第3回中性子星核物質研究会

BCS-BEC クロスオーバーの状態方程式
(The EOS in the BCS-BEC crossover)

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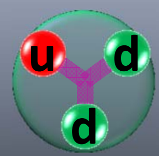


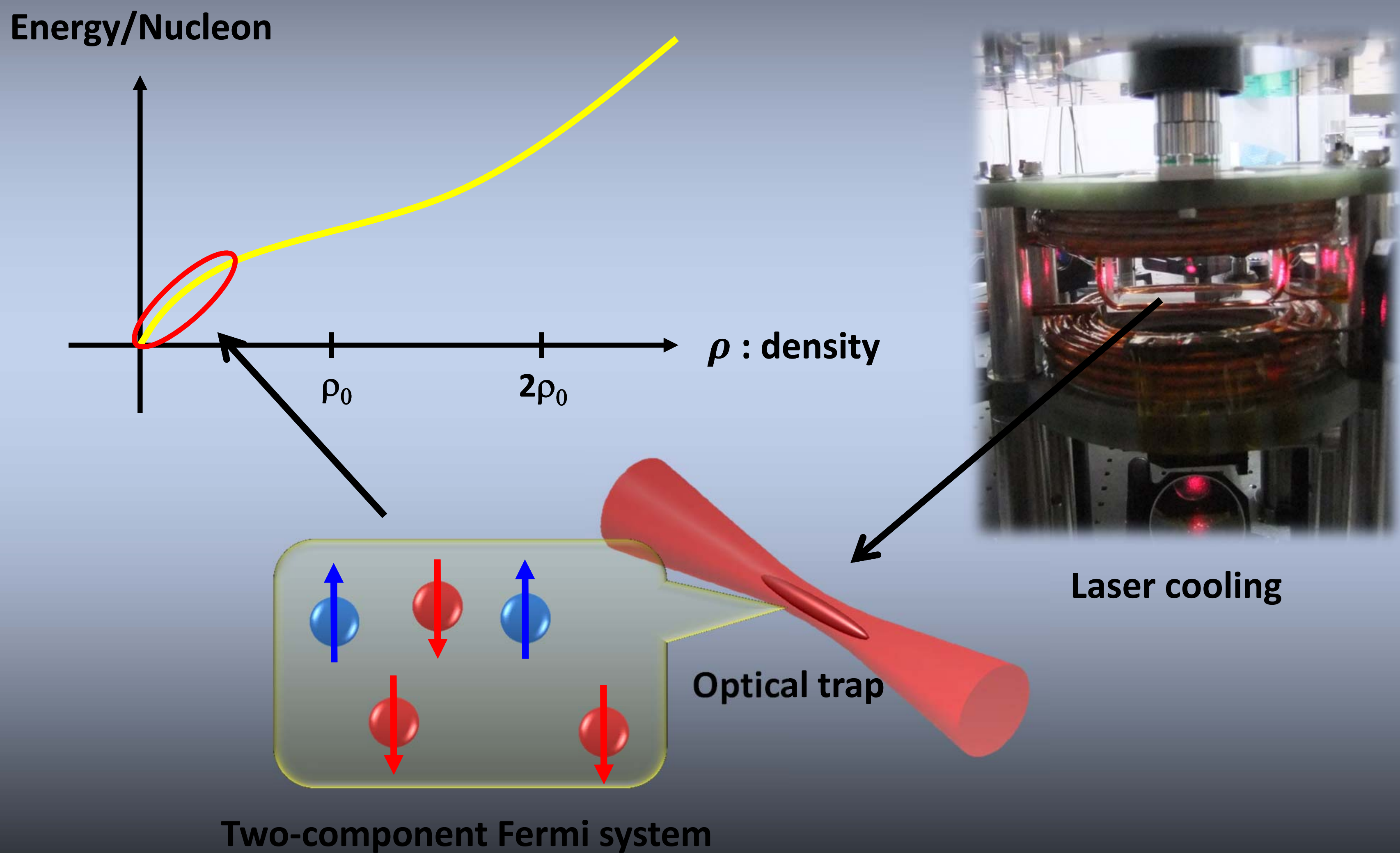
2014/9/25@熱川温泉

Contents

- **Goal of this project**

- **Construction of the EOS at the unitarity**
  - **Principle**
  - **Experimental**

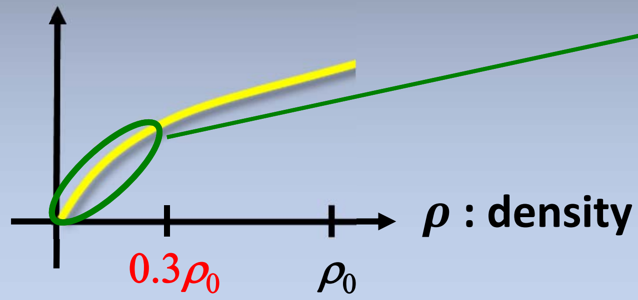
Goal : Direct determination of the EOS for dilute neutron matter using cold Fermi gases

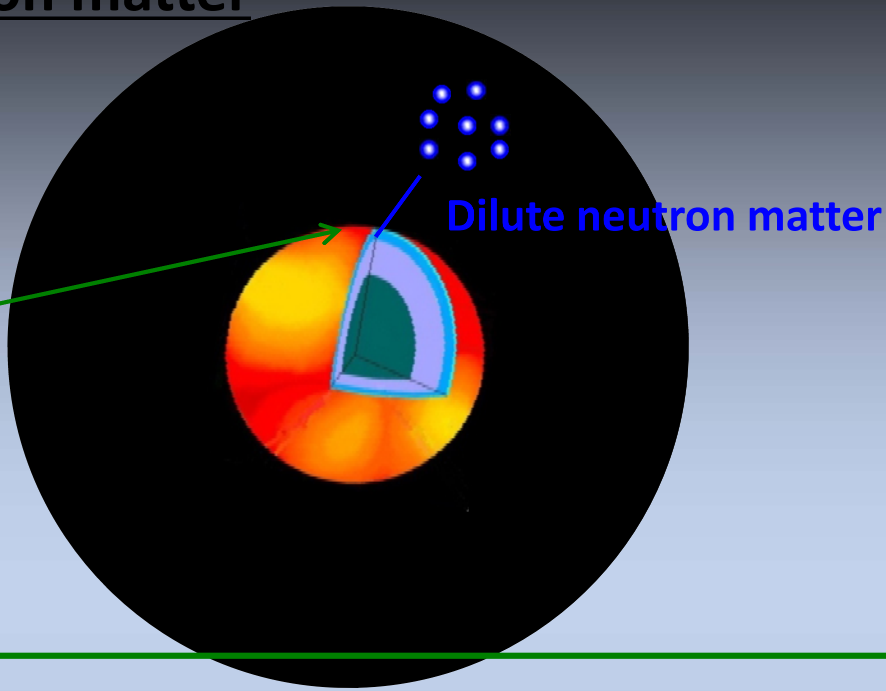


Dilute neutron matter

Neutron matter

$E = f(\rho, (n_N))$





Mass $(= mc^2)$: **940MeV**

Temperature (T) : **$10^5 \sim 10^7$K**

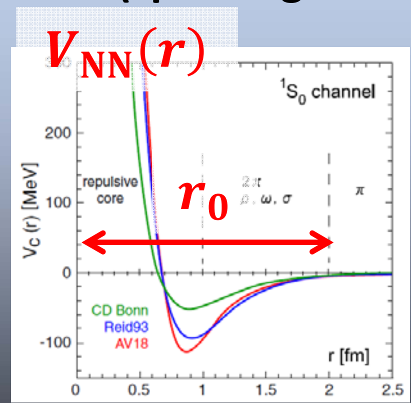
Density (ρ) : **$0.001 \sim 0.3\rho_0$**

Inter-particle spacing (d) : **3~16fm**

Interaction : **Nuclear force (short range)**

Potential size (r_0) : **~2fm**

NN potential (spin singlet and s-wave)



