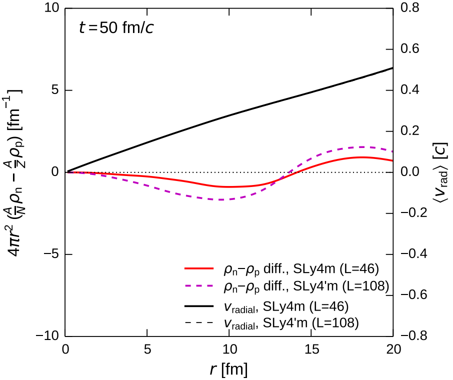


- Difference of angle-averaged densities

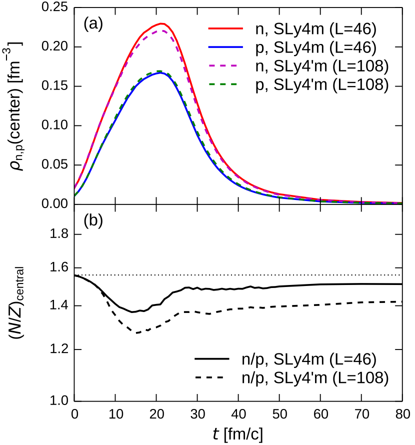
$$4\pi r^2 \left[\frac{A}{N} \rho_n(r) - \frac{A}{Z} \rho_p(r) \right]$$

- Average radial velocity $v_{\rm rad}(r)$

Dynamics in Compression and Expansion



^{132}Sn + ^{124}Sn, $E/A = 300$ MeV, $b \sim 0$



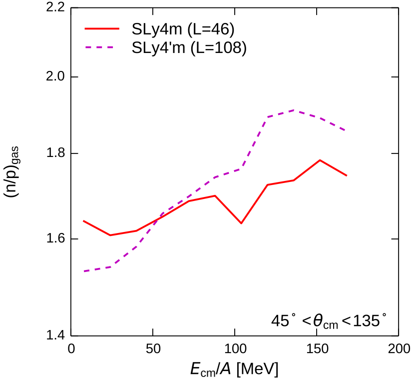
- Difference of angle-averaged densities

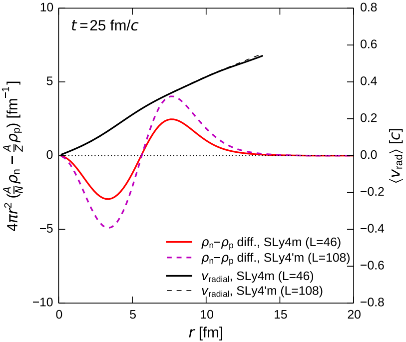
$$4\pi r^2\left[\frac{A}{N}\rho_n(r) - \frac{A}{Z}\rho_p(r)\right]$$

- Average radial velocity $v_{\text{rad}}(r)$

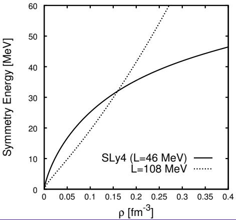
The effect at compression remains until later times.
⇒ In observables?

N/Z Spectrum Ratio — an observable





$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)}$$

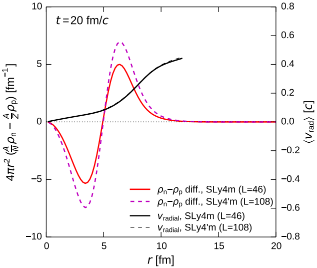


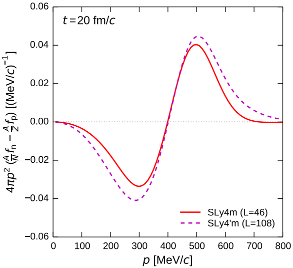
The N/Z spectrum ratio seems similar to the difference of $\rho_n(r)$ and $\rho_p(r)$ in the early stage.

