

arXiv 1402.4567

Evolution of Magnetized NSs (2D stars)

Chiba Institute of Technology
Nobutoshi Yasutake

Nuclear physics

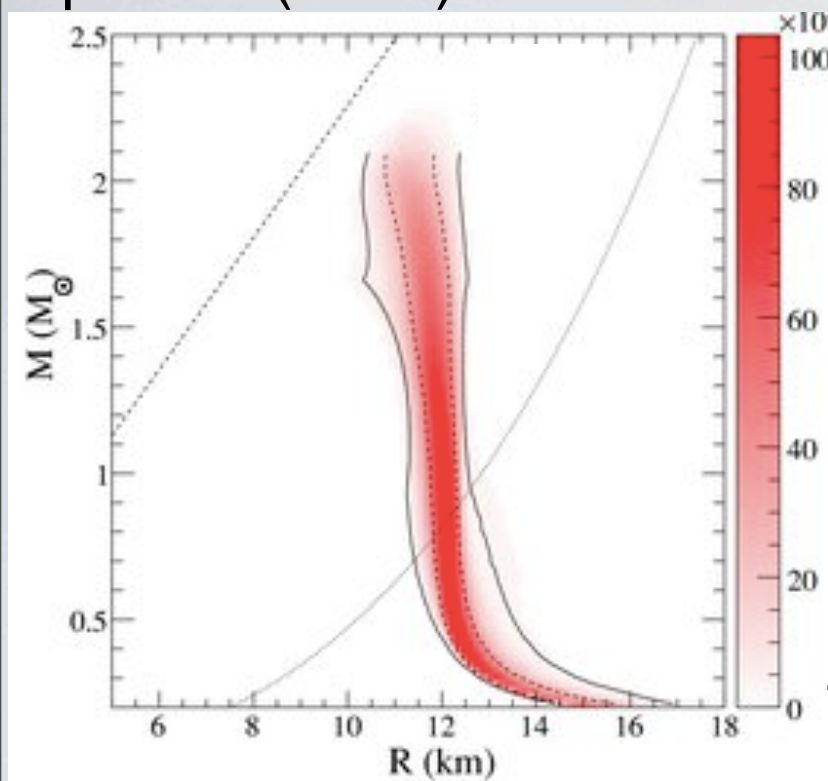
T. Maruyama(JAEA), **T.Sasaki**, **M.Yahiro**(Kyushu univ.), **T.Tatsumi**(Kyoto univ.),
Y.Yamamoto(RIKEN), **M.Kohno**(Kyushu dental univ.),
T. Furumoto(Ichinoseki National college of Technology),
H.Kouno(Saga univ.), **T. Rijken**(Nijmegen univ.),
D. Blaschke(Wroclow univ.), **S. Benic**(Zagreb univ.), **Chen**(Wuhan)

Astrophysics

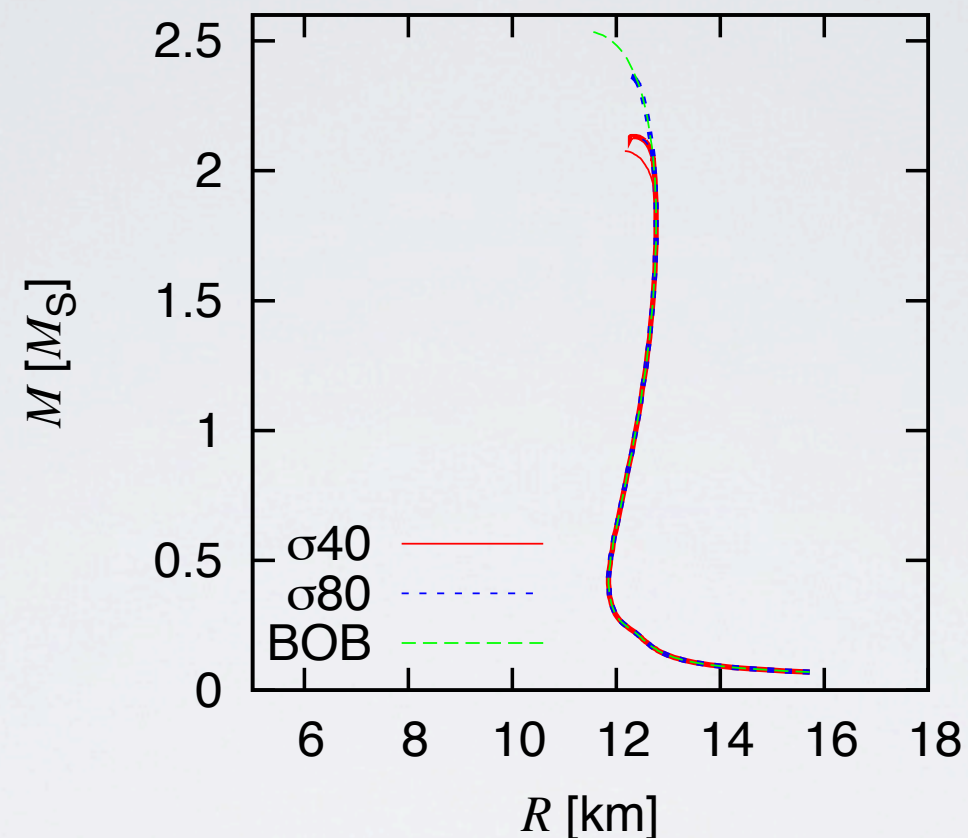
K.Fujisawa, **S.Yamada**(Waseda univ.), **M. Kutsuna**, **T.Shigeyama**(Univ. of Tokyo),
K. Kotake(Fukuoka univ.),

Observations

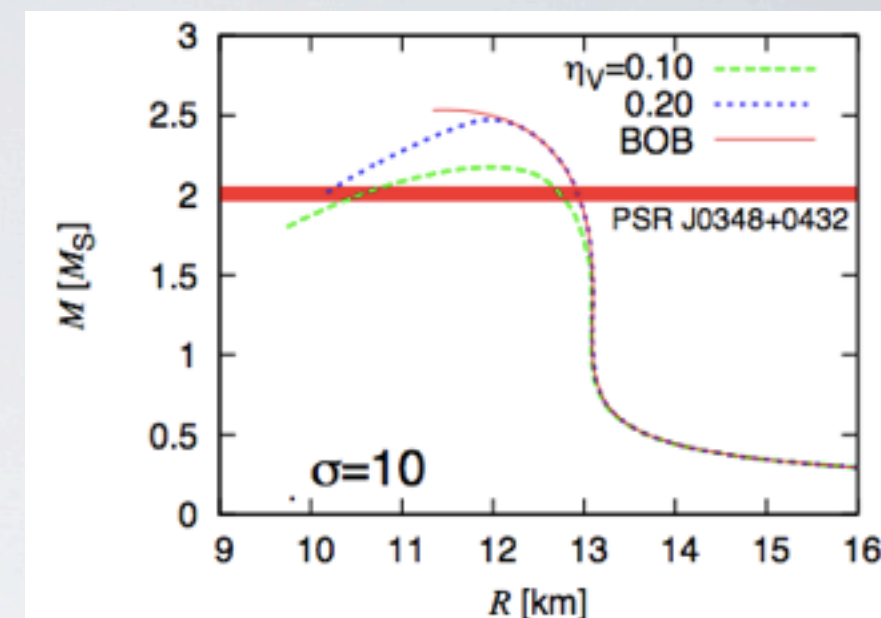
A.W.Steiner, J.M.Lattimer, E.F.Brown
ApJ **722** (2010) 33



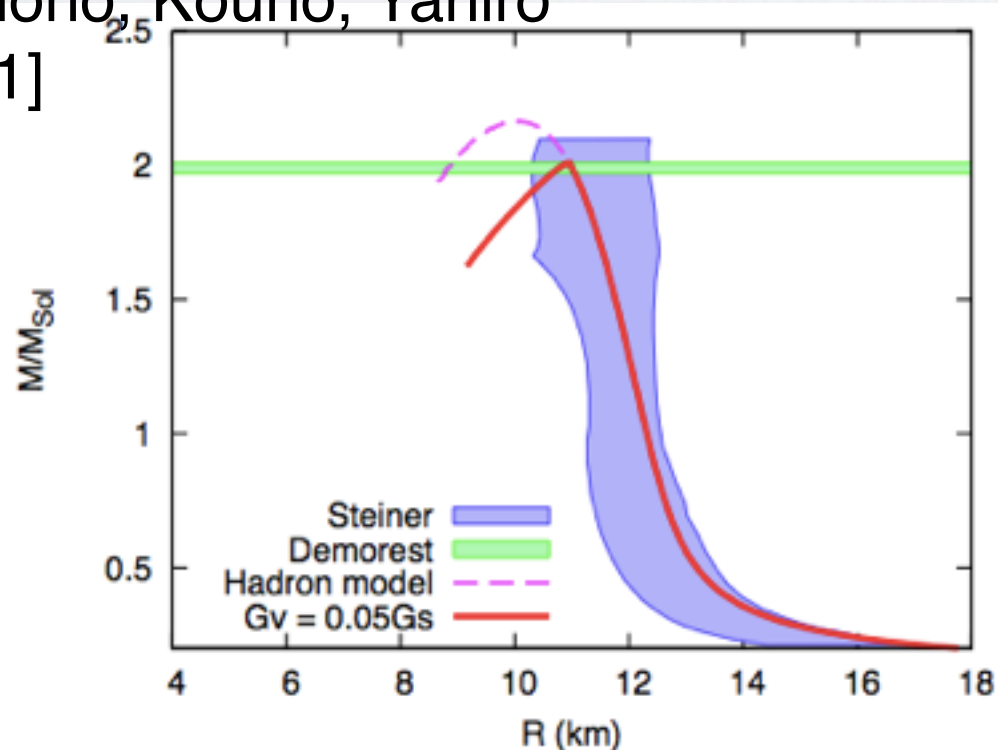
DS+BHF(BOB+TBF)
with pasta [in prep.]



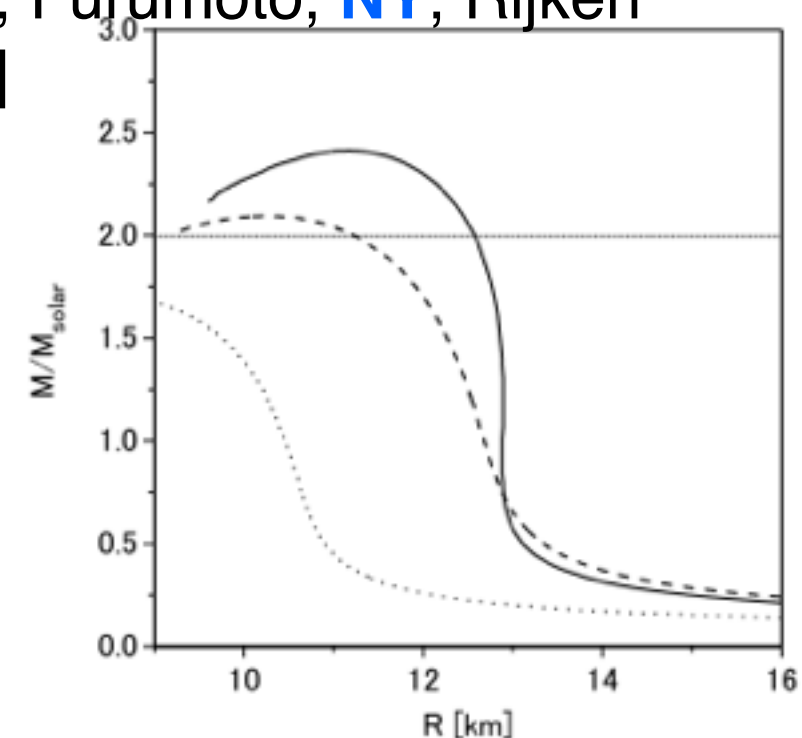
NLNJL+BHF(BOA) with pasta
[[NY](#), Lastwiecki, Benic,
Blaschke, Maruyama, Tatsumi
PRC(2014)]



EPNJL+chiPT
[Sasaki, [NY](#), Khono, Kouno, Yahiro
arXiv:1307.0681]

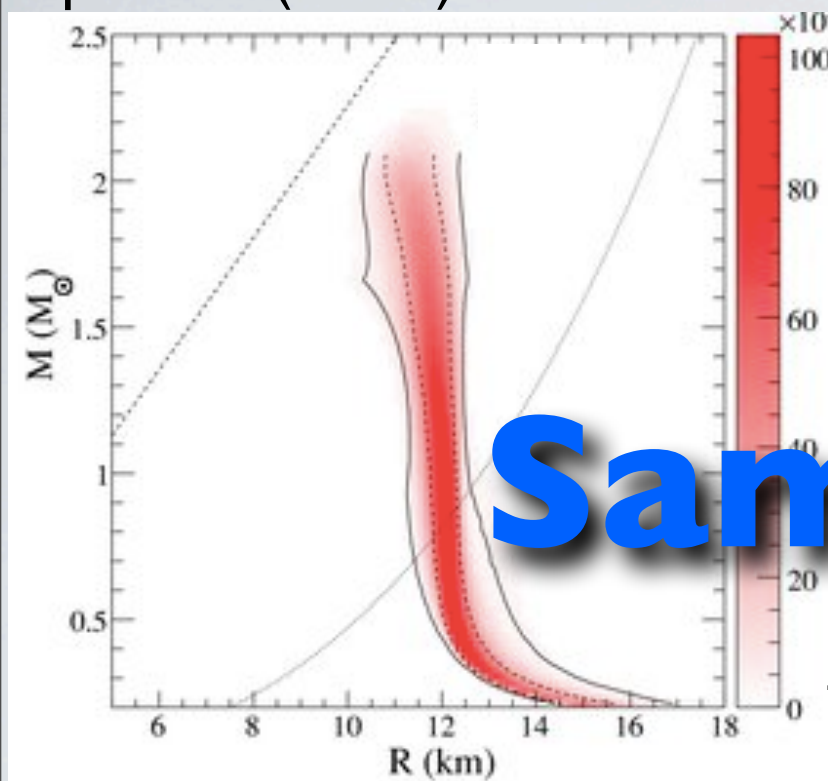


BHF(UTBF by Pomeron)
[Yamamoto, Furumoto, [NY](#), Rijken
PRC(2013)]

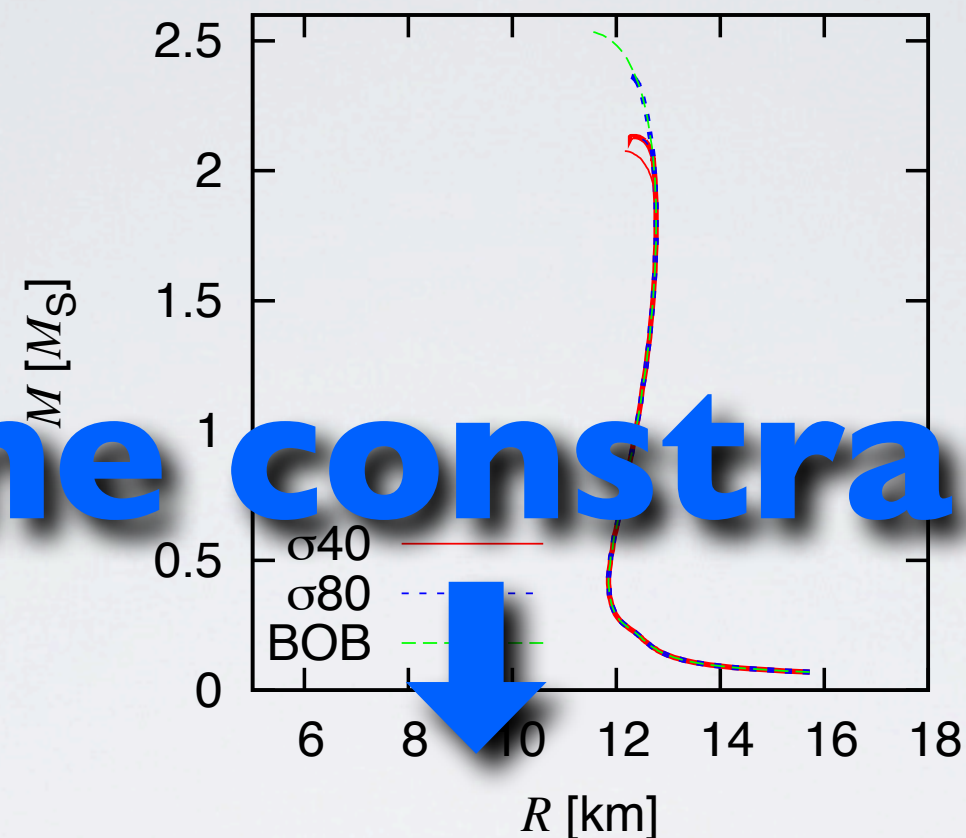


Observations

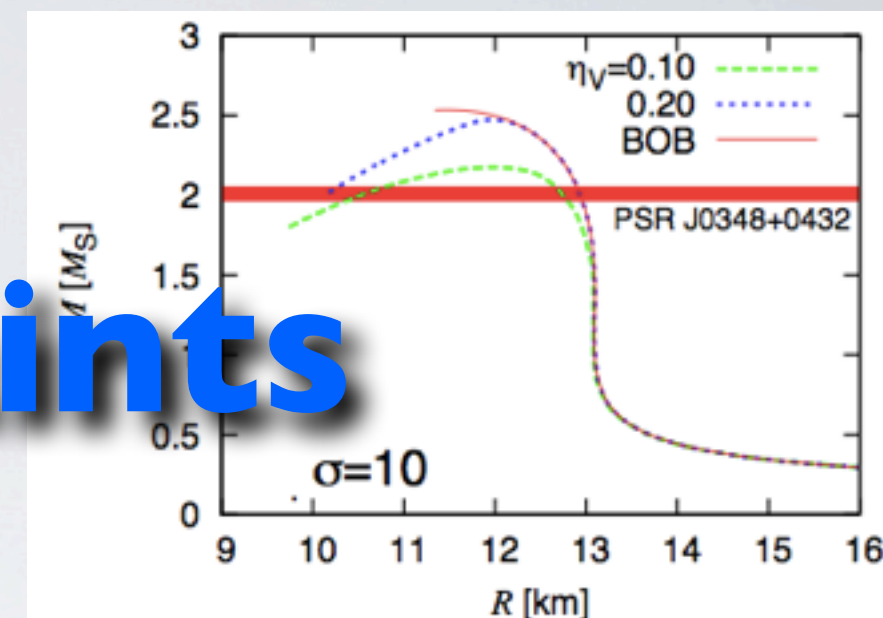
A.W.Steiner, J.M.Lattimer, E.F.Brown
ApJ **722** (2010) 33



DS+BHF(BOB+TBF)
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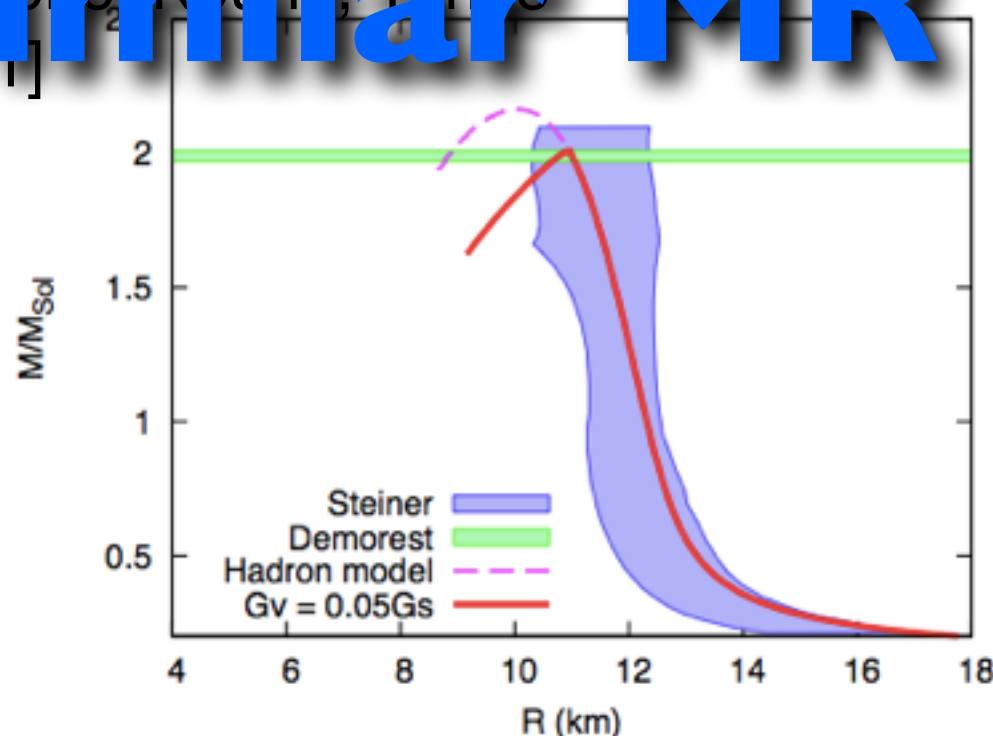


NLNJL+BHF(BOA) with pasta
[NY, Lastwiecki, Benic,
Blaschke, Maruyama, Tatsumi
PRC(2014)]



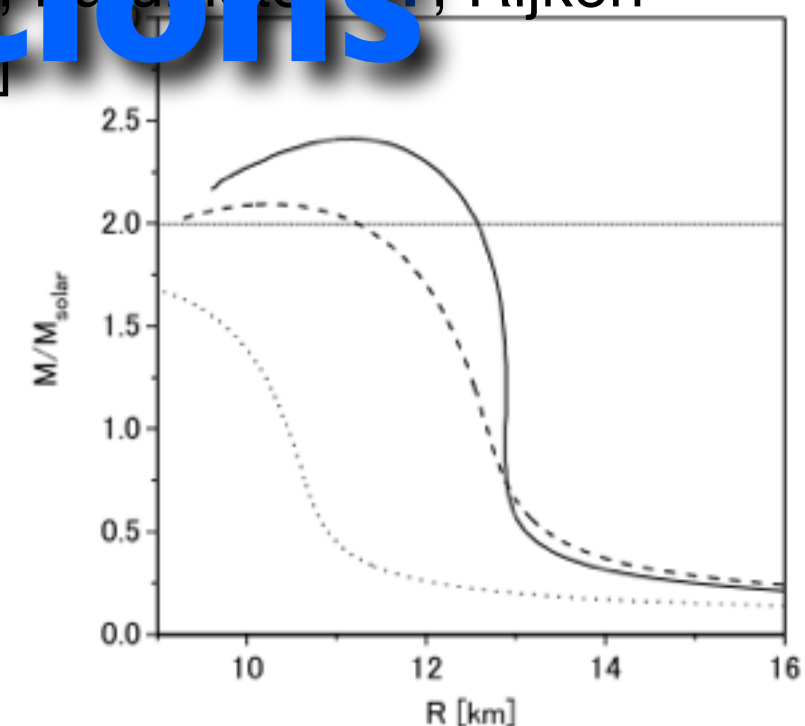
EPNJL+χPT

[Sasaki, NY, Khopra, et al.,
arXiv:1307.0681]



BHF(UTBF by Pomeron)

[Koranda, Furukawa, Rijken
PRC(2013)]



Same constraints

Similar MR relations

COOLING OF NEUTRON STARS

MR relation, etc...

→ only hardness of EOS

Cooling...