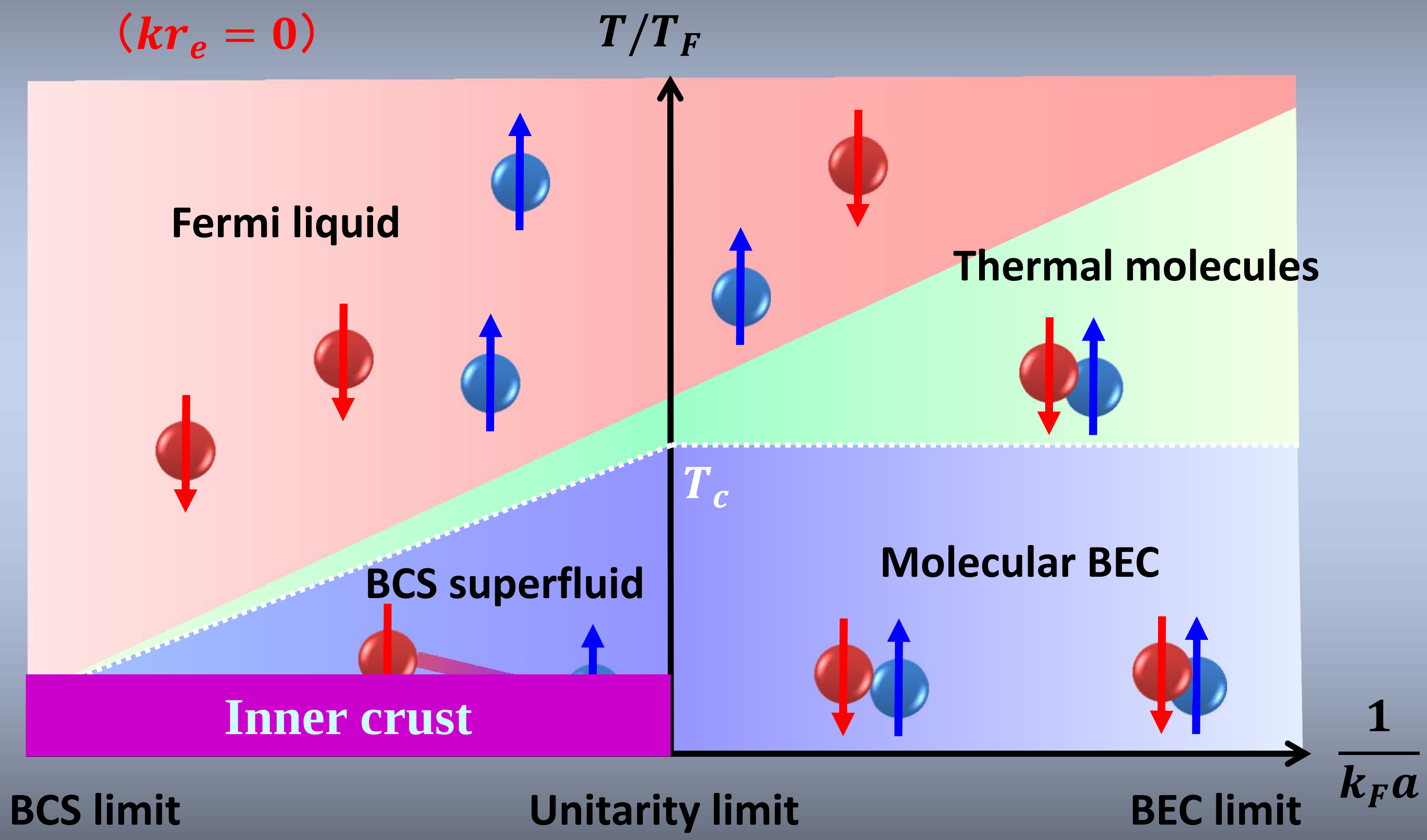
[N.Ishii, Phys. Rev. Lett. 99, 022001 (2007)]

Cold atom (^{6}Li) and dilute neutron matter (inner crust)

Ultracold Fermi system interacting with s-wave

	Cold Fermi atom	Neutron matter
Interaction	s-wave	s-wave
Temperature : T	$\sim 10^{-7}$K	$10^5 \sim 10^7$K
Fermi temperature : T_F	$\sim 10^{-6}$K	$10^{10} \sim 10^{11}$K
Interparticle distance : d	250nm	$3 \sim 16$fm
Scattering length : a	$-\infty \sim \infty$(Feshbach resonance)	-18.95fm
Effective range : r_e	4.7nm	2.75fm
↓	↓	↓
Temperature : T/T_F	$10 \sim 0.05$	~ 0
Interaction : $1/k_F a$	$-\infty$(BEC limit)$\sim +\infty$(BCS limit)	$-0.05 \sim -0.3$
Effective range : $k_F r_e$	0.05	$0.5 \sim 3$

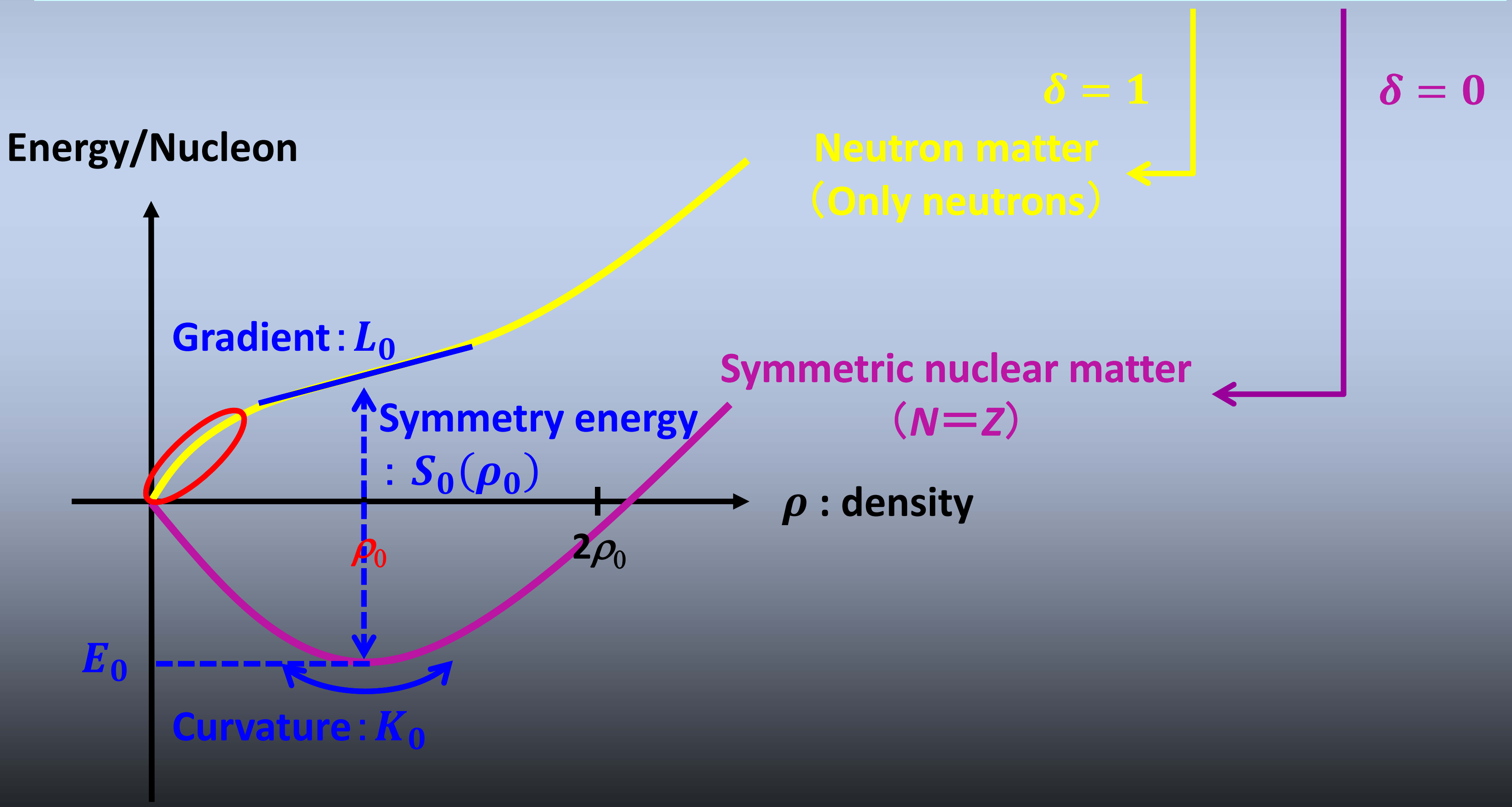
Neutron stars in the cold atom BCS-BEC crossover



Connection between cold atom and nuclear physics

Phenomenological EOS for nuclei：

$$E(\rho,\delta) = E_0 + \frac{K_0}{18\rho_0^2}(\rho-\rho_0)^2 + \left[S_0 + \frac{L_0}{3\rho_0}(\rho-\rho_0)\right]\delta^2, \qquad \left(\delta = \frac{N-Z}{A}\right)$$



From cold atom to the neutron star M-R curve

Cold Fermi atoms

$$\frac{E}{E_{\mathrm{F}}}\left(\frac{T}{T_{\mathrm{F}}}, -\frac{1}{k_{\mathrm{F}}a}\right)$$

