

# $\sigma$ AND $\kappa$ MESONS AS BROAD DYNAMICAL RESONANCES IN ONE-MESON-EXCHANGE MODEL

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2. One meson exchange potential
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# Introduction

- Many model of meson-meson interaction
  - Quark Model
  - Chiral perturbation
    - J.A.Oller, E.Oset, Nucl.Phys A620, 438-456(1997).
    - J.A.Oller, E.Oset, Phys. Rev. D 59, 074001 (1999).
    - A.G.Nicola and J.R.Pelaez arXiv:help-ph/0109056v2 (2001).
  - LQCD
    - Silas R. Beane, Thomas C. Luu et al, Phys. Rev D 77, 094507 (2008).
  - Meson exchange model
    - D. Lohse et al, Nuc. Phys. A516, 513-548 (1990).
- In meson-meson interaction  $\pi\pi$ ,  $\pi K$  scatterings have been investigated both in experiments and theories.
- Construct a unified hadron-hadron potential model which appropriate for all BB, MB and MM interactions.
- Meson-exchanged models keeps its validity as a suitable effective description of the h-h interactions in the low-energies region.

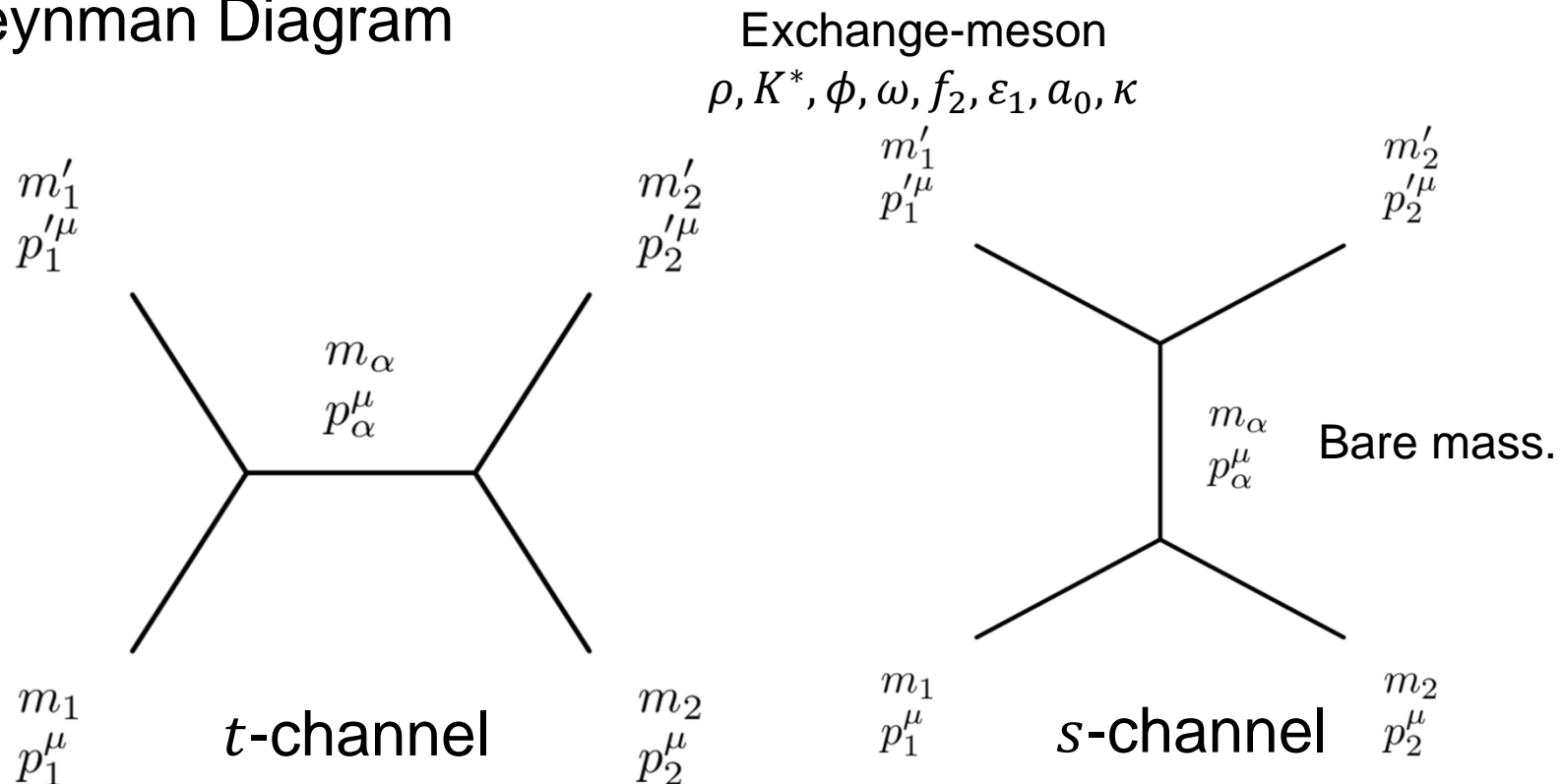
# Introduction

- By  $SU(3)$ -symmetric one-meson exchange mechanisms, we construct:
  - $K^{bar} K^{bar}$  ( $S = -2$ )
  - $\pi K^{bar} - \eta K^{bar}$  ( $S = -1$ )
  - $\pi\pi - KK^{bar} - \eta\pi - \eta\eta$  ( $S = 0$ )
  - $\pi K - \eta K$  ( $S = 1$ )
  - $KK$  ( $S = 2$ )

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# Meson exchange potential

- Feynman Diagram



# Meson exchange potential

- Interaction Lagrangians
- Interaction Hamiltonian  $W = - \int L d^3x$