→ other physical properties

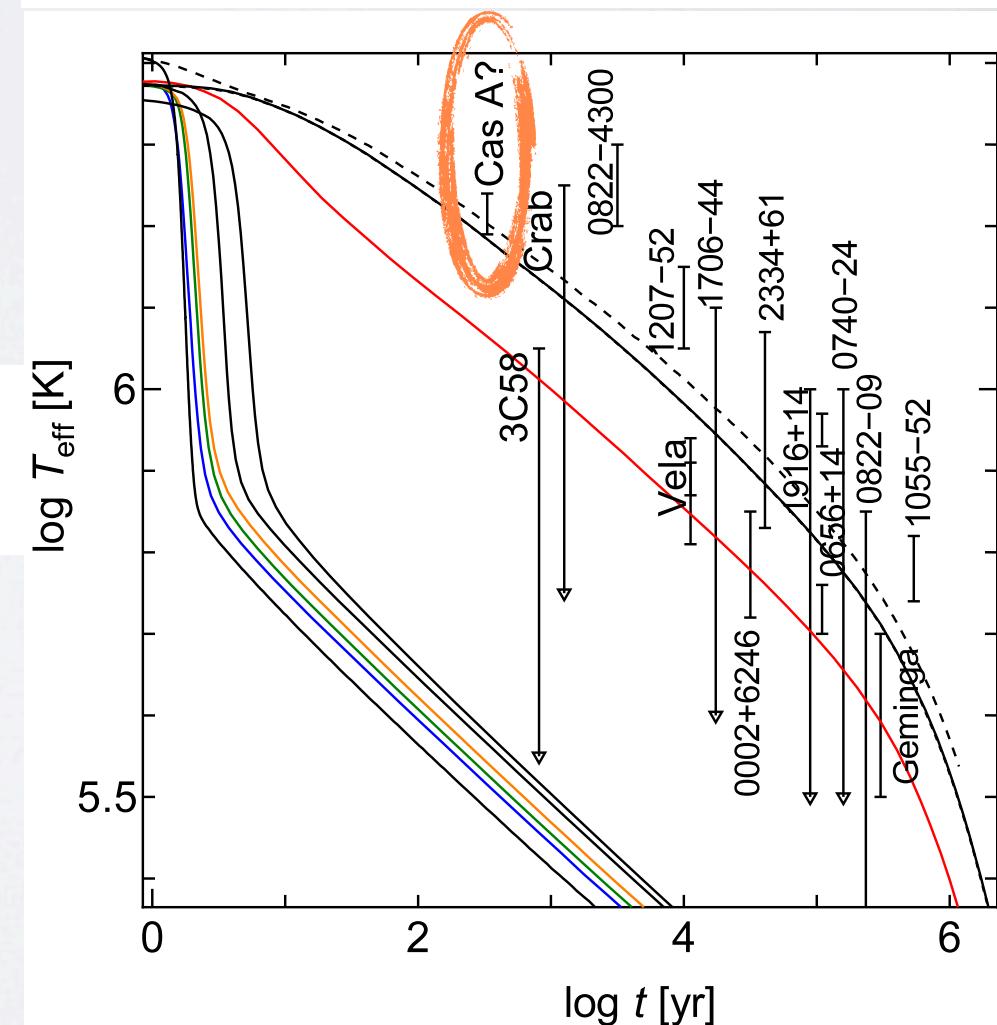
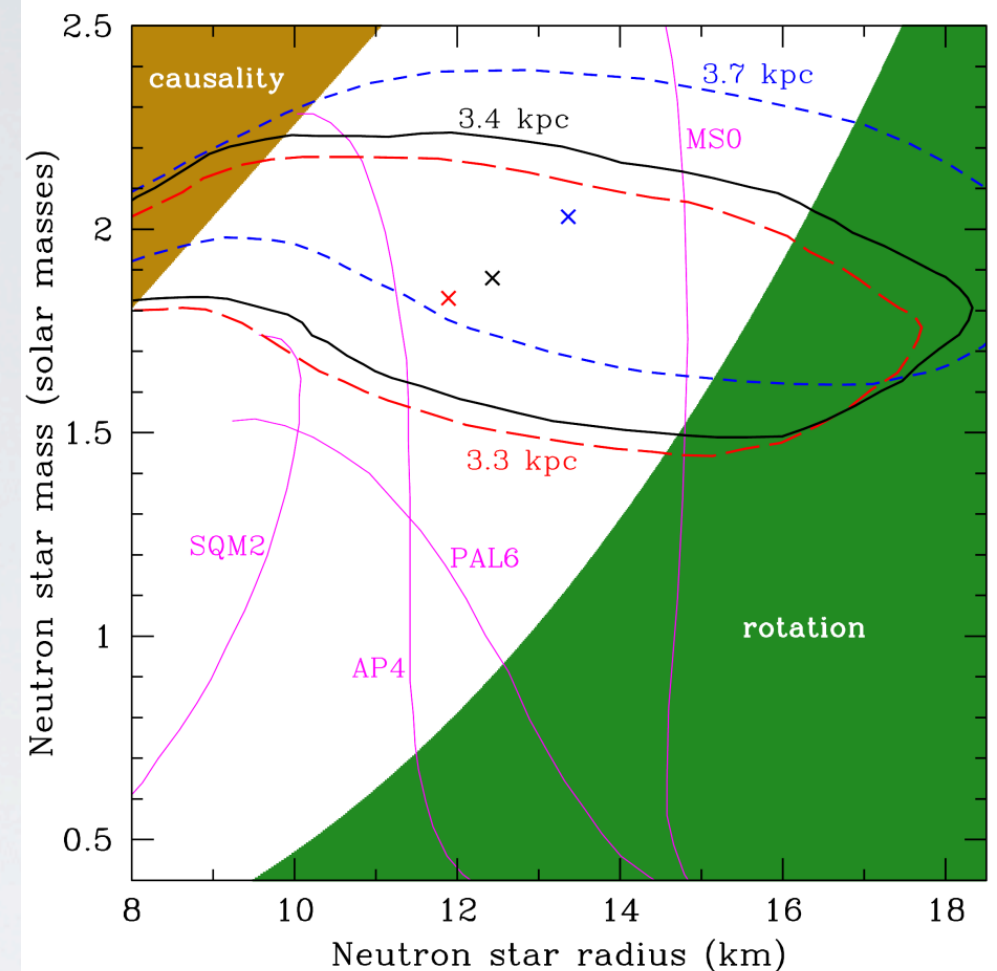
$$c_v e^\Phi \frac{\partial T}{\partial t} + \nabla \cdot (e^{2\Phi} \mathbf{F}) = e^{2\Phi} Q$$

thermal diffusion eq.

heat capacity

flux
(thermal conductivity)

cooling rate (neutrino)
+
heating rate (magnetic field)



Temperature distribution

NY, Kotake, Kutsuna, Shigeyama (2014) PASJ

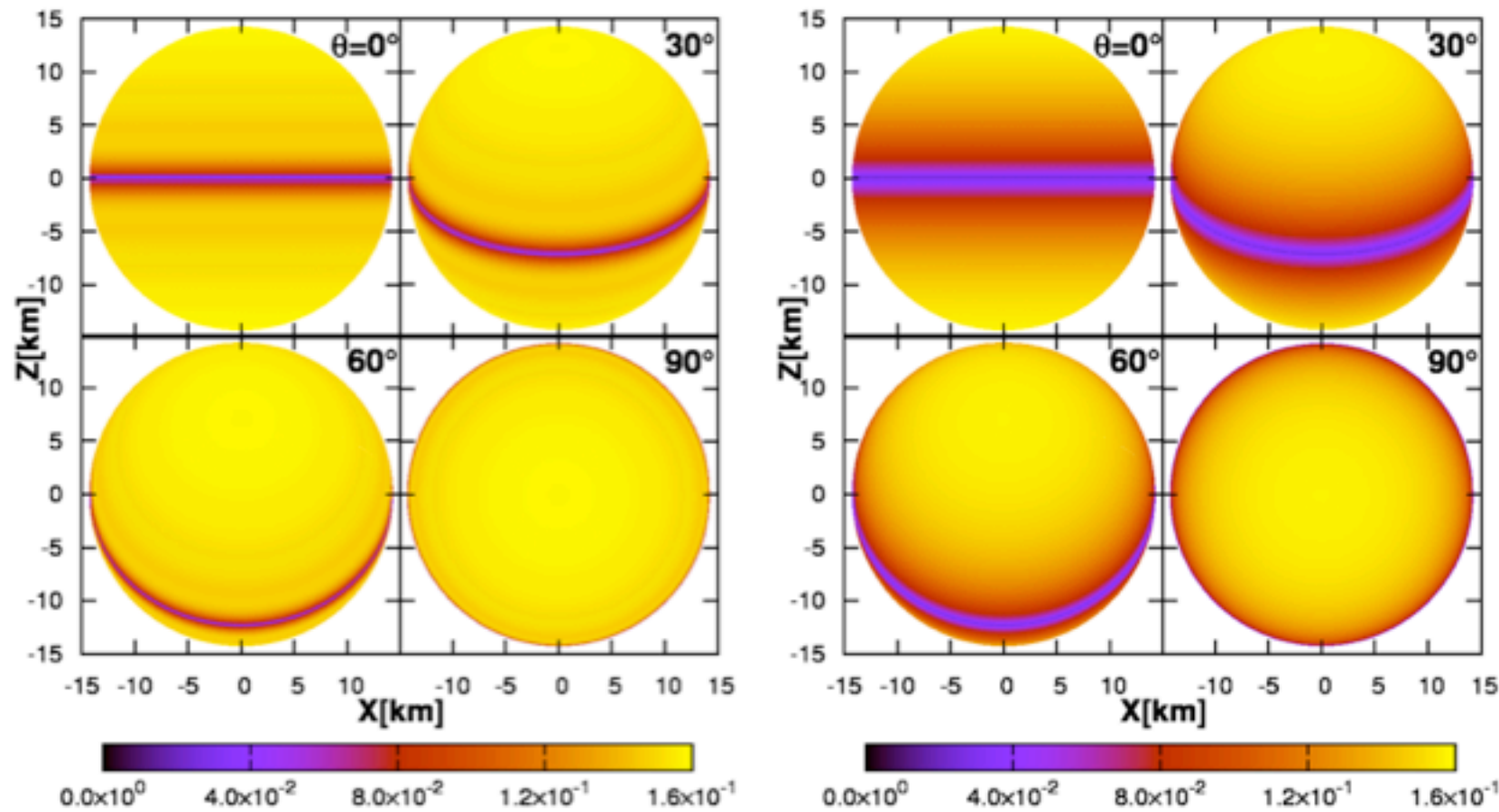
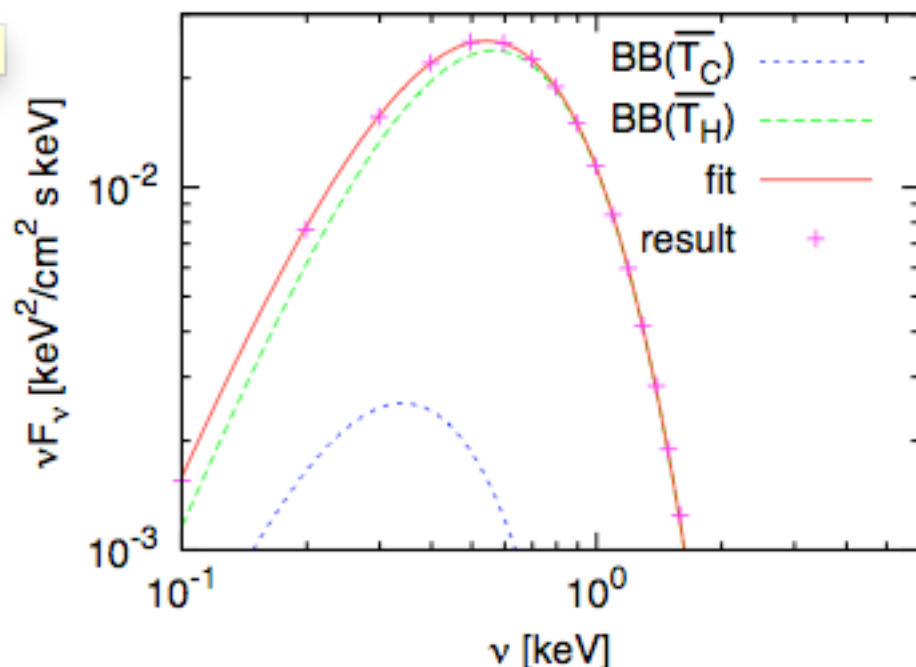


FIG. 7: (Color online) Temperature distribution for model "mSUK" after 10^4 years depended on the inclination angle θ . The unit of color contour is [keV].

移動



Our results

NY, Kotake, Kutsuna, Shigeyama
(2014) PASJ

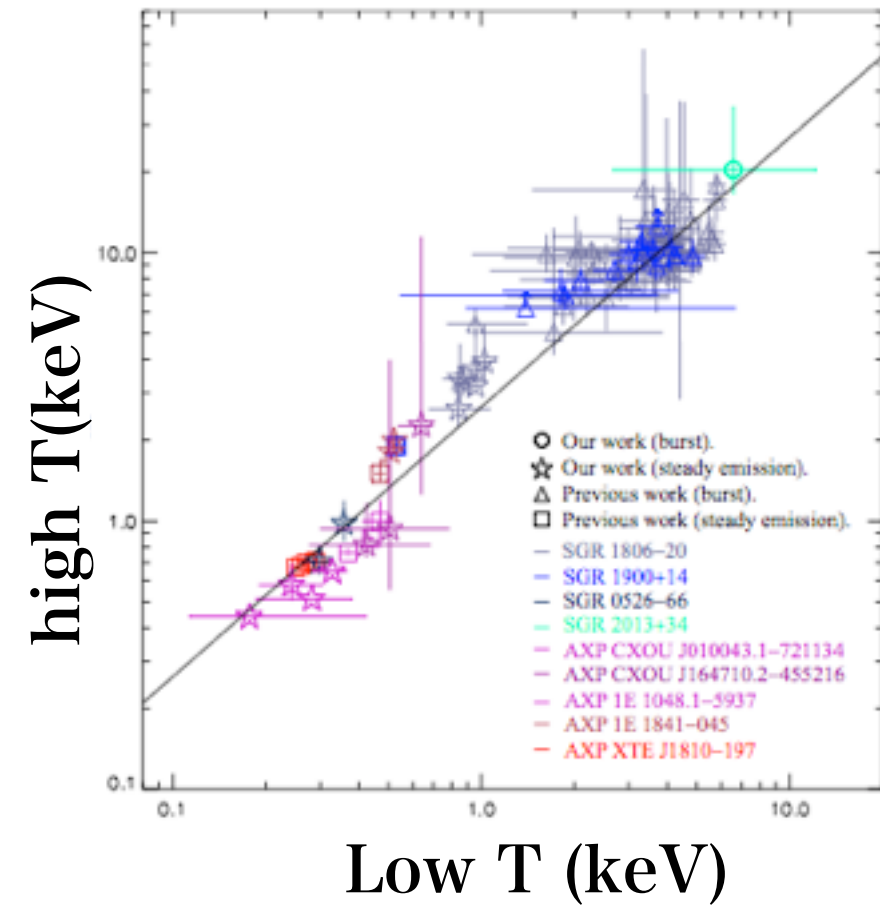
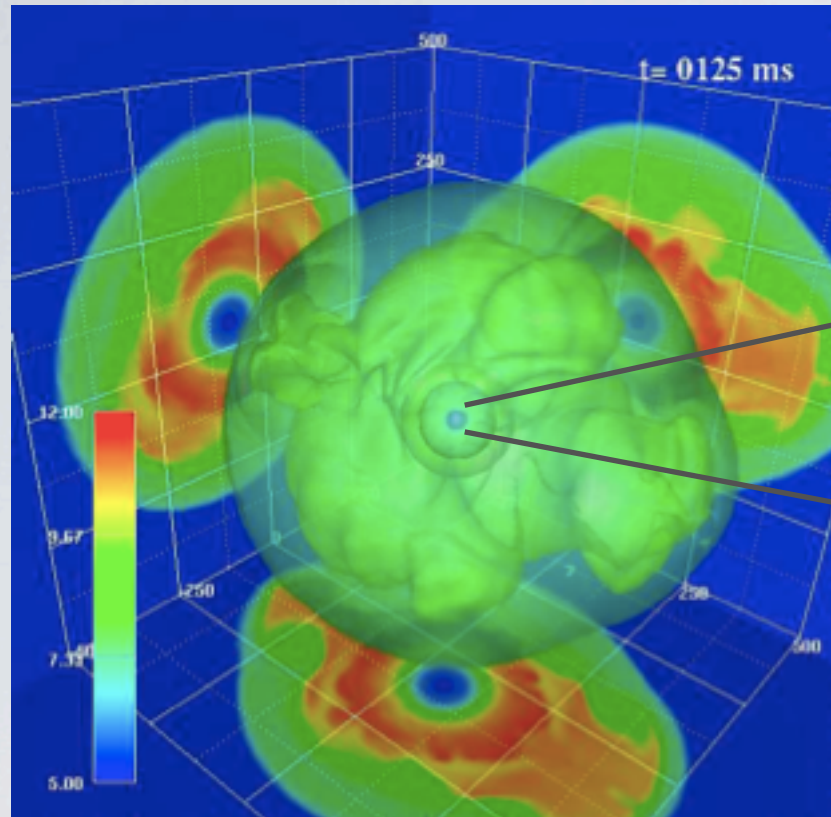


Fig. 3. Relationship between the 2BB temperatures kT_{LT} and kT_{HT} . The triangles and squares denote the previous work on the bursts (Feroci et al. 2004; Olive et al. 2004; Götz et al. 2006a; Nakagawa et al. 2007) and the quiescent emission (Morii et al. 2003; Gotthelf et al. 2004; Gotthelf & Halpern 2005; Tiengo et al. 2005; Mereghetti et al. 2006a), respectively. The circles and stars denote our work on the bursts and the quiescent emission, respectively. The line represents the best-fit power law model.

Observation, Yujin, et al. (2009) PASJ

Study on 2D stellar structures

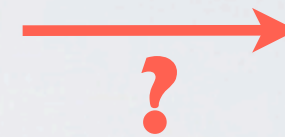
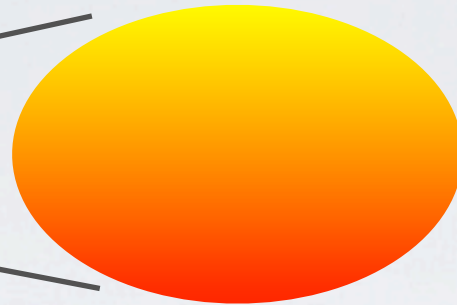


K.Kotake, K.Sumiyoshi, S.Yamada,
T.Takiwaki, T.Kuroda, Y.Suwa,
H.Nagakura (2012) PTEP

multi-dimensional evolution

PNSs($R \sim 50\text{km}$)

NSs($R \sim 10\text{km}$)



c.f. See the reference;(2010) Physics Today

**Chandrasekhar's role
in 20th-century
science**

Freeman Dyson

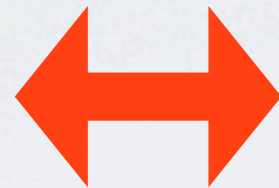
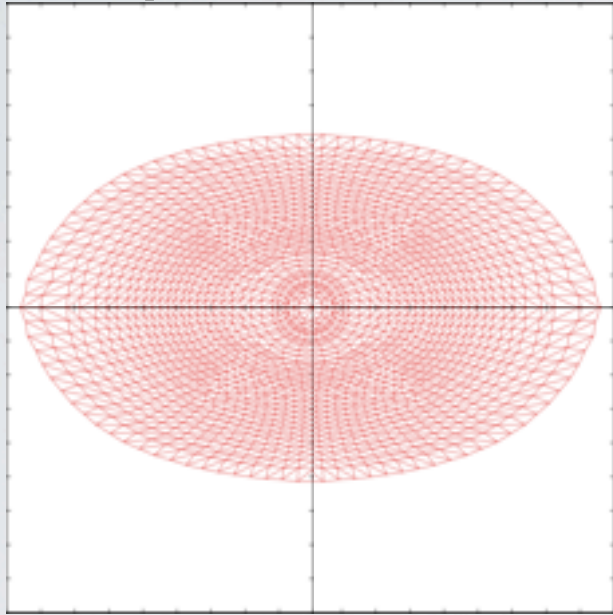
Others >> C. Jacobi, R. Dedekind, P.L. Dirichlet, B. Riemann...

cf.) one-dimensional evolution

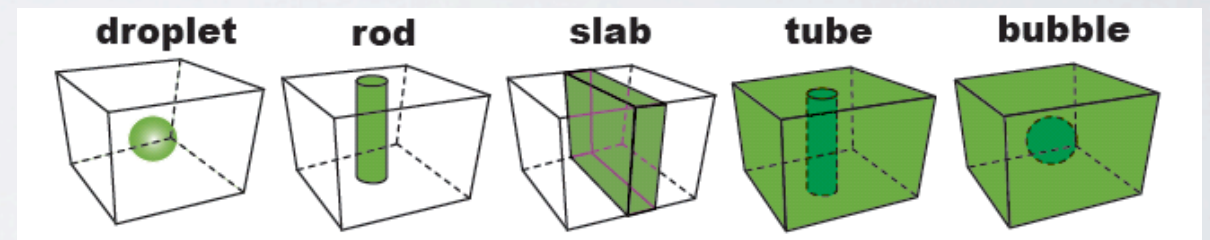
Heyney method (1964)

“DEFORMED STAR/PASTA DUALITY”

Deformed stars
(non-spherical stars)

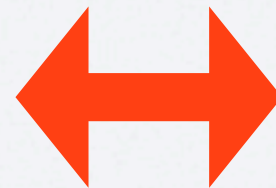


Pasta structures which are non-uniform structures in phase transitions; neutron drip, quark-hadron phase transition, etc.)



Hydro-static equilibrium

$$\frac{\delta E[\xi]}{\delta \xi} = \nabla P + \rho \nabla \phi - \frac{\rho j^2}{\varpi^3} \mathbf{e}_\varpi = 0.$$

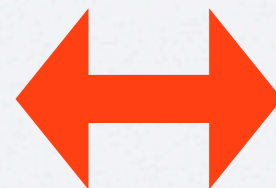


Chemical equilibrium

$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_C^Q, & \mu_d &= \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_C^Q, \\ \mu_n &= \mu_\Lambda = \mu_B, & \mu_p &= \mu_B + \mu_{C,H}, & \mu_{\Sigma^-} + \mu_p &= 2\mu_B, \\ \mu_L^{H(Q)} &= \mu_{\nu_e}^{H(Q)}, & \mu_C^{H(Q)} &= \mu_L^{H(Q)} - \mu_e^{H(Q)}, \end{aligned}$$

ex) quark-hadron phase transition

repulsion = pressure and rotation
attraction = gravitation



repulsion = Coulomb interaction
attraction = surface tension

“NODES & EDGES”

“NODES & EDGES”



Put some nodes, numbering them randomly.

