Physics Department, Tohoku University,
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Nuclear Forces

- Lecture 5 -

EFT for Low-energy QCD and nuclear two-body forces

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Lecture 5: EFT for low-energy QCD and nuclear two-body forces

- An EFT for low-energy QCD: Why?
- An EFT for low-energy QCD: How?
- Power counting
- The Lagrangians
- The Diagrams: Hierarchy of nuclear force
- The new generation of chiral NN potentials

Knowing the symmetries, we can now apply …

Weinberg's "Folk Theorem"

If one writes down the most general possible Langrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Langrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition, and the assumed symmetry principles.

Physica **96A**, *327 (1979)*

Weinberg's Folk Theorem is the stepping stone to what has become known as an Effective Field Theory (EFT).
Here, in short, the reasoning:

QCD at low energy is non-perturbative and, therefore, not solvable analytically in terms of the "fundamental" degrees of freedom (dof); quarks and gluons are ineffective dof at low energy.

Below the chiral symmetry breaking scale $\Lambda_\chi \approx 1 \text{ GeV}$ pions and nucleons are the appropriate dof.

Moreover, pions are Goldstone bosons which typically interact weakly. Thus, there is hope that a perturbative approach might work.

To ensure the connection with QCD, we must observe the same symmetries as low-energy QCD, particularly, spontaneously broken chiral symmetry, such that our chiral EFT **can be indentified with** low-energy QCD.

How to do this EFT program?
Take the following steps:

- Write down the most general Lagrangian including **all** terms consistent with the assumed symmetries, particularly, spontaneously broken chiral symmetry. (Note: There will be infinitely many terms.)
- Calculate Feynman diagrams. (Note: There will be infinitely many diagrams.)
- Find a scheme for assessing the importance of the various diagrams (because we cannot calculate infinitely many diagrams).

The organizational scheme

"Power Counting"

Organize the contributions in terms of $\left(\frac{Q}{\Lambda_\chi}\right)^\nu$;

where Q denotes a momentum (derivative) or a pion mass (m_π);

Λ_χ is the chiral symmetry breaking scale, $\Lambda_\chi \approx 1$ GeV;

and $\nu \geq 0$.

⟺ **Chiral Perturbation Theory (ChPT)**

The Lagrangian

$$\mathcal{L} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN}$$

with

$$\mathcal{L}_{\pi\pi} = \mathcal{L}_{\pi\pi}^{(2)} + \ldots$$

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \ldots$$

$$\mathcal{L}_{NN} = \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \mathcal{L}_{NN}^{(4)} + \ldots$$

where the superscript refers to the number of derivaties or pion mass insertions.

The pi-pi Lagrangian

At order two (leading order)

- Derivative part

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4} \, \mathrm{Tr}\left[\partial_\mu U \, \partial^\mu U^+\right]$$

with $U = \exp(i\vec{\tau} \bullet \vec{\pi} \,/\, f_\pi)$ (any number of pion fields).

Goldstone bosons can only interact when they carry momentum → derivative coupling.

Lorentz invariance → even number of derivatives.

- Plus symmetry breaking mass term:

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4} \, \mathrm{Tr}\left[\partial_\mu U \, \partial^\mu U^+ + m_\pi^2 \left(U + U^+\right)\right]$$

The pi-N Lagrangian with one derivative (=lowest order or "leading" order)

$$\mathcal{L}_{\pi N}^{(I)} = \bar{\Psi}_N \left(i\gamma^\mu D_\mu - M_N + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu \right) \Psi_N$$

with

$$\begin{aligned}
D_\mu &= \partial_\mu + \Gamma_\mu \\
\Gamma_\mu &= (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)/2 \approx i\boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi})/4f_\pi^2 + \ldots \\
u_\mu &= i(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \quad \approx -\boldsymbol{\tau} \cdot \partial_\mu \boldsymbol{\pi}/f_\pi + \ldots \\
U &= \xi\xi \\
\xi &= e^{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/2f_\pi} \qquad \approx 1 + \frac{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{2f_\pi} - \frac{\boldsymbol{\pi}^2}{8f_\pi^2} + \ldots
\end{aligned}$$

and

$$\begin{aligned}
f_\pi &= 92.4 \text{ MeV} \\
g_A &= g_{\pi NN}\, f_\pi / M_N = 1.29
\end{aligned}$$

equivalent to $g_{\pi NN}^2/4\pi = 13.67$

Non-linear realization of chiral symmetry; Callan, Coleman, Wess, and Zumino, PR 177, 2247 (1969).

pi-N Lagrangian, cont'd

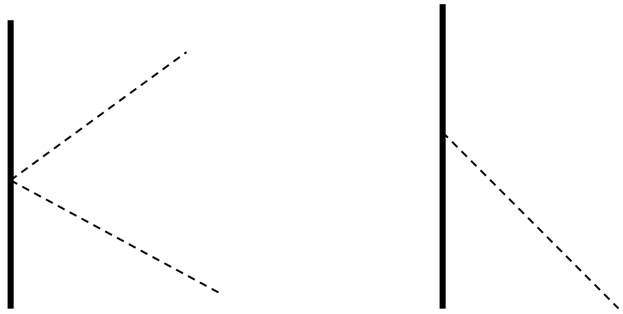
The M_N term is a problem.