- Transition operator  $T(z)$  are represented by a series expansion defined by all diagrams containing an incoming and outgoing two-meson state.
- One meson potential can determined

# Coupling constants

- SU(3) Symmetric Lagrangians

$$\mathcal{L}_{pps} = \frac{f_{pps}}{m_\pi} \mathbf{Tr}[\partial^\mu P \partial_\mu PS]$$

$$\mathcal{L}_{ppv} = g_{ppv} \mathbf{Tr}[(\partial^\mu P)P - P(\partial^\mu P)]V$$

$$\mathcal{L}_{ppt} = g_{ppt} \frac{2}{m_\pi} \mathbf{Tr}[(\partial_\mu P \partial^\nu P)T^{\mu\nu}]$$

- $f_{pps}$ ,  $g_{ppv}$ ,  $g_{ppt}$ : coupling constants
- P is  $3 \times 3$  matrix representation of the pseudo-scalar octet.
- S is  $3 \times 3$  matrix representation of the scalar octet.
- V is  $3 \times 3$  matrix representation of the vector octet.

- $P_8 = \begin{pmatrix} \frac{\pi_0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi_0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix};$

- $V_8 = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\phi}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\phi}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\phi \end{pmatrix};$

- $S_8 = \begin{pmatrix} \frac{a_0}{\sqrt{2}} + \frac{f_0}{\sqrt{6}} & a^+ & \kappa^+ \\ a^- & -\frac{a_0}{\sqrt{2}} + \frac{\phi}{\sqrt{6}} & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & -\sqrt{\frac{2}{3}}\phi \end{pmatrix};$

$$S_1 = \sigma$$

# Form Factors

- Monopole Type

- $t$ -channel

- $F^{(t)}(q_\alpha^2) = \frac{\Lambda^2 - m_\alpha^2}{(\Lambda^2 + q_\alpha^2)}$

- $s$ -channel

- $F^{(s)}(\omega_p^2) = \frac{\Lambda^2 + m_\alpha^2}{(\Lambda^2 + \omega_p^2)}$

- Forth-order ( $s$ -channel)