“NODES & EDGES”

Step 1

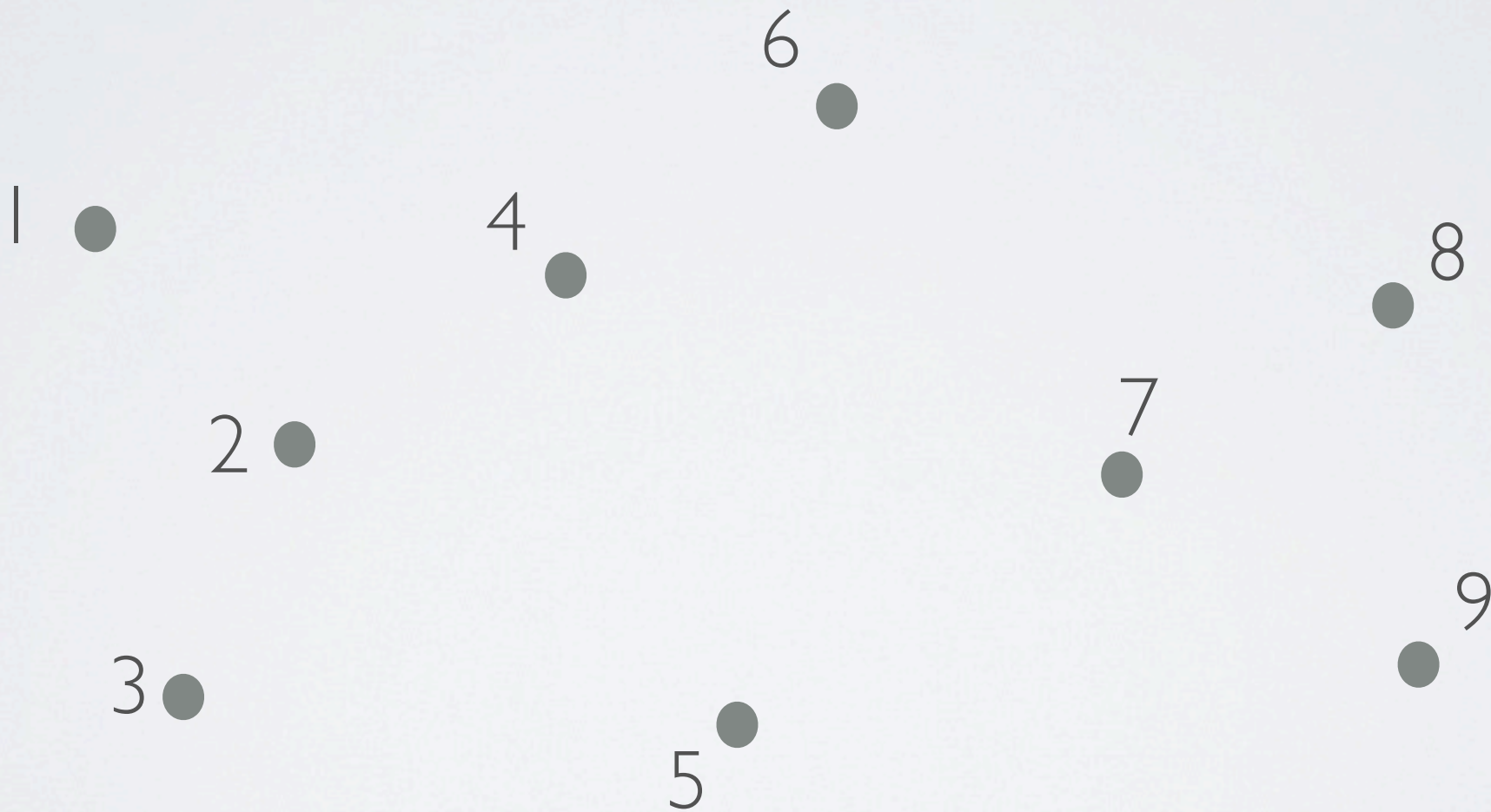
Put some nodes, numbering them randomly.



“NODES & EDGES”

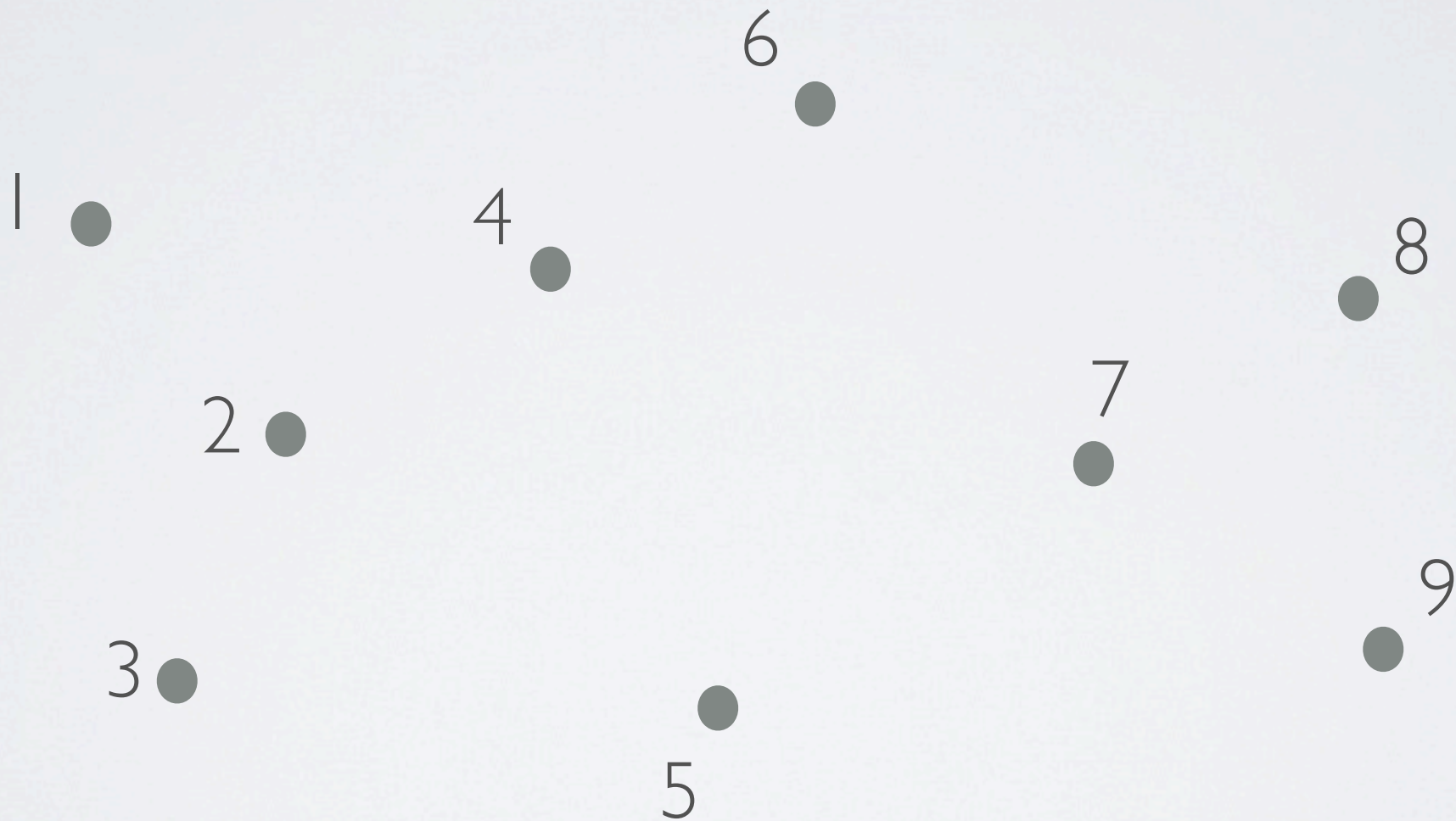
Step 1

Put some nodes, numbering them randomly.



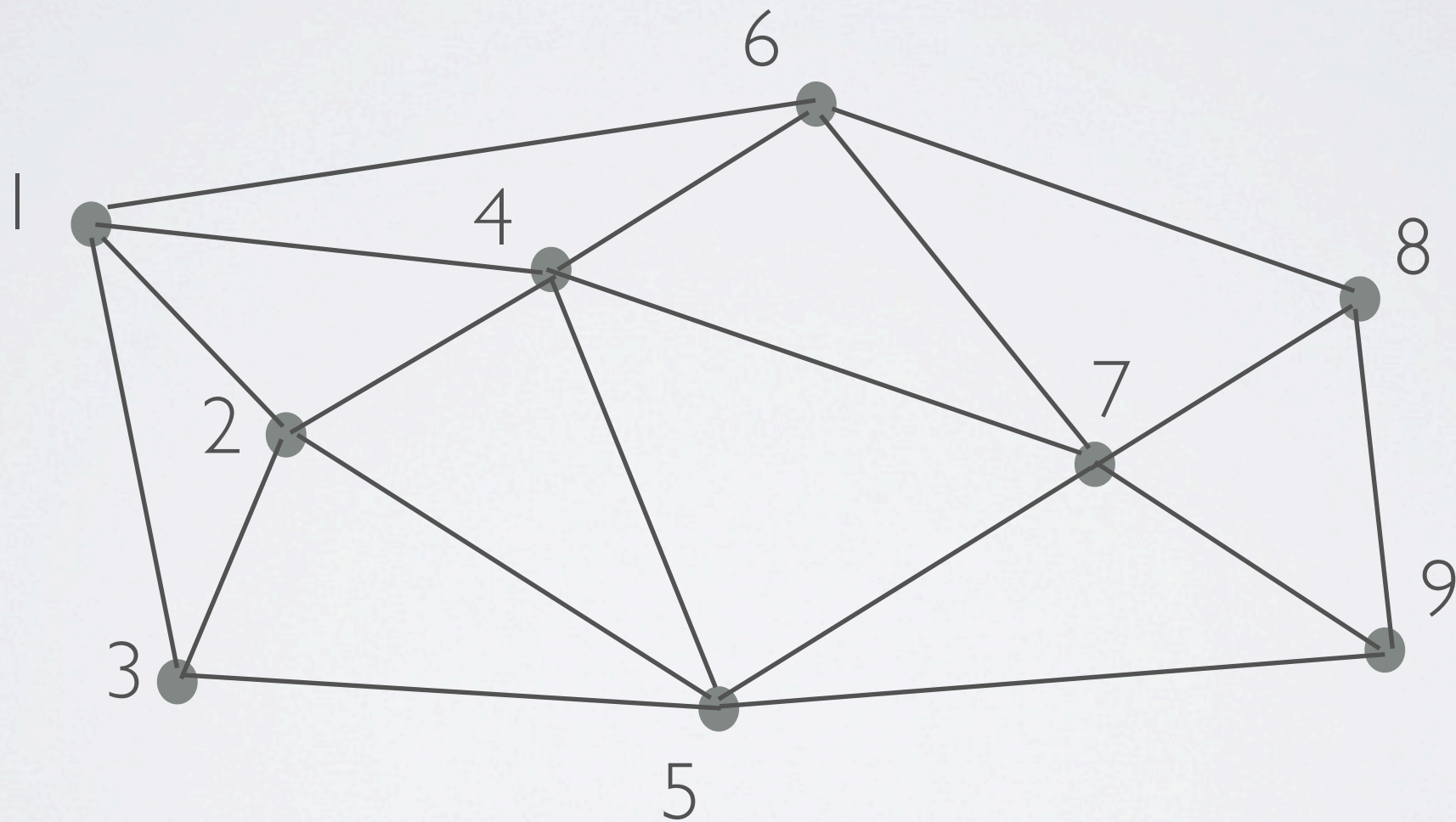
Step II

Connect the nodes with edges making triangles as you want.



Step II

Connect the nodes with edges making triangles as you want.



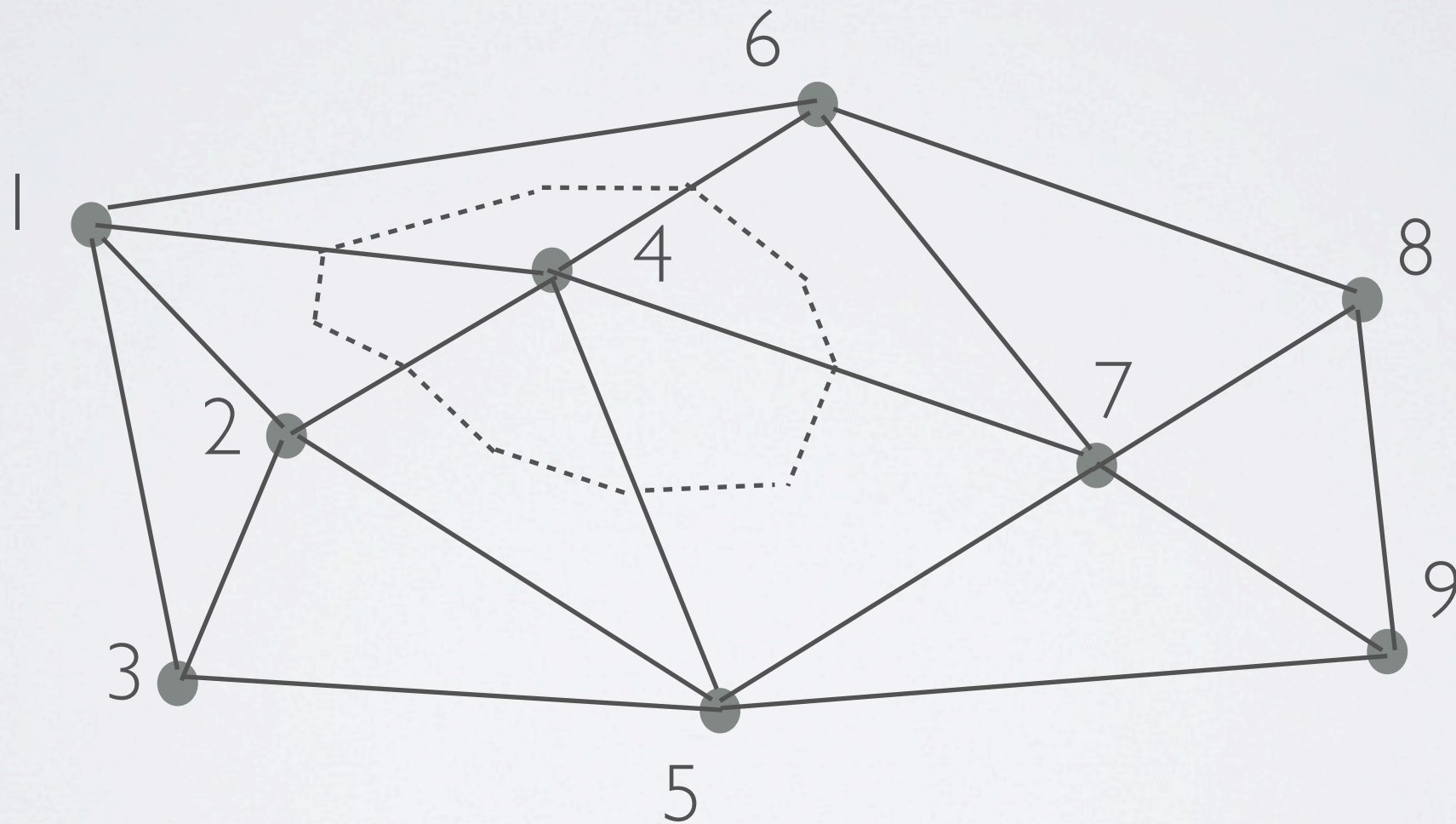
StepIII

Make the matrix which shows the relationship between the nodes; if they have an edge between two nodes, put “1”, otherwise put “0” in the matrix. Then, you will get a beautiful symmetric matrix, which is known as an “adjacency matrix” in *the graph theory*.

	1	2	3	4	5	6	7	8	9
1	0	1	1	1	0	1	0	0	0
2	1	0	1	1	1	0	0	0	0
3	1	1	0	0	1	0	0	0	0
4	1	1	0	0	1	1	1	0	0
5	0	1	1	1	0	0	1	0	1
6	1	0	0	1	0	0	1	1	0
7	0	0	0	1	1	1	0	1	1
8	0	0	0	0	0	1	1	0	1
9	0	0	0	0	1	0	1	1	0

Step IV

If you give each node the physical quantities; position, and Lagrange values (mass, angular momentum, entropy, fractions ...), you can get Eulerian values (partial volume, density, ...) considering with the relation between neighboring nodes (and/or the adjacency matrix).



each node: $x, y, m, j, s, Y_e, Y_n, Y_p, Y_{He} \dots \rightarrow dv, \boldsymbol{\rho}, P, T, u, \dots$

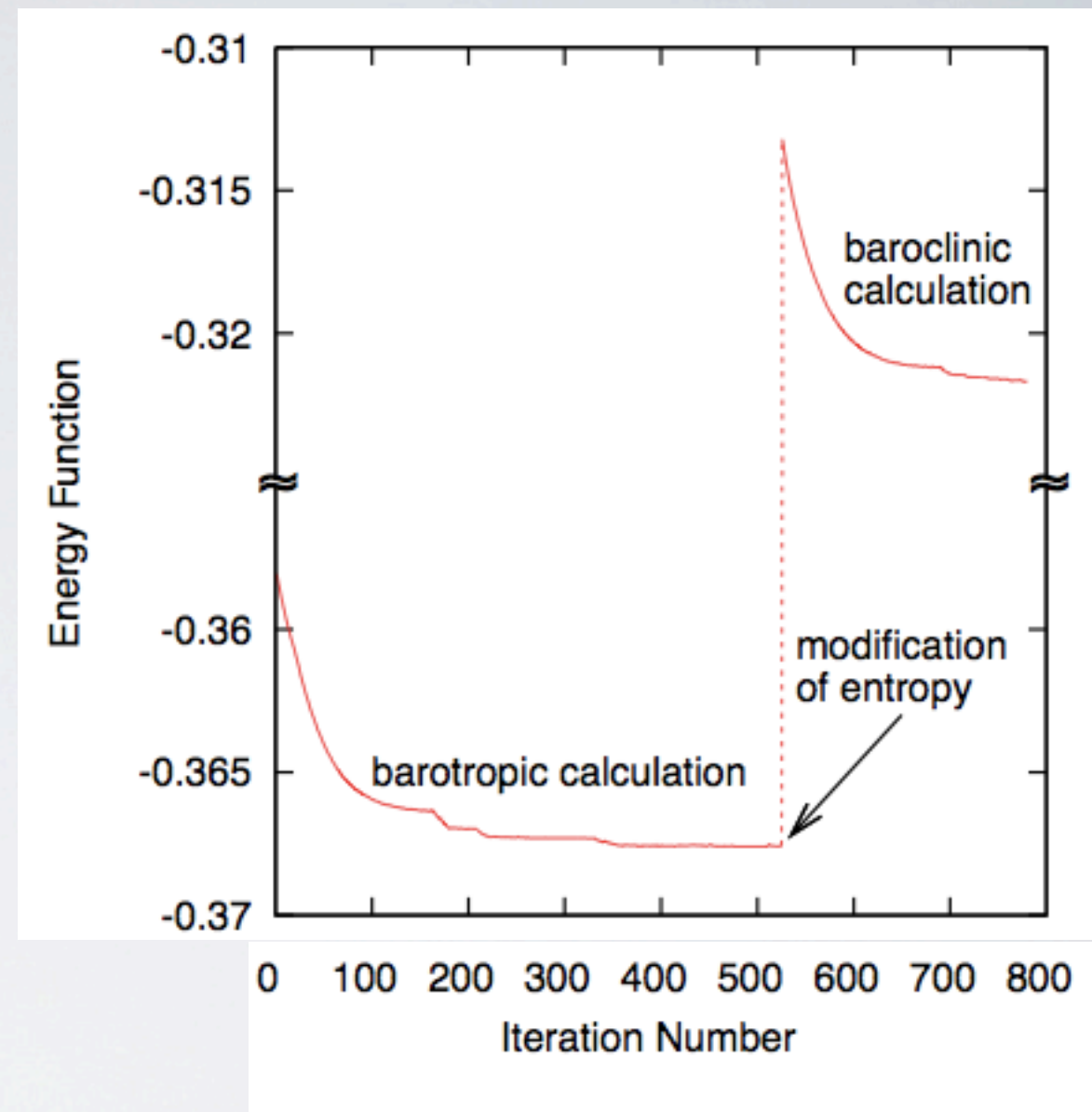
Step V

Find the most optimal arrangement by changing the positions of nodes finding the minimum total energy.

$$E_{\text{FEM}}(\mathbf{r}_i) = \sum_i \varepsilon_i m_i + \frac{1}{2} \sum_i \phi_i m_i + \sum_i \frac{1}{2} \left(\frac{j_i}{\varpi_i} \right)^2 m_i.$$

Caution

Now, you may think that this is about two dimension. Of course, if you give (x_i, y_i) as the positions on nodes, it becomes a two-dimensional topic. But, if you give $(x_i, y_i, z_i, \dots)$, it becomes the multi-dimensional.