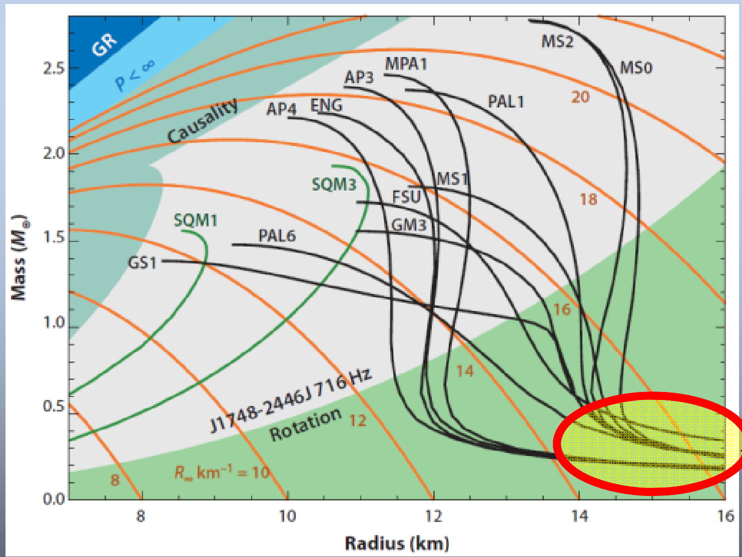
Correction

$$\frac{\partial E}{\partial(k_F r_e)} \sim 0.1$$

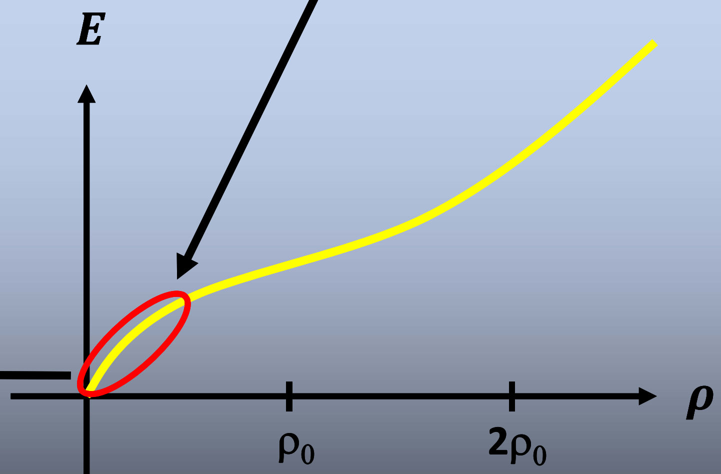
$$\frac{E}{E_F}\left(\frac{T}{T_F}, -\frac{1}{k_F a}, k_{\mathrm{F}} r_{\mathrm{e}}\right)$$

Neutron stars

$E = f(\rho, (n_n))$

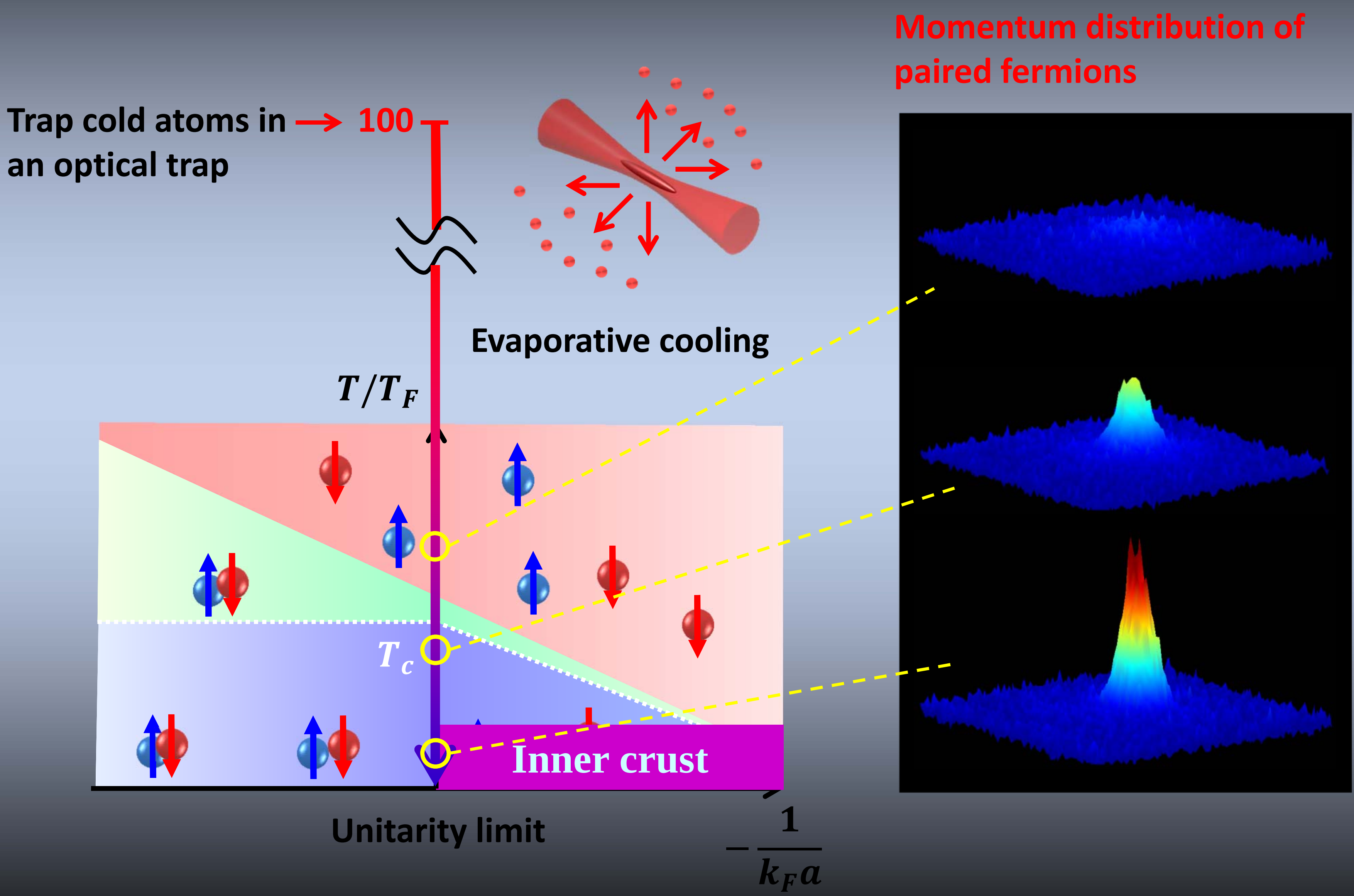


TOV



Constrain in these regions

Previous meeting : Superfluid transition at the unitarity



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The universal thermodynamic function in the crossover

General thermodynamic function for a two-component Fermi gas :

$$p(T, \mu_\uparrow, \mu_\downarrow, a_s, r_s, a_p, r_p, R_{vdw}, \cdots)$$

↓ No impurity, balanced system, ultracold, dilute, two-body short range collision, s-wave Feshbach resonance

$$p(T, \mu, a_s)$$

↓ Extended Gibbs-Duhem relation by Tan's theory

$$dp = sdT + nd\mu + Ida^{-1}$$

$I = \frac{\hbar^2}{4\pi m} C$, (C : Tan's contact density)

$$p(T, \mu, a_s^{-1}) \longrightarrow p\frac{\Lambda_T^3}{k_B T} = f\left(\frac{\mu}{k_B T}, \frac{\Lambda_T}{a_s}\right)$$

Dimensional analysis using $k_B T$

Normalization by ideal Fermi gas

$$\longrightarrow h\left(\frac{\mu}{k_B T}, \frac{\Lambda_T}{a_s}\right) \equiv \frac{p}{p^{(0)}} = \frac{f\left(\frac{\mu}{k_B T}, \frac{\Lambda_T}{a_s}\right)}{f^{(0)}\left(\frac{\mu}{k_B T}\right)}$$

The universal thermodynamic function

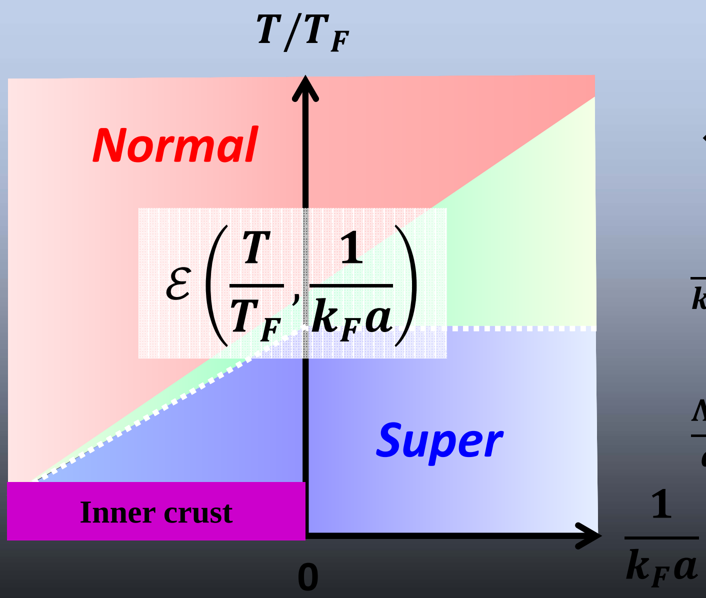
Absolute values are quite different between cold atoms and neutron star, but relative values among them are same

The universal thermodynamic function in the crossover

How to use $h\left(z \equiv \frac{\mu}{k_B T}, x \equiv \frac{\Lambda_T}{a_s}\right)$?