$\Rightarrow$ Perform a $1/M_N$ expansion to remove it.

"Heavy Baryon Chiral Perturbation Theory"

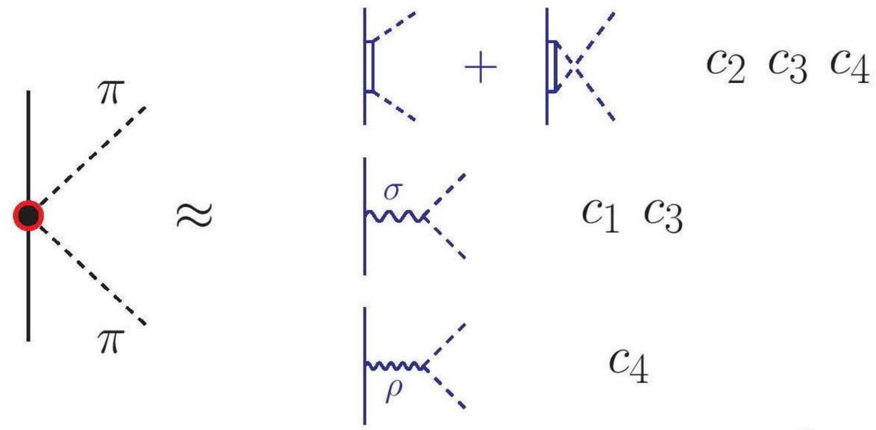
$$\mathcal{L}_{\pi N}^{(I)} = \bar{\Psi}_N \left(i\gamma^\mu D_\mu - M_N + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu \right) \Psi_N$$
$$\approx \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N, M_N}^{(2)} + \mathcal{L}_{\pi N, M_N}^{(3)} + \ldots$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N} \left(iD_0 - \frac{g_A}{2} \vec{\sigma} \cdot \vec{u} \right) N$$
$$\approx \bar{N} \left[i\partial_0 - \frac{1}{4f_\pi^2} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \partial_0 \boldsymbol{\pi}) - \frac{g_A}{2f_\pi} \boldsymbol{\tau} \cdot (\vec{\sigma} \cdot \vec{\nabla}) \boldsymbol{\pi} \right] N + \ldots$$



pi-N Lagrangian with two derivatives
("next-to-leading" order)

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\pi N, c_i} \quad = \quad & \bar{N}\Big[2\, c_1\, m_\pi^2\, (U + U^\dagger) \\
& + \left(c_2 - \frac{g_A^2}{8M_N}\right) u_0^2 \\
& + c_3\, u_\mu\, u^\mu \\
& + \frac{i}{2}\left(c_4 + \frac{1}{4M_N}\right) \vec{\sigma}\cdot(\vec{u}\times\vec{u})\Big] N
\end{aligned}
$$



The N-N Langrangian ("contact terms")

- At order zero (leading order):

$$\mathcal{L}_{NN}^{(0)} = -\frac{1}{2} C_S \bar{N} N \bar{N} N - \frac{1}{2} C_T \bar{N} \vec{\boldsymbol{\sigma}} N \bar{N} \vec{\boldsymbol{\sigma}} N$$

- At order two:

$$\begin{aligned}
\mathcal{L}_{NN}^{(2)} = & -C_1'[(\bar{N}\vec{\nabla}N)^2 + (\overline{\vec{\nabla}N}N)^2] - C_2'(\bar{N}\vec{\nabla}N)\cdot(\overline{\vec{\nabla}N}N) - C_3'\bar{N}N[\bar{N}\vec{\nabla}^2N + \overline{\vec{\nabla}^2N}N] \\
& -iC_4'[\bar{N}\vec{\nabla}N\cdot(\overline{\vec{\nabla}N}\times\vec{\sigma}N) + \overline{(\vec{\nabla}N)}N\cdot(\bar{N}\vec{\sigma}\times\vec{\nabla}N)] - iC_5'\bar{N}N(\overline{\vec{\nabla}N}\cdot\vec{\sigma}\times\vec{\nabla}N) \\
& -iC_6'(\bar{N}\vec{\sigma}N)\cdot(\overline{\vec{\nabla}N}\times\vec{\nabla}N) \\
& -(C_7'\delta_{ik}\delta_{jl} + C_8'\delta_{il}\delta_{kj} + C_9'\delta_{ij}\delta_{kl})[\bar{N}\sigma_k\partial_iN\bar{N}\sigma_l\partial_jN + \overline{\partial_iN}\sigma_kN\overline{\partial_jN}\sigma_lN] \\
& -(C_{10}'\delta_{ik}\delta_{jl} + C_{11}'\delta_{il}\delta_{kj} + C_{12}'\delta_{ij}\delta_{kl})\bar{N}\sigma_k\partial_iN\overline{\partial_jN}\sigma_lN \\
& -(\frac{1}{2}C_{13}'(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{kj}) + C_{14}'\delta_{ij}\delta_{kl})[\overline{\partial_iN}\sigma_k\partial_jN + \overline{\partial_jN}\sigma_k\partial_iN]\bar{N}\sigma_lN,
\end{aligned}$$

- At order

$$\mathcal{L}_{NN}^{(4)} = \quad \ldots$$

Why contact terms?