Dynamics in Momentum Space

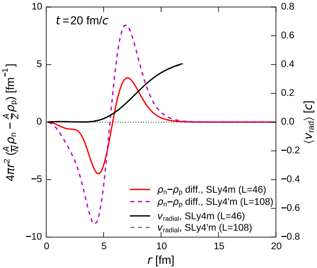
Coordinate space | Momentum space

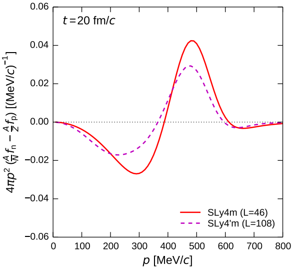
With Cluster





Without Cluster

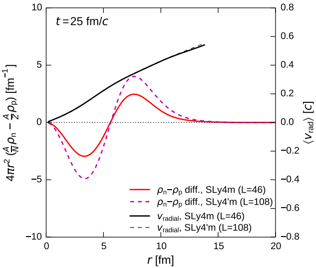


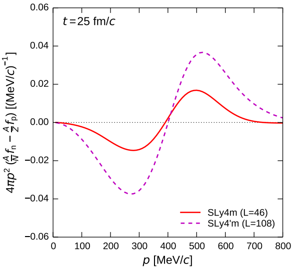


Dynamics in Momentum Space

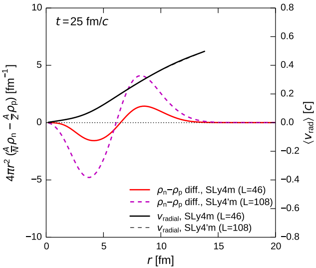
Coordinate space | Momentum space

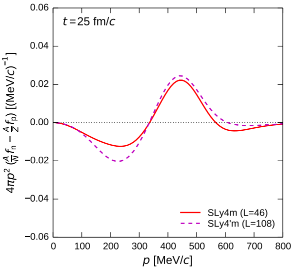
With Cluster





Without Cluster

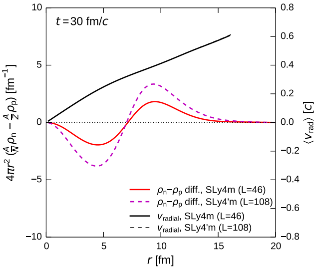


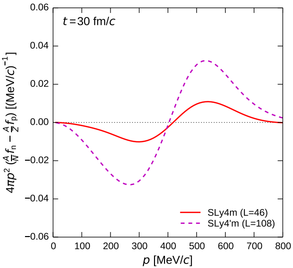


Dynamics in Momentum Space

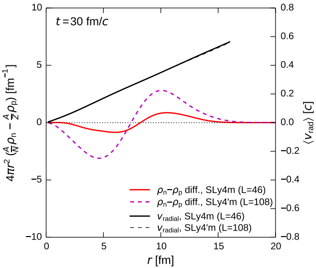
Coordinate space | Momentum space

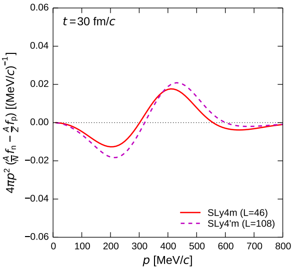
With Cluster





Without Cluster

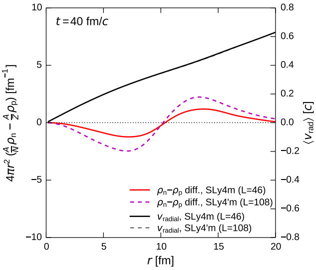


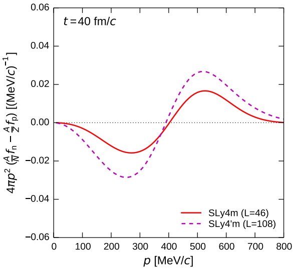


Dynamics in Momentum Space

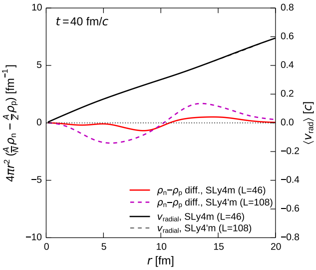
Coordinate space | Momentum space

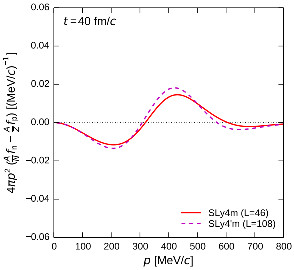
With Cluster





