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Nuclear Forces
- Lecture 3 -

The Meson Theory of Nuclear Forces

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Lecture 3: The Meson Theory of Nuclear Forces

- The mesons
- How do those mesons contribute to the NN interaction?
- The One-Boson-Exchange Potential
- Closing remarks

The mesons:
Have a look at the Particle Data Group (PDG) Table

Citation: S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004) (URL: http://pdg.lbl.gov)

LIGHT UNFLAVORED MESONS
$(S = C = B = 0)$

For $I = 1$ (π, b, ρ, a): $u\bar{d}$, $(u\bar{u} - d\bar{d})/\sqrt{2}$, $d\bar{u}$;
for $I = 0$ (η, η', h, h', ω, ϕ, f, f'): $c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s})$

$\pi^\pm$ $\quad I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57018 \pm 0.00035$ MeV (S = 1.2)
Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s (S = 1.2)
$c\tau = 7.8045$ m

π^0 $\quad I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 134.9766 \pm 0.0006$ MeV (S = 1.1)
$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV
Mean life $\tau = (8.4 \pm 0.6) \times 10^{-17}$ s (S = 3.0)
$c\tau = 25.1$ nm

η $\quad I^G(J^{PC}) = 0^+(0^{-+})$

Mass $m = 547.75 \pm 0.12$ MeV [f] (S = 2.6)
Full width $\Gamma = 1.29 \pm 0.07$ keV [g]

$f_0(600)$ [i] or σ $\quad I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400–1200)$ MeV
Full width $\Gamma = (600–1000)$ MeV

$f_0(600)$ DECAY MODES	Fraction (Γ_i/Γ)
$\pi\pi$	dominant

$\rho(770)$ [j] $\quad I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.8 \pm 0.5$ MeV
Full width $\Gamma = 150.3 \pm 1.6$ MeV
$\Gamma_{ee} = 7.02 \pm 0.11$ keV

$\rho(770)$ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
$\pi\pi$	$\sim$ 100 %	

$\omega(782)$ $\quad I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 782.59 \pm 0.11$ MeV (S = 1.7)
Full width $\Gamma = 8.49 \pm 0.08$ MeV
$\Gamma_{ee} = 0.60 \pm 0.02$ keV

$\omega(782)$ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
$\pi^+\pi^-\pi^0$	(89.1 ±0.7) %	S=1.1

Citation: S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004) (URL: http://pdg.lbl.gov)

LIGHT UNFLAVORED MESONS
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for $I = 0$ (η, η', h, h', ω, ϕ, f, f'): $c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s})$

$\pi^{\pm}$ $I^G(J^P) = 1^-(0^-)$

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π^0 $I^G(J^P$

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$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ M
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pseudo scalar

η $I^G(J^{PC}) = 0^+(0^{-+})$

Mass $m = 547.75 \pm 0.12$ MeV [f] (S = 2.6)
Full width $\Gamma = 1.29 \pm 0.07$ keV [g]

$f_0(600)$ [i] or σ $I^G(J^{PC}) = 0^+(0^{++})$

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Full width $\Gamma = (600\text{–}1000)$ MeV

f_0(600) DECAY MODES	Fraction (
$\pi\pi$	dominant

scalar

$\rho(770)$ [j] $I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.8 \pm 0.5$ MeV
Full width $\Gamma = 150.3 \pm 1.6$ MeV
$\Gamma_{ee} = 7.02 \pm 0.11$ keV

ρ(770) DECAY MODES	Fraction $(\Gamma_i/\Gamma$
$\pi\pi$	$\sim$ 100

vector

$\omega(782)$ $I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 782$.59 ± 0.1 … = 1.7)
Full width Γ … eV
Γ_{ee} … 02 ke

ω … DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
$\pi^+\pi^-\pi^0$	(89.1 ±0.7) %	S=1.1