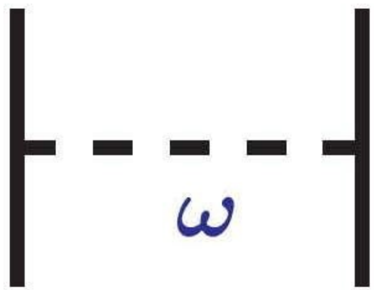
Two reasons:

- Renormalization:
  Absorb infinities from loop integrals.

- A physics reason (see next slide).

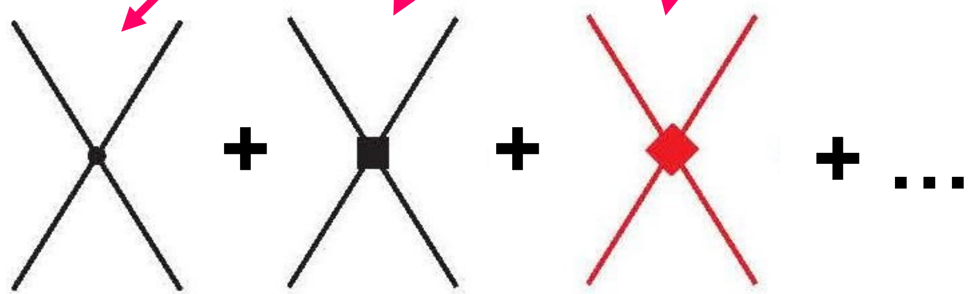
What is the physics of contact terms?

Consider the contribution from the exchange of a heavy meson



$$\frac{1}{m_\omega^2 + Q^2} \approx \frac{1}{m_\omega^2}\left[1 - \frac{Q^2}{m_\omega^2} + \frac{Q^4}{m_\omega^2} - \dots\right.$$

$\approx$



Contact terms take care of the short range structures without resolving them.

Remember the homework we have to do

- Write down the most general Lagrangian
  including **all** terms consistent with the assumed symmetries, particular spontaneously broken chiral symmetry. (Note: There will be infinitely m terms.)
- Calculate Feynman diagrams. (Note: There will be infinitely many diagrams.)
- Find a scheme for assessing the importance of the various diagrams (se we cannot calculate infinitely many diagrams).

The Diagrams

Power counting for Feynman diagrams

$$\text{Power} = -2 + 2A - 2C + 2L + \sum_{\text{all vertices}} \Delta_i$$

with

$A =$ number of nucleons;

$C =$ number of seperately connected pieces;

$L =$ number of loops;

$$\Delta_i = d_i + \frac{n_i}{2} - 2,$$

where

$d_i =$ number of derivatives,

$n_i =$ number of nucleon operators.

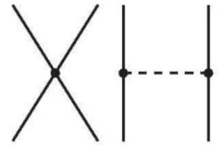
	2N forces	3N forces	4N forces

The Hierarchy of Nuclear Forces

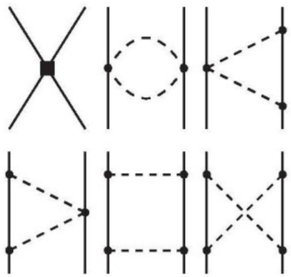
$$\text{Power} = -2 + 2A - 2C + 2L + \sum_{\text{all vertices}} \Delta_i$$

$$\text{with } \Delta_i = d_i + \frac{n_i}{2} - 2$$

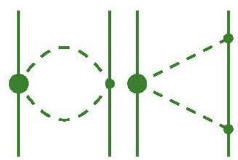
Leading Order — Q^0 LO



Next-to Leading Order — Q^2 NLO



Next-to-Next-to Leading Order — Q^3 N^{2}LO



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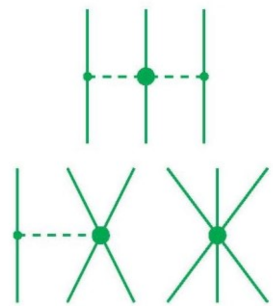
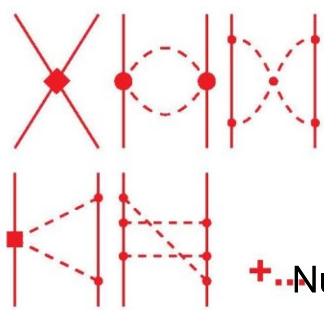