- $F_4(\omega_p^2) = \frac{\Lambda^4 + m_\alpha^4}{(\Lambda^4 + \omega_p^4)}$

$\Lambda$  : cut-off parameters

$q_\alpha$  : momentum

$m_\alpha$  : mass of exchanged mesons

$\omega_\alpha$  : total energy



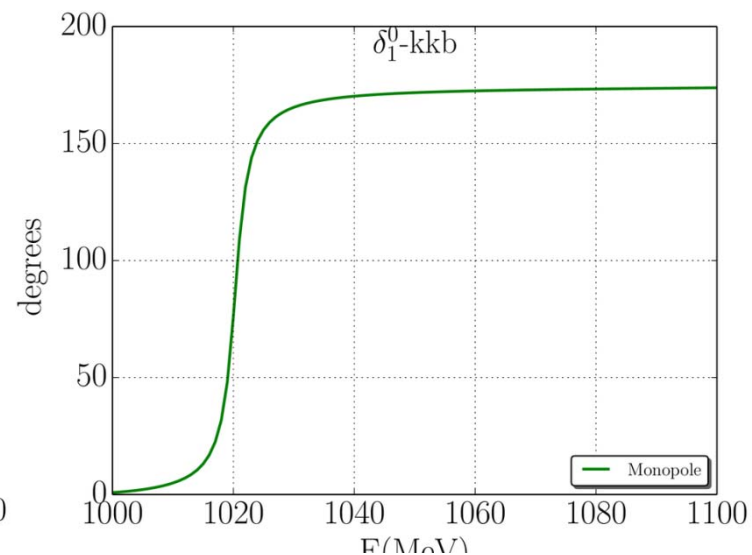
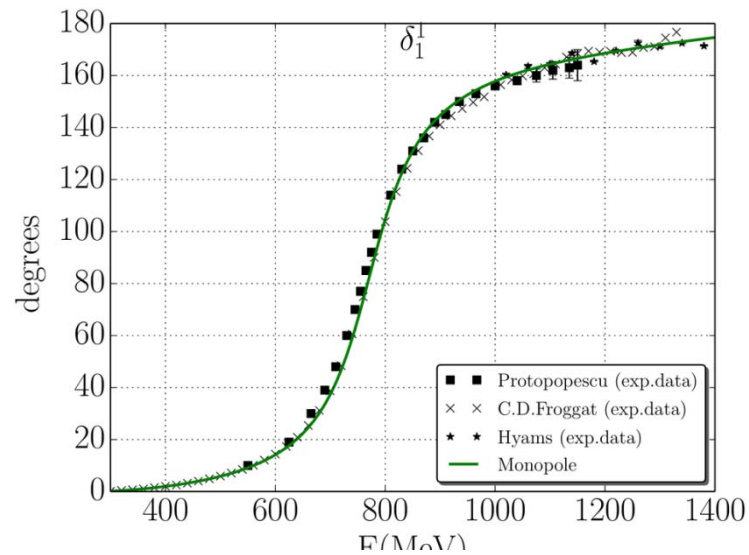
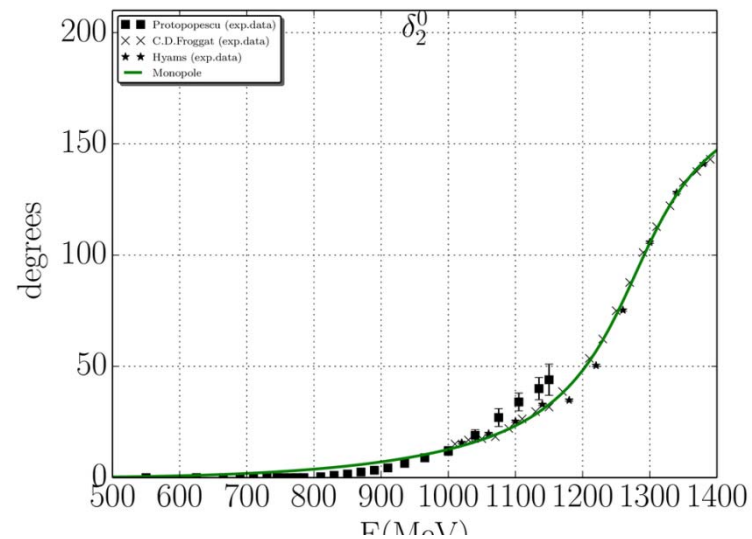
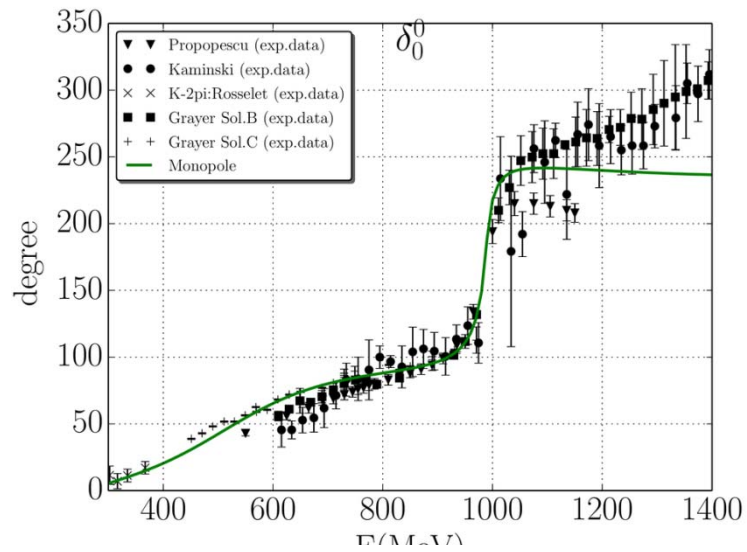
# Parameters