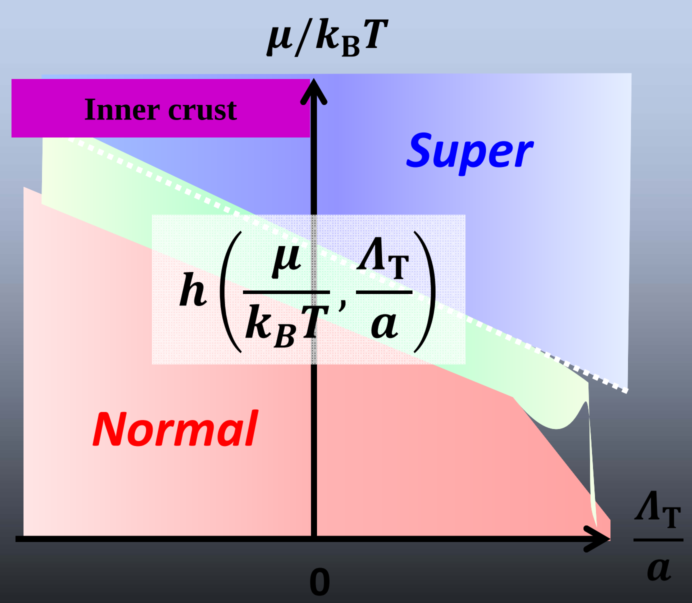
- **Pressure**: $p(T, \mu, a_s^{-1}) = p^{(0)}(T, \mu) \cdot h(z, x)$
- **Density**: $n(T, \mu, a_s^{-1}) = \left.\frac{\partial p(T, \mu, a_s^{-1})}{\partial \mu}\right|_{T, a_s^{-1}} = n^{(0)}(T, \mu) h(z, x) + p^{(0)}(T, \mu) \frac{z}{k_B T} \frac{\partial h(z, x)}{\partial z}$
- **Entropy, internal energy, Tan's contact, ･･･**

This is the EOS we determine

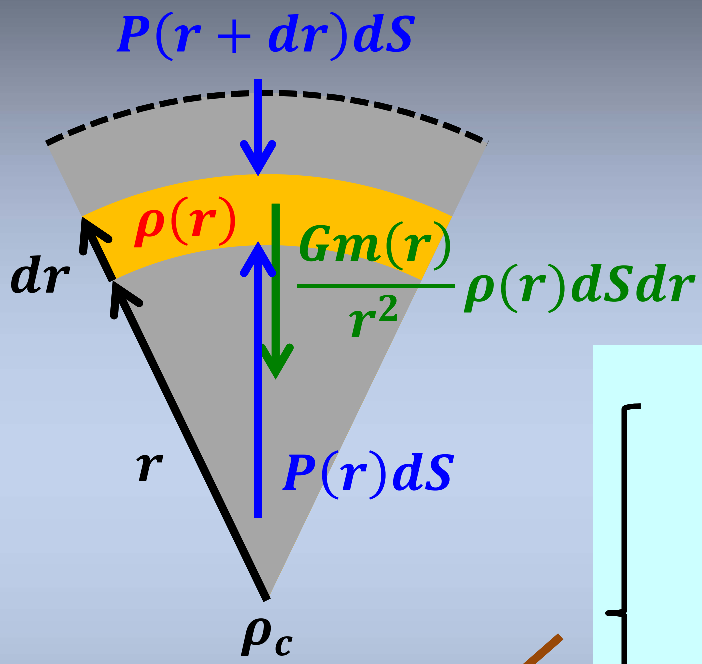


$$\frac{\mu}{k_B T} = \left(\frac{\mu}{\varepsilon_F}\right) \Big/ \left(\frac{T}{T_F}\right)$$

$$\frac{\Lambda_T}{a} = 2\sqrt{\pi}\left(\frac{1}{k_F a}\right) \Big/ \sqrt{\frac{T}{T_F}}$$



The M-R curve and the EOS



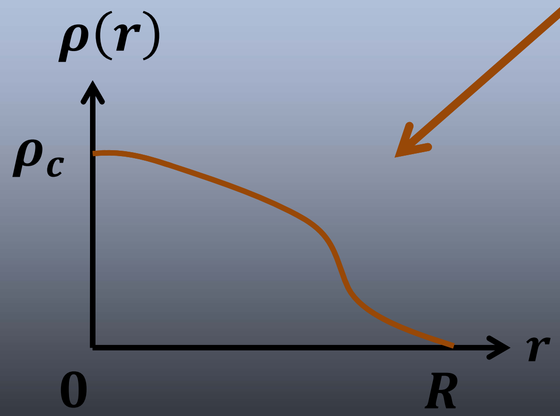
Force balance: $\frac{dP(r)}{dr} = -\frac{Gm(r)}{r^2}\rho(r)$

↓ Relativistic correction

TOV : $\frac{dP}{dr} = -\frac{G\{m(r) + 4\pi P r^3/c^2\}\{\rho(r) + P/c^2\}}{r^2\{1 - 2Gm(r)/c^2 r\}}$

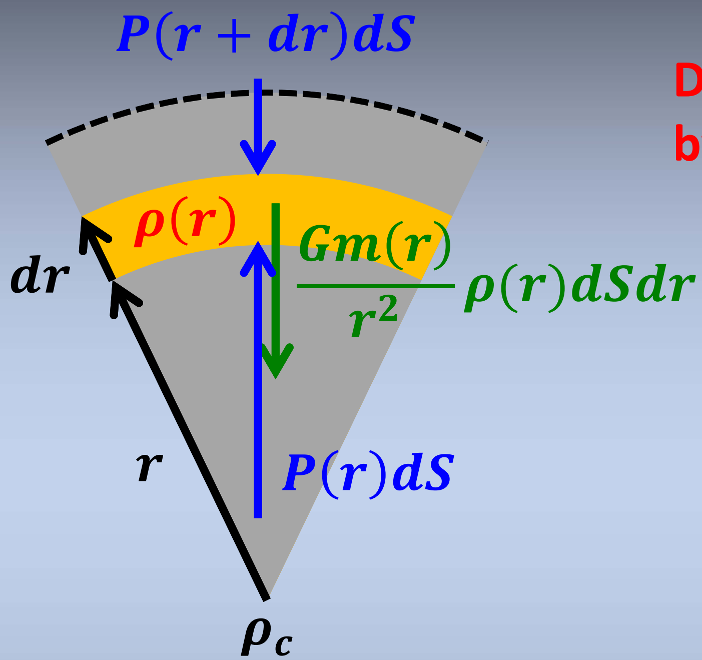
EOS : $P = f_P(\rho) = \rho^2 \frac{\partial f_E(\rho)}{\partial \rho}$

Mass : $m(r) = \int_o^r 4\pi r^2 \rho(r) dr$



M-R関係： $M = m(R) < \frac{Rc^2}{2G}$ （Black hole）

Principle of determination of the EOS from cold atom experiment



Density distribution is similarly determined by a force balance and the EOS

Star : **(Newton's level)** $$\frac{dP(\rho(r))}{dr} = -\frac{Gm(r)}{r^2}\rho(r)$$

$\rho(r)$: mass density[g/cm^3]