Repulsive

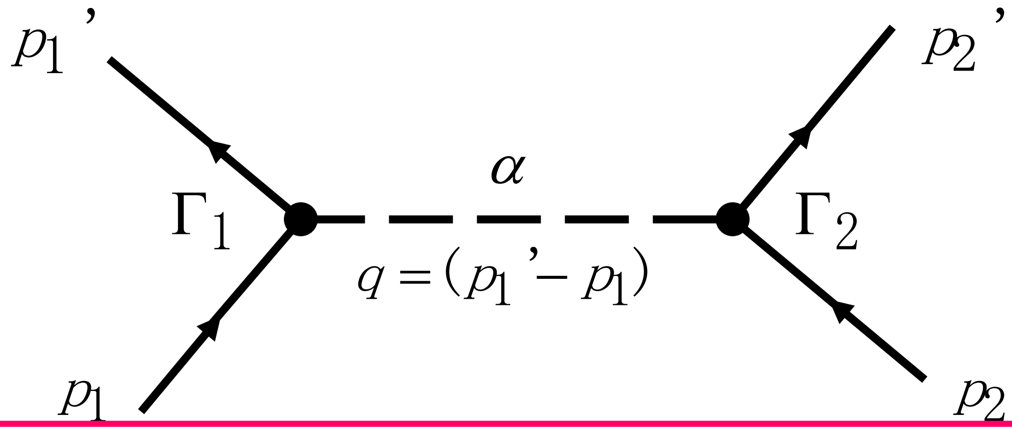
STOP

What do those mesons do to the NN interaction?

To find out, we have to do some calculations. Proper calculations are done in the framework of Quantum Field Theory. That means, we have to take the following steps:

- Write down appropriate Lagrangians for the interaction of the mesons with nucleons.
- Using those interaction Lagrangians, calculate Feynman diagrams that contribute to NN scattering.

Feynman diagram for NN scattering



$$\text{Amplitude: } F_\alpha(p',p) = \frac{\bar{u}_1' \Gamma_1 u_1 \; P_\alpha \; \bar{u}_2' \Gamma_2 u_2}{q^2 - m_\alpha^2}$$

with Dirac spinor $u(p,s) = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M}\chi_s \end{pmatrix} \approx \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M}\chi_s \end{pmatrix} \approx \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}$

where $E = \sqrt{\vec{p}^2 + M^2}$ and χ_s is a two-component Pauli spinor.

Pseudo-vector coupling of a pseudo-scalar meson

Lagrangian: $$\mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma^\mu \gamma_5 \vec{\tau} \psi \cdot \partial_\mu \vec{\varphi}^{(\pi)}$$

Vertex: i times the Lagrangian stripped off the fields (for an incoming pion)

$$\Gamma_{\pi NN} = (i)^2 \frac{f_{\pi NN}}{m_\pi} \gamma^\mu \gamma_5 \vec{\tau} \, q_\mu \qquad \approx \frac{f_{\pi NN}}{m_\pi} (\vec{\sigma} \cdot \vec{q}) \vec{\tau}$$

Potential: i times the amplitude $(P_\pi = i, \; q^2 \approx -\vec{q}^{\,2})$

$$V_\pi = i F_\pi \approx -\frac{f_{\pi NN}^2}{m_\pi^2} \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^{\,2} + m_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2$$

Pseudo-vector coupling of a pseudo-scalar meson, cont'd

Using the operator identity

$$(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) = \frac{\vec{q}^{\,2}}{3}\left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12}(\hat{q})\right]$$

with $S_{12}(\hat{q}) \equiv 3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$ ("Tensor operator"),

the one-pion exchange potential (OPEP) can be written as

$$V_\pi = \frac{f_{\pi NN}^2}{3m_\pi^2} \frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + m_\pi^2}\left[-\vec{\sigma}_1 \cdot \vec{\sigma}_2 - S_{12}(\hat{q})\right] \vec{\tau}_1 \cdot \vec{\tau}_2$$

Scalar coupling

Lagrangian: $$\mathcal{L}_{\sigma NN} = -g_\sigma \bar{\psi}\psi\, \varphi^{(\sigma)}$$

