Physics Department, Tohoku University,
June 30 – July 2, 2014

Nuclear Forces

- Lecture 6 -

Nuclear many-body forces from chiral EFT

R. Machleidt
University of Idaho

Nuclear Forces
- Overview of all lectures -

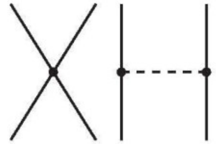
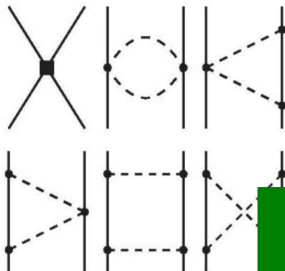
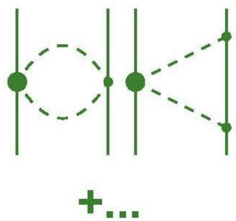
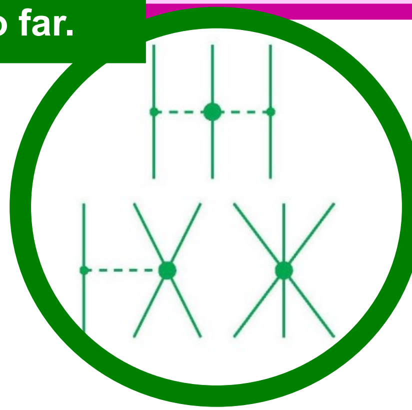
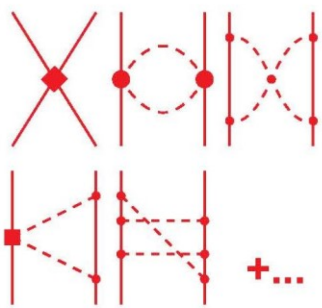
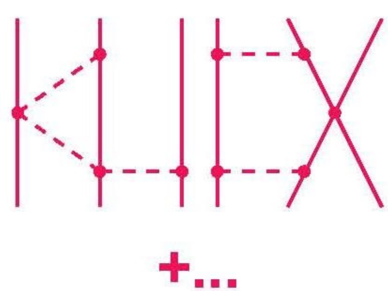
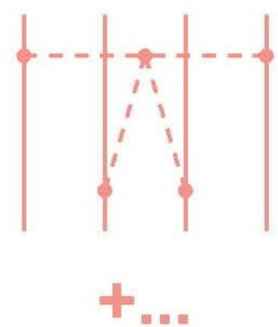
- Lecture 1: Historical perspective
- Lecture 2: Properties and phenomenology of the nuclear force
- Lecture 3: The meson theory of nuclear forces
- Lecture 4: QCD and nuclear forces; the symmetries of low-energy QCD
- Lecture 5: Effective field theory (EFT) for low-energy QCD and nuclear two-body forces
- Lecture 6: Nuclear many-body forces from chiral EFT

Lecture 6: Nuclear many-body forces

- Beyond the two-nucleon force: 3NFs
- Applications in nuclear structure
- More on many-nucleon forces
- Conclusions

Three-nucleon forces (3NF)

The Hierarchy of Nuclear Forces

		2N forces	3N forces	4N forces
Leading Order	Q^0 LO			
Next-to Leading Order	Q^2 NLO			
Next-to-Next-to Leading Order	Q^3 N^2LO			
Next-to-Next-to-Next-to Leading Order	Q^4 N^3LO			

$$\text{Power} = -2 + 2A - 2C + 2L + \sum_{\text{all vertices}} \Delta_i$$

ith $\Delta_i = d_i + \frac{n_i}{2} - 2$

The 3NF at NNLO; used so far.

Chiral 3N Force

Now, showing only 3NF diagrams.

$$\text{Power} = -2 + 2A - 2C + 2L + \sum_{\text{all vertices}} \Delta_i$$

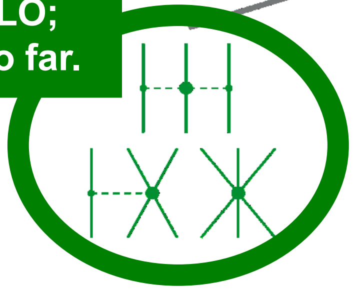
$$\text{with } \Delta_i = d_i + \frac{n_i}{2} - 2$$

LO
$(Q/\Lambda_\chi)^0$

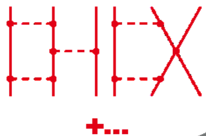
NLO
$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.

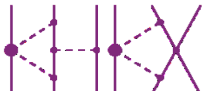
NNLO
$(Q/\Lambda_\chi)^3$



N^{3}LO
$(Q/\Lambda_\chi)^4$

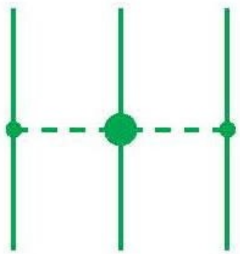


N^{4}LO
$(Q/\Lambda_\chi)^5$



Three-nucleon forces at NNLO

TPE-3NF



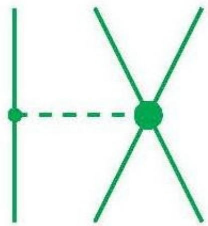
$$V^{\rm 3NF}_{\rm TPE} = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{1}{2} \sum_{i\neq j\neq k} \frac{(\vec{\sigma}_i\cdot\vec{q}_i)(\vec{\sigma}_j\cdot\vec{q}_j)}{(q_i^2+m_\pi^2)(q_j^2+m_\pi^2)} F^{\alpha\beta}_{ijk}\,\tau_i^\alpha \tau_j^\beta$$

with $\vec{q}_i \equiv \vec{p_i}' - \vec{p}_i$, where $\vec{p}_i$ and $\vec{p_i}'$ are the initial and final momenta of nucleon i, respectively, and

$$F^{\alpha\beta}_{ijk} = \delta^{\alpha\beta}\left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2}\,\vec{q}_i\cdot\vec{q}_j\right] + \frac{c_4}{f_\pi^2}\sum_\gamma \epsilon^{\alpha\beta\gamma}\,\tau_k^\gamma\,\vec{\sigma}_k\cdot[\vec{q}_i\times\vec{q}_j]$$

No new parameters!

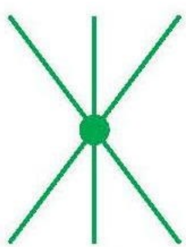
OPE-3NF



$$V^{\rm 3NF}_{\rm OPE} = D\,\frac{g_A}{8f_\pi^2} \sum_{i\neq j\neq k} \frac{\vec{\sigma}_j\cdot\vec{q}_j}{q_j^2+m_\pi^2}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)(\vec{\sigma}_i\cdot\vec{q}_j)$$

One new parameter, D.

Contact-3NF



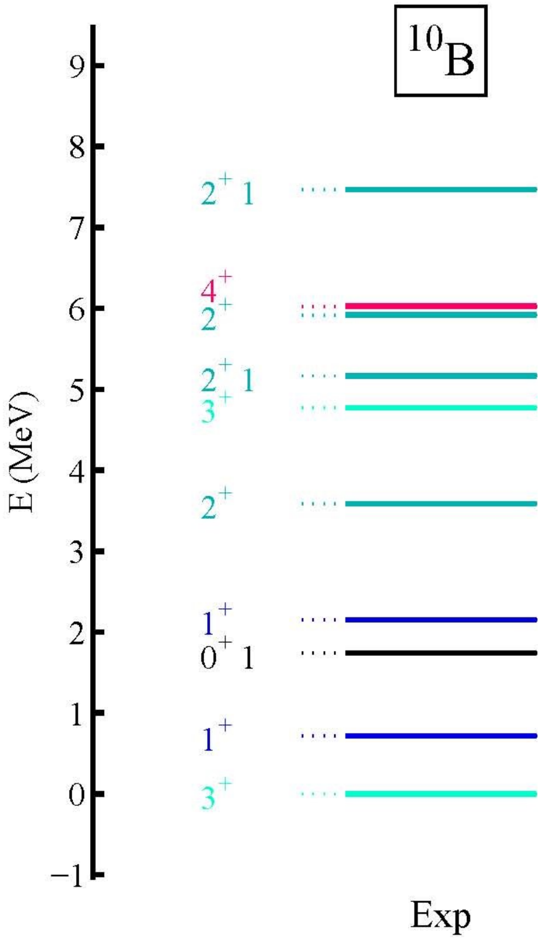
$$V^{\rm 3NF}_{\rm ct} = E\,\frac{1}{2}\sum_{j\neq k} \boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k\,.$$

One new parameter, E.