$m_0$	Mono.
$\rho$	1126.66877
$\varepsilon_1$	2112.07666
$K^*$	1403.13526
$\kappa$	1522.01264
$f_2$	1367.63563

	Mono.
$\Lambda_{\eta KK^*}$	864.0841
$g_{KK^{bar}a_0}$	0.0408802
$\Lambda_{KK^{bar}a_0}$	1233.61523
$m_0(a_0)$	1235.73076
$\Lambda_{KK^{bar}\phi}$	2137.02377
$m_0(\phi)$	1150.23241

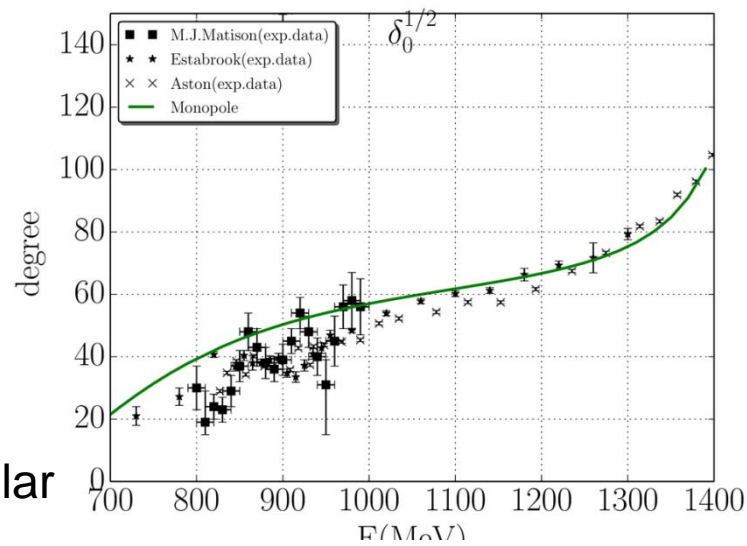
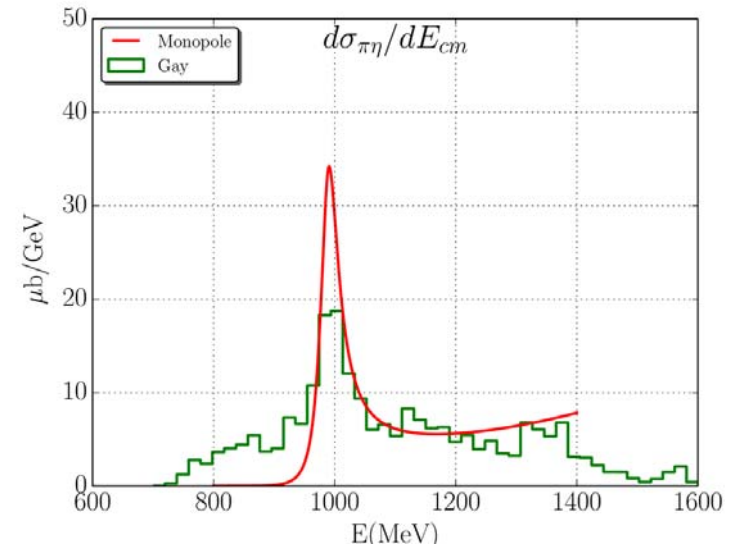
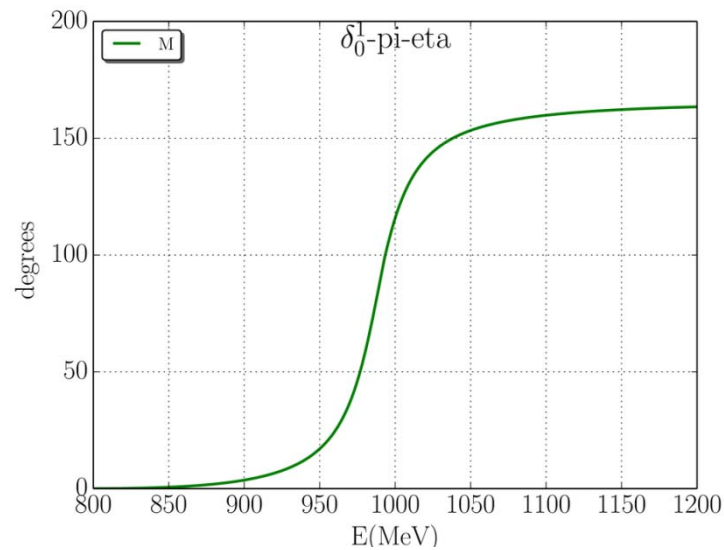
	Monopole
$g_{\pi\pi\rho}$	0.50101
$g_{\pi\pi f_2}$	0.02843626
$g_{\pi\pi\varepsilon_1}$	0.01620557
$g_{\pi\pi\kappa}$	$0.191683 \times 10^{-3}$