Next-to-Next-to-Next-to Leading Order — Q^4 N^{3}LO



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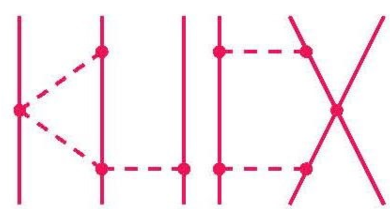

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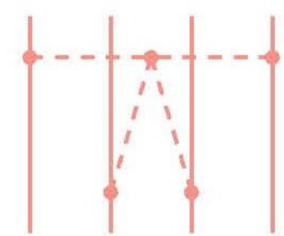

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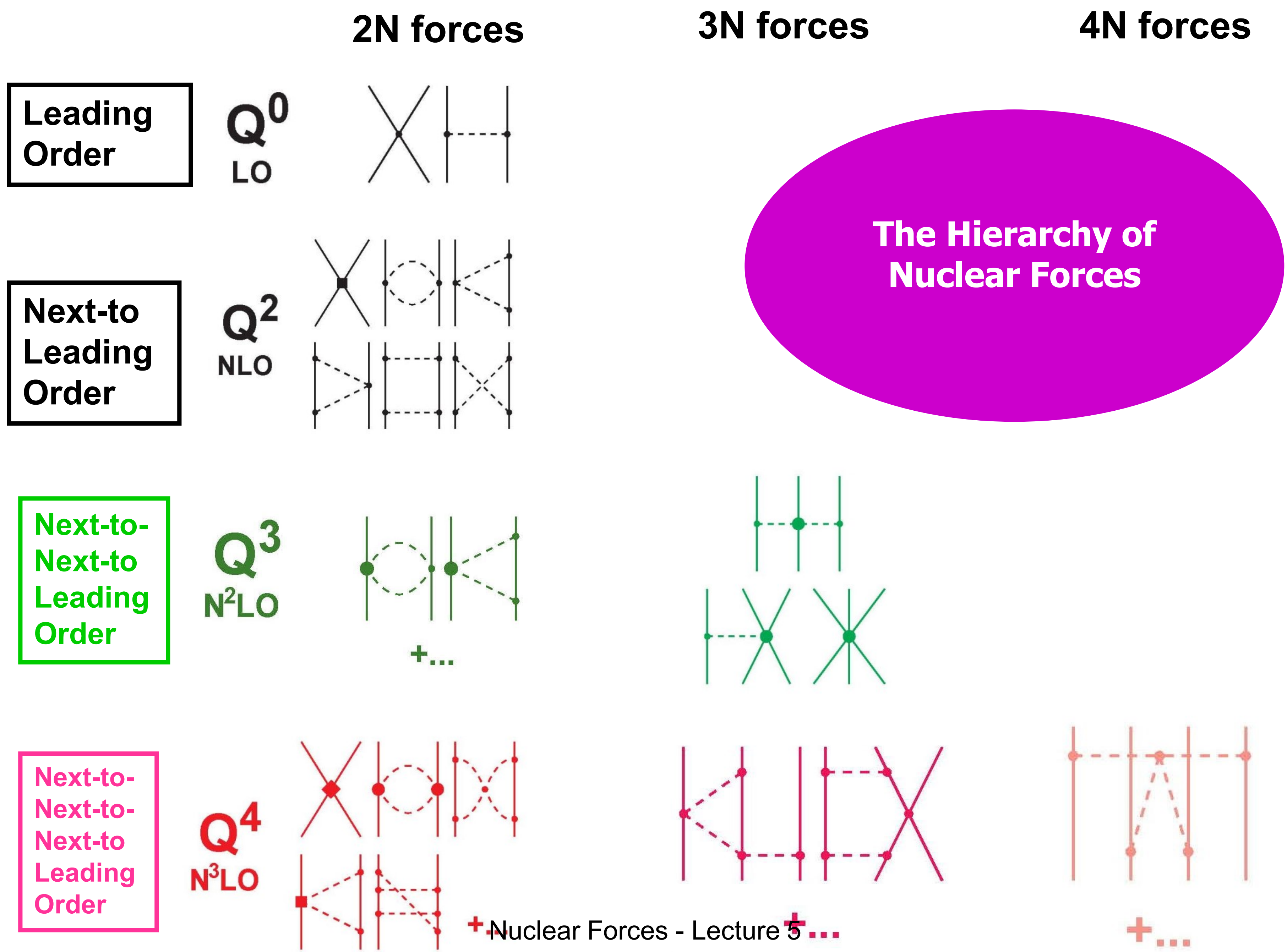
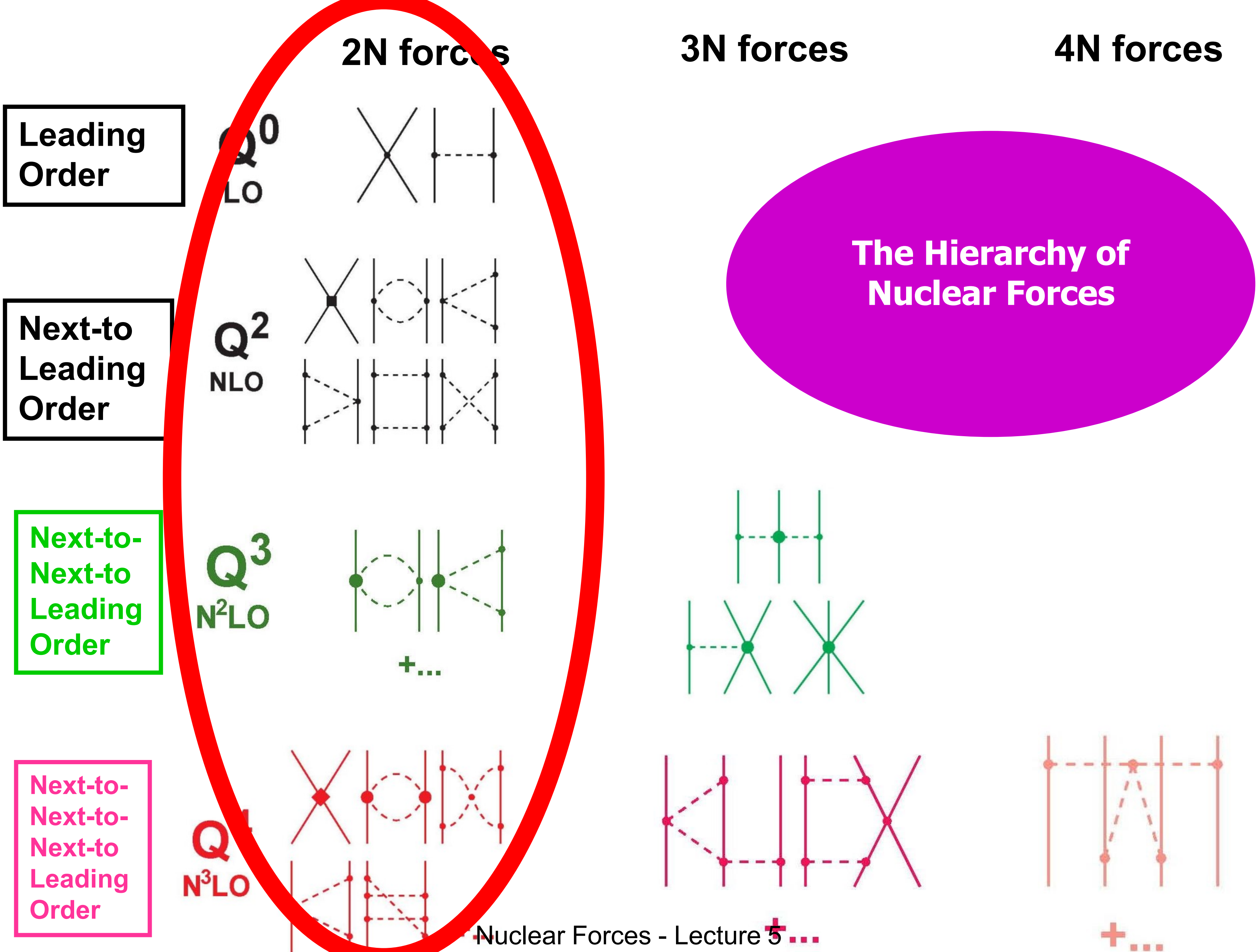
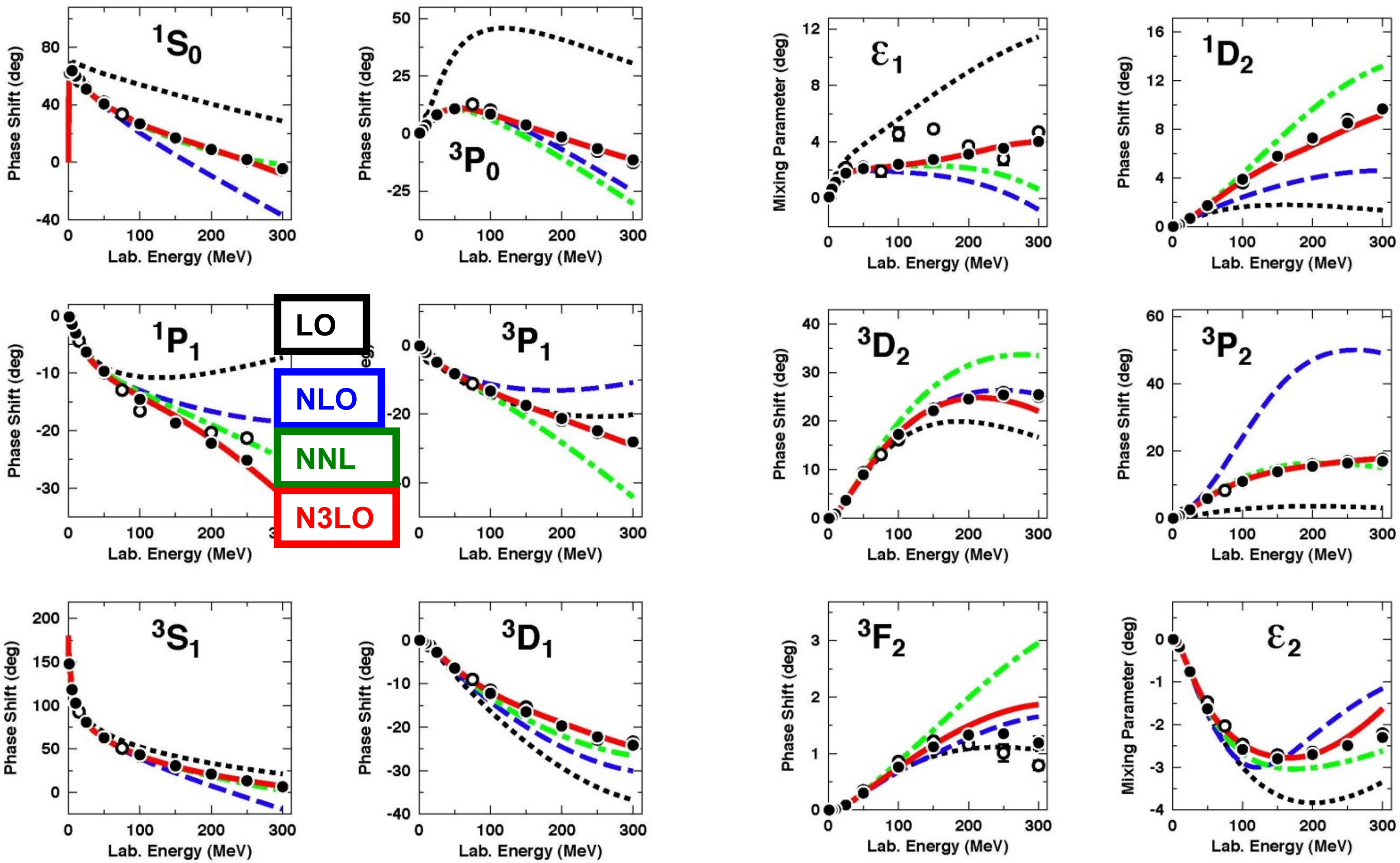