Strategy: Adjust D and E to two few-nucleon observables, e.g., the triton and alpha-particle binding energies. Then predict properties of other light nuclei.

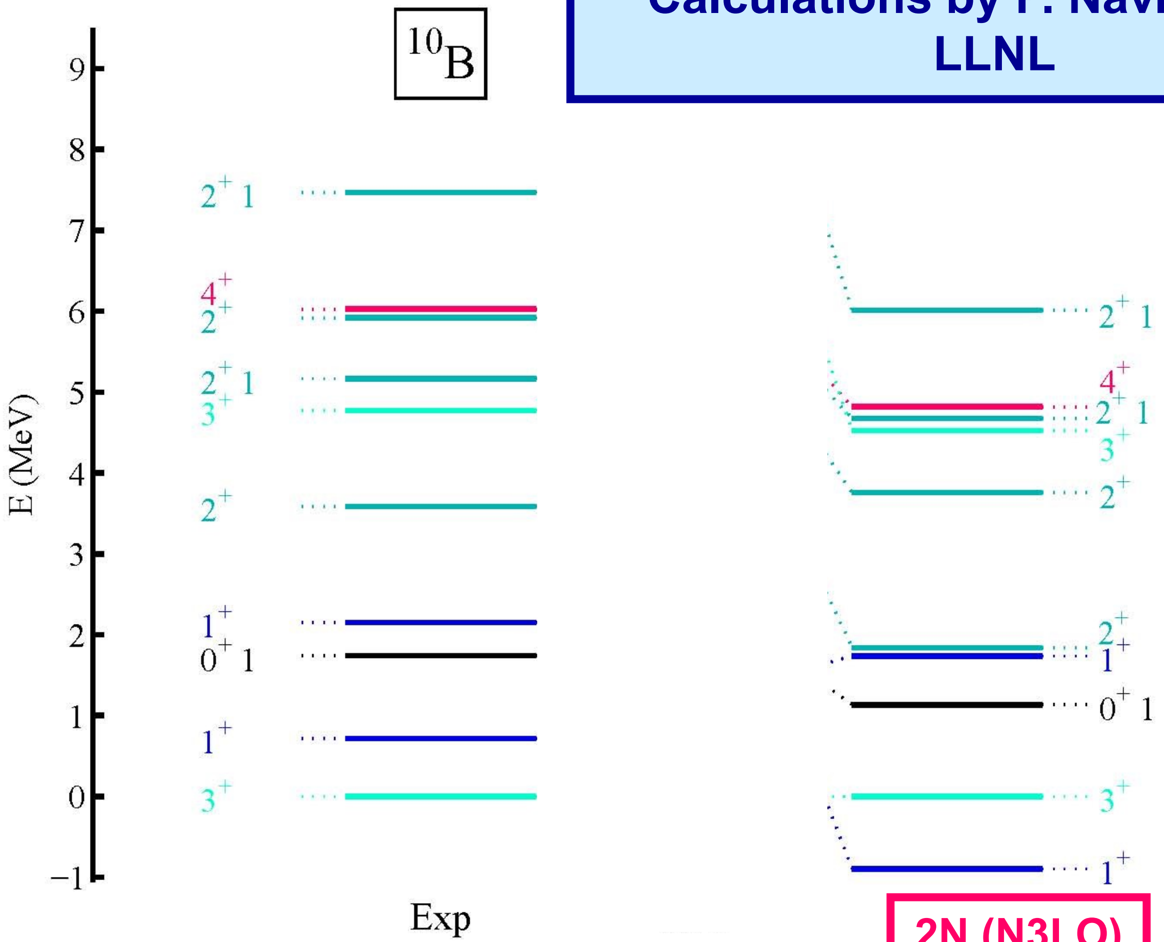
Calculating the properties of light nuclei using chiral 2N and 3N forces

"No-Core Shell Model " Calculations by P. Navratil et al., LLNL



Calculating the properties of light nuclei using chiral 2N and 3N forces

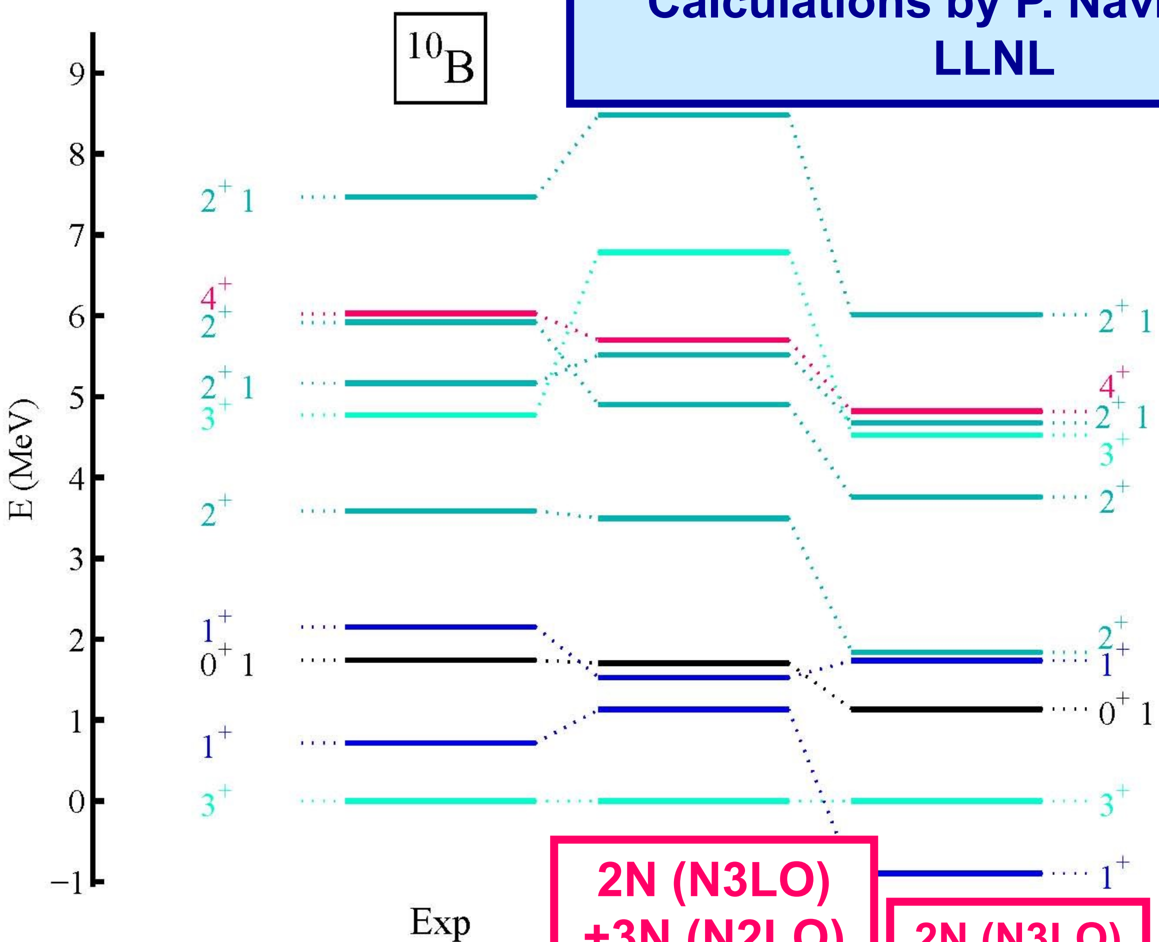
"No-Core Shell Model " Calculations by P. Navratil et al., LLNL