“APPLICATIONAL RESULT”

Stellar structures without Høiland criteria

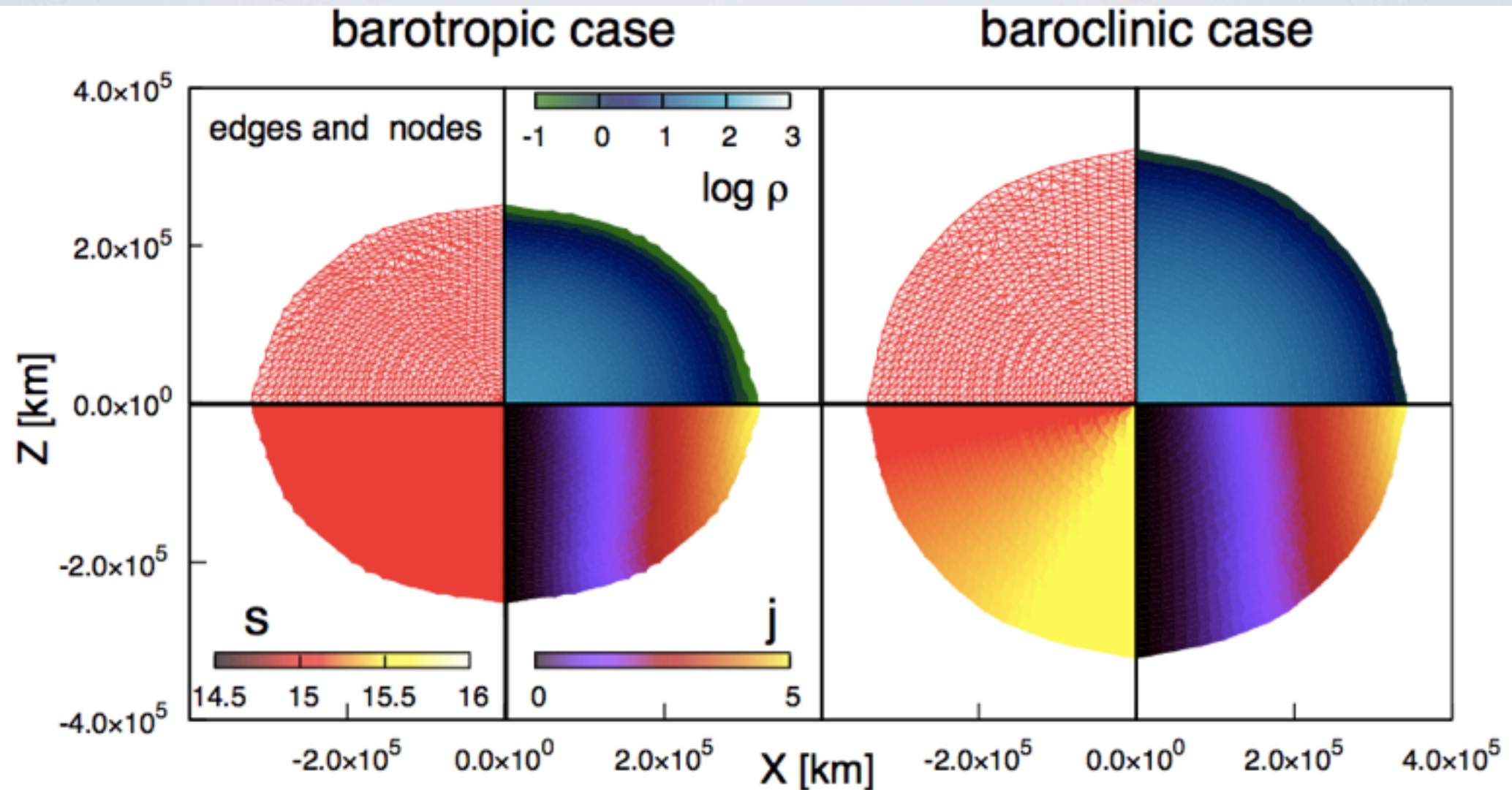
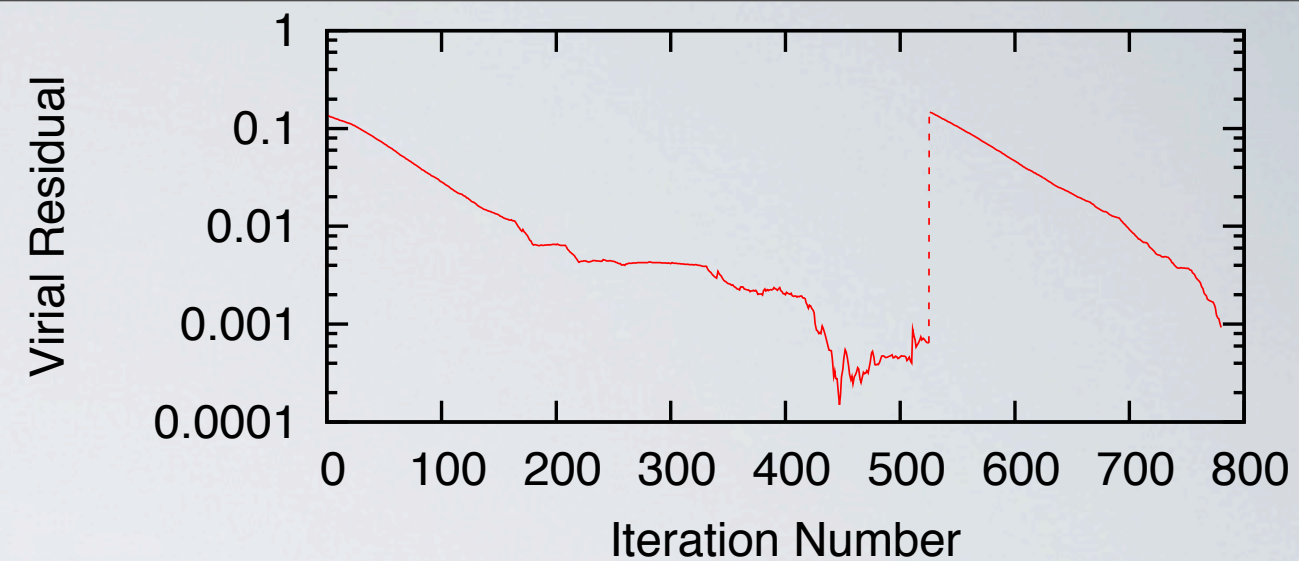
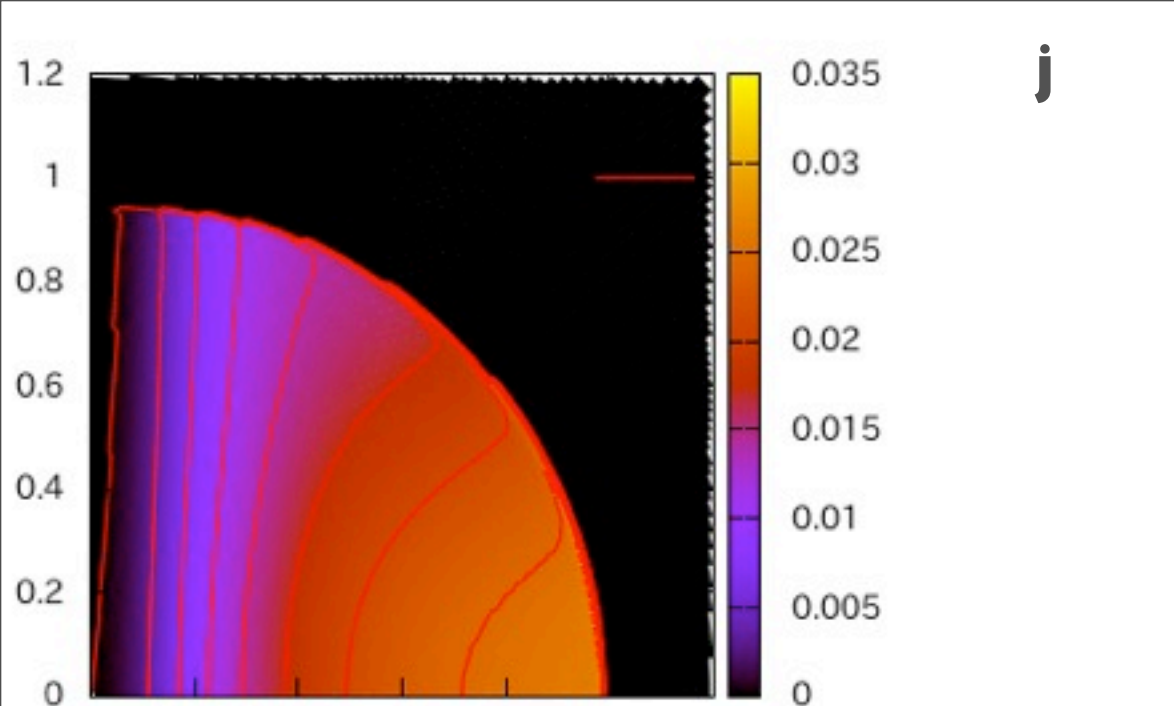


Figure 1. (color on line). Structures of a star in rotational equilibria for barotropic (left four panels) and baroclinic (right four panels) EOS's. The upper left quadrants show the nodes and edges in the triangulated mesh. The other panels display clockwise the color contours of logarithmic density in g/cm^3 , specific entropy in k_B and specific angular momentum in $10^{18}\text{cm}^2/\text{s}$. The color scales are identical for both cases.

We succeeded to get **hydro-static equilibriums for stars with realistic (baroclinic) EOS in Lagrange coordinate**. The distribution of specific angular momentum j is not cylindrical as shown in the right panel.

→ **Bjerkness theorem.**

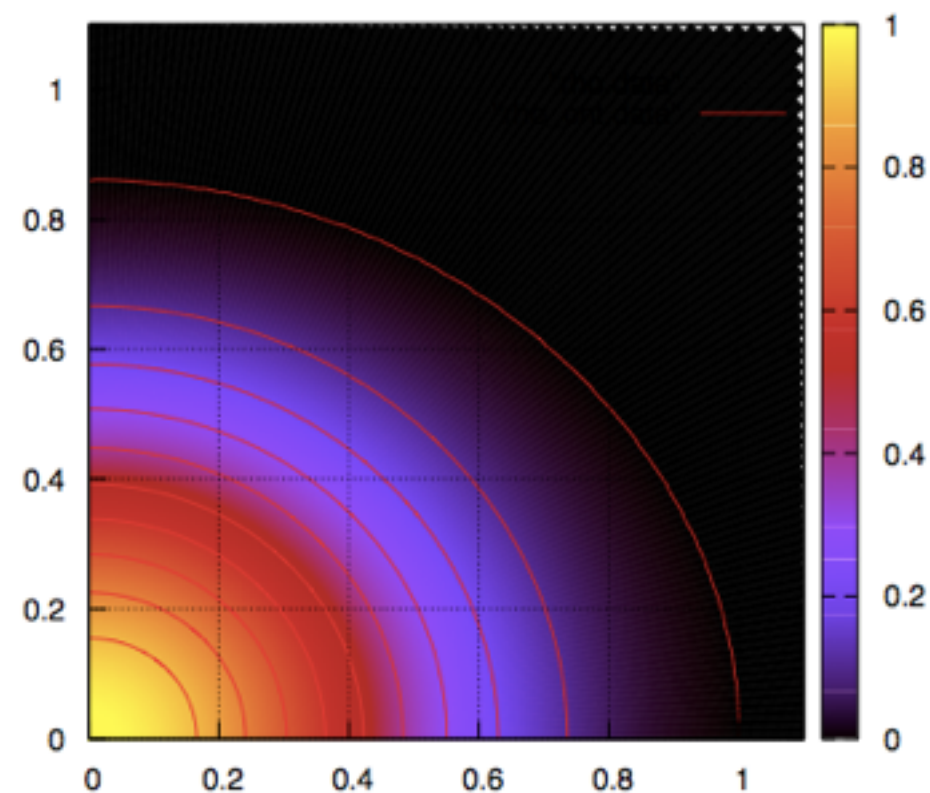
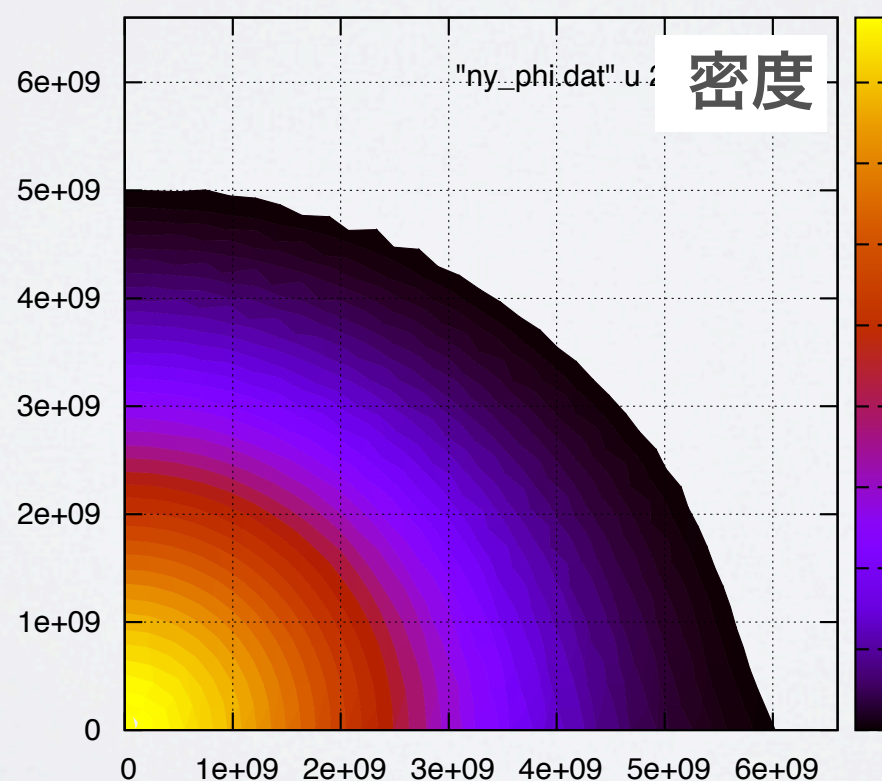


$$V_C = \left| \frac{2T + W + 3 \int P dV}{W} \right|, \sim \mathcal{O}(10^{-4})$$

Virial constant → OK

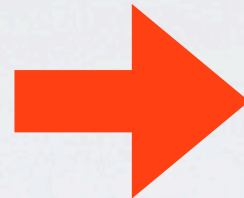
**Now checking
Uryu&Eriguchi method,
Fujisawa method (new!)**

Consistent with Hachisu method



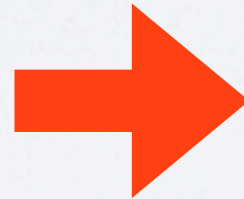
“STRONG ADVANTAGES”

Lagrange coordinate



Very easy to apply to stellar evolutions.

Variational principle



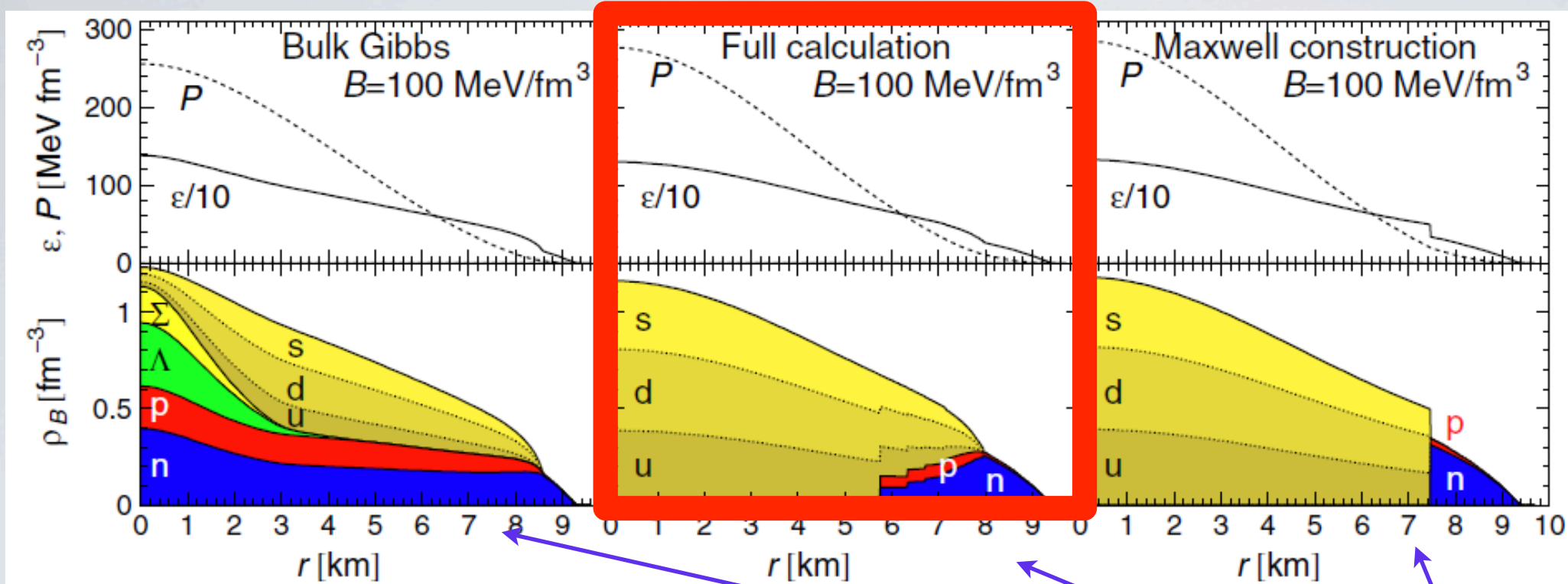
Very easy to check the stability.

***mixing(convection)**

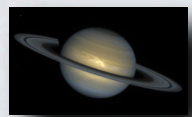
***mass shedding
(mass loss)**

These two are the most important for stellar evolution.

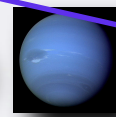
Structures “NS Structures with mixed phase”



11



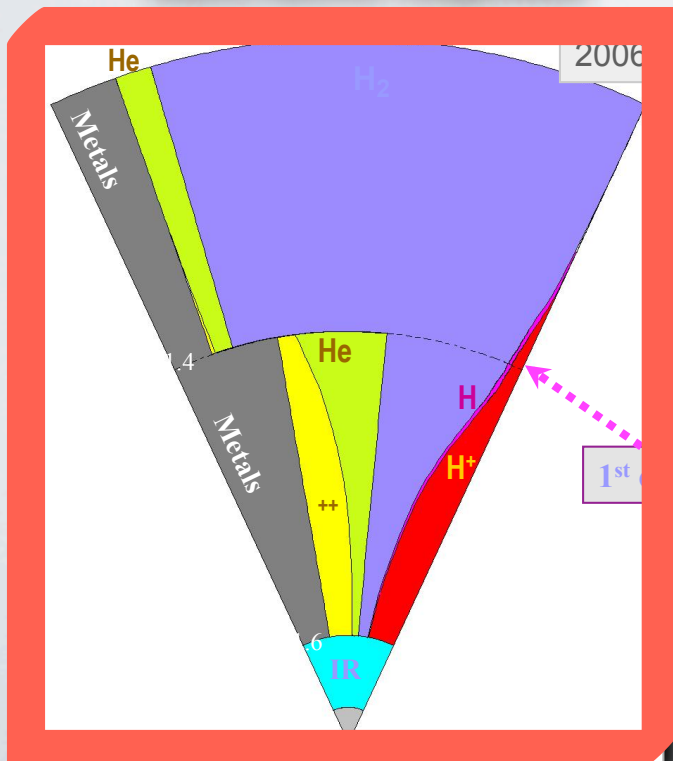
Giant planets interior composition



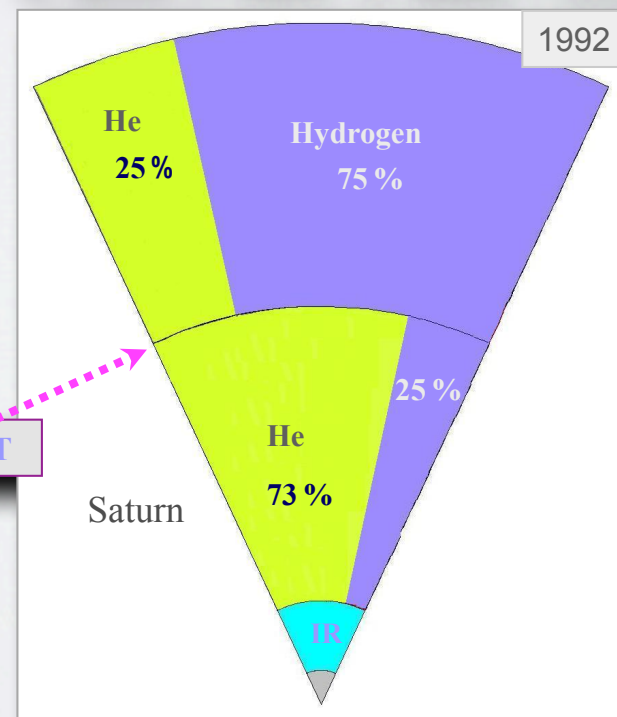
“fraction”

→ Cooling process etc.

Saturn interior composition



Optimized models of Jupiter and Saturn
(D. Saumon, G. Chabrier, W. Hubbard, J. Lunine)

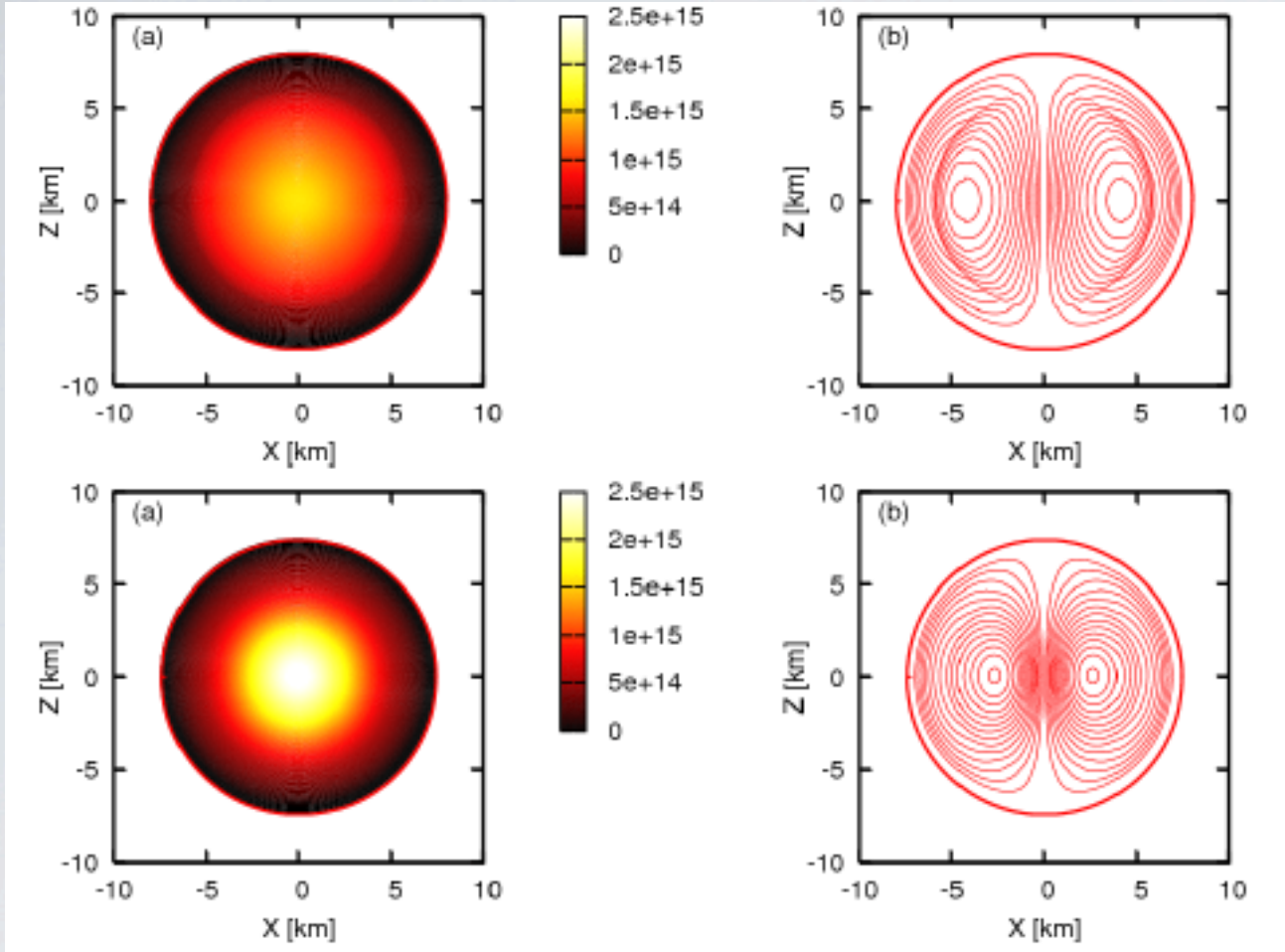


“Saturn-Structure with mixed phase”