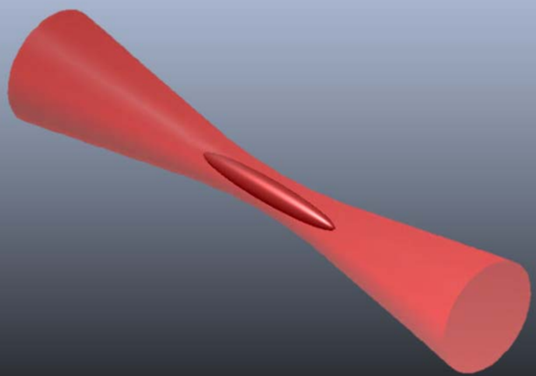
Magnetic field : B

Temperature : T

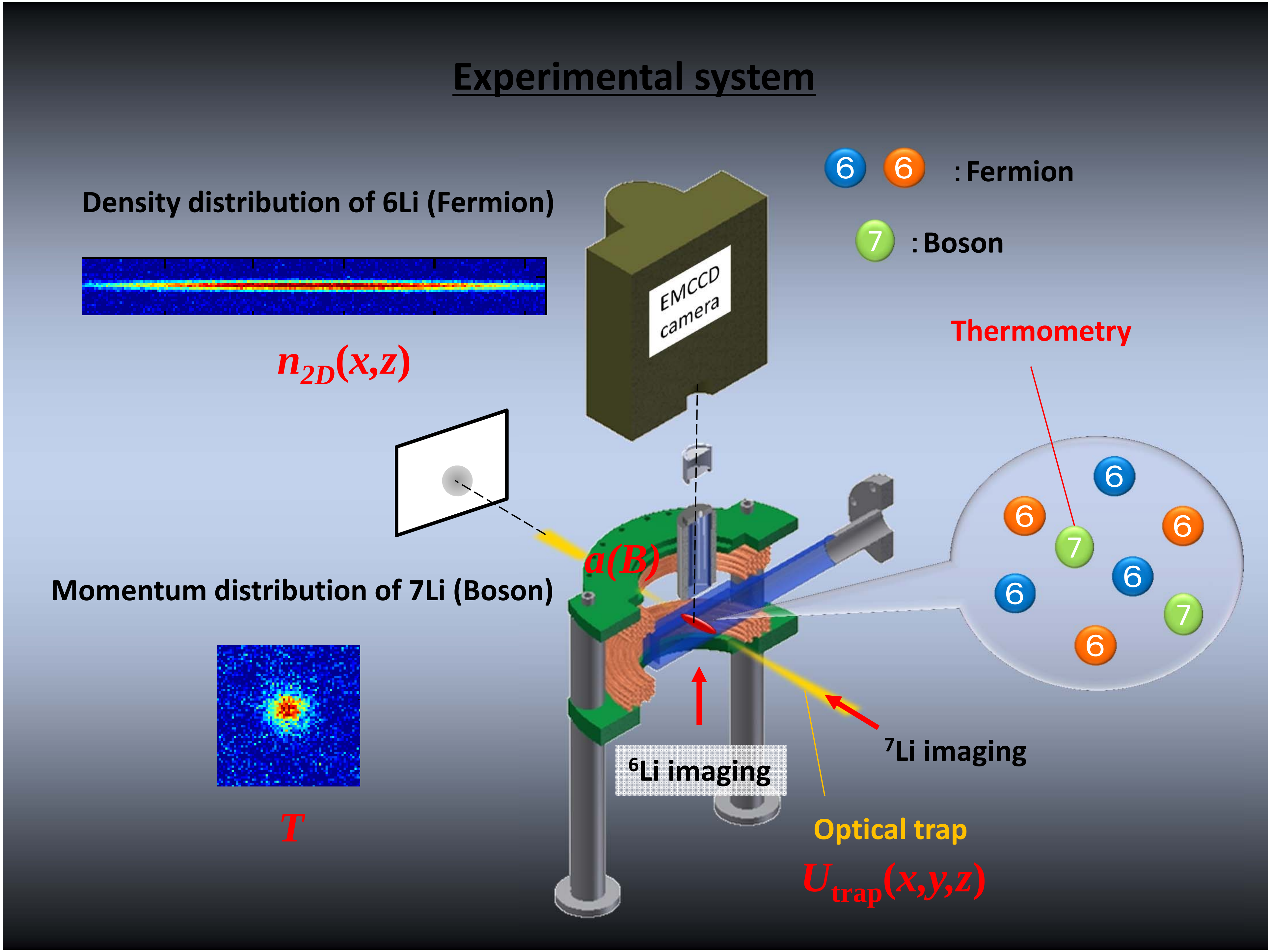
Measurement

Cold atom in a trap : $$\nabla P(n(r)) = -n(r)\nabla U_{trap}(r)$$

$n(r)$: number density[/cm^3]



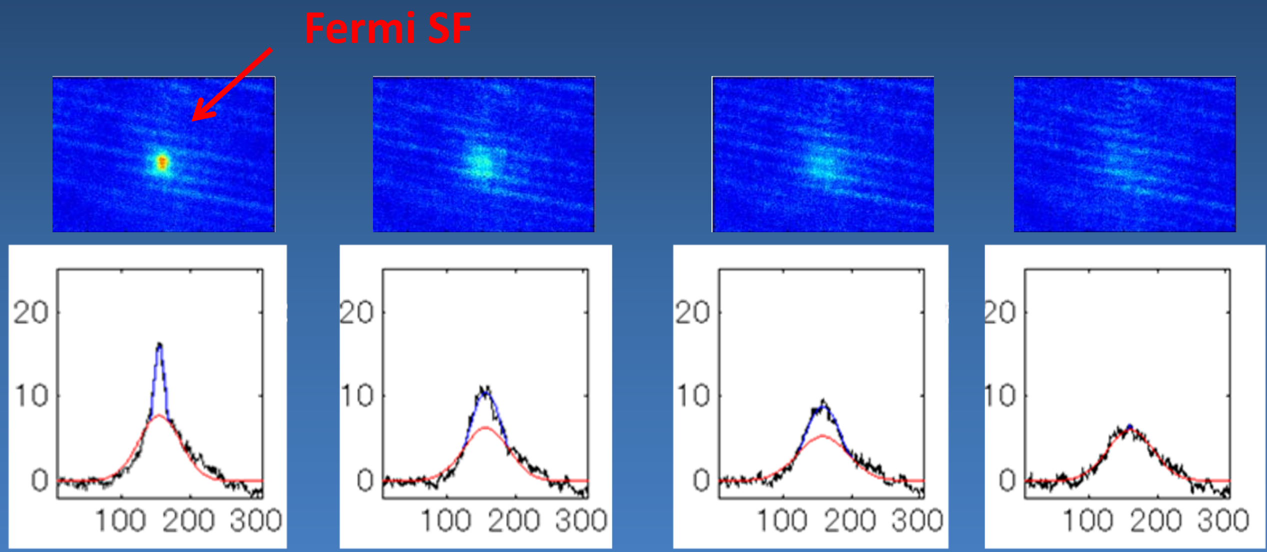
EOS : $P\left(T, \mu, a(B)^{-1}\right)$


Experimental system
6
6
: Fermion
7
: Boson
Density distribution of 6Li (Fermion)
$n_{2D}(x,z)$
EMCCD camera
Thermometry
$a(B)$
Momentum distribution of 7Li (Boson)
T
^{6}Li imaging
^{7}Li imaging
Optical trap
$U_{trap}(x,y,z)$

Hot topic **: Bose-Fermi superfluid mixture !!**

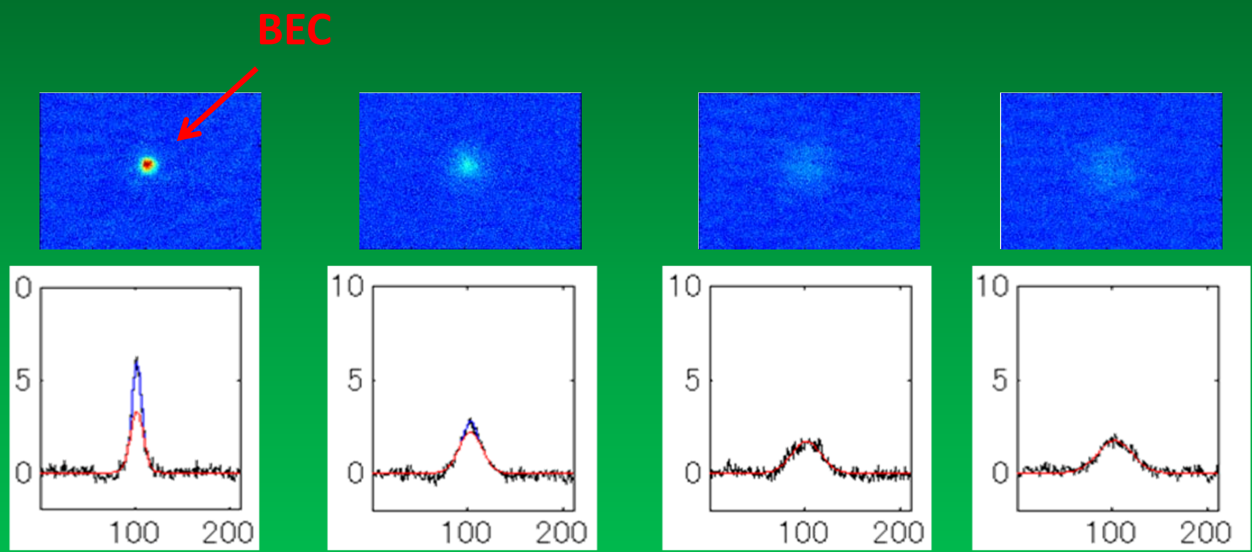
^{6}Li (Fermi)

Momentum distribution of paired fermions



^{7}Li (Bose)

Momentum distribution of bosons



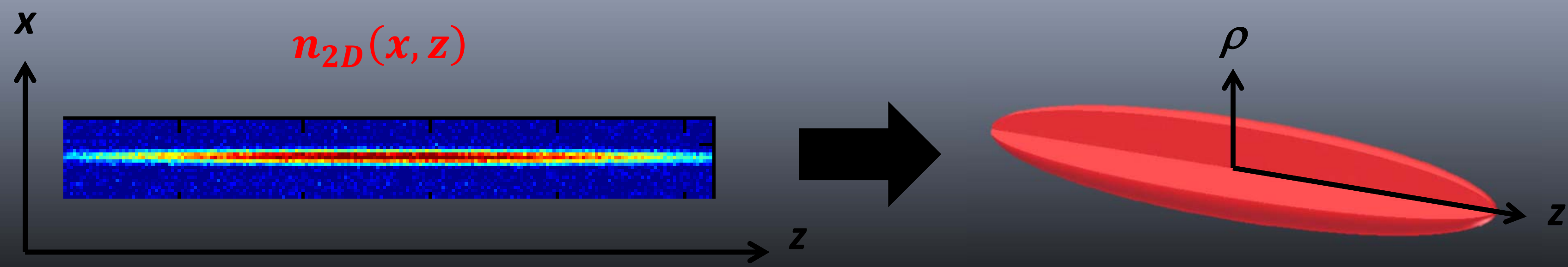
Local pressure from inhomogeneous systems

Gibbs-Duham : $$P = \int_{-\infty}^{\mu} n \, d\mu' = \int_{\rho}^{\infty} n(\rho') \frac{\partial U_{trap}(\rho')}{\partial \rho} d\rho'$$