Without Cluster



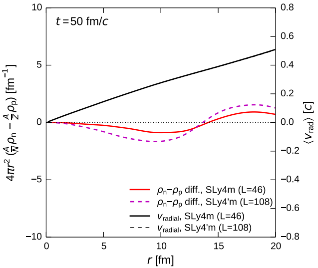


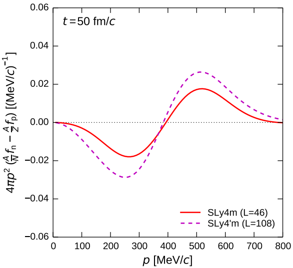
Dynamics in Momentum Space

Coordinate space

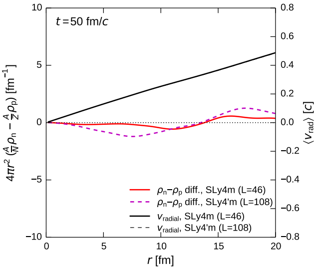
Momentum space

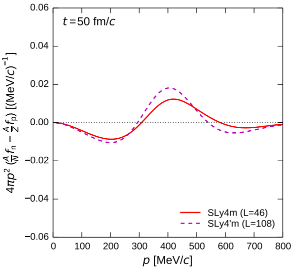
With Cluster





Without Cluster



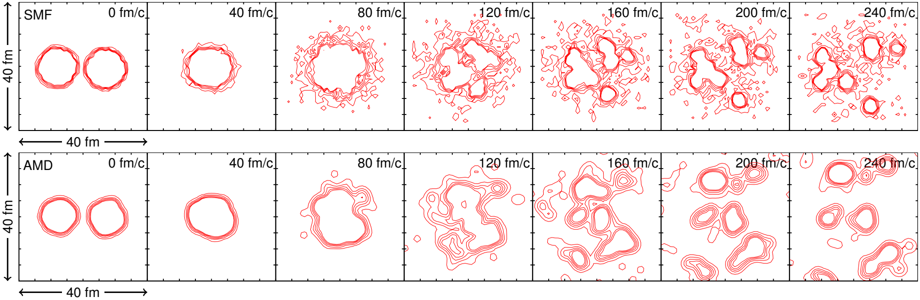


Comparison of AMD and SMF — density evolution

Rizzo, Colonna, Ono, PRC76 (2007) 024611.

Colonna, Ono, Rizzo, PRC82 (2010) 054613.

- SMF = Stochastic Mean Field model
- AMD = Antisymmetrized Molecular Dynamics



Central Collisions of ^{112}Sn + ^{112}Sn at 50 MeV/nucleon

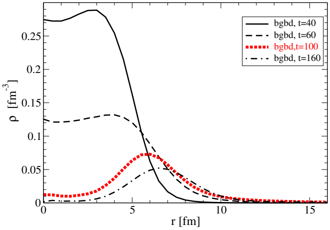
Used the same σ_{NN} and very similar effective interactions in both models.

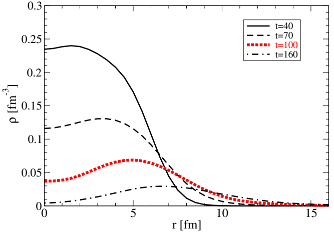
Comparison of AMD and SMF — radial expansion

SMF

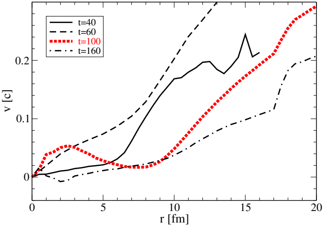
AMD

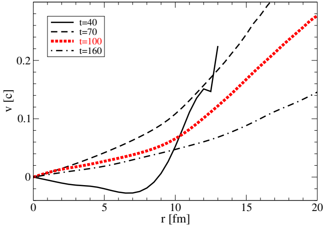
Density distribution $\langle\rho\rangle(r)$