2N (N3LO) force only

Calculating the properties of light nuclei using chiral 2N and 3N forces

"No-Core Shell Model " Calculations by P. Navratil et al., LLNL



2N (N3LO) +3N (N2LO) forces

2N (N3LO) force only

PRL **108,** 242501 (2012) PHYSICAL REVIEW LETTERS week ending 15 JUNE 2012

Continuum Effects and Three-Nucleon Forces in Neutron-Rich Oxygen Isotopes

G. Hagen,[1,2] M. Hjorth-Jensen,[3,4,5] G. R. Jansen,[3] R. Machleidt,[6] and T. Papenbrock[2,1]

PRL **109,** 032502 (2012) PHYSICAL REVIEW LETTERS week ending 20 JULY 2012

Evolution of Shell Structure in Neutron-Rich Calcium Isotopes

G. Hagen,[1,2] M. Hjorth-Jensen,[3,4] G. R. Jansen,[3] R. Machleidt,[5] and T. Papenbrock[1,2]

Oxygen

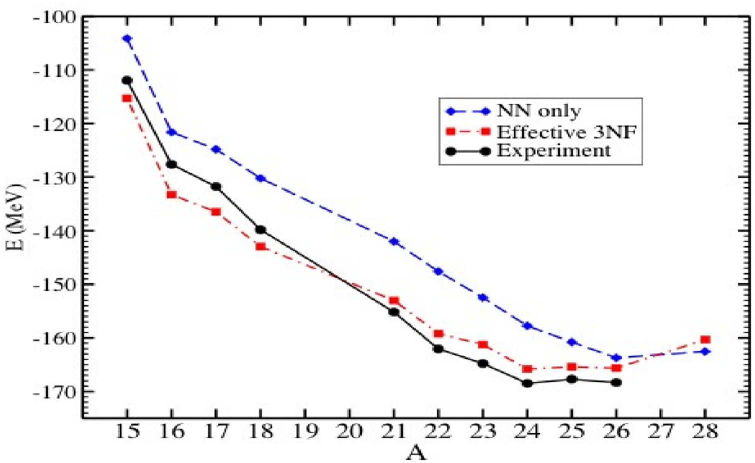


FIG. 1: (Color online) Ground-state energy of the oxygen isotope ^{A}O as a function of the mass number A. Black circles: experimental data; blue diamonds: results from nucleon-nucleon interactions; red squares: results including the effects of three-nucleon forces.

Calcium

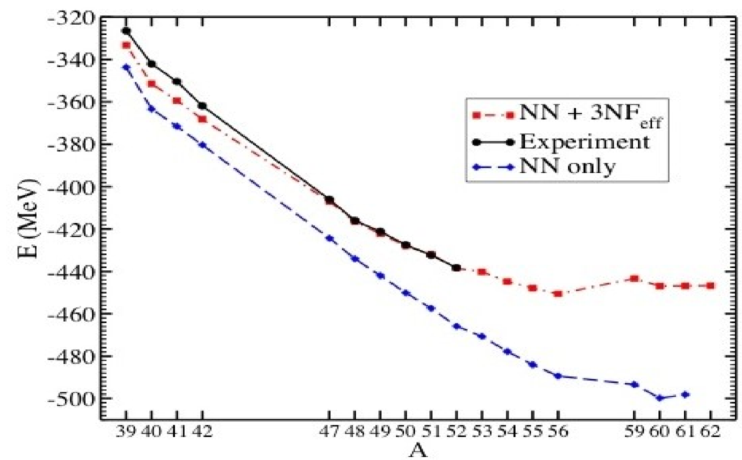


FIG. 1: (Color online) Ground-state energy of the calcium isotopes as a function of the mass number A. Black circles: experimental data; red squares: theoretical results including the effects of three-nucleon forces; blue diamonds: predictions from chiral NN forces alone. The experimental results for ^{51,52}Ca are from Ref. [34].

Analyzing Power Ay

p-d

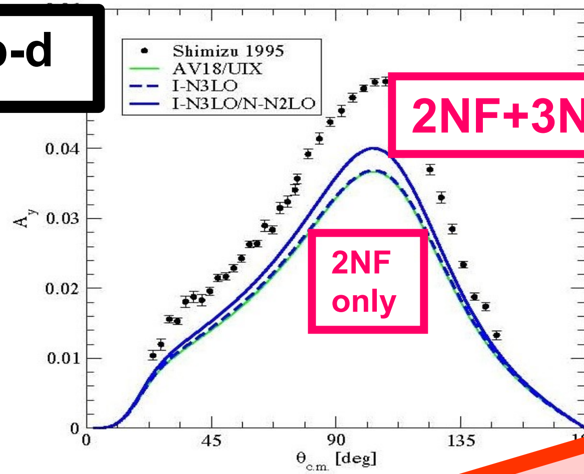


Fig. 9. $p-d$ observable A_y at E_p = 3 M
N3LO (blue dashed line), I-N3L
AV18/UIX (thin green
MeV. The exp

Calculations by the Pisa Group

The Ay puzzle is NOT solved by the 3NF at NNLO.

p-^{3}He

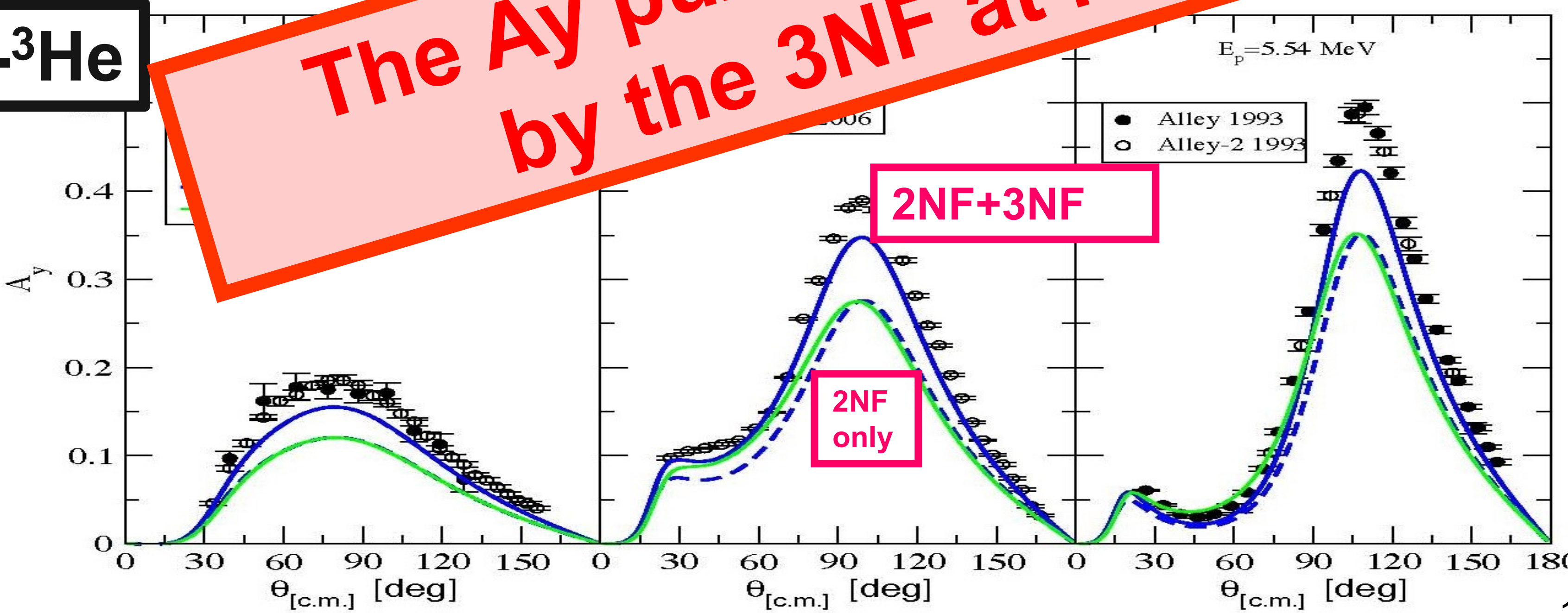


Fig. 6. $p-{}^3$He A_y observable calculated with the I-N3LO (blue dashed line), the I-N3LO/N-N2LO (blue solid line), and the AV18/UIX (thin green solid line) interaction models for three different incident proton energies. The experimental data are from Refs. [37,22,36].

And so,
we need 3NFs beyond NNLO, because …

- **The 2NF is N3LO;**
  consistency requires that all contributions are included up to the same order.