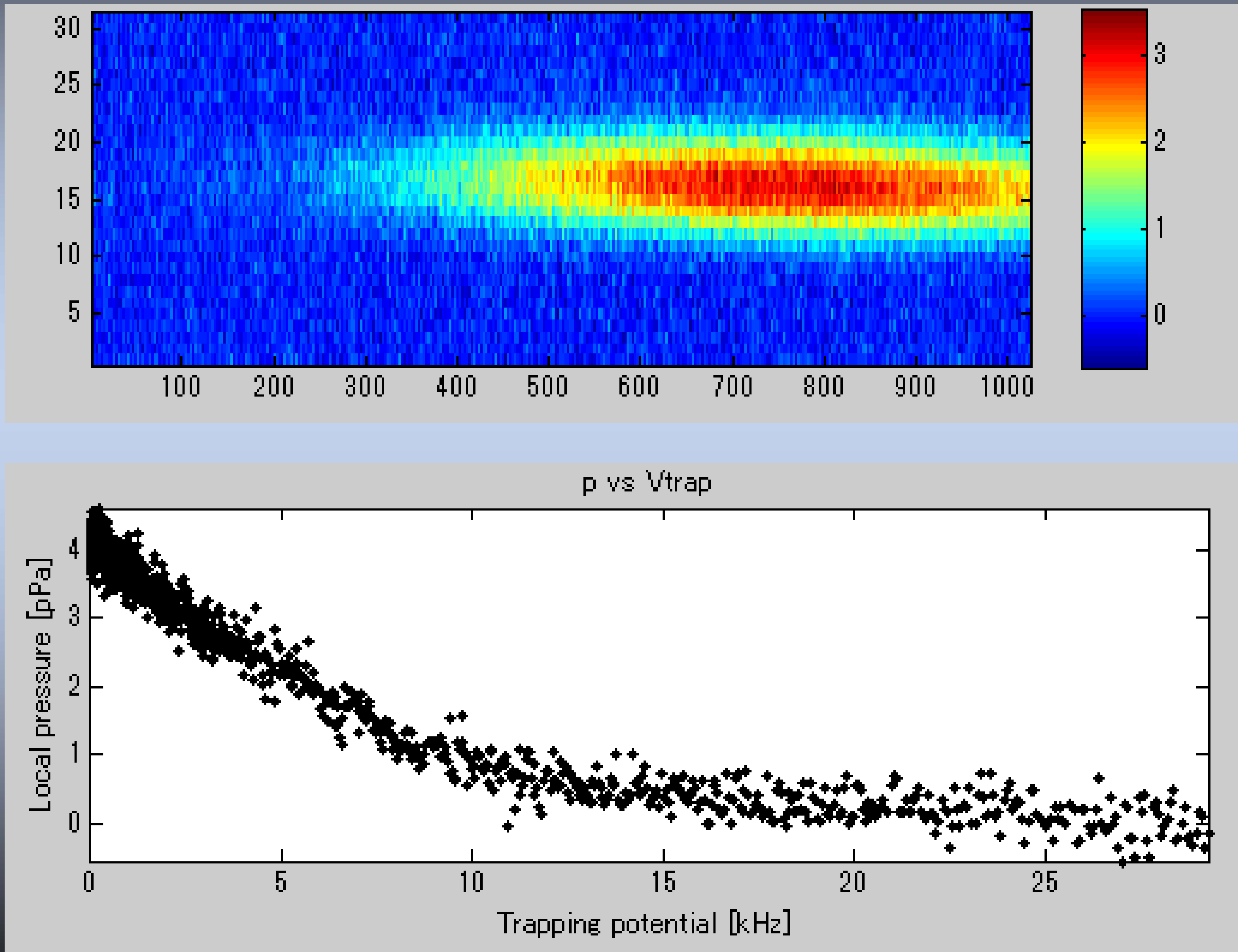
(LDA : $\mu(\rho, z) = \mu_g - U_{trap}(\rho, z)$)

Inv-Abel transformation : $$n(\rho, z) = -\frac{1}{\pi} \int_{\rho}^{\infty} \frac{1}{(x^2 - \rho^2)^{1/2}} \frac{\partial n_{2D}(x, z)}{\partial x} dx$$

$$P(\rho, z) = \frac{1}{\pi} \int_{\rho}^{\infty} dx \, n_{2D}(x, z) \left[\frac{\partial U_{trap}(x, z)/\partial \rho}{(x^2 - \rho^2)^{1/2}} + \int_{\rho}^{x} \frac{\rho' \frac{\partial U_{trap}(x, z)}{\partial \rho} - x \frac{\partial U_{trap}(\rho', z)}{\partial \rho}}{\left(x^2 - \rho'^2\right)^{3/2}} \right]$$



Local pressure from inhomogeneous systems

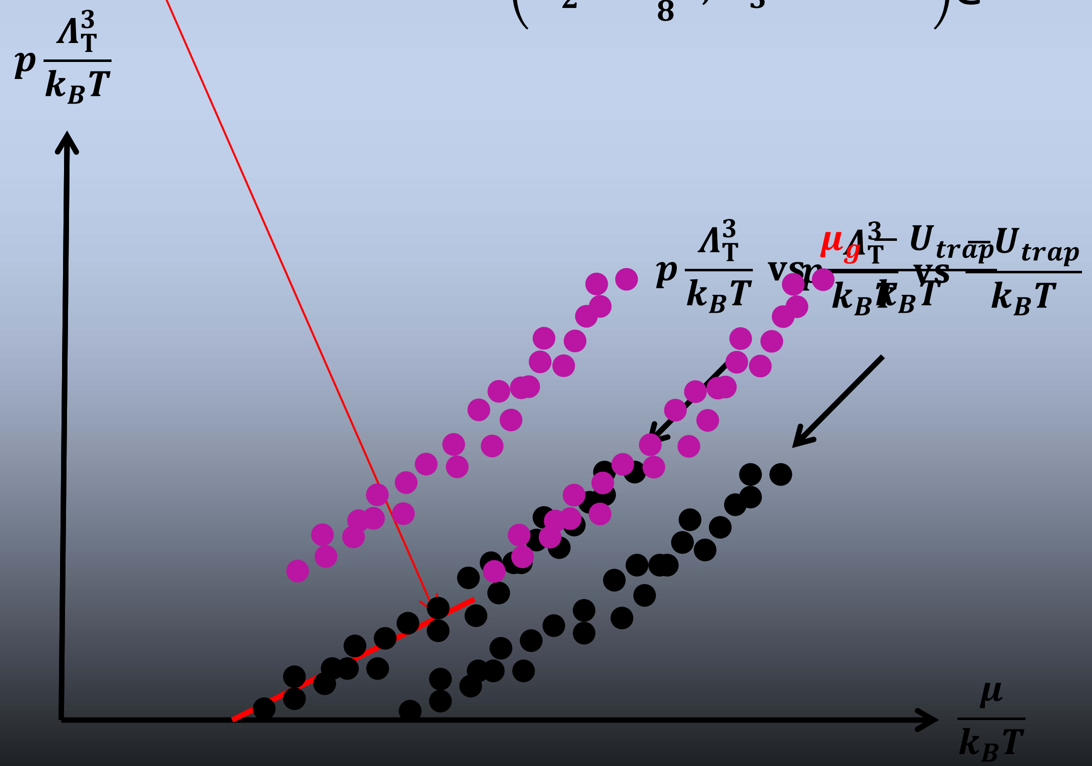


Construction of the EOS at uniitarity $(a_s^{-1} = 0)$

$$\left(p, T, \mu = \mu_g - U_{trap}\right) \longrightarrow p\frac{\Lambda_T^3}{k_B T} = f\left(\frac{\mu = \mu_g - U_{trap}}{k_B T}\right)$$

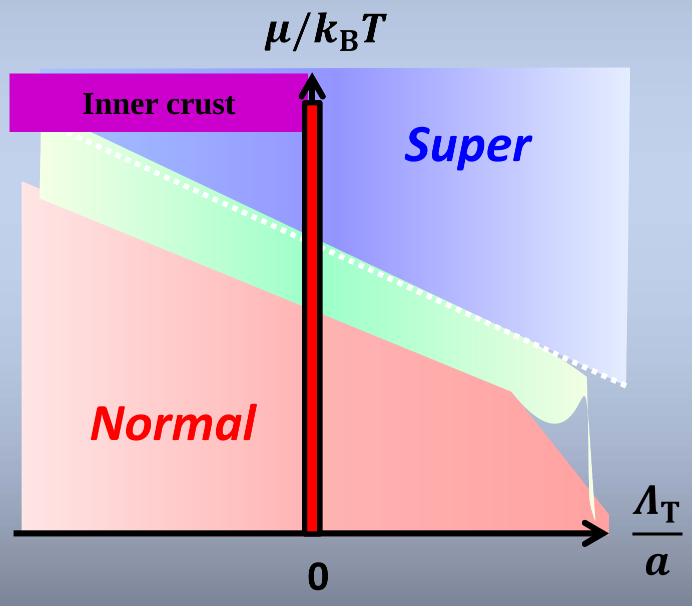
High T limit : $p\frac{\Lambda_T^3}{k_B T} = e^{\frac{\mu}{k_B T}} + B_2 e^{2\frac{\mu}{k_B T}} + B_3 e^{3\frac{\mu}{k_B T}} + \cdots$