Collective momentum $\left\langle \frac{\mathbf{r}}{|\mathbf{r}|} \cdot \mathbf{p} \right\rangle(r)$





r = distance from the center of the system

Summary

中性子過剰核の重イオン衝突 ^{132}Sn + ^{124}Sn（$E/A = 300$ MeV, $b \sim 0$）において，圧縮・膨張する系の陽子・中性子のダイナミクスを AMD を用いて調べた．
特に，高密度での対称エネルギーがどのように反映されているか．

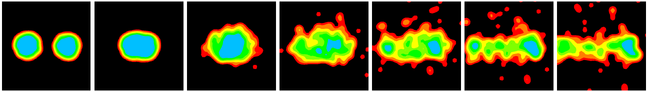
- 圧縮時に高密度での対称エネルギーを反映して，中心付近（および外側）の陽子・中性子比が決まる．
- クラスター相関を取り入れた計算では，膨張が単純なようである．中性子と陽子の差の空間分布と運動量分布が，単純に関係している．
  ⇒ 圧縮時の陽子・中性子比が，観測される陽子・中性子スペクトルに反映される．

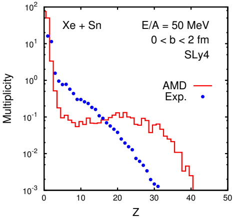
  対称エネルギーが硬い ⇔ 高密度部分の N/Z が下がる ⇔ 高運動量部分が中性子過剰
- クラスター相関を無視した計算では，膨張が単純でないため，圧縮時の陽子・中性子比と，最終的なスペクトルとの関係は単純でない．

Effect of Cluster and C-C Correlations

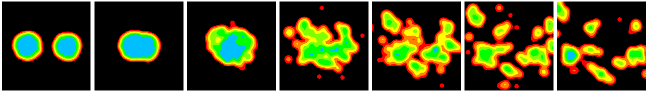
Xe + Sn central collisions at 50 MeV/nucleon

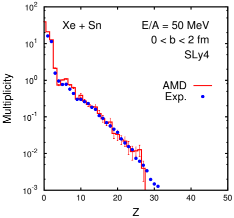
Without cluster correlations (AMD with NN collisions)





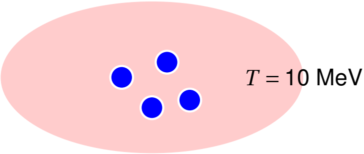
With cluster and cluster-cluster correlations





Effects of clusters on the bulk properties and dynamics

Example: Four nucleons in the gas at $T = 10$ MeV



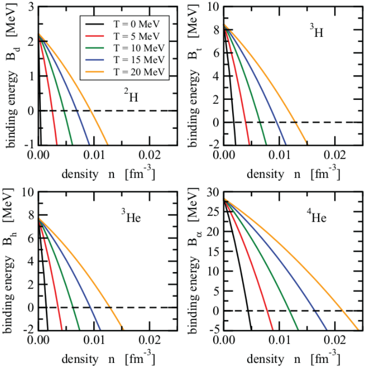
- Uncorrelated: $\langle E \rangle = \frac{3}{2} T \times 4 = 60$ MeV
- α cluster: $\langle E \rangle = \frac{3}{2} T \times 1 - 28.3\,\mathrm{MeV} = -13.3$ MeV

The energy difference is large enough to change the bulk properties.

Cluster correlations should be considered in a way consistent with the collision dynamics.