Vertex: $$\bar{u}(p')\Gamma_{\sigma NN}\, u(p) = -ig_\sigma\, \bar{u}(p')\, u(p) \approx -ig_\sigma \left(1 - \frac{(\vec{\sigma}\cdot\vec{p}\,')(\vec{\sigma}\cdot\vec{p})}{(E'+M)(E+M)}\right)$$

$$= -ig_\sigma \left(1 - \frac{\vec{p}\,'\cdot\vec{p} + i\vec{\sigma}\cdot(\vec{p}\,'\times\vec{p})}{(E'+M)(E+M)}\right) \approx -ig_\sigma \left(1 - \frac{\vec{k}^2 - \frac{1}{4}\vec{q}^2 - \vec{\sigma}\cdot\vec{L}}{4M^2}\right)$$

Potential: keeping all terms up to Q^2/M^2 $\left[P_\sigma = i,\ \vec{k} \equiv \frac{1}{2}(\vec{p}\,'+\vec{p}),\ \vec{L}\cdot\vec{S} = -\frac{i}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot(\vec{q}\times\vec{k})\right]$

$$\boxed{V_\sigma = iF_\sigma \approx \frac{g_\sigma^2}{\vec{q}^2 + m_\sigma^2}\left[-1 + \frac{\vec{k}^2}{2M^2} - \frac{\vec{q}^2}{8M^2} - \frac{\vec{L}\cdot\vec{S}}{2M^2}\right]}$$

Vector coupling of a vector meson

Lagrangian: $$\mathcal{L}_{\omega NN} = -g_\omega \bar{\psi}\gamma^\mu \psi \, \varphi_\mu^{(\omega)}$$

Vertex:

$$\mu = 0: \quad \bar{u}(p')\Gamma^0_{\omega NN} u(p) = -ig_\omega \, \bar{u}(p')\gamma^0 u(p) \quad \approx -ig_\omega \left(1 + \frac{(\vec{\sigma}\cdot\vec{p}\,')(\vec{\sigma}\cdot\vec{p})}{(E'+M)(E+M)}\right)$$

$$\approx -ig_\omega \left(1 - \frac{\vec{\sigma}\cdot\vec{L}}{4M^2}\right), \text{ keeping only the } \vec{\sigma}\cdot\vec{L} \text{ term.}$$

Potential, including also the $\vec{\gamma}$ terms: $\left[P_\omega = -ig_{\mu\nu} + \ldots\right]$

$$V_\omega = iF_\omega \approx \frac{g_\omega^2}{\vec{q}^{\,2} + m_\omega^2}\left[1 - 3\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$

Tensor coupling of a vector meson

Lagrangian:

$$\mathcal{L}_{\rho NN}^{(\text{tensor})} = -\frac{f_\rho}{4M}\bar{\psi}\sigma^{\mu\nu}\vec{\tau}\psi \cdot (\partial_\mu \vec{\varphi}_\nu^{(\rho)} - \partial_\nu \vec{\varphi}_\mu^{(\rho)})$$

Vertex (incoming meson)

$$\Gamma_{\rho NN}^{(\text{tensor})} = -\frac{f_\rho}{4M}\sigma^{\mu\nu}(q_\mu - q_\nu)\vec{\tau} = -\frac{f_\rho}{2M}\sigma^{\mu\nu}q_\mu\vec{\tau} \approx -\frac{f_\rho}{2M}(\vec{\sigma}\times\vec{q})\vec{\tau}$$

Potential: $\left[P_\rho = -ig_{\mu\nu} + \dots\right]$

$$V_\rho^{(\text{tensor})} = iF_\rho^{(\text{tensor})} = -\frac{f_\rho^2}{4M^2}\frac{(\vec{\sigma}_1\times\vec{q})(\vec{\sigma}_2\times\vec{q})}{\vec{q}^{\,2}+m_\rho^2}\vec{\tau}_1\cdot\vec{\tau}_2$$