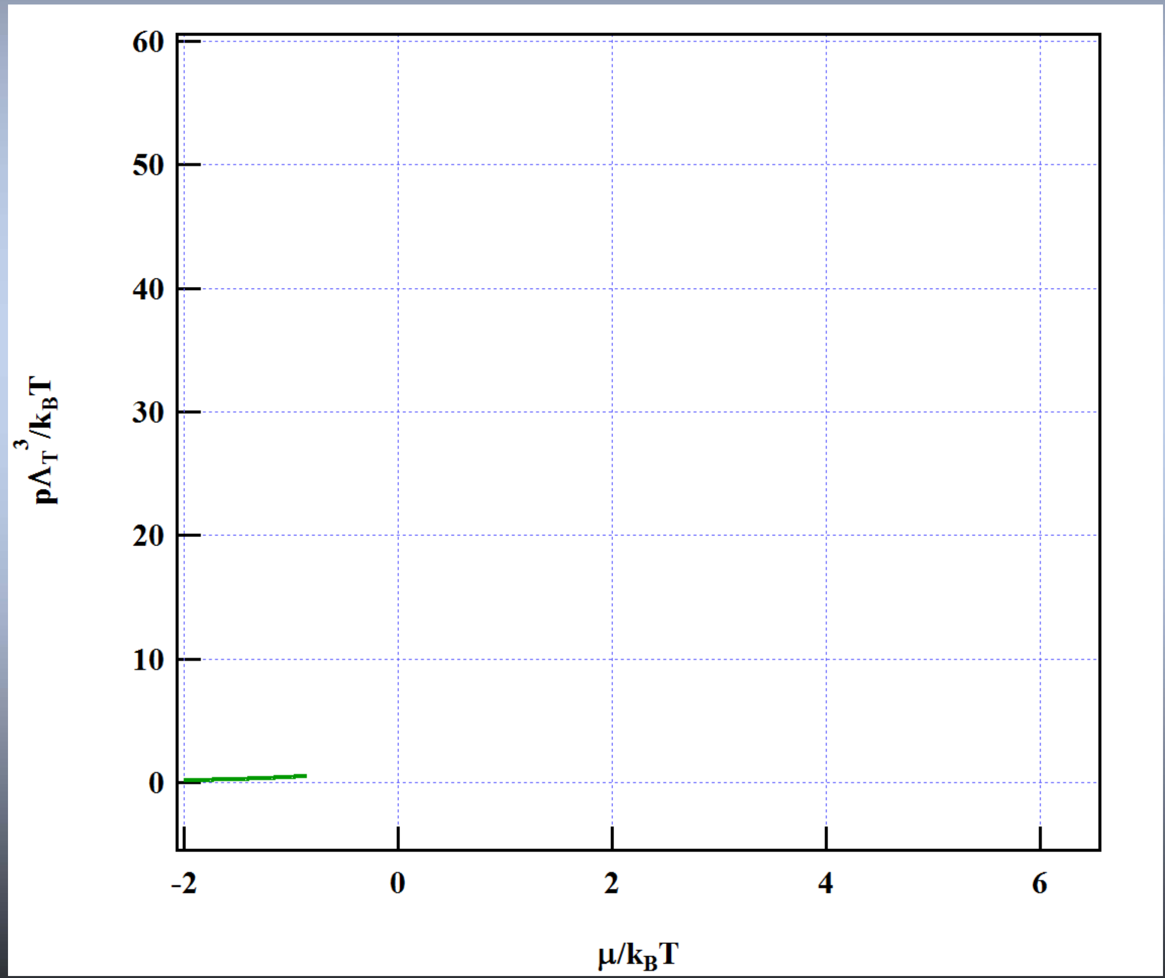
$\left(B_2 = \frac{3\sqrt{2}}{8}, B_3 = -0.29\right)$@Unitarity



Construction of the EOS at uniitarity ($a_s^{-1} = 0$)

$$p\frac{\Lambda_T^3}{k_B T} = f\left(\frac{\mu}{k_B T}\right)$$

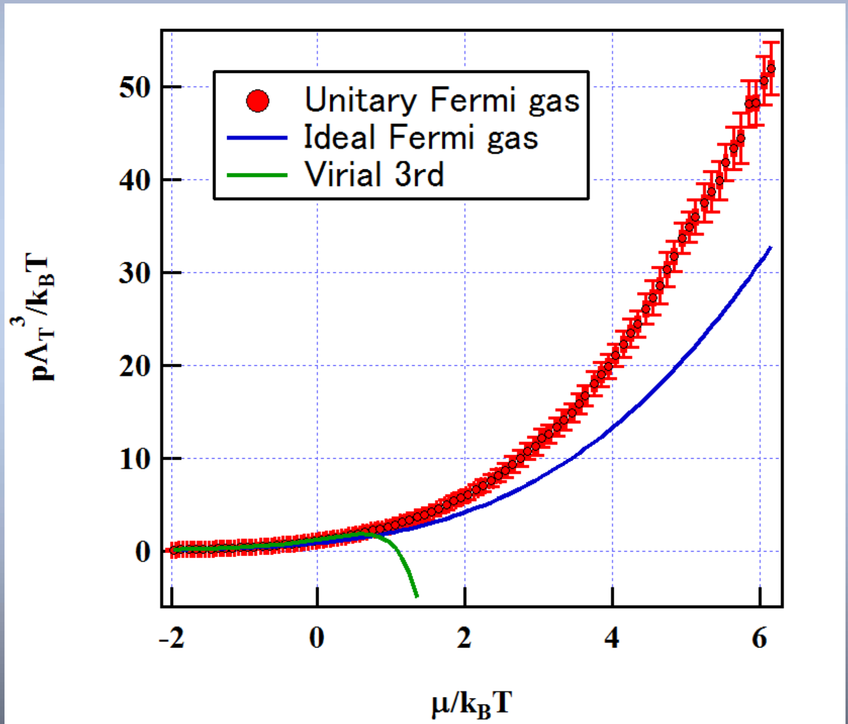


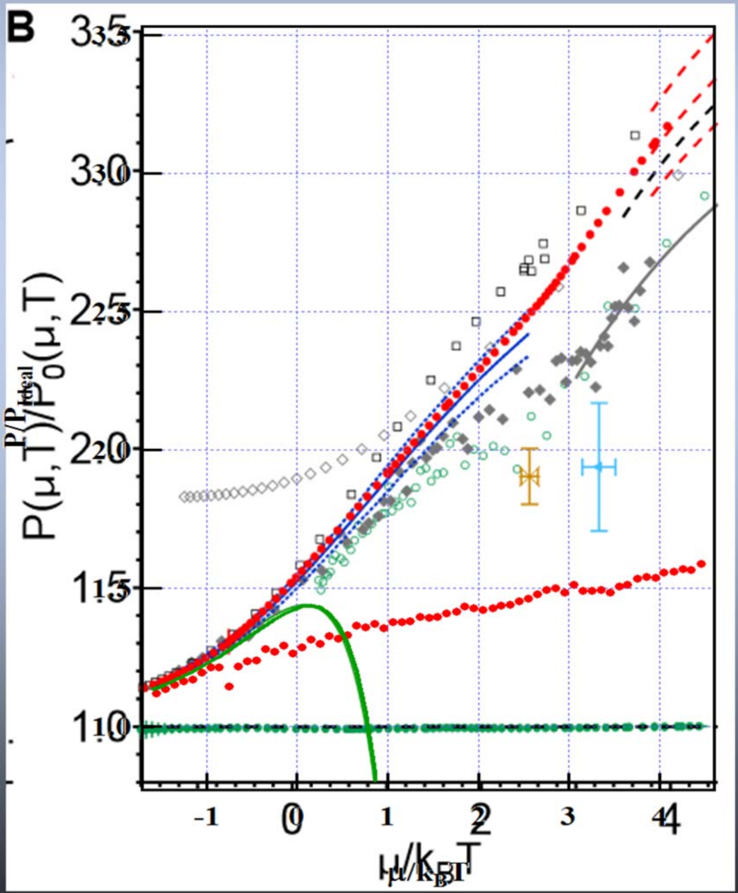


Evaluation

$$p\frac{\Lambda_T^3}{k_BT} = f\left(\frac{\mu}{k_BT}\right)$$

$$h\left(\frac{\mu}{k_BT}\right) \equiv \frac{p(\mu/k_BT)}{p^{(0)}(\mu/k_BT)}$$



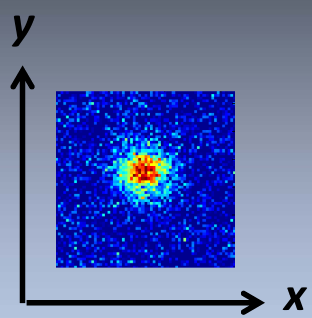


Large discrepancy

[Mark Ku, Science 335, 563 (2012)]

Possibility of the deviation(1)

- ^{7}Li thermometry



B.E distribution : $n(x, y\,; t_{TOF}) = A \cdot \mathrm{PolyLog}_2\left[z \exp\left(-\frac{x^2}{\sigma_x(t_{TOF})^2} - \frac{y^2}{\sigma_y(t_{TOF})^2}\right)\right]$

Temperature : $T = \frac{m_{7Li}}{2k_B}\left(\frac{\omega_i^2}{1+\omega_i^2 t_{TOF}^2}\right)\sigma_i(t_{TOF})^2$