Clusters in nuclear medium

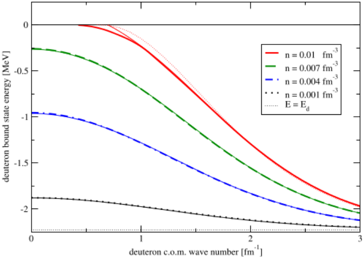
Typel et al, PRC81(2010)015803



- $\mathbf{P}=0$: Clusters at rest (relative to medium)
- T: temperature of medium

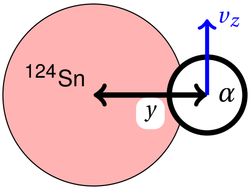
Equation for a deuteron in medium

$$\left[e(\tfrac{1}{2}\mathbf{P}+\mathbf{p})+e(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\right]\tilde{\psi}(\mathbf{p})$$
$$+\left[1-f(\tfrac{1}{2}\mathbf{P}+\mathbf{p})-f(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\right]\int\frac{d\mathbf{p}'}{(2\pi)^3}\langle\mathbf{p}|v|\mathbf{p}'\rangle\tilde{\psi}(\mathbf{p}')$$
$$=E\tilde{\psi}(\mathbf{p})$$



$\mathbf{P}$-dependence of the deuteron binding energy in medium
Röpke, NPA867 (2011) 66.

Cluster put into a nucleus (AMD)



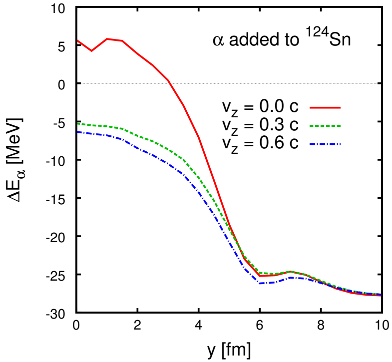
α cluster $|\alpha, \mathbf{Z}\rangle$: Four wave packets with different spins and isospins placed at the same phase space point $\mathbf{Z}$.

$$E_\alpha : \quad \mathscr{A}\, |\alpha, \mathbf{Z}\rangle |^{124}\mathrm{Sn}\rangle$$

$$E_\mathrm{N} : \quad \mathscr{A}\, |\mathbf{Z}\rangle |^{124}\mathrm{Sn}\rangle \quad (\mathrm{N} = p\uparrow, p\downarrow, n\uparrow, n\downarrow)$$

$$-B_\alpha = \Delta E_\alpha = E_\alpha - (E_{p\uparrow} + E_{p\downarrow} + E_{n\uparrow} + E_{n\downarrow})$$

(Energies are defined relative to $|^{124}\mathrm{Sn}\rangle$.)



$$\frac{\mathrm{Re}\,\mathbf{Z}}{\sqrt{\nu}} = (0, y, 0),$$

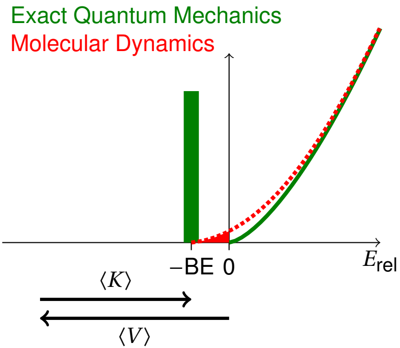
$$\frac{2\hbar\sqrt{\nu}\,\mathrm{Im}\,\mathbf{Z}}{M} = (0, 0, v_z)$$

- Dependence on y
  $\approx$ Dependence on density
- Dependence on $P_\alpha = M_\alpha v_z$
- Due to the density dependence of the Skyrme force, the interaction between nucleons in the α cluster is weakened in the nucleus.

Energy is OK, but probability is . . . .

Clusters have to be handled in a special way

Two-body level density



Classical phase space is not consistent with quantum mechanics. (Bound phase space in AMD is too small compared to h^3.)

⇓

In a two-nucleon collision, the probability of cluster formation in the final state is too small.

Two-nucleon collision:

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

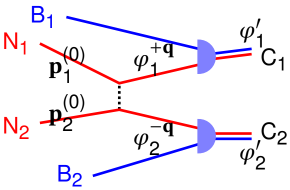
Clustered states should be explicitly included in the set of final states $|\Psi_f\rangle$.