2N forces
3N forces
4N forces
Leading Order
Q0
LO
The Hierarchy of Nuclear Forces
Next-to Leading Order
Q2
NLO
Next-to-Next-to Leading Order
Q3
N2LO
+...
Next-to-Next-to-Next-to Leading Order
Q4
N3LO
+...
+...
+...


2N forces
3N forces
4N forces
Leading Order
Q^0
LO
Next-to Leading Order
Q^2
NLO
Next-to-Next-to Leading Order
Q^3
N^2LO
+...
Next-to-Next-to-Next-to Leading Order
Q^4
N^3LO
+...
+...
The Hierarchy of Nuclear Forces

NN phase shifts up to 300 MeV

Red Line: N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
Green dash-dotted line: NNLO Potential, and
blue dashed line: NLO Potential
by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).



χ^2/datum for the reproduction of the 1999 *np* database

Bin (MeV)	# of data	N^3LO	NNLO	NLO	AV18
0–100	1058	1.05	1.7	4.5	0.95
100–190	501	1.08	22	100	1.10
190–290	843	1.15	47	180	1.11
0–290	2402	1.10	20	86	1.04

N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
NNLO and NLO Potentials by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).

Summary: χ^2/datum

- NLO: ≈ 100
- NNLO: ≈ 10
- N3LO: ≈ 1

Great rate of convergence of ChPT!

Parameters

- pi-pi Lagrangian: $\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4} \operatorname{Tr}\left[\partial_\mu U \partial^\mu U^+ + m_\pi^2 \left(U + U^+\right)\right];$

  m_π and f_π are fixed ($f_\pi = 92.4$ MeV). No free parameters.

- pi-N Lagrangians: $\mathrm{L}^{(1)}_{\pi N} = \bar{N}\left[i\partial_0 - \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_0\boldsymbol{\pi}) - \frac{g_A}{2f_\pi}\boldsymbol{\tau}\cdot(\vec{\sigma}\cdot\vec{\nabla})\boldsymbol{\pi}\right]N$

  $g_A = 1.29$, no free parameters.

  $\mathrm{L}^{(2)}_{\pi N}$: 4 parameters.

  $\mathrm{L}^{(3)}_{\pi N}$: 4 parameters.

  } In principal fixed by pi-N data; but cf. Table on next slide.

- N-N Lagrangian ("Contacts"): 2+7+15=24 essentially free parameters.

The free parameters are used to adjust the potential to the empirical NN phase shifts and data.

Parameters, cont'd
pi-N Lagrangian parameters

TABLE I. Low-energy constants applied in the N^{3}LO NN potential (column 'NN'). The c_i belong to the dimension-two πN Lagrangian and are in units of GeV^{-1}, while the $\bar{d}_i$ are associated with the dimension-three Lagrangian and are in units of GeV^{-2}. The column 'πN' shows values determined from πN data.