- **There are unresolved problems in 3N and 4N scattering, and nuclear structure.**

Chiral 3N Force

Δ-less

LO
$(Q/\Lambda_\chi)^0$

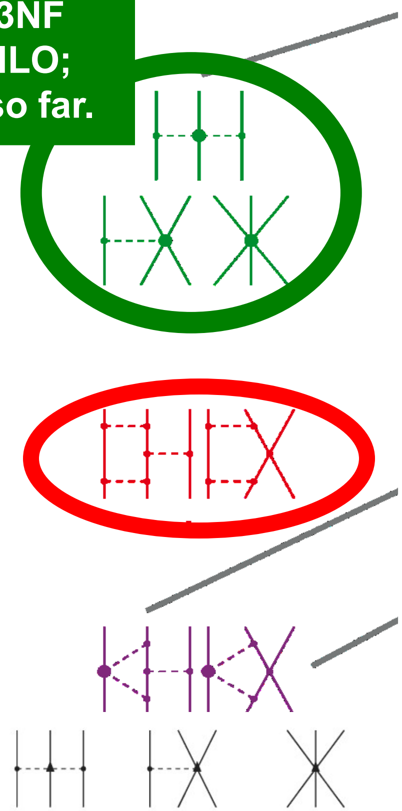
NLO
$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.

NNLO
$(Q/\Lambda_\chi)^3$

N^{3}LO
$(Q/\Lambda_\chi)^4$

N^{4}LO
$(Q/\Lambda_\chi)^5$



Chiral 3N Force

Δ-less

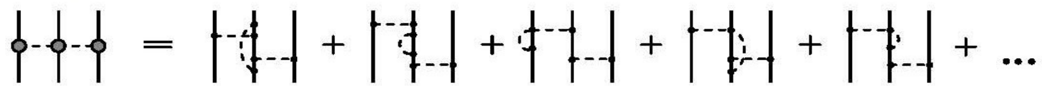
LO

$(Q/\Lambda_\chi)^0$

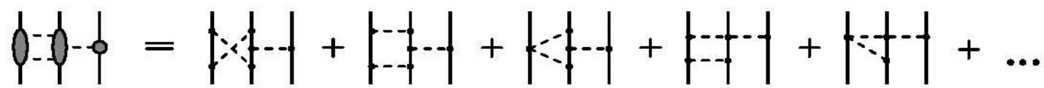
NLO

$(Q/\Lambda_\chi)^2$

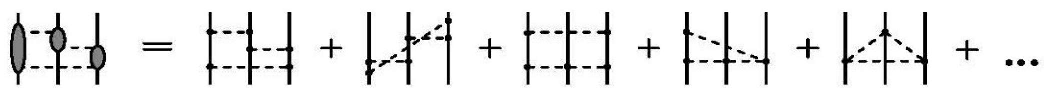
2π−exchange



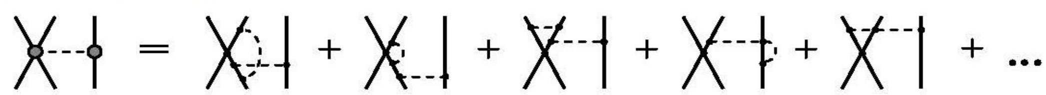
2π-1π−exchange



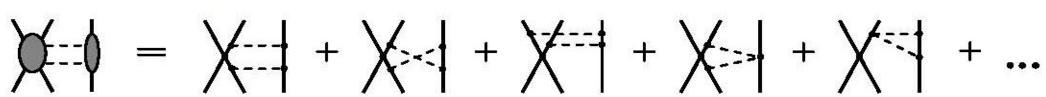
ring diagrams



contact-1π−exchange



contact-2π−exchange



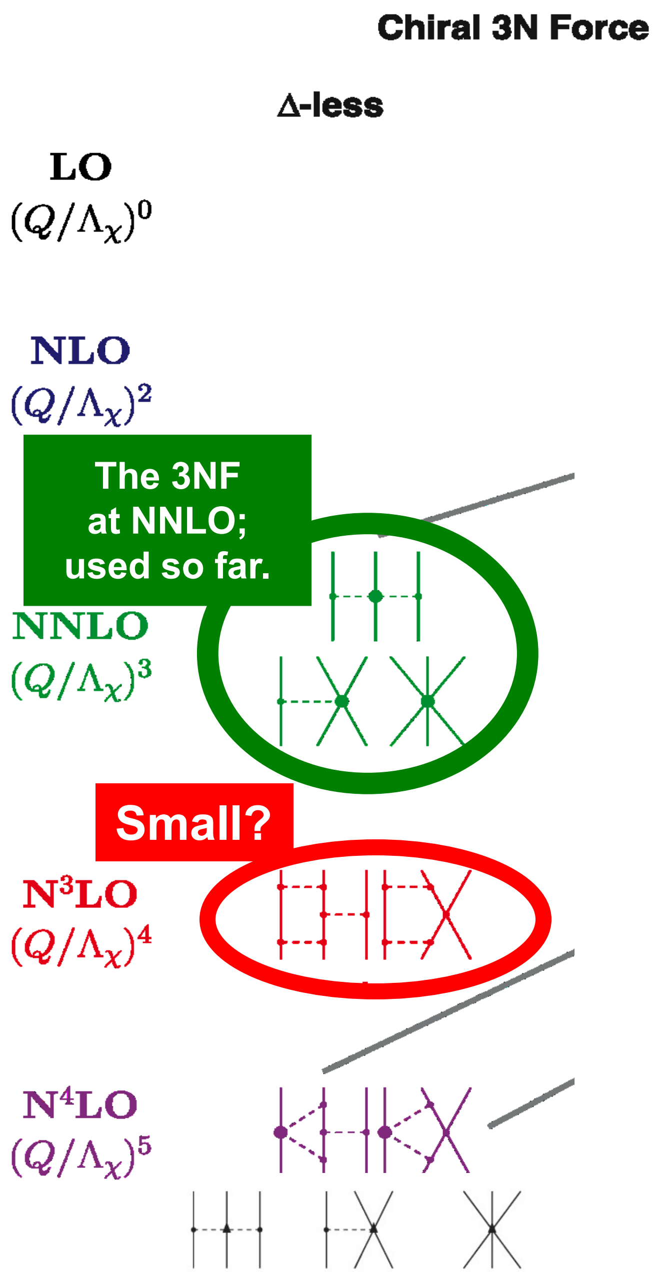
R. Machleidt

Apps of N3LO 3NF:

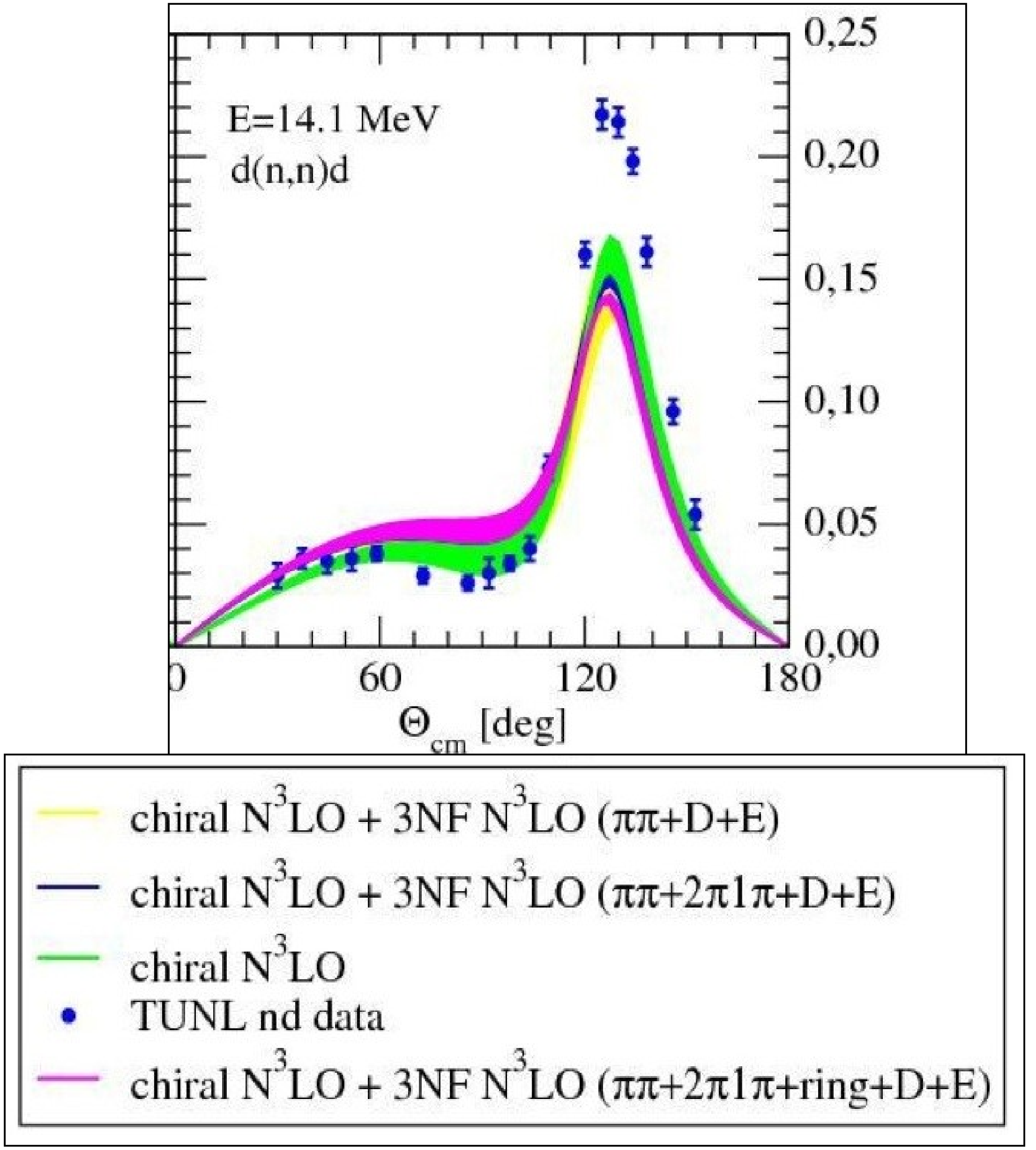
Triton: Skibinski et al., PRC 84, 054005 (2011).
Not conclusive.