Effects of EOS on magnetic field

OTHER TOPICS

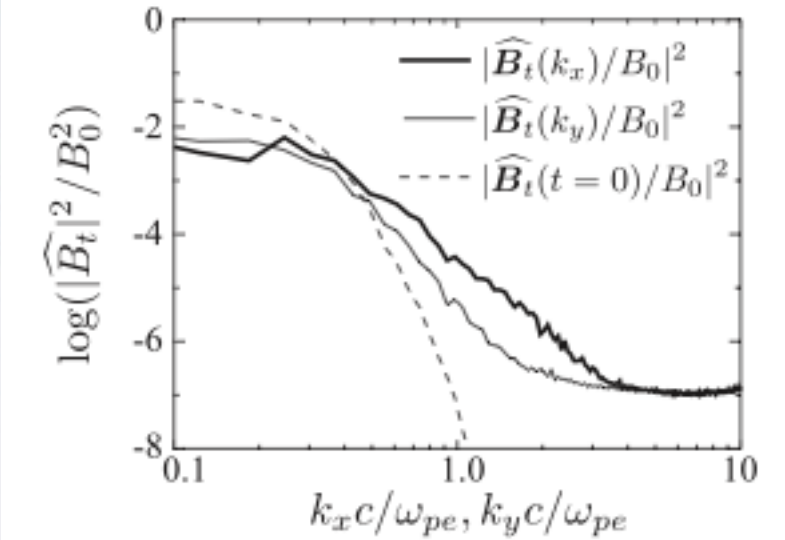
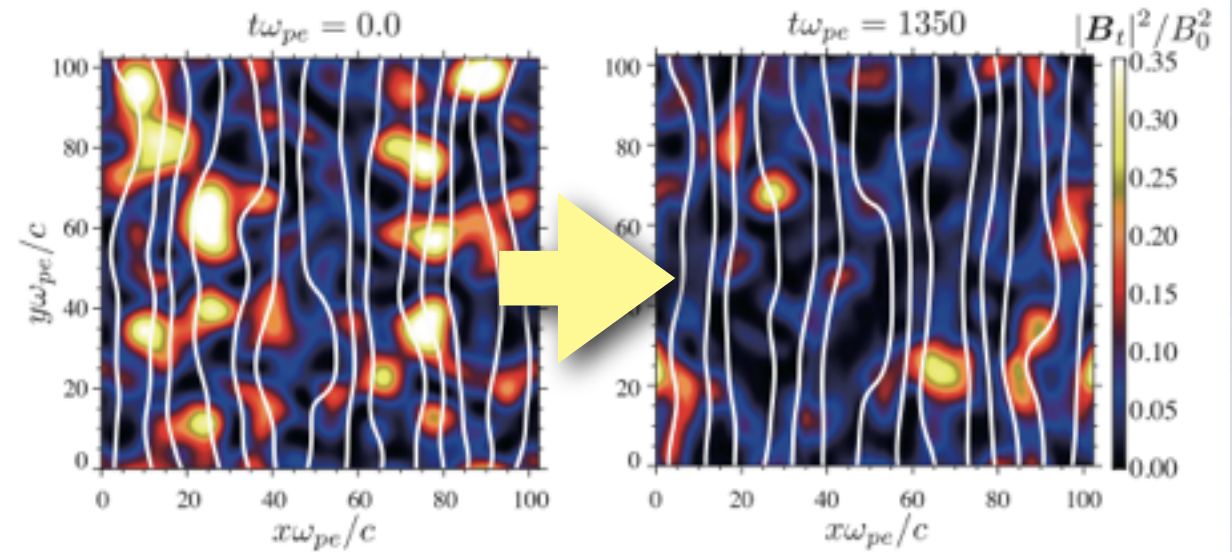
Diffusion of magnetic field in the crust



NY, Kiuchi, Kotake (2010) MNRAS

	Comp.	$\frac{dcL_z/dt}{dE_T/dt}$	$\dot{P}/P \text{ (s}^{-1}\text{)}$		
			$\rho_B = \rho_0$	$\rho_B = \rho_0/10$	MDR
Mag-A	p, n	66.9	7.82×10^{-2}	1.03×10^{-2}	7.76×10^{-12}
	p, n, Λ	109	1.13×10^{-1}	1.11×10^{-2}	9.86×10^{-12}
Mag-B	p, n	9.64	1.13×10^{-3}	4.50×10^{-5}	7.76×10^{-12}
	p, n, Λ	7.81	8.07×10^{-4}	2.29×10^{-5}	9.86×10^{-12}

Maruyama, Hidaka, Kajino, NY, Kuroda, Ryu, Cheoun, Mathews (2014) PRC 89 035801, (2012)PRD 86.123003



Takahashi, Kotake, NY (2011) ApJ

Anisotropy of neutrino emission from magnetars

- * We have developed an entirely new formulation to obtain self-gravitating, axisymmetric configurations in permanent rotation.**
- * It is based on the Lagrangian variational principle and, as a consequence, will allow us to apply it to stellar evolution calculations rather easily.**
- * We adopt a Monte Carlo technique, which is analogous to those employed in other fields, e.g. nuclear physics, in minimizing the energy functional.**
- * Possible applications are not limited to main sequence stars but will be extended to e.g. compact stars, proto-stars and planets.**

MATHEMATICAL/NUMERICAL PROBLEMS

全てのnodeを一機に動かして解を探そうとすると回転の自由度は残るためうまくいかない。

LAGRANGIAN PERTURBATION THEORY OF NONRELATIVISTIC FLUIDS*

JOHN L. FRIEDMAN

Department of Physics, University of Wisconsin-Milwaukee

AND

BERNARD F. SCHUTZ

Department of Applied Mathematics and Astronomy, University College, Cardiff, Wales

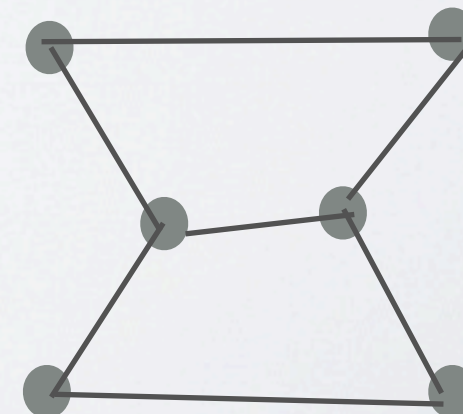
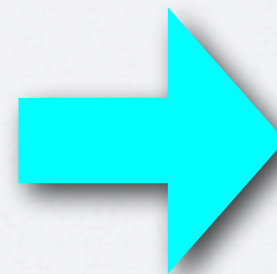
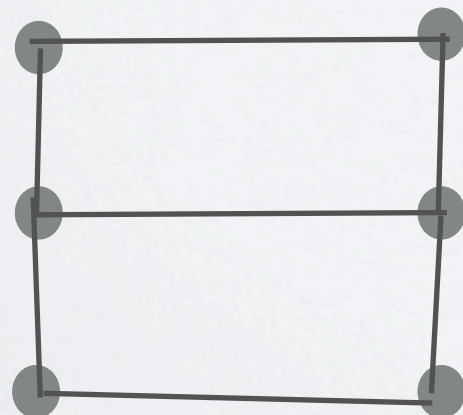
Received 1977 July 11; accepted 1977 November 7

ABSTRACT

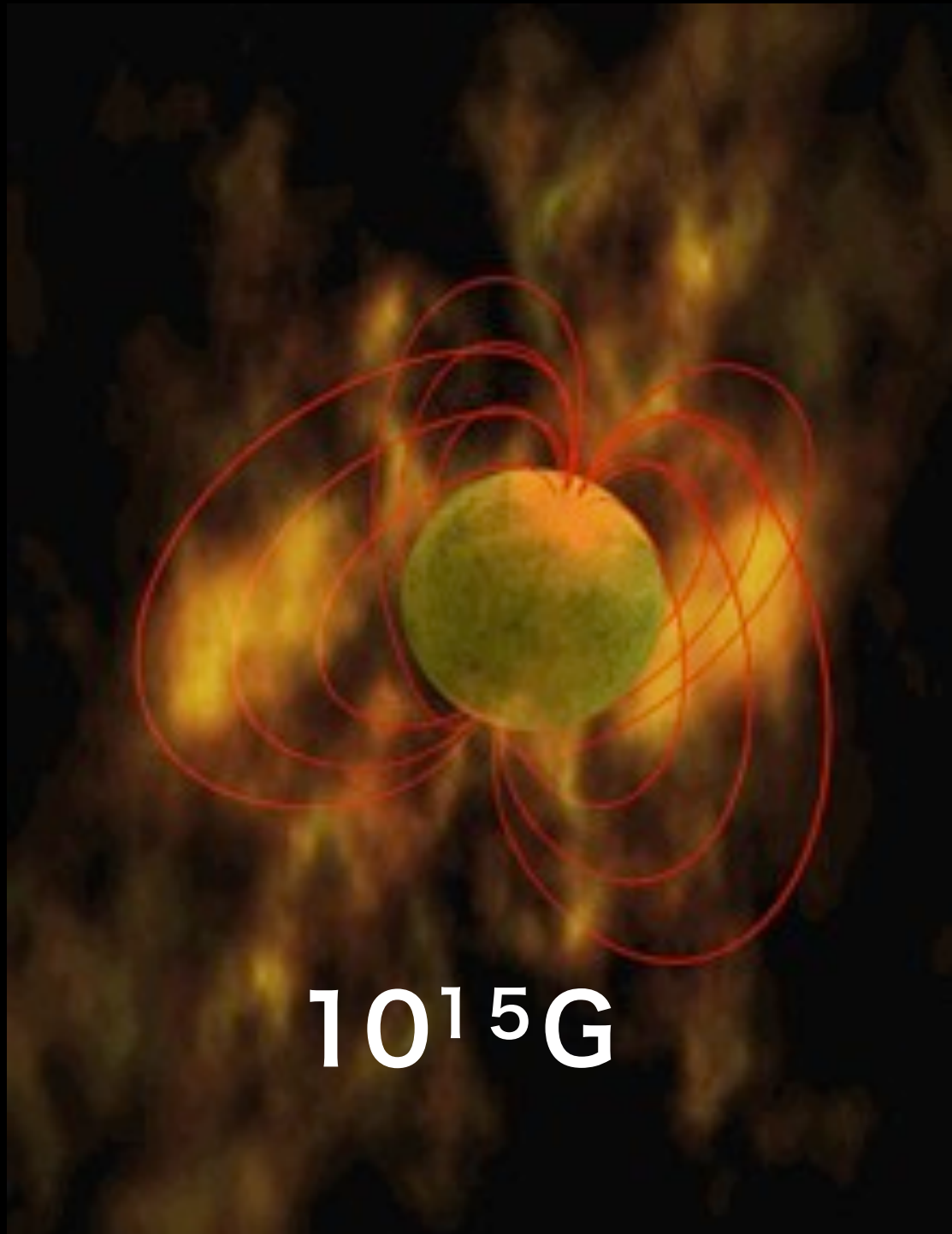
In this paper the conventional description of adiabatic perturbations of stationary fluids in terms of a Lagrangian displacement is reexamined, to take account of certain difficulties that have been overlooked in other treatments. A class of displacements—called trivials—that leave the physical variables unchanged is identified; these define “gauge” transformations of the initial data in the Lagrangian picture. The conserved canonical energy E_c (Hamiltonian) and angular momentum J_c (in the case of axisymmetric unperturbed fluids) associated with the dynamical equations are shown not to be invariant under these gauge transformations. Since E_c has formed the basis of previous criteria for secular stability of stars, it is necessary to eliminate the gauge freedom in order to regain a meaningful criterion. To this end a conserved inner product (the symplectic structure) is introduced and used to define a dynamically invariant class of “canonical” displacements orthogonal to the trivials. In general, canonical displacements obey the extra

有限要素法における砂時計問題

等ポテンシャル面→



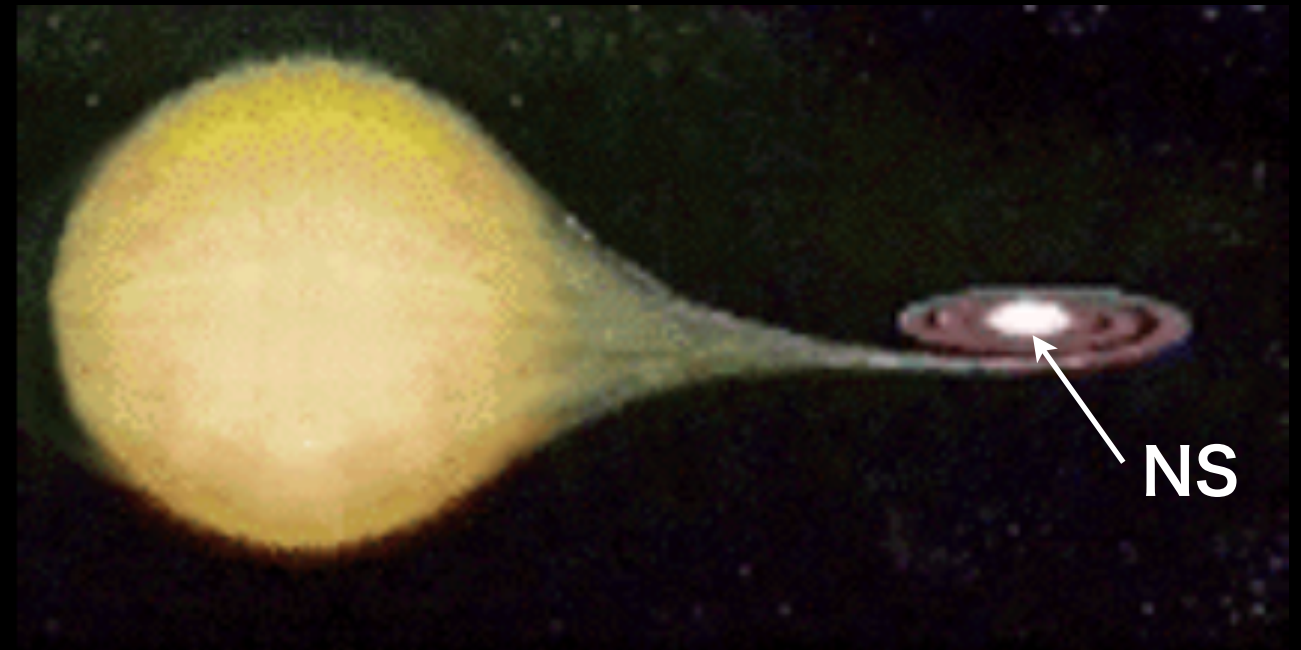
Matter
(EOS)
+
Cooling of
magnetised NSs
(2D evolution)



1979~

observation of magnetars

→ **What is the mechanism?**
What does exist inside?



2007~

Some X-ray transits have
 strong cooling mechanism.

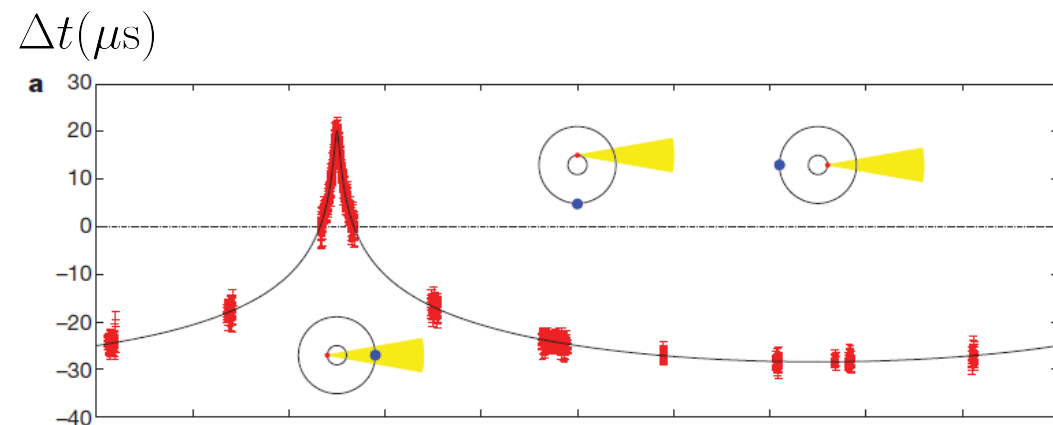
→ **Exotic matter**

Matter inside of NSs

CONSTRAINTS ON EOS

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest¹, T. Pennucci², S. M. Ransom¹, M. S. E. Roberts³ & J. W. T. Hessels^{4,5}

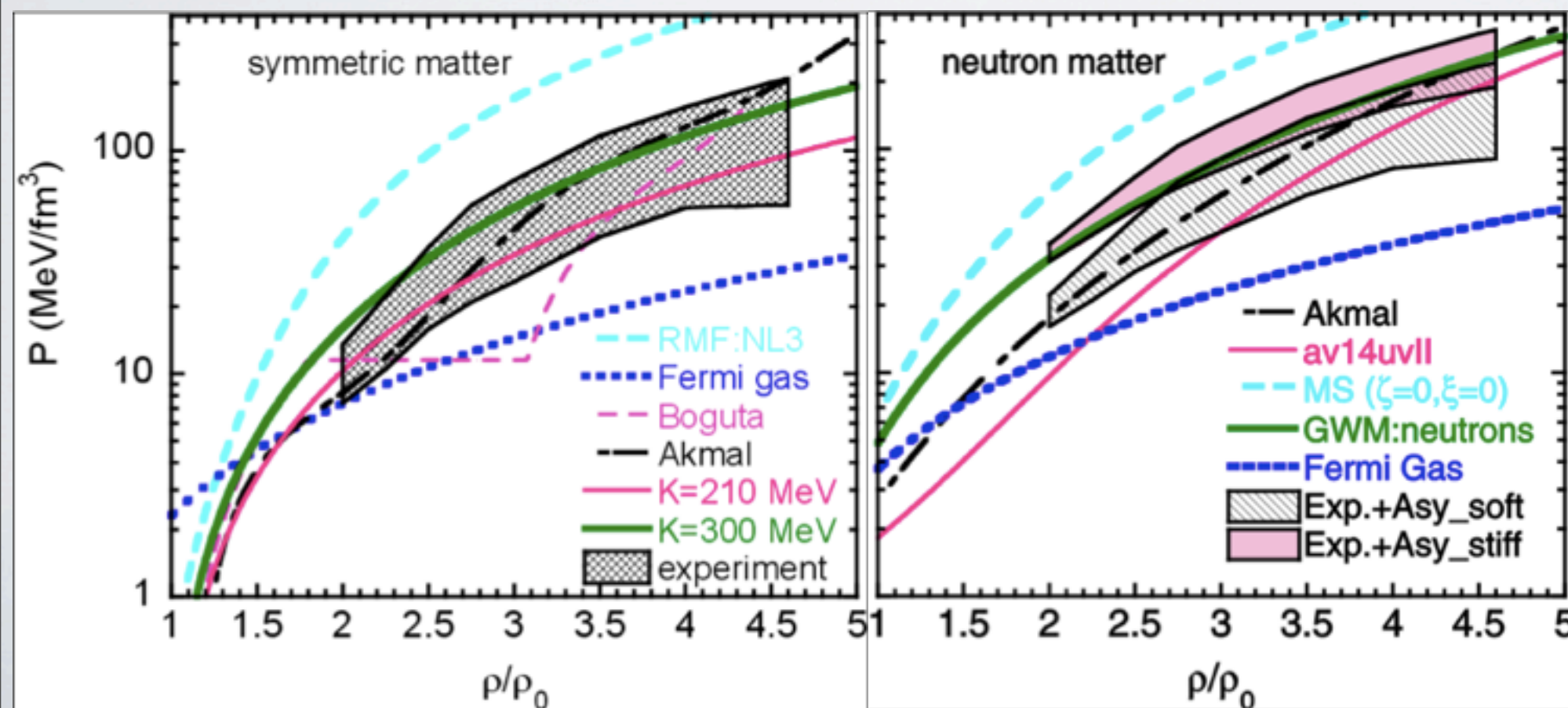


Demorest et al. 2010 nature

$$M \sim 1.97 M_{\odot}$$

“Shapiro delay”

Radar signals passing near a massive object take slightly longer to travel to a target and longer to return than they would if the mass of the object were not present.



Danielewicz et al. 2012 science

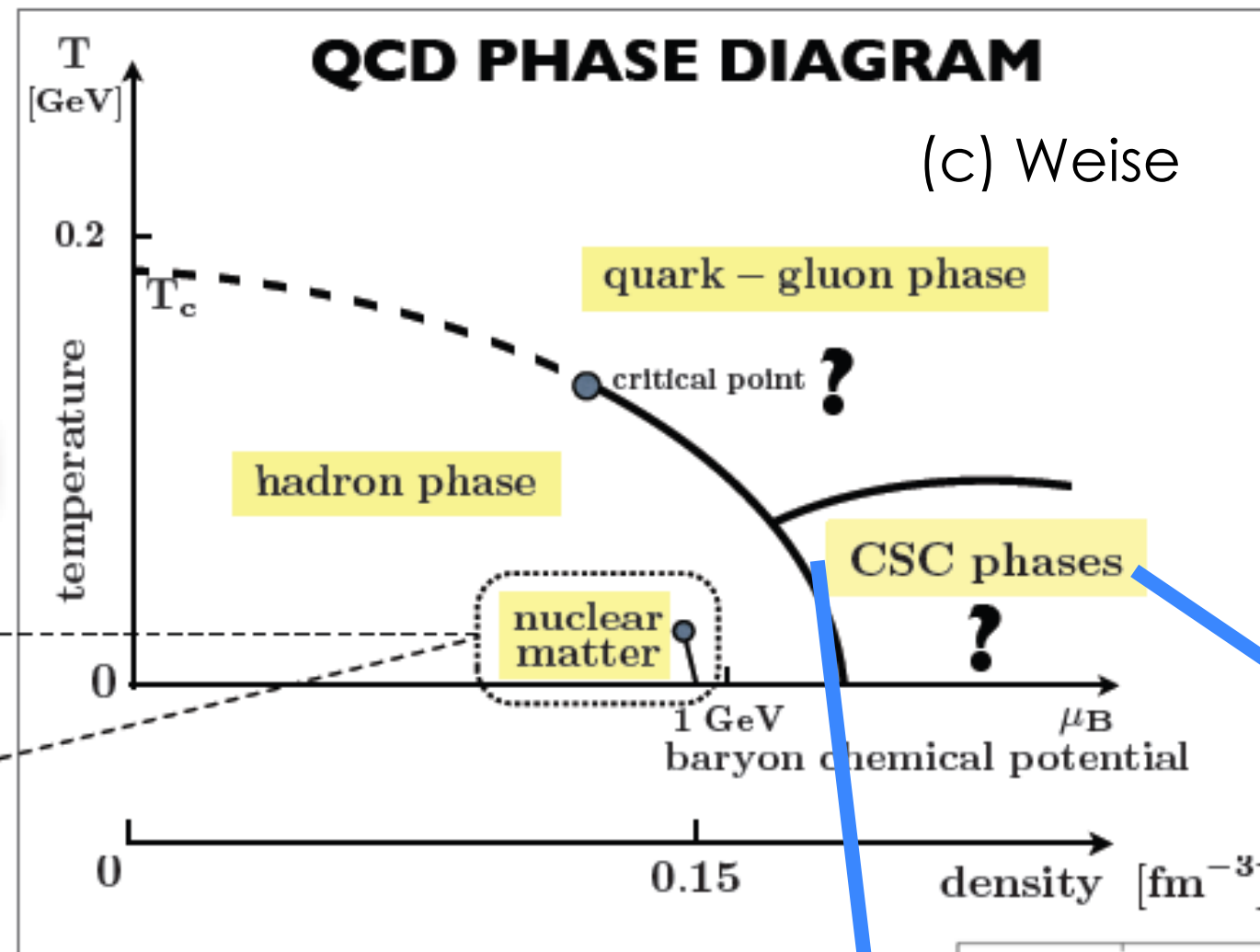
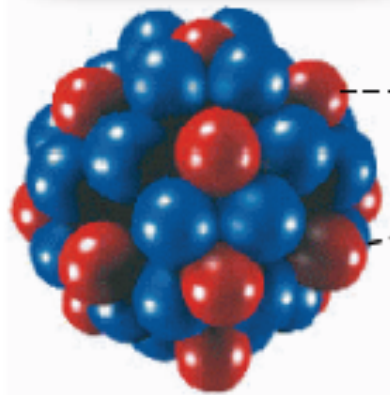
“Constraint by Experiment”

There is the upper limit of hard EOS.