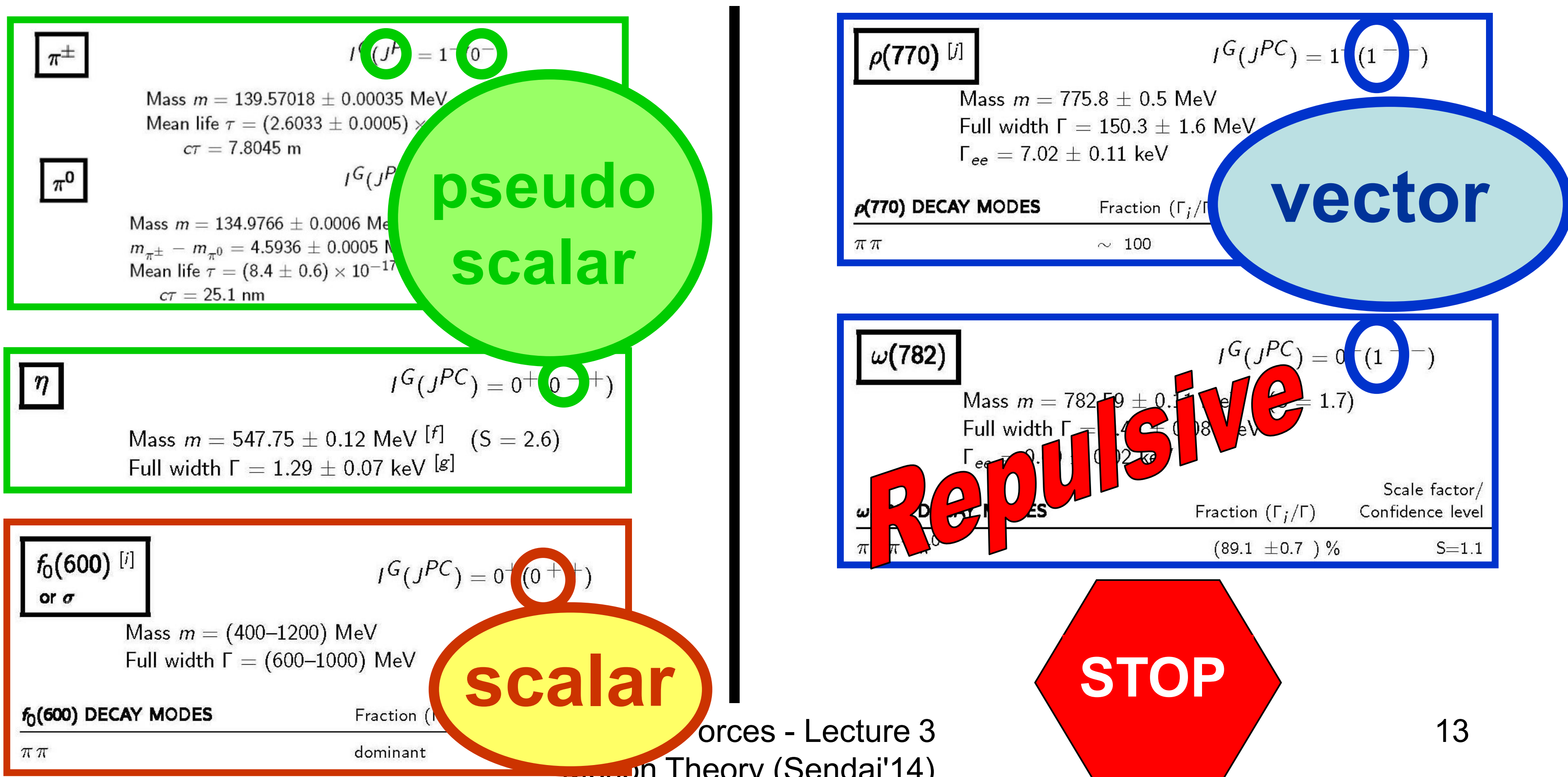
$$= -\frac{f_\rho^2}{4M^2}\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2\,\vec{q}^{\,2} - (\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{\vec{q}^{\,2}+m_\rho^2}\vec{\tau}_1\cdot\vec{\tau}_2$$

$$= \frac{f_\rho^2}{12M^2}\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\rho^2}\left[-2\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

Recall: We found the mesons below in PDG Table and asked:

What do they do?

Now, we have the answer. Let’s summarize.