Neutron matter:
Hebeler, Schwenk and co-workers,
PRL 110, 032504 (2013).
Not small!(?)

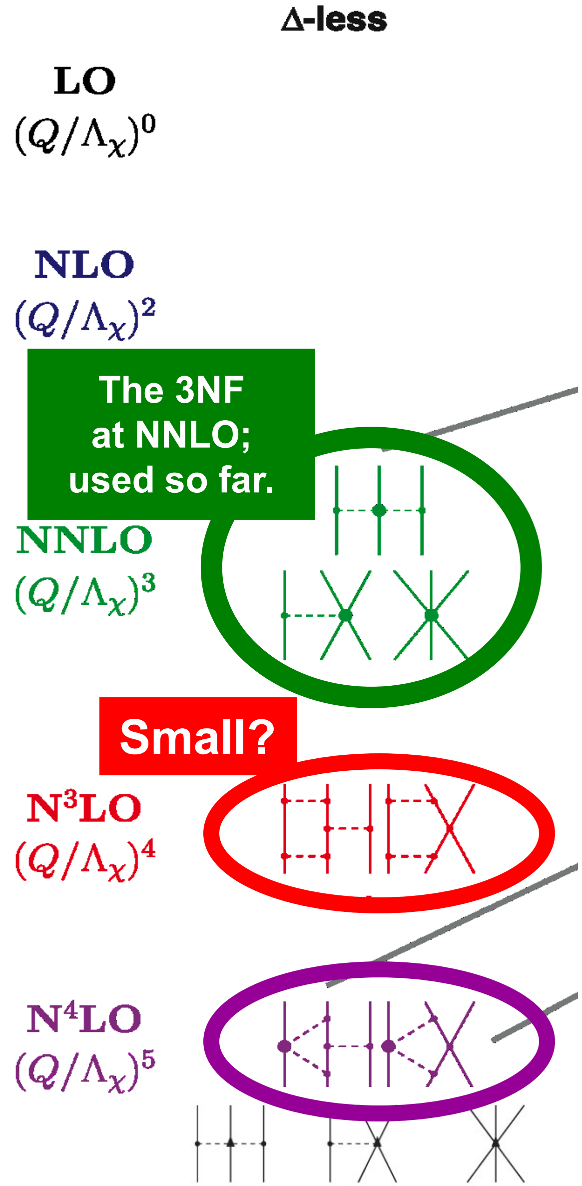
N-d scattering (Ay):
Witala et al.
Small!



N-d A_y calculations by Witala et al.



Chiral 3N Force



Chiral 3N Force

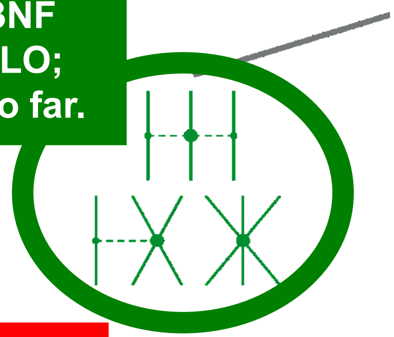
Δ-less

LO
$(Q/\Lambda_\chi)^0$

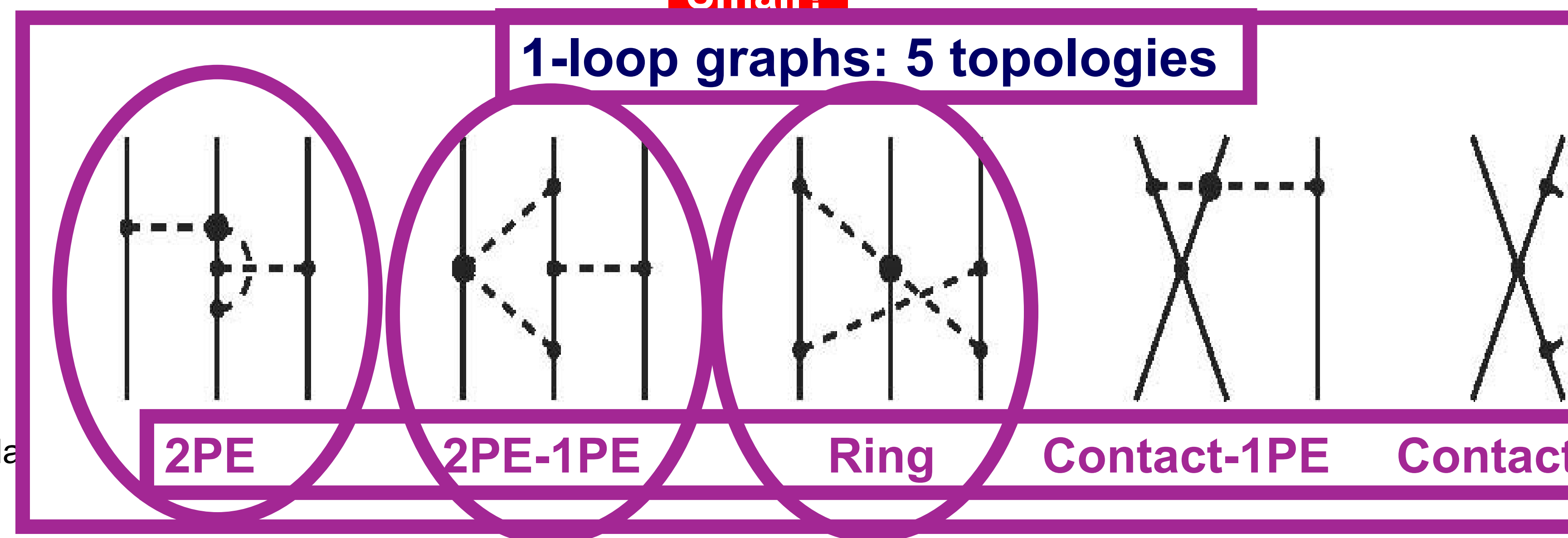
NLO
$(Q/\Lambda_\chi)^2$

The 3NF
at NNLO;
used so far.

NNLO
$(Q/\Lambda_\chi)^3$



Small?