$\Lambda_{\pi\pi\rho}$	2896.41242
$\Lambda_{\pi KK^*}$	2301.47525
$\Lambda_{K\bar{K}\rho}$	3923.01907
$\Lambda_{K\bar{K}\omega,\phi}$	4520.8249
$\Lambda_{\pi\pi\rho-sch}$	2757.75065
$\Lambda_{\pi\pi f_2-sch}$	1481.43534
$\Lambda_{\pi\pi\varepsilon_1-sch}$	1159.73371
$\Lambda_{\pi KK^*-sch}$	4061.81573
$\Lambda_{\pi K\kappa-sch}$	3622.67379

# Phase shift results

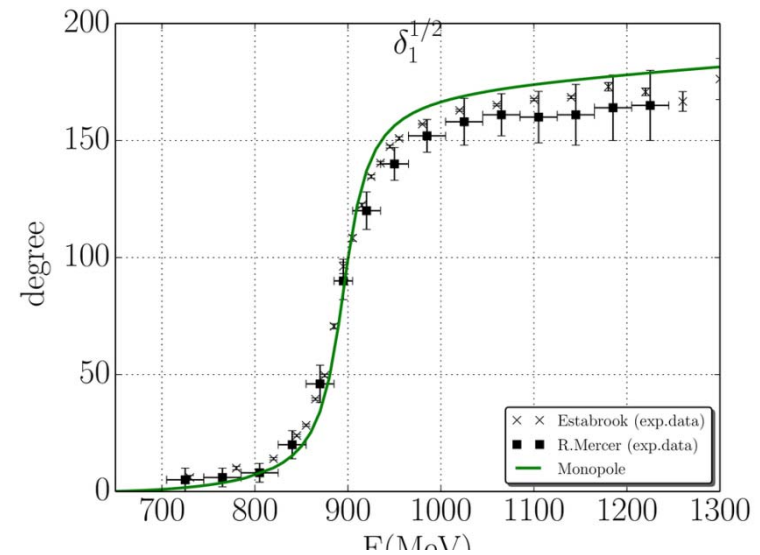


$\delta_J^I$   
 $I$ : isospin  
 $J$ : total angular momentum

# Phase shift results



$\delta_J^I$   
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# Resonances