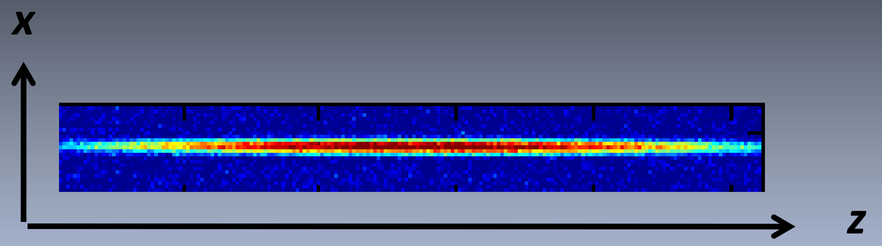
Interactions of ^{7}Li-^{7}Li and ^{6}Li-^{7}Li are enough small ⟶ **Ideal Bose gas**

($a_{77} = 61.0a_0$, $a_{67} = 40.8a_0$)

It looks no problem

Possibility of the deviation(2)

- ^{6}Li pressure



$$n_{2D}(x,z) = -\frac{1}{\sigma_{abs}}\ln(T_{abs}(x,y))$$

$$p(\rho=0,z) = C_{ODT}\frac{4P}{\pi^2 w_x^2(z) w_y^2(z)}\int_{-\infty}^{+\infty} n_{2D}(x,z)\left\{1-2\sqrt{2}\frac{x}{w_x(z)}D\left(\sqrt{2}\frac{x}{w_x(z)}\right)\right\}dx$$

Dawson function

Absorption cross-section : $\sigma_0 = \frac{3\lambda^2}{2\pi}$

Resonant component in a probe laser beam : >96%

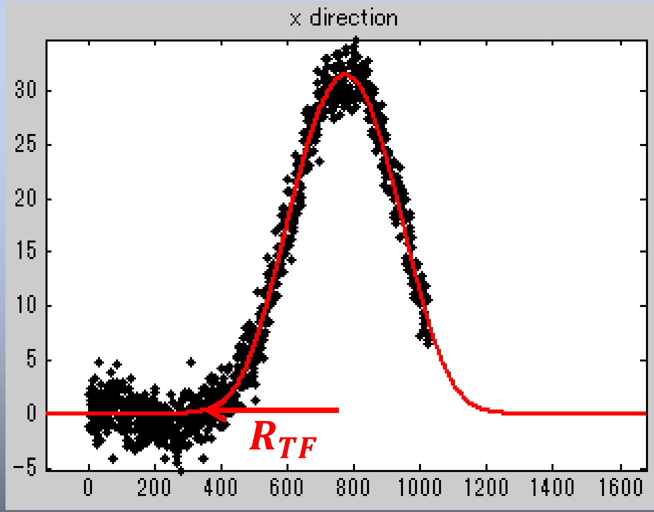
Although it also looks no problem, we tried calibration of the number of atoms

$$n_{2D}(x,z) = -\frac{\alpha}{\sigma_{abs}}\ln(T_{abs})$$

Caliblation of the number of atoms

Total absorption : $N_{abs} = \iint n_{2D}(x,z)\,dxdz = -\frac{1}{\sigma_{abs}} \iint \ln(T_{abs}(x,y))\,dxdy$

Fermi radius : (Chemical potential) $E_F = \hbar\,\bar{\omega}(6N)^{1/3} = \frac{m_{6Li}}{2}\omega_z^2 R_{TF}^2 \longrightarrow N = \frac{m_{6Li}^3 \omega_z^6 R_{TF}^6}{48\hbar^3\bar{\omega}^3}$

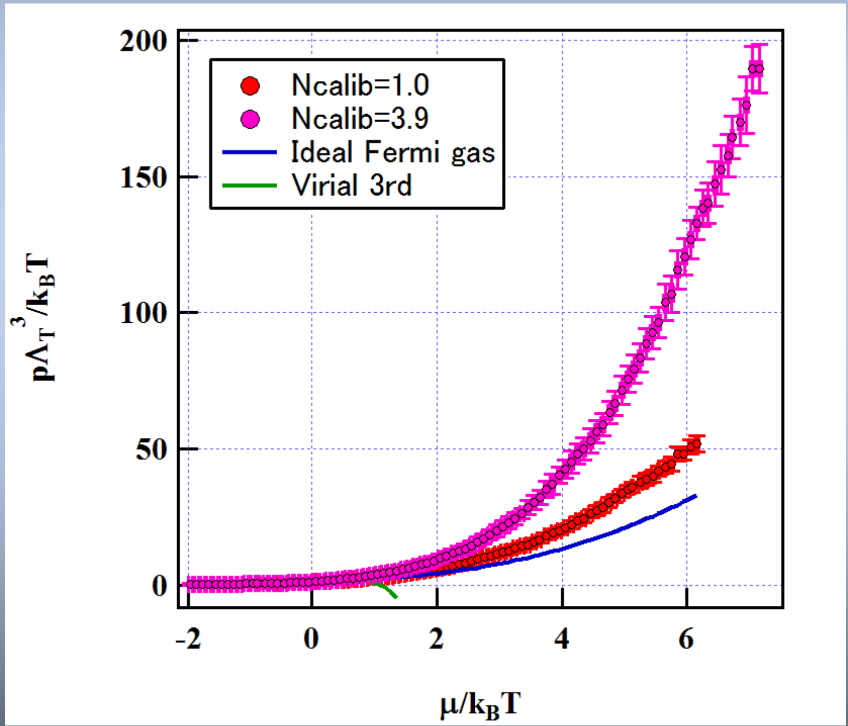


$\alpha = \frac{N}{N_{abs}} = 3.9$ ※Unrealistic value

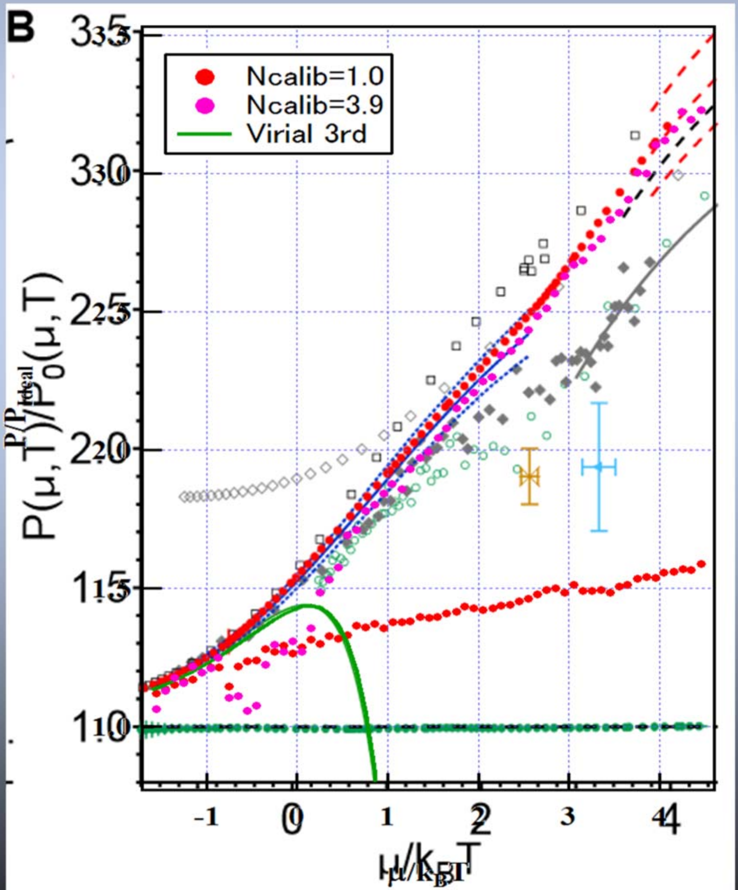
Effect of the calibration

$$p\frac{\Lambda_T^3}{k_BT} = f\left(\frac{\mu}{k_BT}\right)$$

$$h\left(\frac{\mu}{k_BT}\right) \equiv \frac{p(\mu/k_BT)}{p^{(0)}(\mu/k_BT)}$$



It looks nice agreement,
but I can not believe it



[Mark Ku, Science 335, 563 (2012)]

Possibility of the deviation(3)

Validity of LDA

So far, data have been is analized under the LDA that "**local physical quantities are given only by the local density**"

In the case of a large aspect ratio, "**density gradient should be considered ?**"

Internal energy density : $\boldsymbol{\mathcal{E}(n(r), \nabla n) = \mathcal{E}\left(n(r)\right) + \lambda \frac{\hbar^2}{8m}\frac{[\nabla n(r)]^2}{n(r)}}$ $(\boldsymbol{\lambda \sim 0.25})$

Density distribution can be modified?

Aspect ratio of trapping potentials

MIT experiment : 5 ⟷ This work : >60

Summary

- ユニタリー極限でEOS構築開始
  - 非現実的な個数校正係数が真か偽か？
  - もし校正係数が正しいならば何が原因か？
  - LDAは成立しているのか？
- ユニタリー極限で信頼性が確認されたらBCS-Unitary領域のEOSを構築
- 希薄中性子物質のEOS構築
- 低密度領域でのM-R曲線の完成を年度内に目指す