Summary

$\pi(138)$

$$V_\pi = \frac{f_{\pi NN}^2}{3m_\pi^2}\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\pi^2}\left[-\vec{\sigma}_1\cdot\vec{\sigma}_2 - S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

Long-ranged tensor force

$\sigma(600)$

$$V_\sigma \approx \frac{g_\sigma^2}{\vec{q}^{\,2}+m_\sigma^2}\left[-1-\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$

intermediate-ranged, attractive central force plus LS force

$\omega(782)$

$$V_\omega \approx \frac{g_\omega^2}{\vec{q}^{\,2}+m_\omega^2}\left[+1-3\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$

short-ranged, repulsive central force plus strong LS force

$\rho(770)$

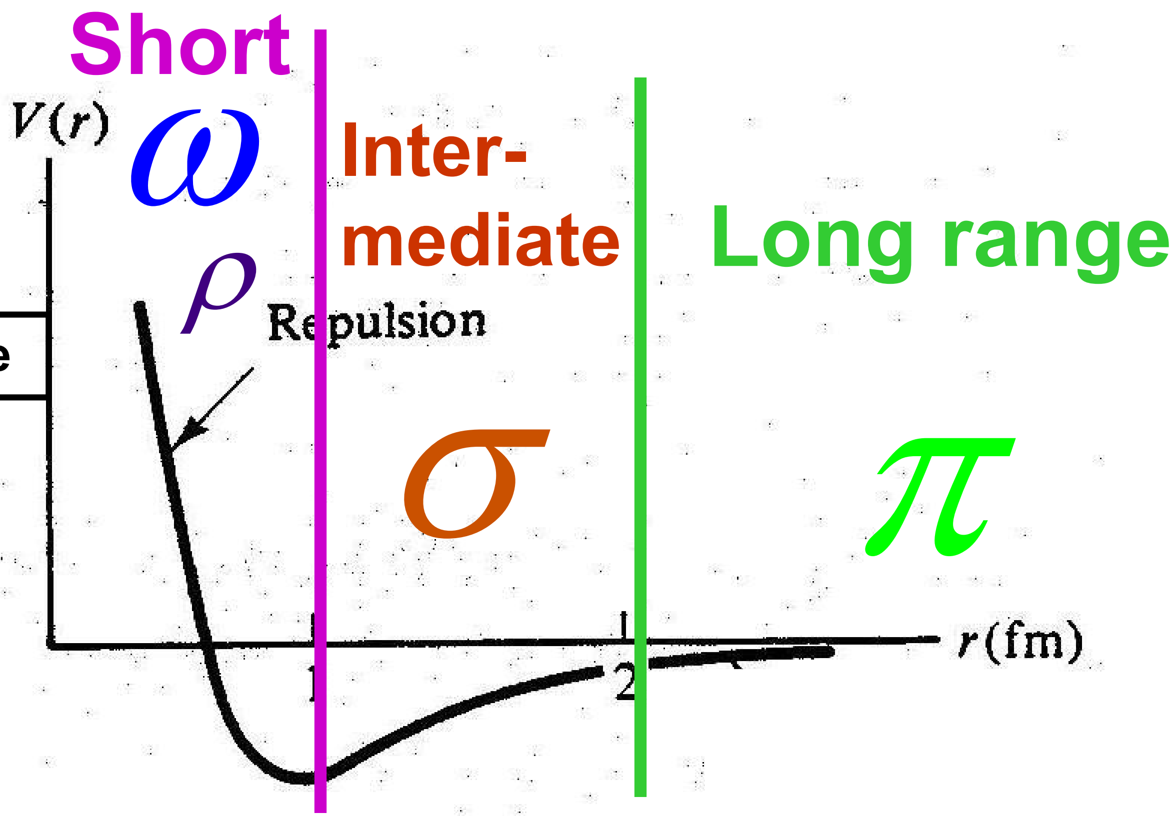
$$V_\rho = \frac{f_\rho^2}{12M^2}\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\rho^2}\left[-2\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

short-ranged tensor force, opposite to pion

It's EVERYTHING we need to describe the nuclear force!

Summary:
Most important parts of the nuclear force

Central force



Tensor force π ρ

Spin-orbit force ω σ

The One-Boson Exchange Potential (OBEP)

$$V_{\mathrm{OBEP}} = \sum_{\alpha=\pi,\sigma,\rho,\omega,\eta,a_0,\ldots} V_\alpha$$

$\eta(548)$ is a pseudo-scalar meson with $I=0,$ therefore, V_η is given by the same expression as V_π, except that V_η carries no $(\vec{\tau}_1 \cdot \vec{\tau}_2)$ factor.