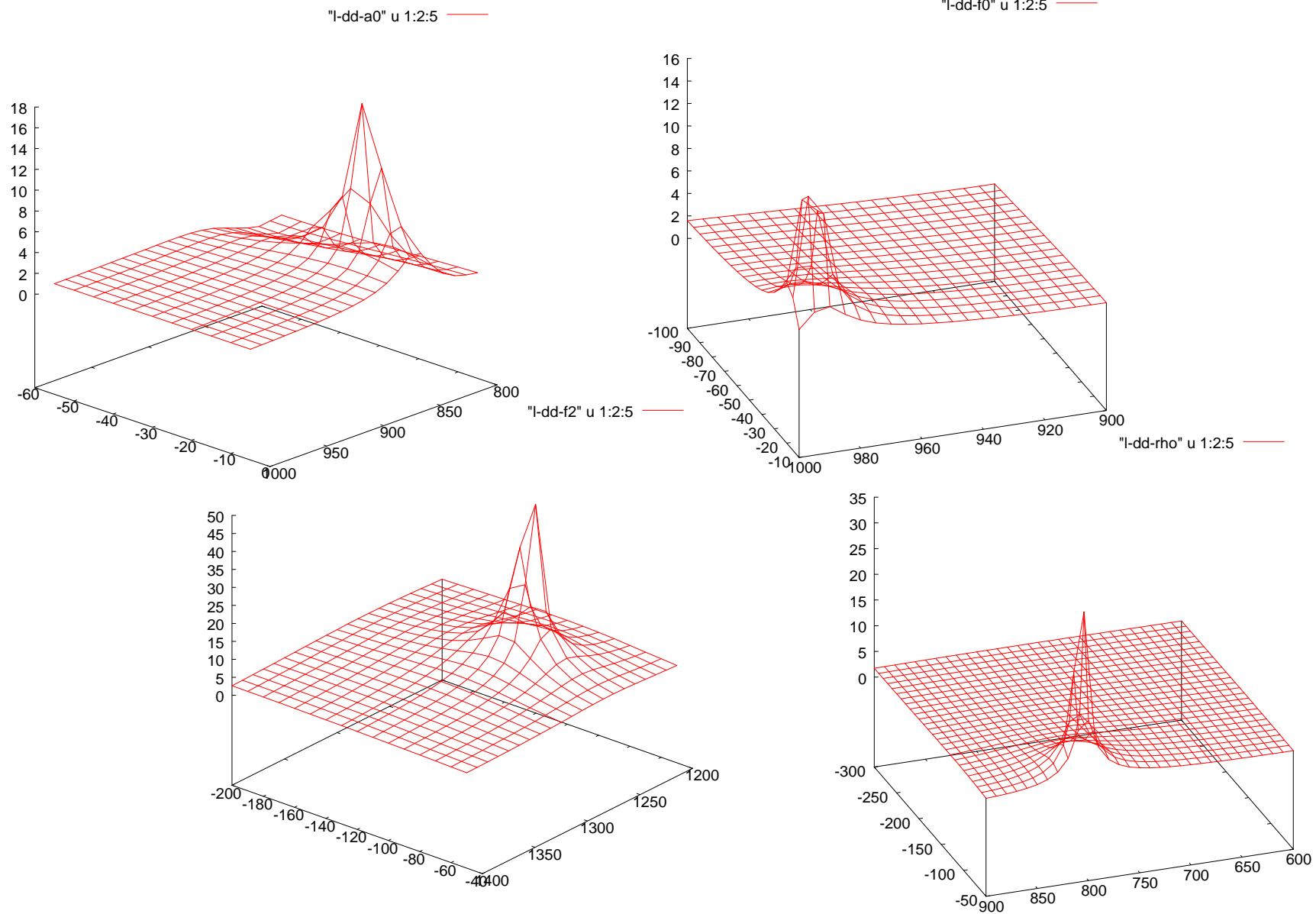
## Poles with s-channel meson exchange

- $V = V_t + V_s$ 
  - $V_s$  contains s-channel diagrams
  - $V_t$  contains t-channel diagrams
  - $V_s(p', p) = \sum_i \gamma_i(p') \Delta_i(P) \gamma_i(p)$  with bare mass and bare coupling constant
- $T = T_t + T_s$ 
  - $T_s = \gamma^*(p') \Delta^*(P) \gamma^*(p)$
  - $\Delta^*(P) = \frac{\Delta(P)}{1 - \Delta(P) \Sigma(p)}$  : dressed propagator
  - $\Sigma(p)$  : self-energy
  - $\gamma^*(p)$  : dressed vertex