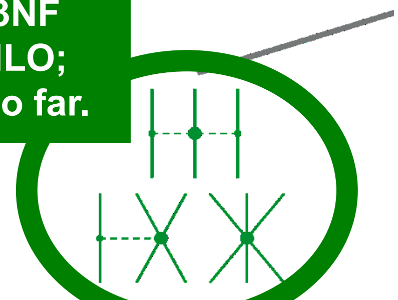
Chiral 3N Force

Δ-less

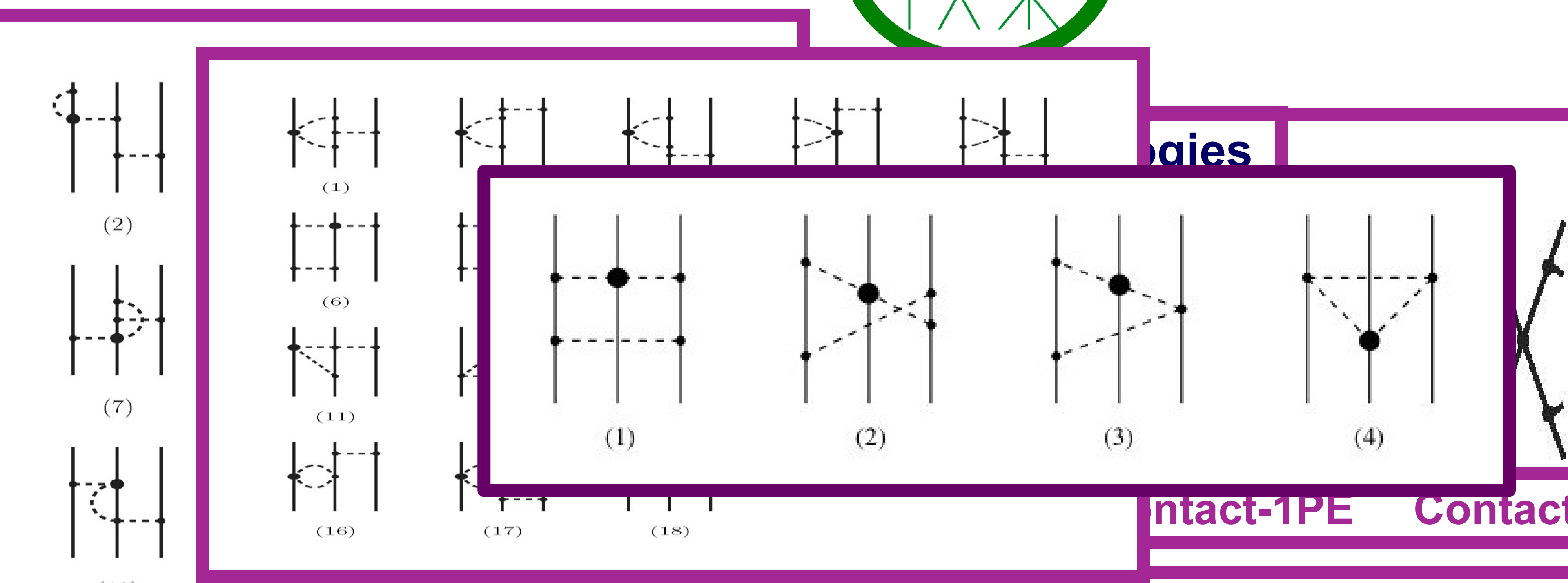
LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.



NNLO
$(Q/\Lambda_\chi)^3$



Chiral 3N Force

Δ-less

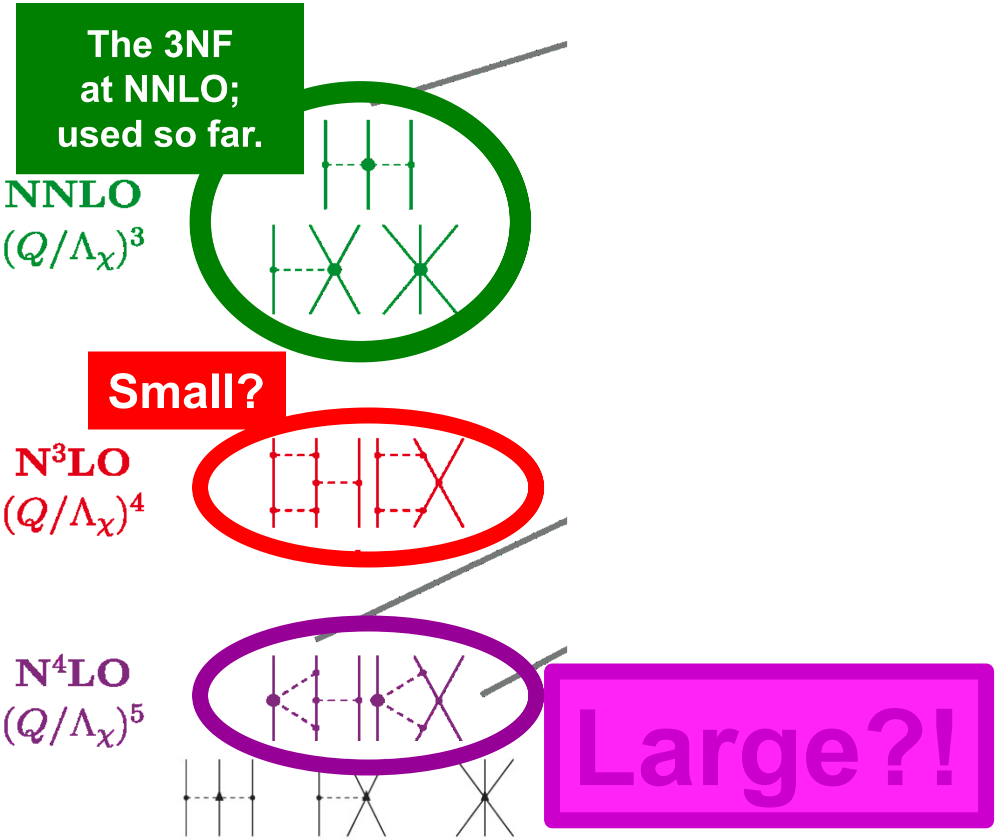
LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