$a_0(980)$ is a scalar meson with $I=1$, therefore, V_{a_0} is given by the same expression as V_σ, except that V_{a_0} carries a $(\vec{\tau}_1 \cdot \vec{\tau}_2)$ factor.

Some comments

- Note that the mathematical expressions for the various V_α given on previous slides are simplified (many approximations) --- for pedagogical reasons.
- For a serious OBEP, one should make few approximations. In fact, it is quite possible to apply essentially no approximations. This is known as the relativistic (momentum-space) OBEP. Examples are the OBEPs constructed by the "Bonn Group", the latest one being the "CD-Bonn potential" (R. M., PRC 63, 024001 (2001)).
- If one wants to represent the OBE potential in r-space, then the momentum-space OBE amplitudes must be Fourier transformed into r-space. The complete, relativistic momentum-space expressions do not yield analytic expressions in r-space after Fourier transform, i.e., it can be done only numerically. However, it is desirable to have analytic expressions. For this, the momentum-space expressions have to be approximated first, e.g., expanded up to Q^2 / M^2, after which an analytic Fourier transform is possible. The expressions one gets by such a procedure are shown on the next slide. Traditionally, the Nijmegen group has taken this approach; their latest r-space OBEPs are published in: V. G. J. Stoks, PRC 49, 2950 (1994).

OBEP expressions in r-space

(All terms up to Q^2/M^2 are included.)

$$V_{ps}(m_{ps},\mathbf{r}) = \frac{1}{12}\frac{g_{ps}^2}{4\pi} m_{ps}\left\{\left(\frac{m_{ps}}{M}\right)^2\left[Y(m_{ps}r) - \frac{4\pi}{m_{ps}^3}\delta^{(3)}(\mathbf{r})\right]\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + Z(m_{ps}\mathbf{r})S_{12}\right\}$$

$$V_s(m_s,\mathbf{r}) = -\frac{g_s^2}{4\pi} m_s\left\{\left[1-\tfrac{1}{4}\left(\frac{m_s}{M}\right)^2\right]Y(m_s r) + \frac{1}{4M^2}[\nabla^2 Y(m_s r) + Y(m_s r)\nabla^2] + \tfrac{1}{2}Z_1(m_s r)\mathbf{L}\cdot\mathbf{S}\right\}.$$

$$V_v(m_v,\mathbf{r}) = \frac{g_v^2}{4\pi} m_v\left\{\left[1+\tfrac{1}{2}\left(\frac{m_v}{M}\right)^2\right]Y(m_v r) - \frac{3}{4M^2}[\nabla^2 Y(m_v r) + Y(m_v r)\nabla^2] + \tfrac{1}{6}\left(\frac{m_v}{M}\right)^2 Y(m_v r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \tfrac{3}{2}Z_1(m_v r)\mathbf{L}\cdot\mathbf{S} - \tfrac{1}{12}Z(m_v r)S_{12}\right\}$$

$$+ \tfrac{1}{2}\frac{g_v f_v}{4\pi} m_v\left[\left(\frac{m_v}{M}\right)^2 Y(m_v r) + \tfrac{2}{3}\left(\frac{m_v}{M}\right)^2 Y(m_v r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - 4Z_1(m_v r)\mathbf{L}\cdot\mathbf{S} - \tfrac{1}{3}Z(m_v r)S_{12}\right]$$

$$+ \frac{f_v^2}{4\pi} m_v\left[\tfrac{1}{6}\left(\frac{m_v}{M}\right)^2 Y(m_v r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \tfrac{1}{12}Z(m_v r)S_{12}\right]$$

with

$$Y(x) = e^{-x}/x$$

$$Z(x) = \left(\frac{m_\alpha}{M}\right)^2\left(1+\frac{3}{x}+\frac{3}{x^2}\right)Y(x)$$

$$Z_1(x) = -\left(\frac{m_\alpha}{M}\right)^2\frac{1}{x}\frac{d}{dx}Y(x) = \left(\frac{m_\alpha}{M}\right)^2\left(\frac{1}{x}+\frac{1}{x^2}\right)Y(x)$$

and

$$S_{12} = 3\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2} - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r - \frac{\mathbf{L}^2}{r^2}$$