## Pure Dynamical Poles (without s-channel exchange)

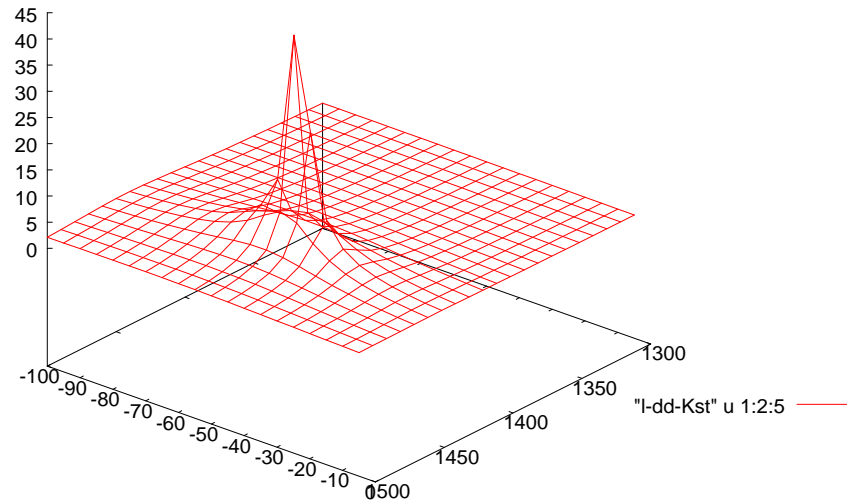
\*This resonance renormalization follow: H.Polinder and Th. A. Rijken Phys. Rev. C 72, 065211 (2005)