		NN	πN
$L^{(2)}_{\pi N}$:	c_1	–0.81	-0.81 ± 0.15[a]
	c_2	2.80	3.28 ± 0.23[b]
	c_3	–3.20	-4.69 ± 1.34[a]
	c_4	5.40	3.40 ± 0.04[a]
$L^{(3)}_{\pi N}$:	$\bar{d}_1 + \bar{d}_2$	3.06	3.06 ± 0.21[b]
	$\bar{d}_3$	–3.27	-3.27 ± 0.73[b]
	$\bar{d}_5$	0.45	0.45 ± 0.42[b]
	$\bar{d}_{14} - \bar{d}_{15}$	–5.65	-5.65 ± 0.41[b]

Parameters, cont' d.

of contact parameters compared to phenomenological fits

NUMBER OF PARAMETERS
for the *np* potential

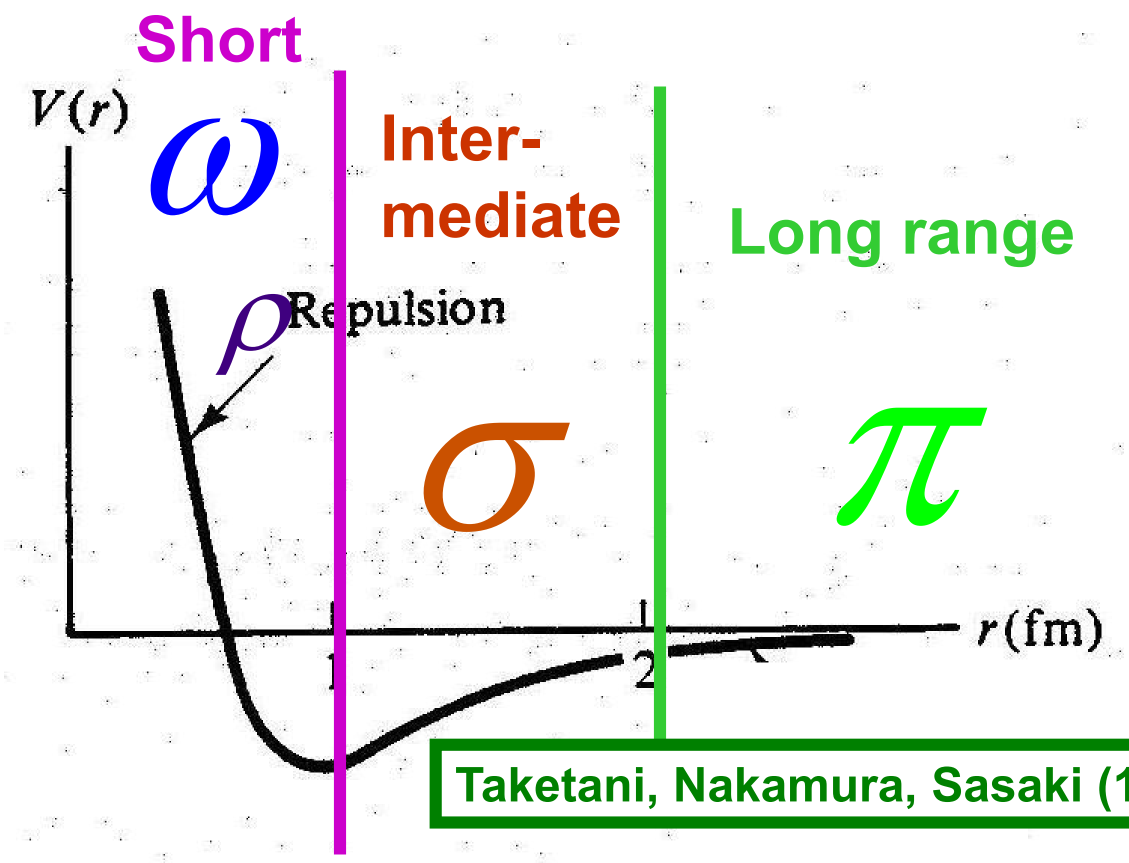
	Nijmegen PWA93	CD-Bonn "high precision"	NLO Q^2 (NNLO)	N^3LO Q^4 (N^4LO)	N^5LO Q^6
$^1S_0$	3	4	2	4	6
$^3S_1$	3	4	2	4	6
$^3S_1$-$^3D_1$	2	2	1	3	6
$^1P_1$	3	3	1	2	4
$^3P_0$	3	2	1	2	4
$^3P_1$	2	2	1	2	4
$^3P_2$	3	3	1	2	4
$^3P_2$-$^3F_2$	2	1	0	1	3
$^1D_2$	2	3	0	1	2
$^3D_1$	2	1	0	1	2
$^3D_2$	2	2	0	1	2
$^3D_3$	1	2	0	1	2
$^3D_3$-$^3G_3$	1	0	0	0	1
$^1F_3$	1	1	0	0	1
$^3F_2$	1	2	0	0	1
$^3F_3$	1	2	0	0	1
$^3F_4$	2	1	0	0	1
$^3F_4$-$^3H_4$	0	0	0	0	0
$^1G_4$	1	0	0	0	0
$^3G_3$	0	1	0	0	0
$^3G_4$	0	1	0	0	0
$^3G_5$	0	1	0	0	0
Total	35	38	9	24	50

How does the chiral EFT approach compare to conventional meson theory?