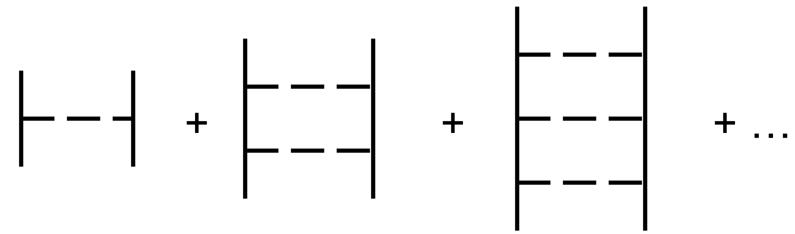
Does the OBE model contain "everything"?

- NO! It contains only the so-called iterative diagrams.

Lippmann-Schwinger eqn: $T = V + V \frac{1}{e} T$

$$T = V + V \frac{1}{e} V + V \frac{1}{e} V \frac{1}{e} V + \ldots$$

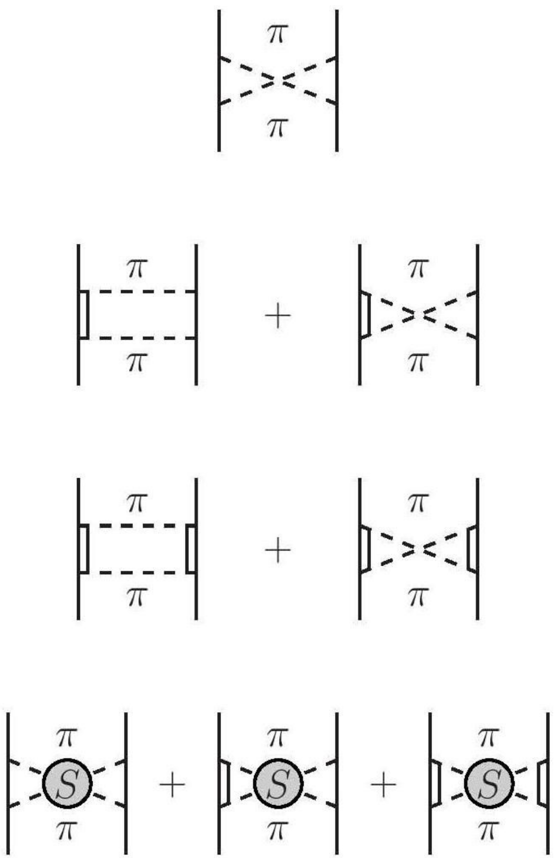
In diagrams: $T =$



"i-t-e-r-a-t-i-v-e"

- However: There are also non-iterative diagrams which contribute to the nuclear force (see next slide).

Some examples for non-iterative meson-exchange contributions not included in the OBE model (or OBEP).



The "Bonn Full Model" (or "Bonn Potential") contains these and other non-iterative contributions. It is the most comprehensive meson-model ever developed (R. M. et al., Phys. Reports 149, 1 (1987)).

The "Paris Potential" is based upon dispersion theory and not on field theory. However, one may claim that, implicitly, the Paris Potential also includes these diagrams; M. Lacombe *et al.,* Phys. Rev. C 21, 861 (1980).

Reviews on Meson Theory

- Pedagogical introduction which also includes a lot of history: R. M., *Advances in Nuclear Physics* **19**, 189-376 (1989).
- The derivation of the meson-exchange potentials in all mathematical details is contained in: R. M., "The Meson Theory of Nuclear Forces and Nuclear Matter", in: *Relativistic Dynamics and Quark-Nuclear Physics,* M. B. Johnson and A. Picklesimer, eds. (Wiley, New York, 1986) pp. 71-173.
- Computer codes for relativistic OBEPs and phase-shift calculations in momentum-space are published in: R. M., "One-Boson Exchange Potentials and Nucleon-Nucleon Scattering", in: *Computational Nuclear Physics 2 – Nuclear Reactions,* K. Langanke, J.A. Maruhn, and S.E. Koonin, eds. (Springer, New York, 1993) pp. 1-29.

End Lecture 3