1 Prelude: **PHASES** and **STRUCTURES** of **QCD**

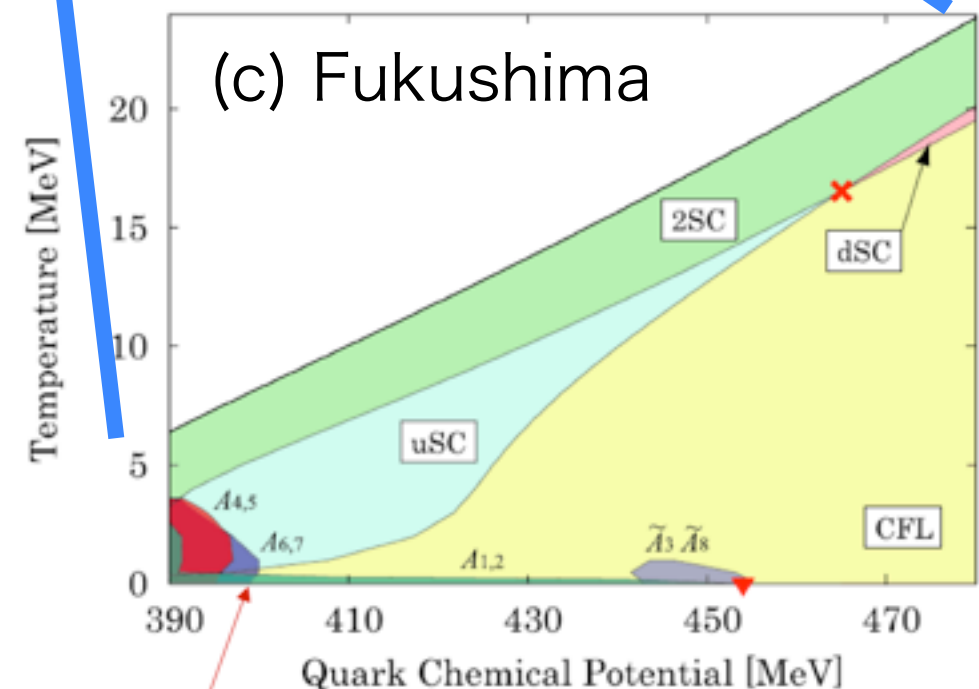
... the goal:

nuclei



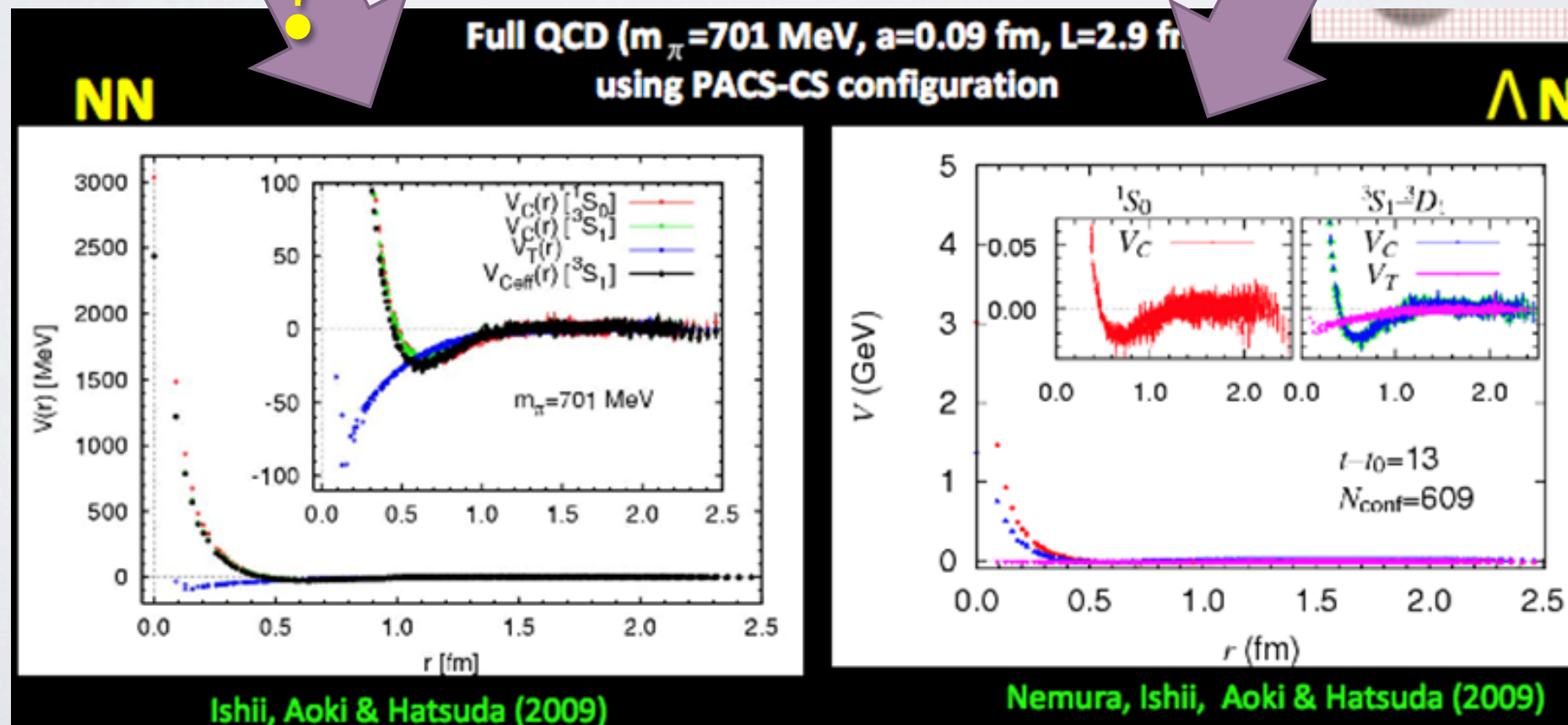
Astrophysical phenomena at $T = 0$.

- Origin of magnetic field ?
- Mechanism of cooling?
- $M > 2 M_s$
- compression modulus:



SITUATIONS OF NUCLEAR PHYSICS

“BARYON-BARYON INTERACTIONS
MAY BE CLEARED IN A FEW YEARS ”



Part I

Non-uniform structures in quark-hadron phase transition “Thermodynamical description”

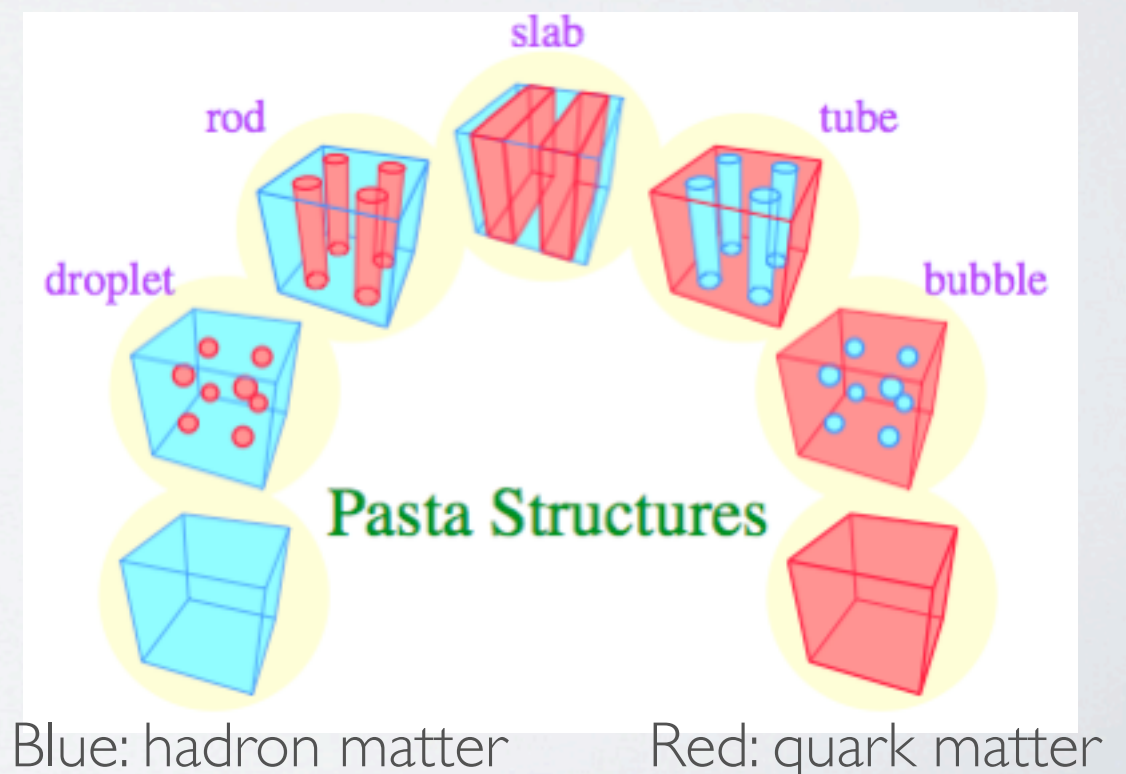
FINITE SIZE EFFECTS IN NON-UNIFORM STRUCTURES

What is “non-uniform structures” ?



Generally, it appears in the phase transition of multi-component system (figure). Namely, we call them as the pasta structures. We find that such structures appear in the hadron-quark(HQ) mixed phase.

Depended on “density” and temperature”, each charged particle clusterizes automatically by “Coulomb interactions” and “surface tensions” ; i.e. **finite size effects**. As a result, they construct non-uniform structures.



FORMALISM

NY, et al. (2013) *Recent Advances in Quarks Research*, Nova, Chap.4, pp.63,
ISBN 9781622579709, arXiv:1208.0427[astro-ph].

Hadron matter

- **Brueckner-Hartree-Fock model** (Baldo et al. 1998, Schulze et al. 1995, Yamamoto et al. 2013)

NN interaction → Argonne V18 potential or Bonn B potential + three body forces
(We will update the interactions by the results of lattice QCD and/or J-PARC.)

- **Chiral Perturbation Theory** (Khono et al. 2011)



Quark matter

- **Dyson-Schwinger method** (Huan et al. 2012, etc.)
- **Extended PNJL model** (Blaschke et al. 2012, Sasaki et al. 2012 etc.)

We assume the non-uniform structures of the mixed phase as droplet, rod, slab, tube, and bubble under Wigner-Seitz cell approximation.

In calculations of mixed phase, we consider

- charge neutrality
- chemical equilibrium
- baryon number conservation
- balance between “surface tension” and “Coulomb interaction”

Changing all of them, we search the minimum free energy.

Dyson-Schwinger method

NY, H. Chen, Maruyama, Tatsumi, in prep.

It provides a continuum approach to QCD that can simultaneously address both “**confinement**” and “**dynamical chiral symmetry breaking**”, solving the quark propagators $S(p; \mu)$;

$$S(p; \mu)^{-1} = Z_2 [i\gamma p + i\gamma_4(p_4 + i\mu) + m_q] + \Sigma(p; \mu),$$

with the renormalized self-energy expressed as

$$\Sigma(p; \mu) =$$

$$Z_1 \int^{\Lambda} \frac{d^4 q}{(2\pi)^4} g^2(\mu) D_{\rho\sigma}(p-q; \mu) \frac{\lambda^a}{2} \gamma_\rho S(q; \mu) \Gamma_\sigma^a(q, p; \mu)$$

Rainbow Approximation

$$\Gamma_\sigma(q, p) = \gamma_\sigma$$

mass function

Gaussian-type interaction

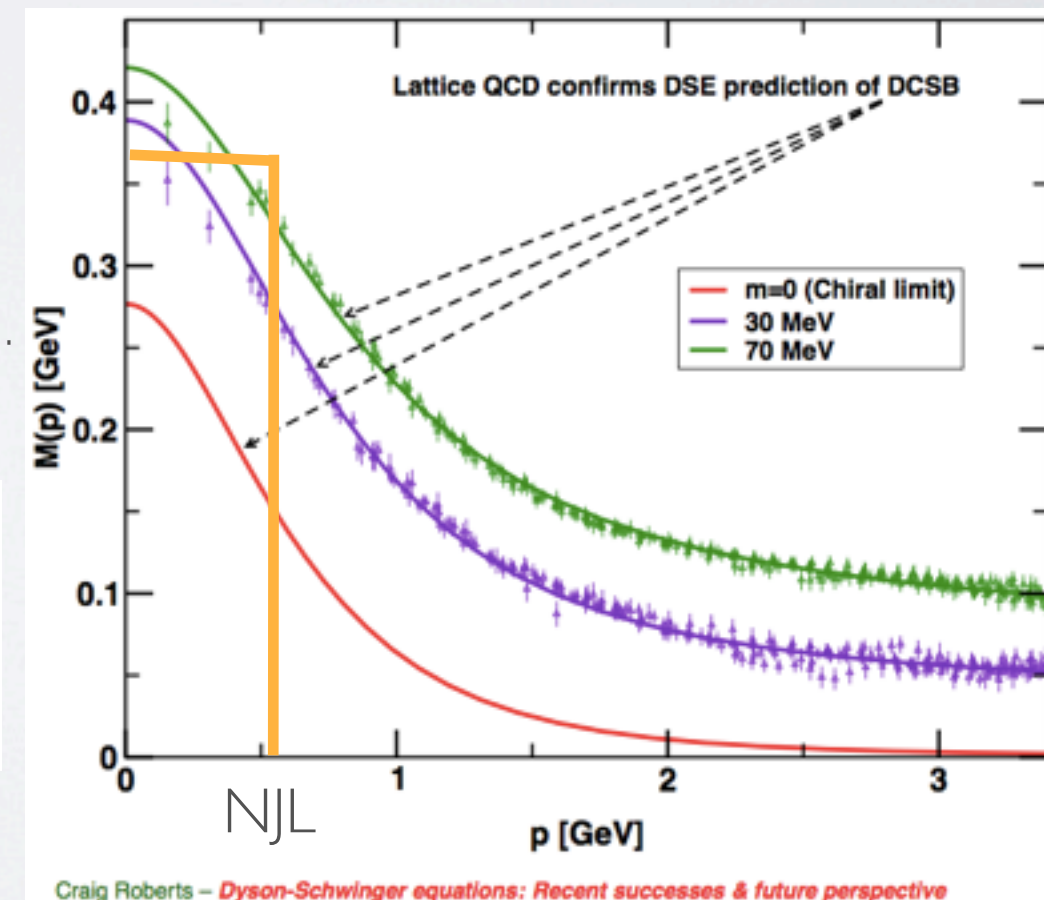
$$\frac{\mathcal{G}(k^2; \mu)}{k^2} = \frac{4\pi^2 D}{\omega^6} e^{-\alpha\mu^2/\omega^2} k^2 e^{-k^2/\omega^2}$$

For example, the caption DS4 means $\alpha=4$. DS model becomes bag model at $\alpha=\infty$.

Ansatz at zero chemical potential

$$Z_1 g^2 D_{\rho\sigma}(p-q) \Gamma_\sigma^a(q, p) = \underline{\mathcal{G}((p-q)^2)} D_{\rho\sigma}^{\text{free}}(p-q) \frac{\lambda^a}{2} \Gamma_\sigma(q, p)$$

Landau gauge free gluon propagator



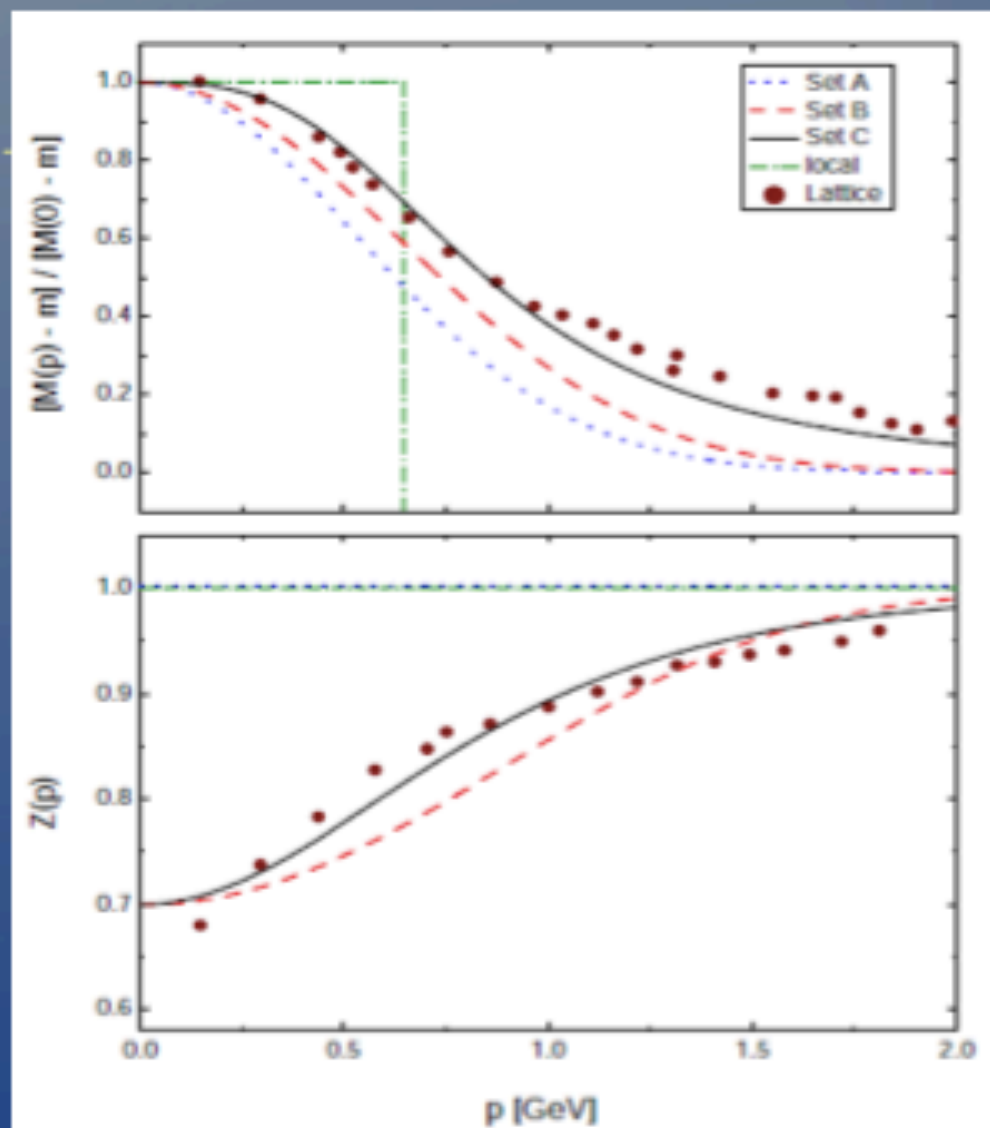
Non-local extended NJL model with WFR

Parameterization w/o WFR

Exponential (Set A)

$$g(p) = e^{-(p^2/\Lambda_0^2)}$$

$$f(p) = 0, \quad \alpha_2 = 0$$



M.B. Parapilly et al., PRD 73 (2006) 054504

Parameterizations with WFR

Exponential (Set B)

$$f(p) = e^{-(p^2/\Lambda_1^2)}$$

$$g(p) = e^{-(p^2/\Lambda_0^2)}$$

Lattice adjusted Lorentzian (Set C)