Main differences

- **Chiral perturbation theory (ChPT) is an expansion in terms of small momenta.**
- **Meson theory is an expansion in terms of ranges (masses).**

The nuclear force in the meson picture



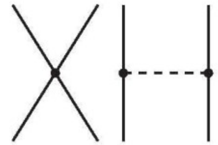
Taketani, Nakamura, Sasaki (1951): 3 ranges

2N forces

Leading Order

Q^0

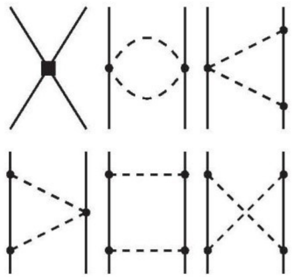
LO



Next-to Leading Order

Q^2

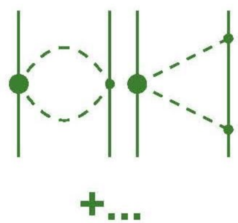
NLO



Next-to-Next-to Leading Order

Q^3

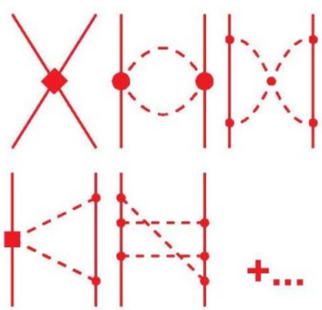
N^2LO

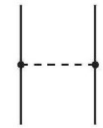
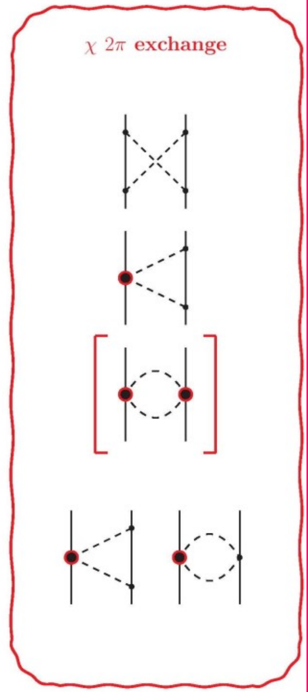
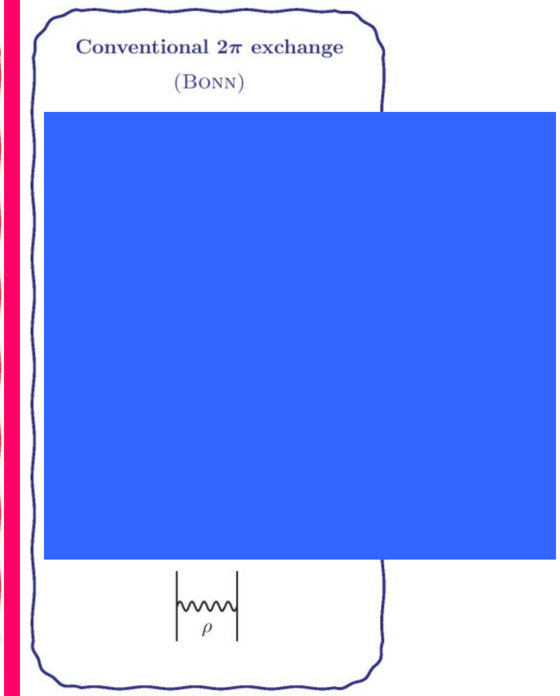
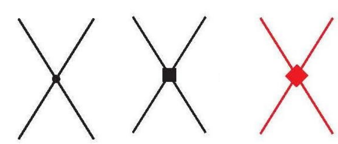
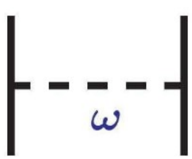


Next-to-Next-to-Next-to Leading Order

Q^4

N^3LO



	ChPT	Conventional meson theory
OPE		
TPE		
Short range		

<u>Question:</u> When everything is so equivalent to conventional meson theory, why not continue to use conventional meson theory?

- **Chiral EFT claims to be a theory, while "meson theory" is a model.**
- **Chiral EFT has a clear connection to QCD, while the QCD-connection of the meson model is more hand-woven.**
- **In ChPT, there is an organizational scheme ("power counting") that allows to estimate the size of the various contributions and the uncertainty at a given order (i.e., the size of the contributions we left out).**
- **Two- and many-body force contributions are generated on an equal footing in ChPT.**

End Lecture 5