Cluster Formation Cross Section

Similar to Danielewicz et al., NPA533 (1991) 712.

$$\mathrm{N}_1 + \mathrm{B}_1 \;+\; \mathrm{N}_2 + \mathrm{B}_2 \;\rightarrow \mathrm{C}_1 + \mathrm{C}_2$$

- $\mathrm{N}_1, \mathrm{N}_2$: Colliding nucleons
- $\mathrm{B}_1, \mathrm{B}_2$: Spectator nucleons/clusters
- $\mathrm{C}_1, \mathrm{C}_2$: N, $(2N)$, $(3N)$, $(4N)$ (up to α cluster)



$$v_{\mathrm{NN}}\, d\sigma(\mathrm{NBNB} \rightarrow \mathrm{CC})$$
$$= |\langle \varphi'_1 | \varphi_1^{+\mathbf{q}} \rangle|^2 \; |\langle \varphi'_2 | \varphi_2^{-\mathbf{q}} \rangle|^2 \; |M|^2 \, \delta(\mathscr{H} - E)\, p_{\mathrm{rel}}^2 \, dp_{\mathrm{rel}}\, d\Omega$$

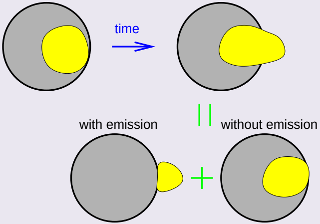
$$\left(v_{\mathrm{NN}}\, d\sigma_{\mathrm{NN}} = |M|^2 \, \delta(\mathscr{H} - E)\, p_{\mathrm{rel}}^2 \, dp_{\mathrm{rel}}\, d\Omega \right)$$

$$\frac{d\sigma}{d\Omega} = F_{\mathrm{kin}} \; |\langle \varphi'_1 | \varphi_1^{+\mathbf{q}} \rangle|^2 \; |\langle \varphi'_2 | \varphi_2^{-\mathbf{q}} \rangle|^2 \left(\frac{d\sigma}{d\Omega} \right)_{\mathrm{NN} \rightarrow \mathrm{NN}}$$

$$\mathbf{p}_{\mathrm{rel}} = \tfrac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) = p_{\mathrm{rel}} \hat{\mathbf{\Omega}}$$
$$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_1^{(0)} = \mathbf{p}_2^{(0)} - \mathbf{p}_2$$
$$\varphi_1^{+\mathbf{q}} = \exp(+i\mathbf{q}\cdot\mathbf{r}_{\mathrm{N}_1})\varphi_1^{(0)}$$
$$\varphi_2^{-\mathbf{q}} = \exp(-i\mathbf{q}\cdot\mathbf{r}_{\mathrm{N}_2})\varphi_2^{(0)}$$

The cross section is given from the NN cross section.

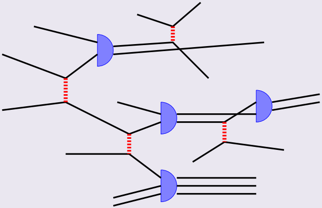
Two directions of extension of AMD



Wave-packet splitting: Give fluctuation to each wave packet centroid, based on the **single-particle motion**.

$$\frac{d}{dt}Z = \{Z, \mathcal{H}\}_{\mathrm{PB}} + \text{(NN Collision)}$$
$$+ \text{(W.P. Splitting)} + \text{(E. Conservation)}$$

AO and Horiuchi, PPNP53 (2004) 501



At each two-nucleon collision, **cluster formation** is considered for the final state.

$$\mathrm{N}_1 + \mathrm{B}_1 \;+\; \mathrm{N}_2 + \mathrm{B}_2 \;\to \mathrm{C}_1 + \mathrm{C}_2$$

$$v\rho\, d\sigma = \frac{2\pi}{\hbar} |\langle \mathrm{CC}|V_{NN}|\mathrm{NBNB}\rangle|^2 \delta(\mathcal{H} - E)\, p_{\mathrm{rel}}^2\, dp_{\mathrm{rel}}\, d\Omega$$

AO, J. Phys. Conf. Ser. 420 (2013) 012103