# Resonances results

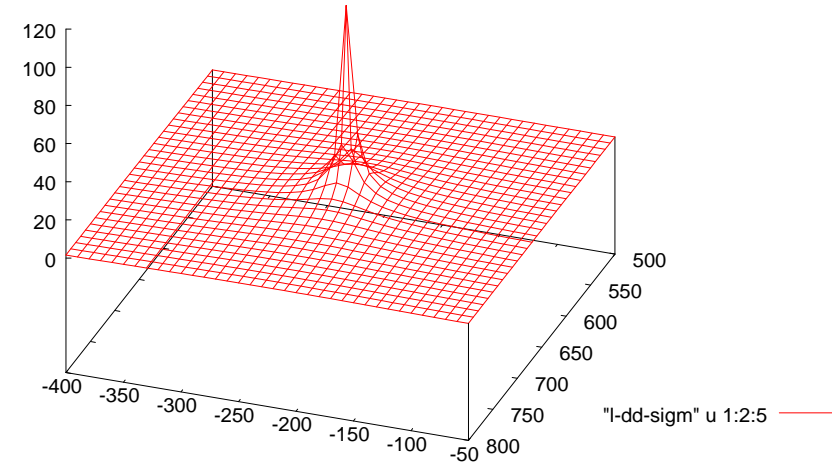


# Resonances results

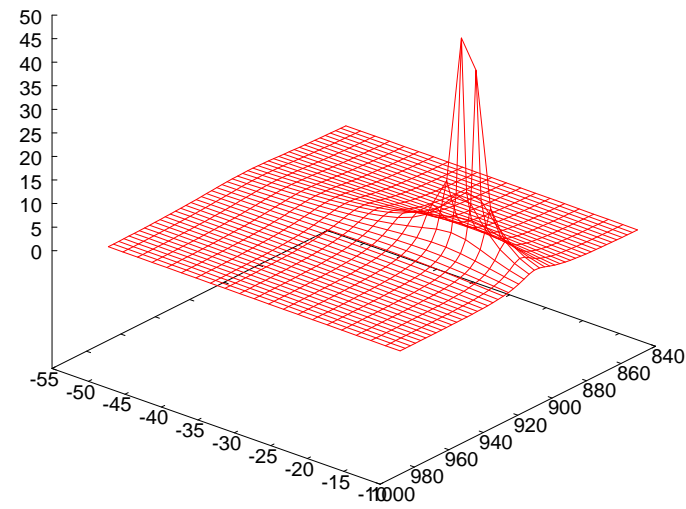
"l-dd-kap14" u 1:2:5



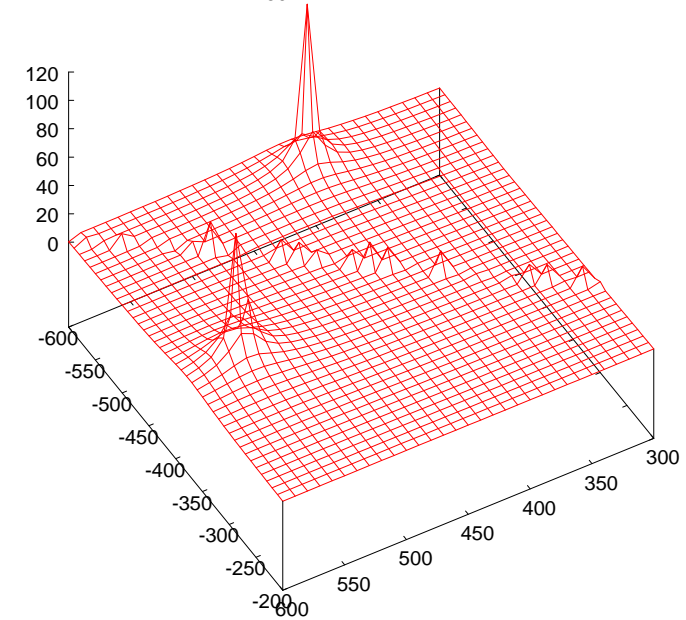
"l-dd-kap7" u 1:2:5



"l-dd-Kst" u 1:2:5



"l-dd-sigm" u 1:2:5



# Resonances

	Exp. Data (PDG)	Monopole
$a_0(980)$	[ 984, -(25 ÷ 50)]	[860, -20]
$f_0(980)$	[ 980, -(20 ÷ 50)]	[980, -70]
$f_2(1270)$	[1275(1.2), -92(2)]	[1280, -100]
$\kappa(1430)$	[1412(6), -147(12)]	[1440, -50]
$\kappa(700)$		[640, -200]
$K^*(892)$	[892, -25]	[910, -20]
$\phi(1020)$	[1019, -2]	[1020, -2]
$\rho(770)$	[771, -75]	[790, -50]
$\sigma_1(600)$	[400 - 1200, -(300 - 500)]	[550, -360]
$\sigma_2$		[410, -550]

✖  $[M, -(\frac{\Gamma}{2})]$

# Conclusion

- This calculation are well reproduce the phase shift, cross-section and pole position in the meson-meson interaction by the meson exchange model.
  - $\pi\pi - KK^{bar} - \pi\eta - \eta\eta$ 
    - $\delta_0^0; \delta_0^1; \delta_1^0; \delta_1^1; \delta_2^0$
    - $\sigma(600), f_0(980), \rho(770), a_0(980), \phi(1020)$
  - $\pi K - \eta K$ 
    - $\delta_0^{\frac{1}{2}}, \delta_1^{\frac{1}{2}}$
    - $\kappa(700), \kappa(1430), K^*(982)$
- The shown results can prove the suitable effective of meson-exchange model on well-produce the interaction of mesons.
- Exist the  $\kappa(700); \sigma(600)$  poles with broad width  $\rightarrow$  experiment???
- **FUTURE PLAN**
  - Refine our model by reasonable bare mass fitting-parameter for  $(\delta_0^0; \delta_0^{\frac{1}{2}}, \delta_1^{\frac{1}{2}})$
  - Calculation with the Gaussian F.F
  - Solve the dynamical 3 body problems to study the existence and properties of the resonances in the hadronic system.



THANK YOU FOR YOUR ATTENTION

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