$$f(p) = \frac{1 + \alpha_z}{1 + \alpha_z f_z(p)} f_z(p)$$

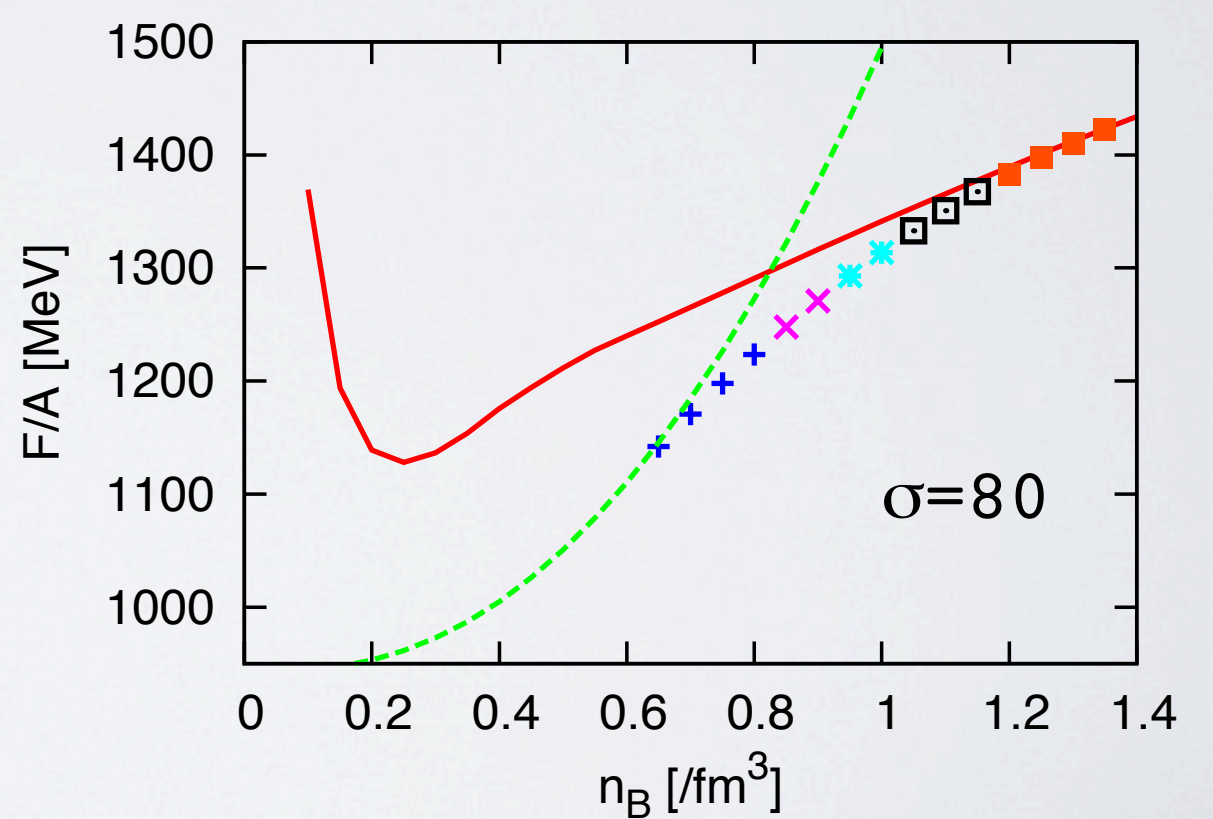
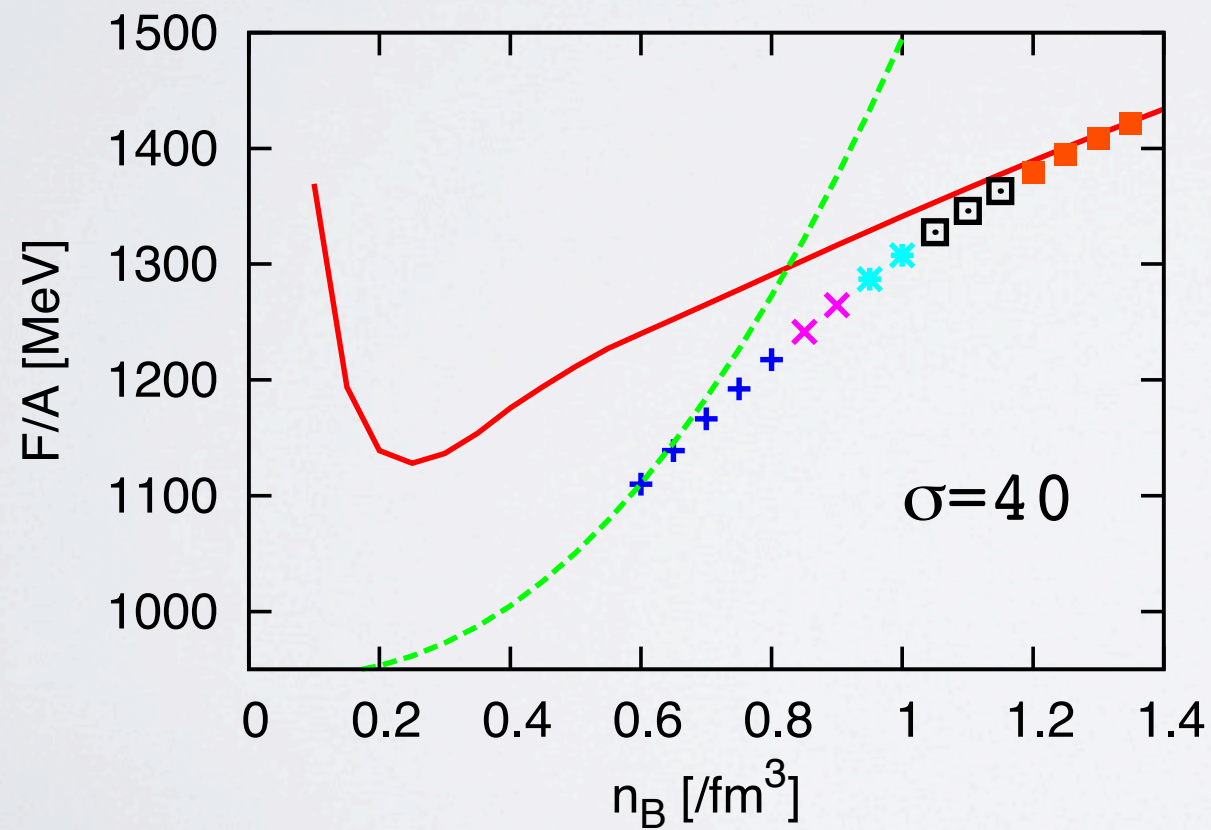
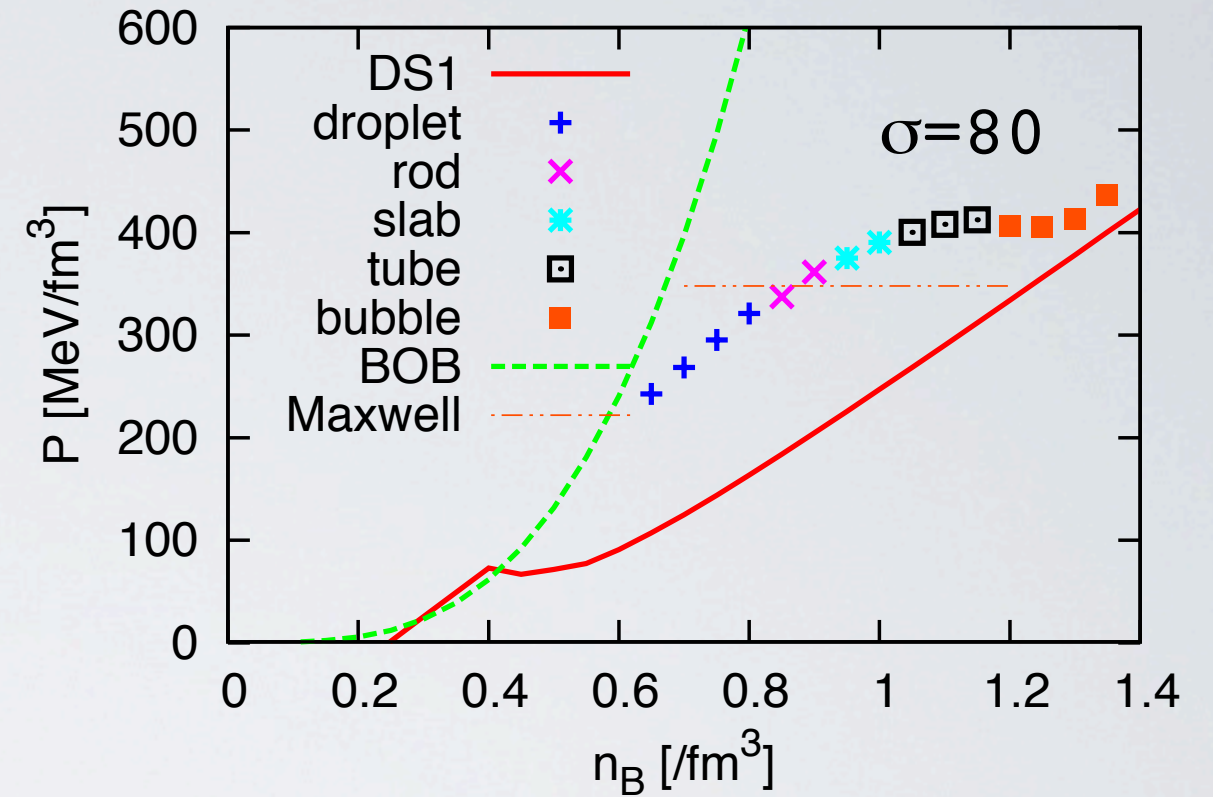
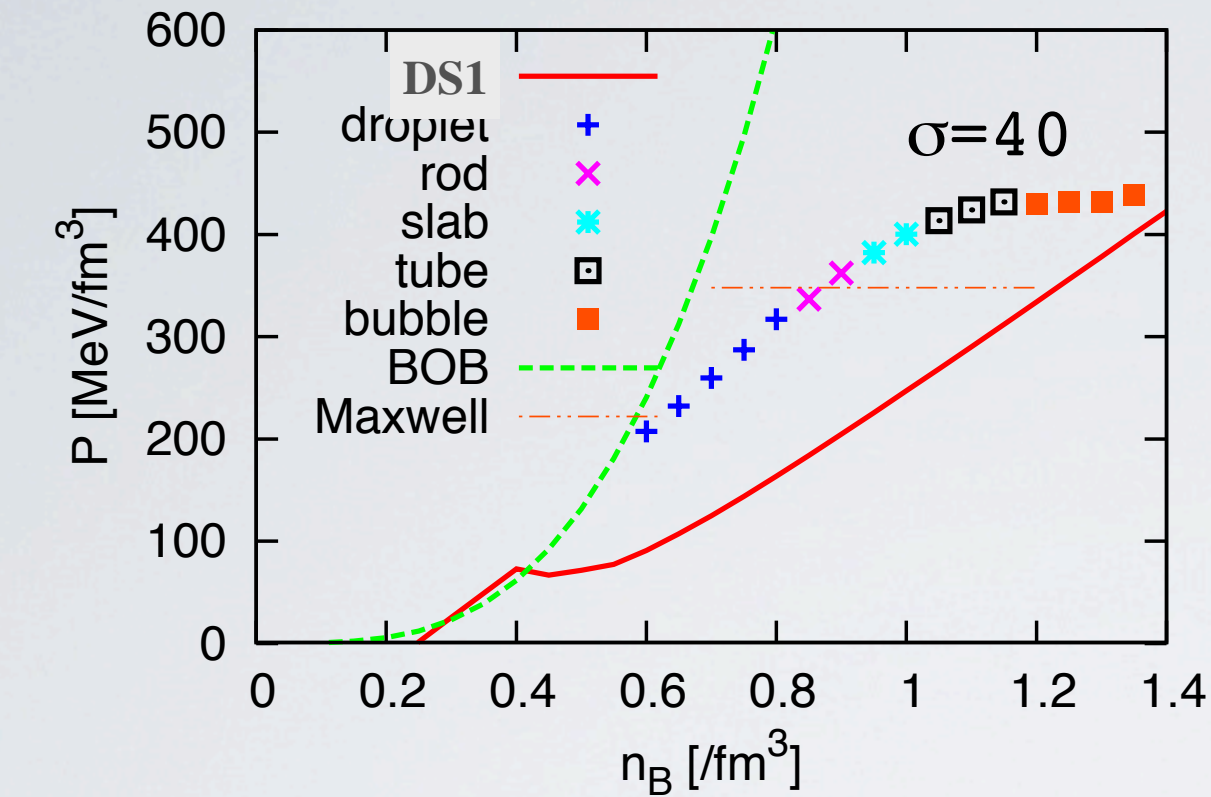
$$g(p) = \frac{1 + \alpha_z}{1 + \alpha_z f_z(p)} \frac{\alpha_m f_m(p) - m_q \alpha_z f_z(p)}{\alpha_m - m_q \alpha_z}$$

$$f_m(p) = [1 + (p^2/\Lambda_0^2)^{3/2}]^{-1}, \quad f_z(p) = [1 + (p^2/\Lambda_1^2)]^{-5/2}$$

$$\alpha_z = -0.3, \quad \alpha_m = 309 \text{ MeV}$$

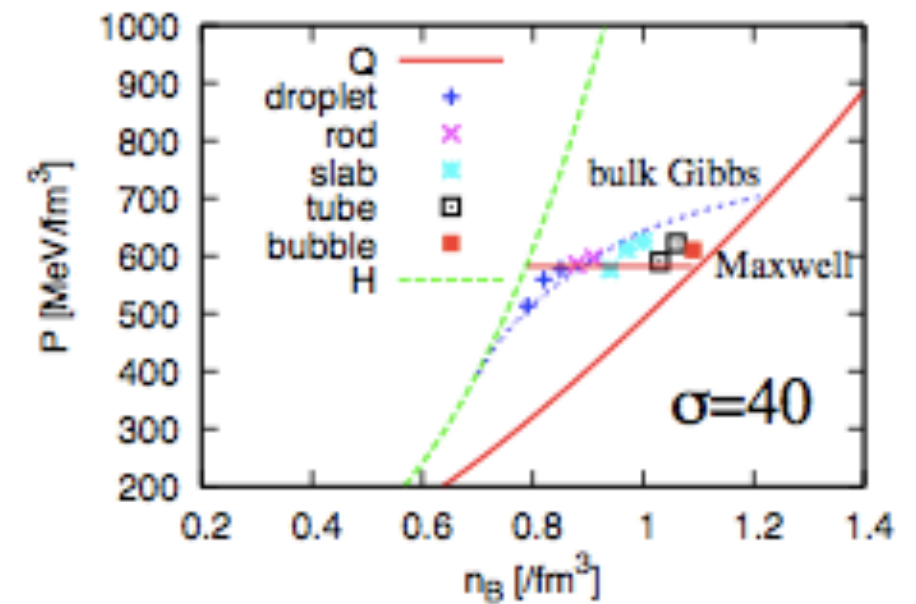
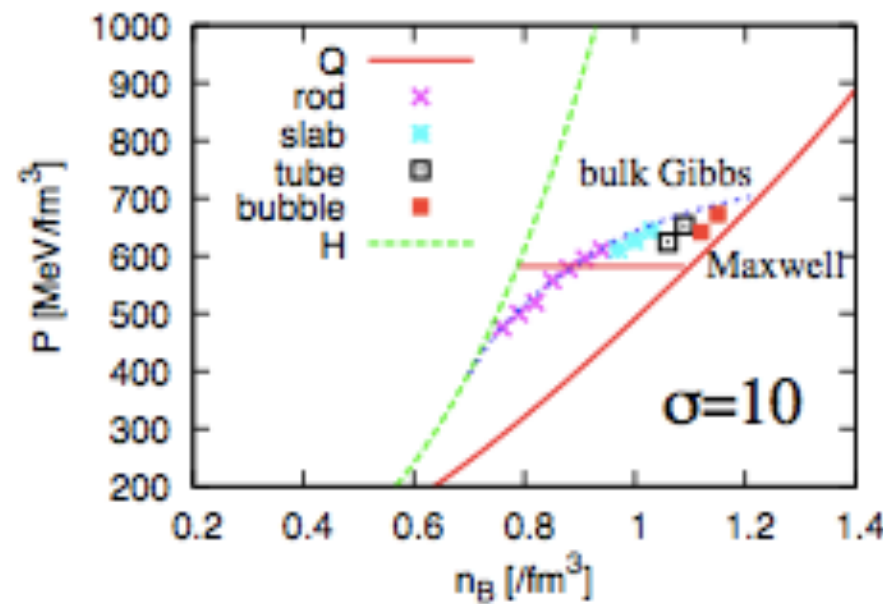
S. Noguera, N. N. Scoccola, Phys.Rev. D78 (2008) 114002

RESULTS①

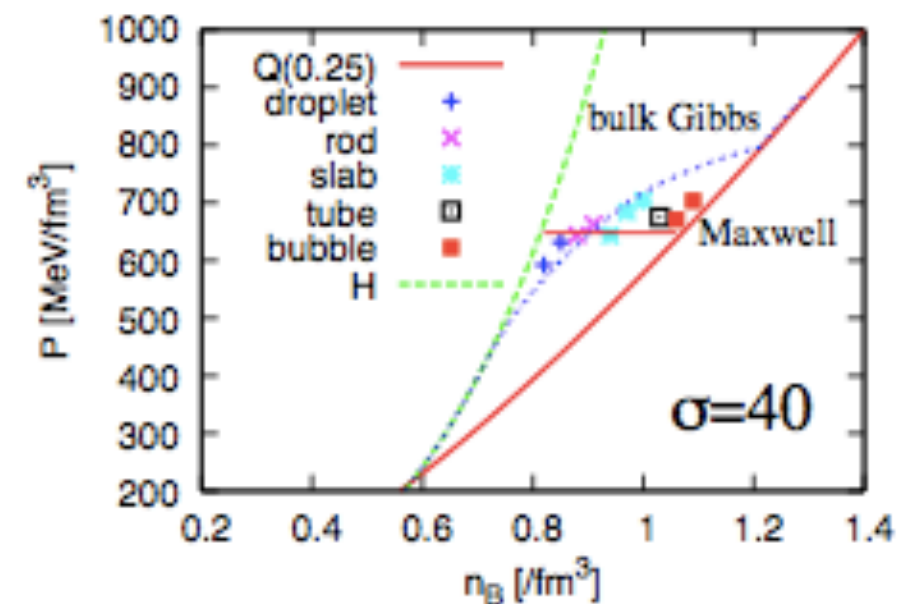
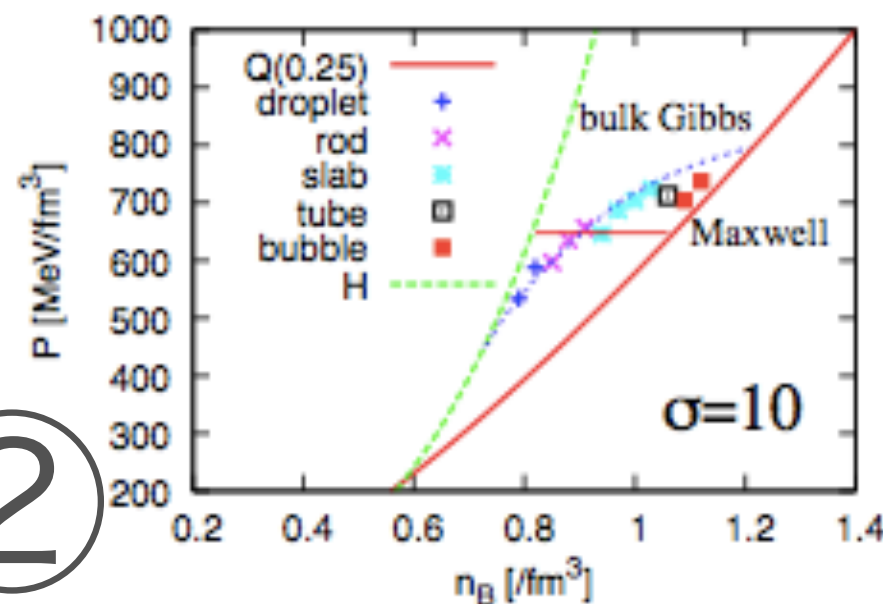
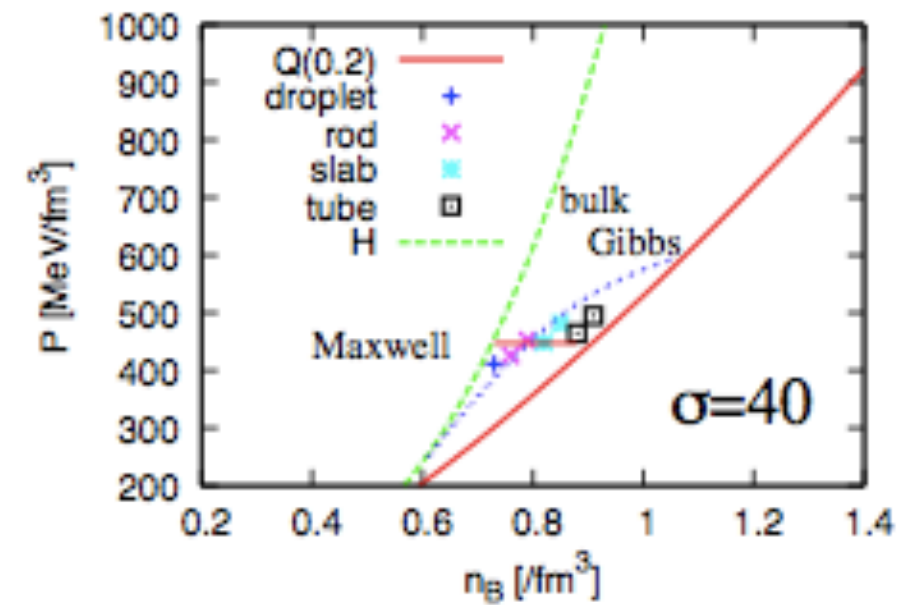
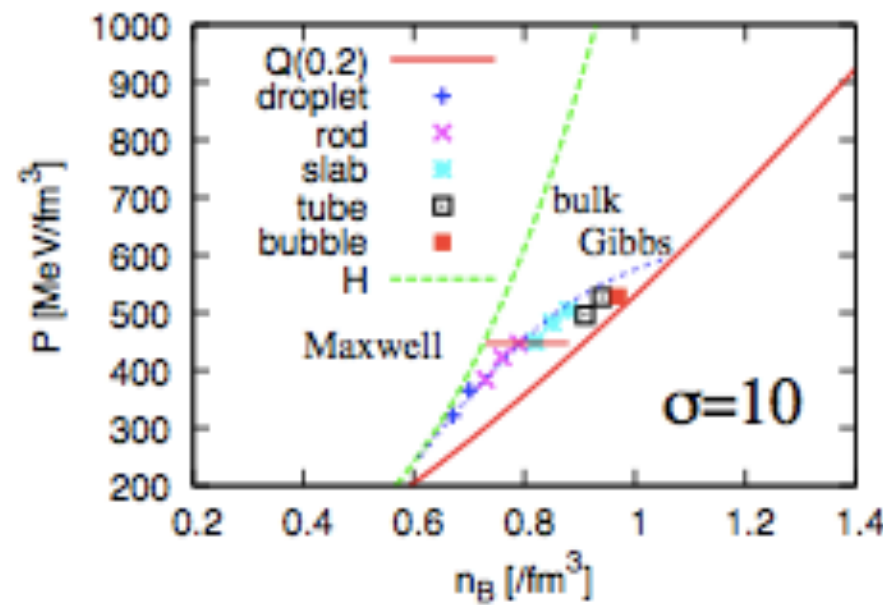


Dyson-Schwinger + BHF(BOB+TBF) with pasta [in prep.]

NJL(Nf=2)
+
BHF(BOB+TBF)
[in prep.]



Non-local NJL(Nf=2)
+
BHF(BOB+TBF)
[in prep.]



RESULTS②

SHORT SUMMARY I

- The non-uniform structures in hadron-quark phase transition are studied.

BHF model, or χ PT for hadron matter (+ finite size effects) + DS model, or Extended PNJL

- (i)the strong surface tension, (ii)finite temperature, (iii)existence of neutrinos,
induce the mechanical instability.
- the EOS becomes to be close
to the one of the pure system. without neutrinos, $\Rightarrow$ density jump(could be...) without neutrinos
to the amorphous state(binary system) with neutrinos. $\Rightarrow$ no density jump with neutrinos
- **MR relations by our models are consistent with observations.**

DISCUSSION

- ① We want to update the data of the NN, NY interactions from Lattice QCD / J-PARC.
- ② With hyperons and universal 3BF?
- ③ DCDW (Nakano & Tatsumi 2005 etc.)?

Part II

Thermal evolution of magnetars/NSs

How to calculate the thermal evolution of compact stars ?

EOS

Quark, hyperon, normal matter,
pion-condensation, kaon-condensation, etc.
(P)NJL, (D)BHF, RMF, variational principle etc.
Landau effects magnetization etc.

structure

w/wo rotation, w/wo magnetic field, axi symmetric etc.

cooling

URCA, MURCA, HURCA, quark beta decay, superconductivity
etc.

heating

Ohmic decay, Hall effect, ambipolar diffusion etc.
vortex etc.

evolution

thermal conduction in strong magnetic field
etc.

atmosphere

Fe including effects strong magnetic field

!!
Comparison with observations

How to calculate the thermal evolution of compact stars ?

EOS

Hardness of EOS
Quark, hyperon, normal matter,
pion condensation, kaon condensation, etc.
Landau effects magnetization etc.

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w/wo rotation, w/wo magnetic field, axi symmetric etc.

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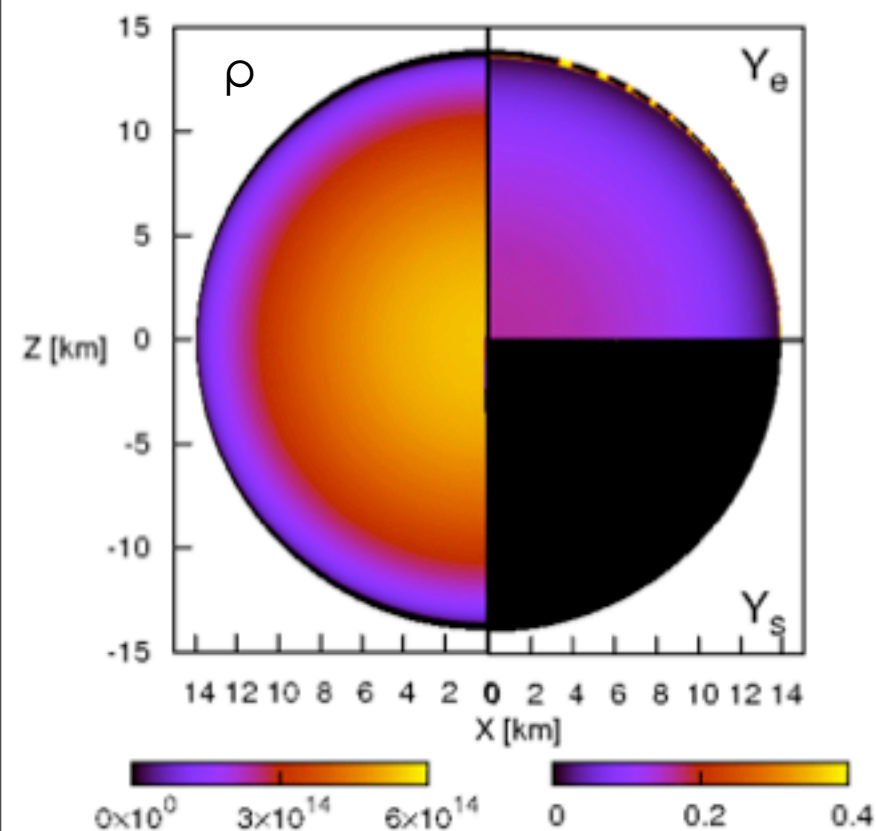
evolution

thermal conduction in strong magnetic field etc.

atmosphere

Fe including effects strong magnetic field

||
Comparison with observations



Tomimura & Eriguchi 2005
 (1) axi symmetric
 (2) equatorial symmetric
 (3) no convection
 etc.

GR correction
 on the gravitational potential
 ref) Mareck et al. 2006

$$-\frac{1}{\rho} \text{grad } p - \text{grad } \Phi_g + \text{grad } \Phi_r + \frac{1}{\rho} (\mathbf{j} \times \mathbf{H}) = 0,$$

$$\Delta \Phi_g = 4\pi G \rho,$$

$$\text{rot } \mathbf{H} = 4\pi \mathbf{j},$$

$$\text{div } \mathbf{H} = 0,$$

↓ integrability condition
 etc.

$$\int \frac{dp}{\rho} = -\Phi_g + \Phi_r + \int \mu(RA_\phi) d(RA_\phi) + C,$$

$$\Phi_g(\mathbf{r}) = -G \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}',$$

$$A_\phi(\mathbf{r}) \sin \phi = -\frac{1}{4\pi} \int \frac{\left(-\frac{\kappa}{R} \int \kappa d(RA_\phi) - 4\pi\mu\rho R'\right)}{|\mathbf{r} - \mathbf{r}'|} \sin \phi' d^3\mathbf{r}',$$

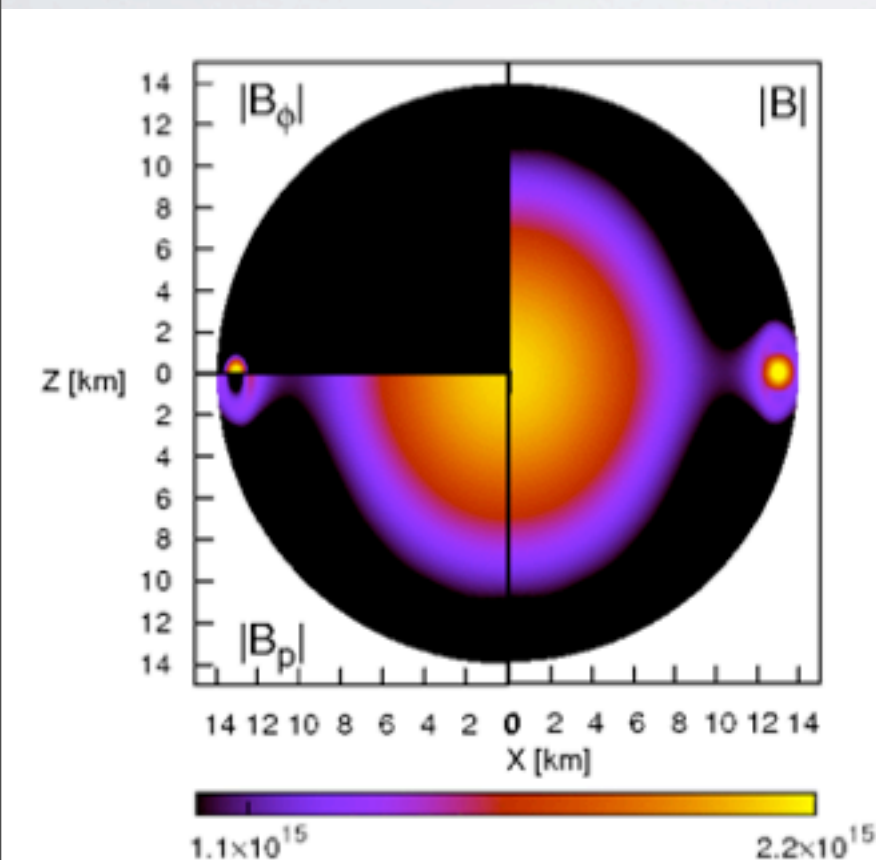
*arbitrary function for magnetic field

$$\mathbf{j} = \frac{\kappa}{4\pi} \mathbf{H} + \mu\rho R_e \mathbf{e}_\phi.$$

$$B_\phi = a(u - u_{\max})^{\kappa+1}/(\kappa + 1).$$

*arbitrary rotational law

Structures with rotation and
 magnetic field



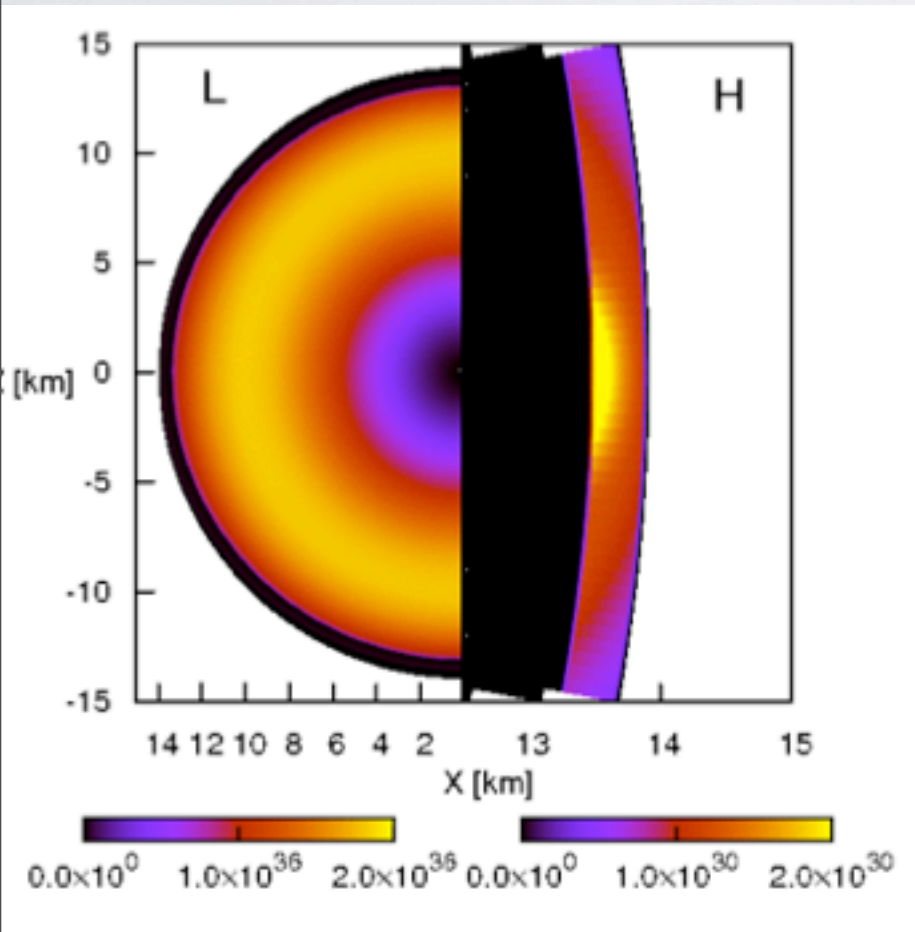
Thermal conduction in strong magnetic field

$$c_v e^\Phi \frac{\partial T}{\partial t} + \nabla \cdot (e^{2\Phi} \mathbf{F}) = e^{2\Phi} Q$$

$$\mathbf{F}_e = -e^\Phi \kappa_e^\perp \left[\nabla \tilde{T} + (\omega_B \tau)^2 (\mathbf{b} \cdot \nabla \tilde{T}) \cdot \mathbf{b} + \omega_B \tau (\mathbf{b} \times \nabla \tilde{T}) \right]$$

- implicit scheme
- operator splitting

cooling rate(L)
& heating rate(H)



Geppert et al.2004

the thermal conductivity

$$\kappa = \begin{pmatrix} \kappa_\perp & \kappa_\wedge & 0 \\ -\kappa_\wedge & \kappa_\perp & 0 \\ 0 & 0 & \kappa_\parallel \end{pmatrix}$$

here

$$\left\{ \begin{array}{l} \kappa_0 = \frac{1}{3} c_v \bar{v}^2 \tau = \frac{\pi^2 k_B^2 T n_e}{3 m_e^*} \tau \\ \kappa_\parallel = \kappa_0 \\ \kappa_\perp = \frac{\kappa_0}{1 + (\omega_B \tau)^2} \\ \kappa_\wedge = \frac{\kappa_0 \omega_B \tau}{1 + (\omega_B \tau)^2} \end{array} \right.$$

TABLE III: Cooling ratio in the cores and crusts we adopt. The details are shown in the references.

process	ratio	reference
Core		
“Modified URCA processes (n-branch)”		
$nn \rightarrow nn\nu\bar{\nu}$		
$pne \rightarrow nn\nu\bar{e}$	$8 \times 10^{21} (n_p)^{1/3} T_9^8$	[31]
“Modified URCA processes (p-branch)”		
$nn \rightarrow nn\nu\bar{\nu}$	$7 \times 10^{19} (n_n)^{1/3} T_9^8$	[31]
$np \rightarrow np\nu\bar{\nu}$	$1 \times 10^{20} (n_p)^{1/3} T_9^8$	[31]
$pp \rightarrow pp\nu\bar{\nu}$	$7 \times 10^{19} (n_p)^{1/3} T_9^8$	[31]
“N – N Bremsstrahlung”		
$nn \rightarrow nn\nu\bar{\nu}$	$7 \times 10^{19} Z n_e^{1/3} T_9^8$	[31]
$np \rightarrow np\nu\bar{\nu}$	$1 \times 10^{20} Z n_e^{1/3} T_9^8$	[31]
$pp \rightarrow pp\nu\bar{\nu}$	$7 \times 10^{19} Z n_e^{1/3} T_9^8$	[31]
Crust		
“e – A Bremsstrahlung”		
$e(A, Z) \rightarrow e(A, Z)\nu\bar{\nu}$	$3 \times 10^{12} Z n_e T_9^8$	[32]
“N – N Bremsstrahlung”		
$nn \rightarrow nn\nu\bar{\nu}$	$7 \times 10^{19} Z n_e^{1/3} T_9^8$	[32]

Temperature distribution

NY, Kotake, Kutsuna, Shigeyama 2013 submitted to PASJ

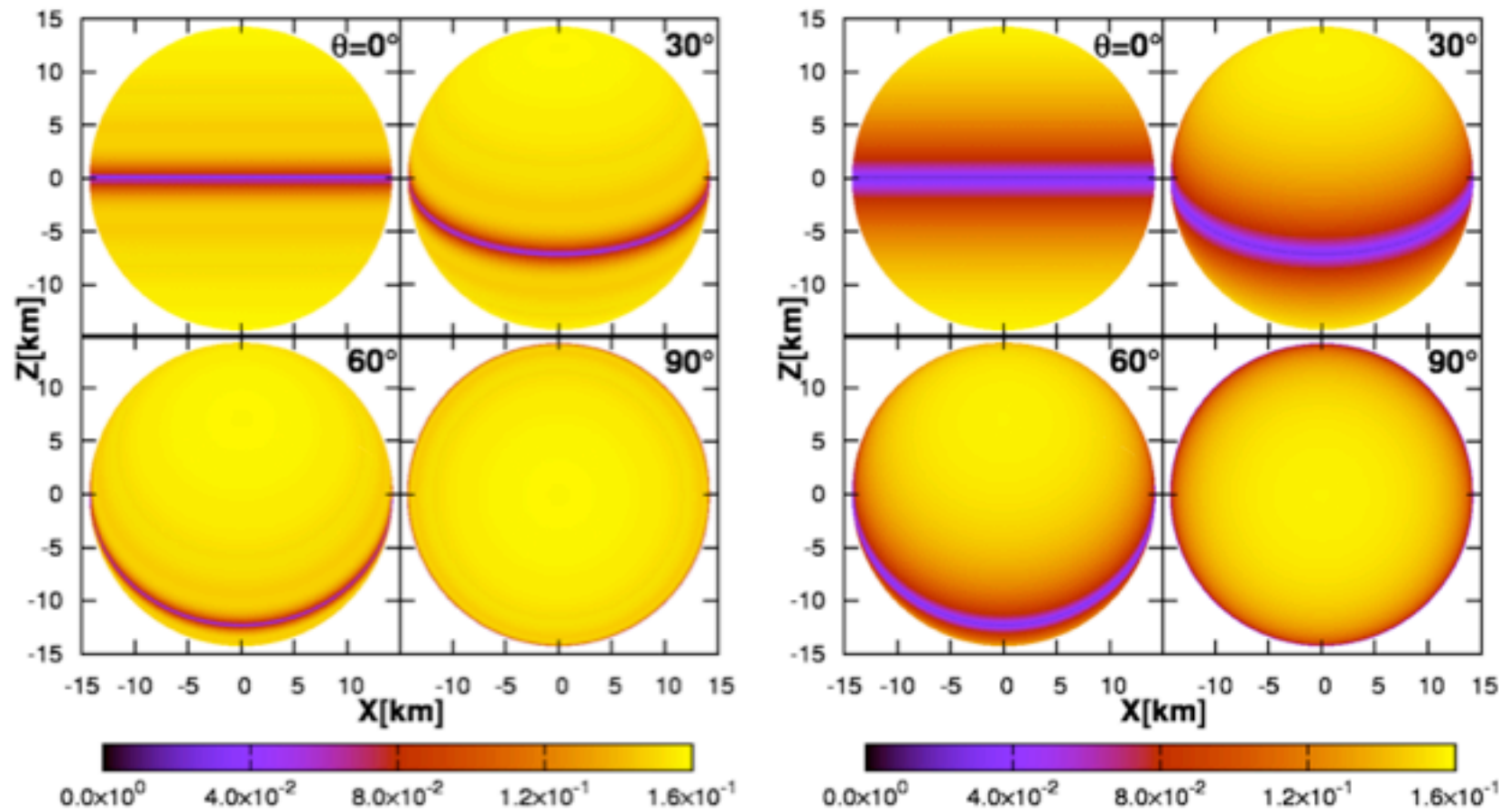
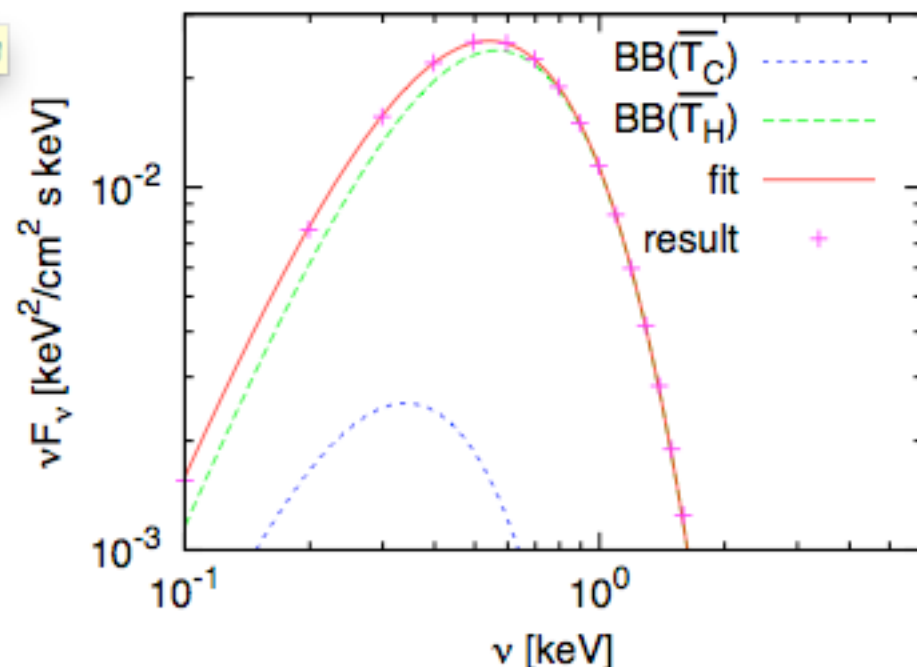


FIG. 7: (Color online) Temperature distribution for model "mSUK" after 10^4 years depended on the inclination angle θ . The unit of color contour is [keV].

移動



Our results

NY, Kotake, Kutsuna, Shigeyama
(2013) PASJ, submitted

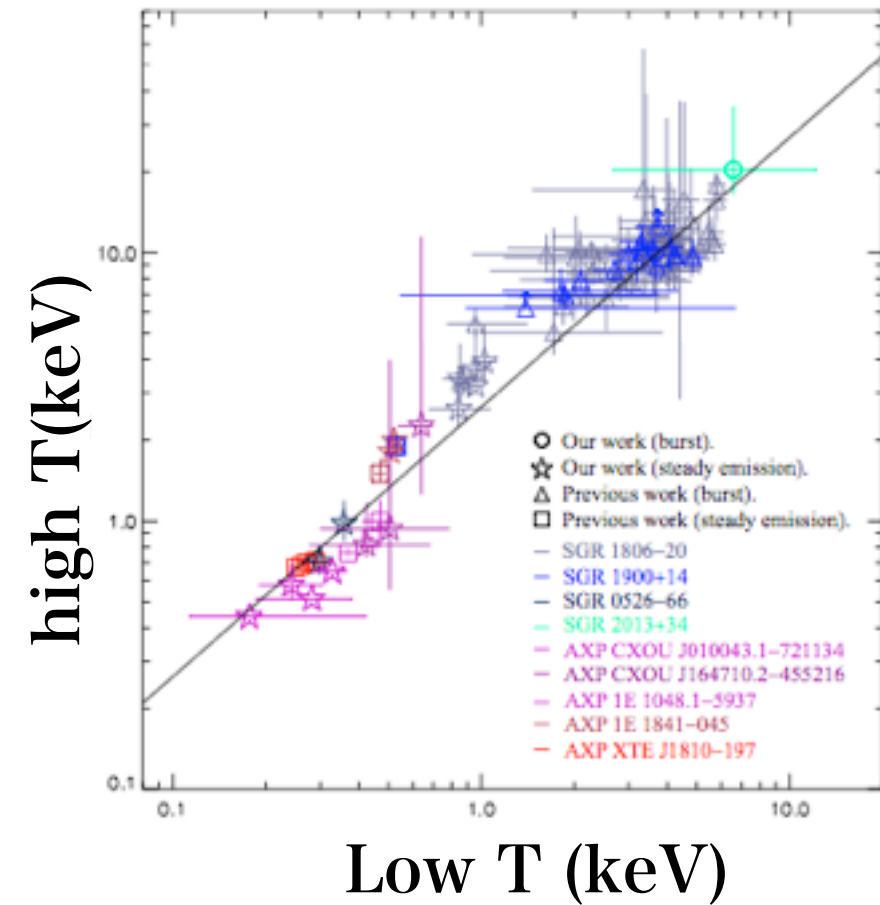


Fig. 3. Relationship between the 2BB temperatures kT_{LT} and kT_{HT} . The triangles and squares denote the previous work on the bursts (Feroci et al. 2004; Olive et al. 2004; Götz et al. 2006a; Nakagawa et al. 2007) and the quiescent emission (Morii et al. 2003; Gotthelf et al. 2004; Gotthelf & Halpern 2005; Tiengo et al. 2005; Mereghetti et al. 2006a), respectively. The circles and stars denote our work on the bursts and the quiescent emission, respectively. The line represents the best-fit power law model.

Observation, Yujin, et al. (2009) PASJ

SHORT SUMMARY II

“Thermal evolution of magnetars”

Our results show the non-uniform temperature of magnetars.
(They are consistent with the observations, qualitatively.)

DISCUSSION

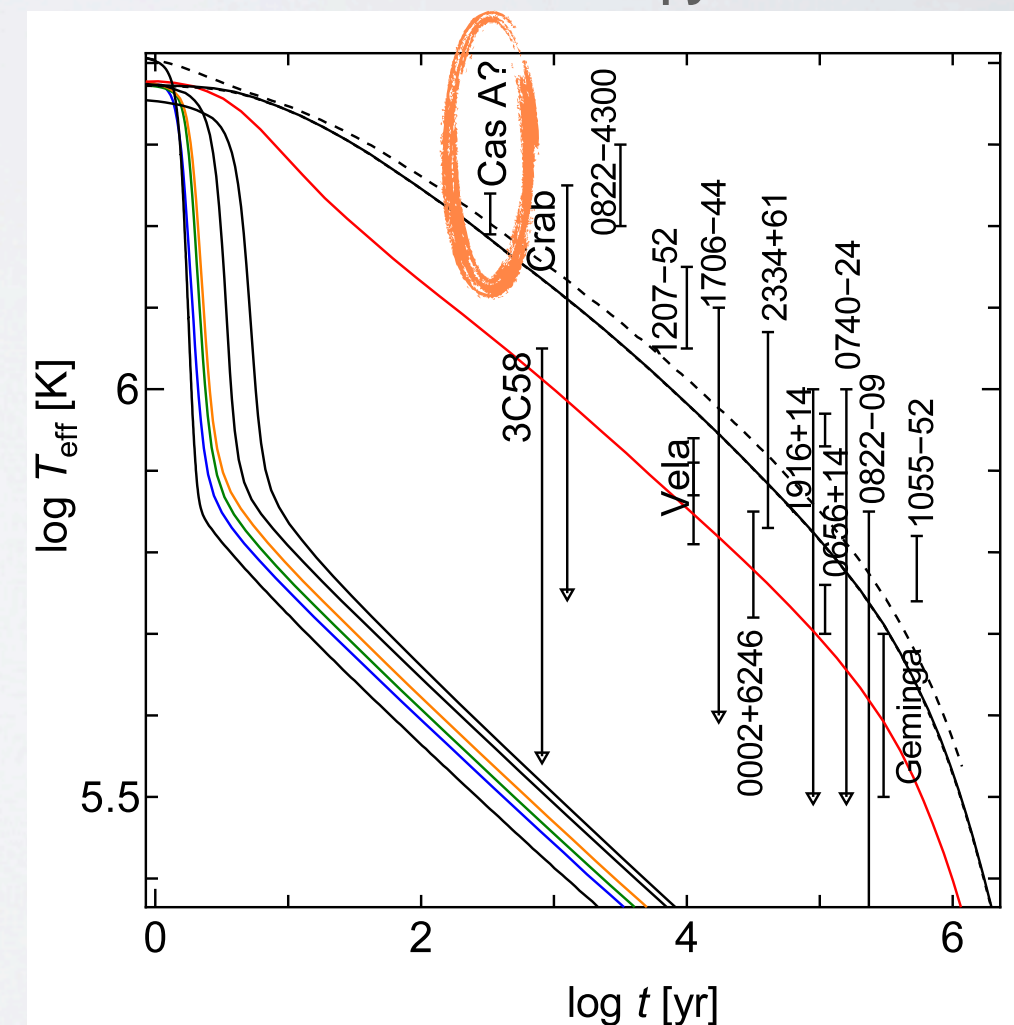
I. Effects of exotic matter?

cf.) Color super conductivity?
superfluid of hadron?
quark-hadron phase transition
pion condensation? ,etc.

2. There is not complete schemes of 2D-evolution in the world now.

cf.) Origin of magnetic field ?
Diffusion of magnetic field ?
Realistic 2D-hydrostatic equilibrium ?
(Beyond Heney method)

Noda, Hashimoto, Matsuo,
NY, Maruyama, Tatsumi, Fujimoto
2012 ApJ



SUMMARY

PART I

- The non-uniform structures in hadron-quark phase transition are studied.
- **MR relations by our models are consistent with observations.**

PART II

- **We could show the non-uniform temperature of magnetars.**
- We should extend our model to the one with exotic matter.
- Our code needs more improvement to show the properties of magnetars.

Future works

- We are trying to make 2D full evolution code. We need a “Break through” like Heney methods in 2D.
- **We made a new method for realistic 2D hydro-static equilibriums.**
- Our technique is based on the variational principle on the mass coordinate.
- It will be useful for all kinds of 2D stellar evolution generally.

SUMMARY

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Fin