NNLO
$(Q/\Lambda_\chi)^3$

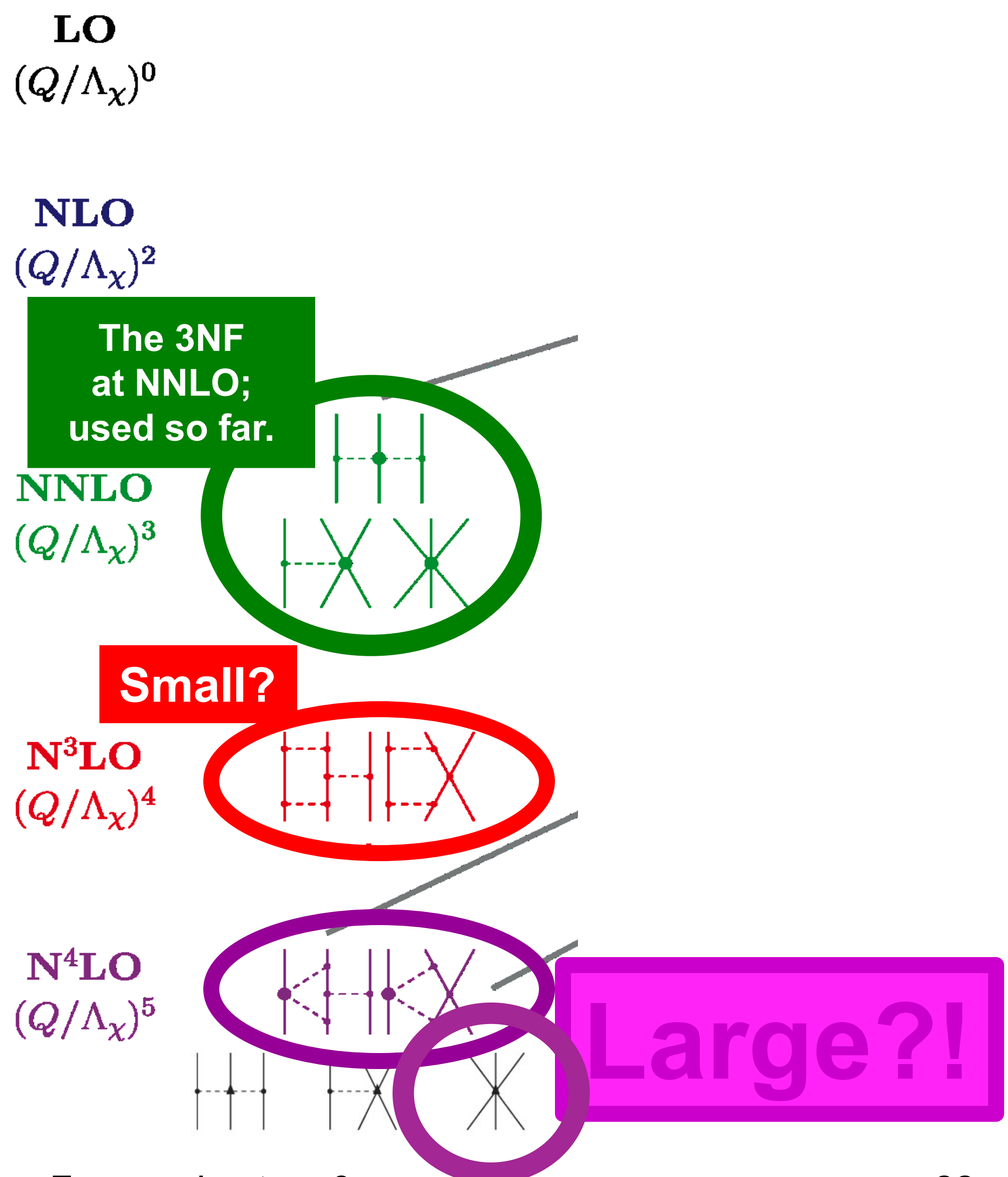
N^{3}LO
$(Q/\Lambda_\chi)^4$

N^{4}LO
$(Q/\Lambda_\chi)^5$



Chiral 3N Force

Δ-less



Chiral 3N Force

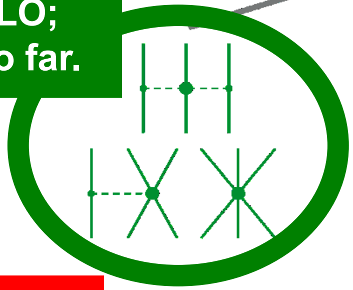
Δ-less

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.

NNLO
$(Q/\Lambda_\chi)^3$



Small?

3NF contacts at N4LO
Girlanda, Kievsky, Viviani, PRC 84, 014001 (2011)

$\mathbf{k}_i = \mathbf{p}_i - \mathbf{p}'_i$ and $\mathbf{Q}_i = \mathbf{p}_i + \mathbf{p}'_i$, $\mathbf{p}_i$ and $\mathbf{p}'_i$ being the initial and final momenta of nucleon i, the potential in mome
space is found to be

$$V = \sum_{i \neq j \neq k} \Big[-E_1 \mathbf{k}_i^2 - E_2 \mathbf{k}_i^2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j - E_3 \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j - E_4 \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$
$$- E_5 (3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j - \mathbf{k}_i^2) - E_6 (3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j - \mathbf{k}_i^2)) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$
$$+ \frac{i}{2} E_7 \mathbf{k}_i \times (\mathbf{Q}_i - \mathbf{Q}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) + \frac{i}{2} E_8 \mathbf{k}_i \times (\mathbf{Q}_i - \mathbf{Q}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k$$

Spin-Orbit Force!

R. Machleidt

Chiral 3N Force

Δ-less

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

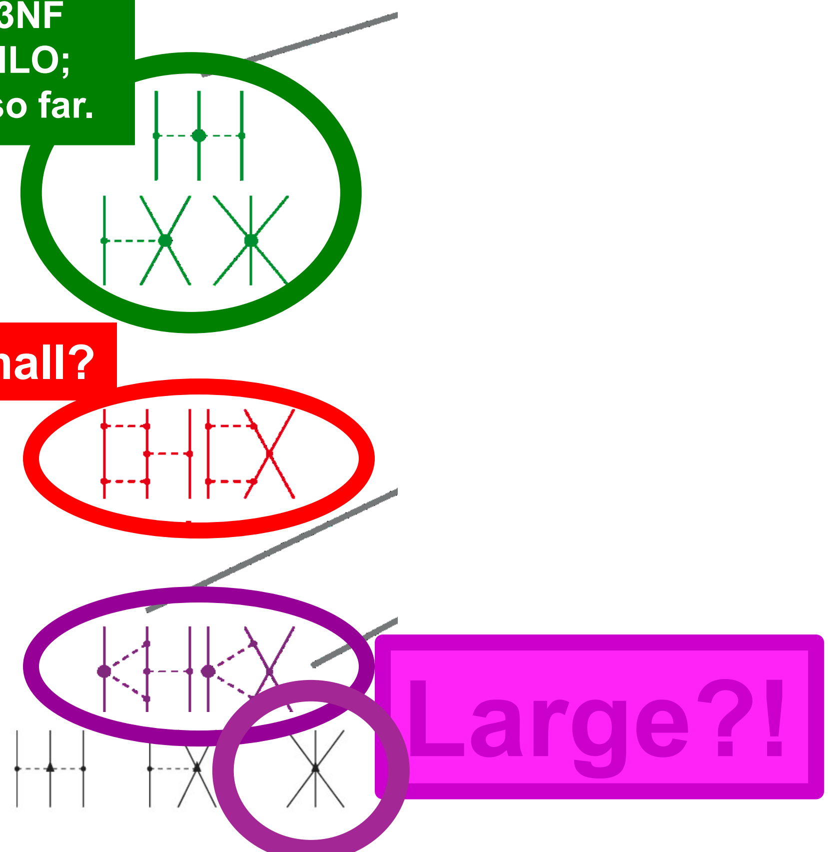
The 3NF at NNLO; used so far.

NNLO
$(Q/\Lambda_\chi)^3$

Small?

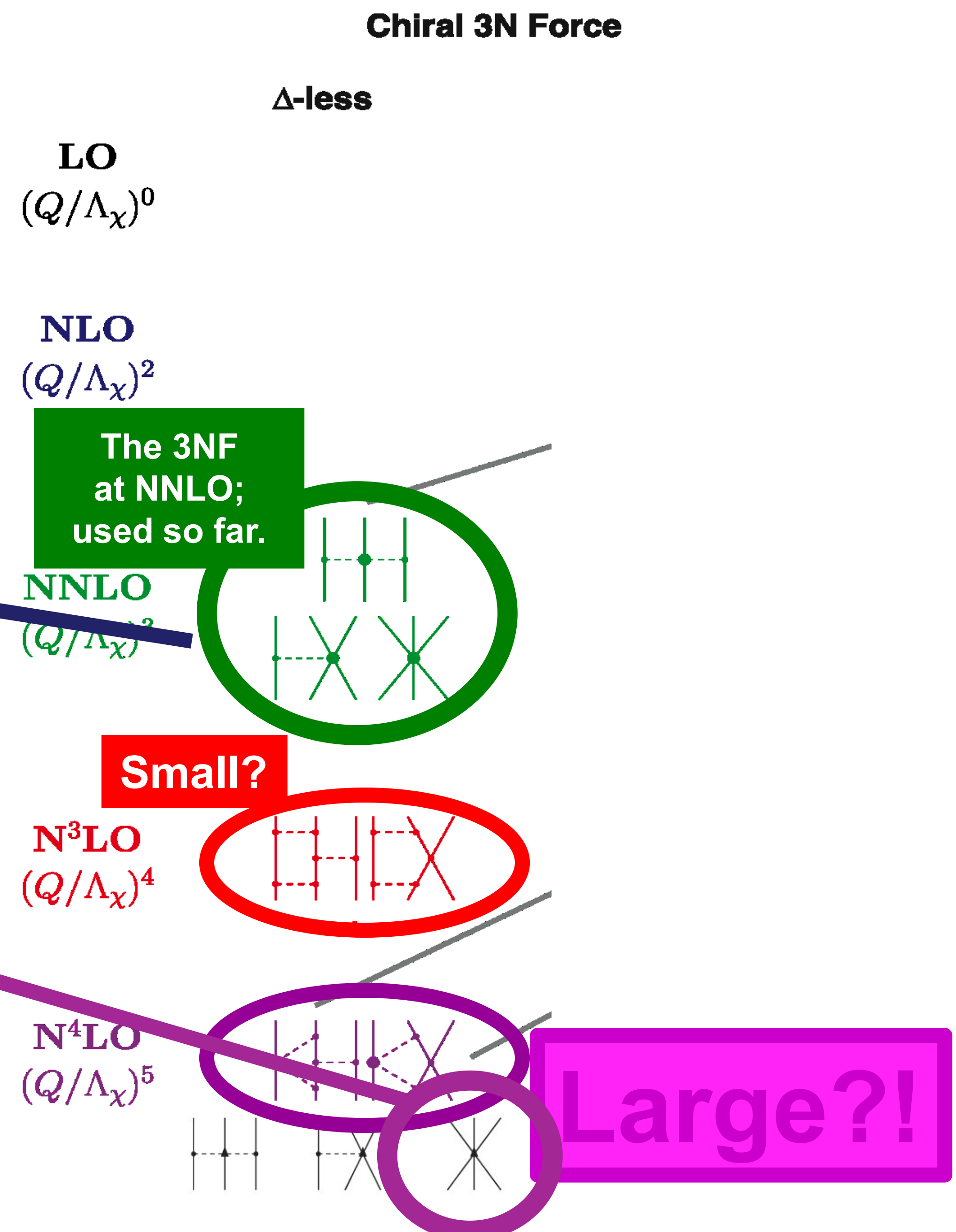
N^{3}LO
$(Q/\Lambda_\chi)^4$

N^{4}LO
$(Q/\Lambda_\chi)^5$



A realistic, investigational approach:

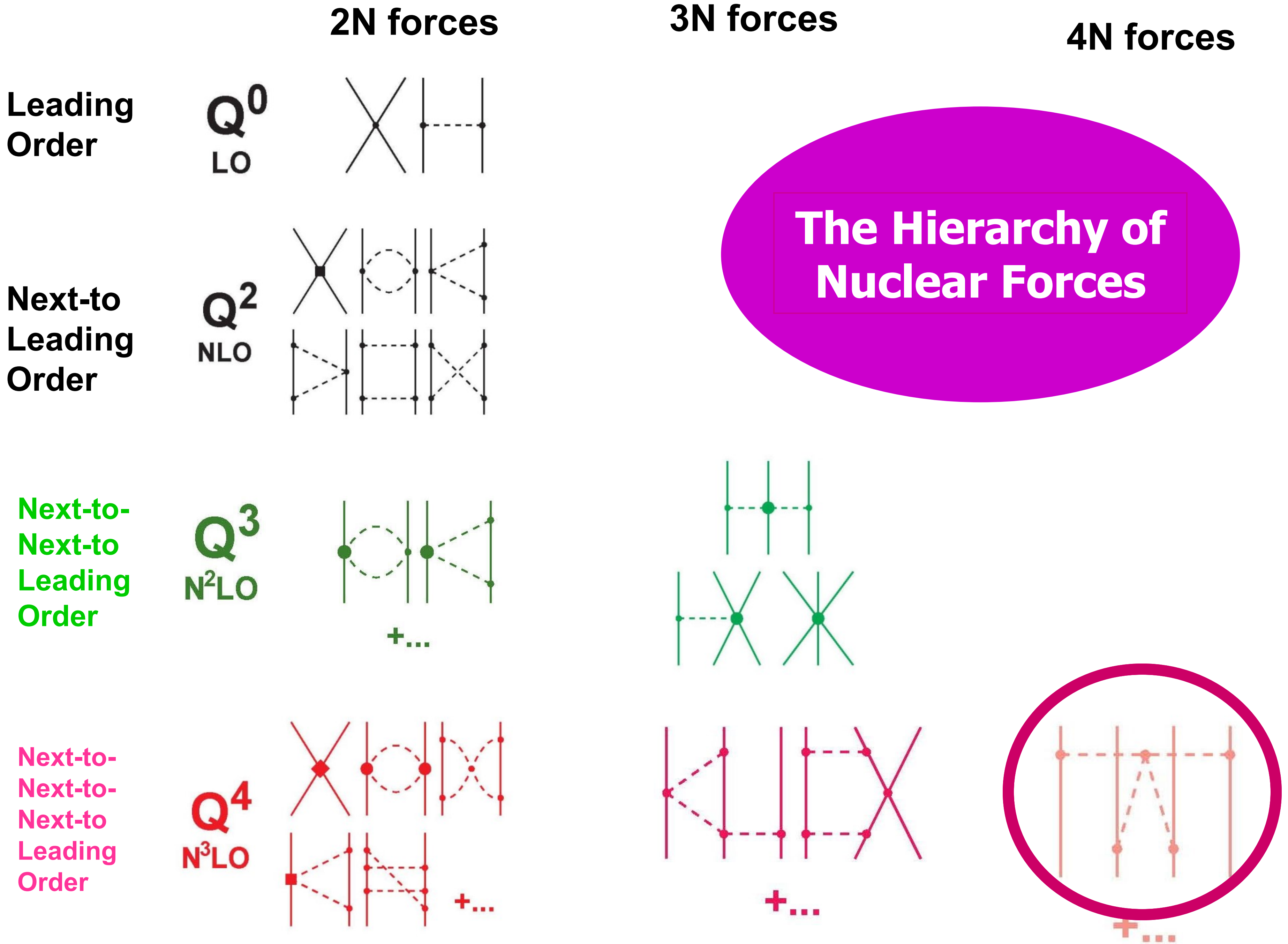
- use Δ-less
- include NNLO 3NF
- skip N3LO 3NF
- at N4LO start with contact 3NF, use one term at a time, e.g. spin-orbit
- that may already solve some of your problems.



Current problems with chiral 3NFs

- “Explosion” of 3NFs at higher orders.
- For the time being, deal with it selectively. Do not try “complete” calculations.
- Many open questions
- Will the chiral 3NF converge with increasing order?
- (This is quite in contrast to the chiral 2NF, where things are under control.)

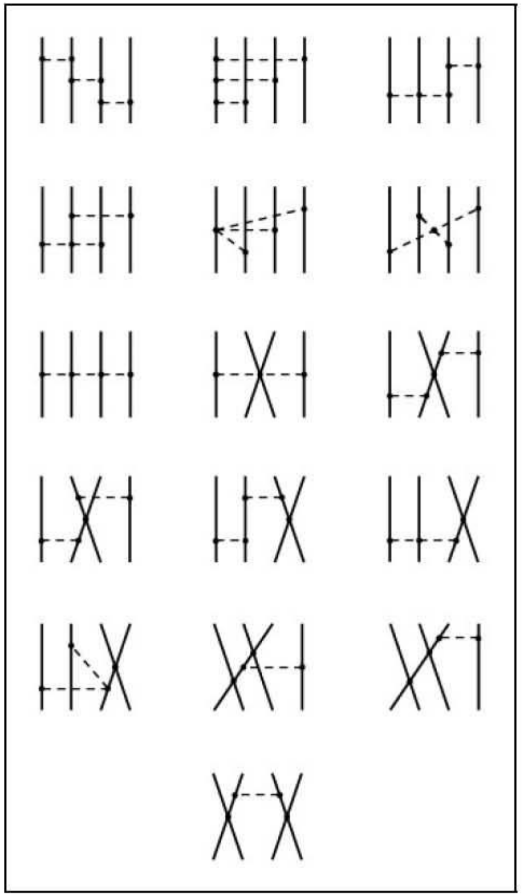
Four-nucleon forces (4NF)



NF from EFT II (Sendai'14)

4NF at N3LO (leading order)

Epelbaum, Phys. Lett. **B639** (2006) 456 [nucl-th//0511025]



Note that only vertices from

$L^{(2)}_{\pi\pi}$, $L^{(1)}_{\pi N}$ and $L^{(0)}_{NN}$ are involved,

- no new parameters,
- weak.

First rough estimate:

$\approx$ 0.1 MeV to α binding.

Conclusions

- After 80 years of struggle, we have now a proper theory for nuclear forces that is based upon the fundamental theory of strong interactions, QCD. It is chiral effective field theory for low-energy QCD, in which the pion plays a fundamental role (Goldstone boson). Thus, the original Yukawa idea that there is ONE MESON that makes the nuclear force is re-instated. But as compared to the early attempts, a crucial constraint has been added: chiral symmetry generates and controls pion-exchanges.
- Based upon chiral perturbation theory (ChPT), quantitative NN potentials have been successfully developed (at N3LO) and are applied in nuclear structure calculations.
- In ChPT, two- and many-body forces are generated on an equal footing. Concerning the 3NF, there are still some open issues including its order-by-order convergence.
- **A new and exciting era has started for microscopic nuclear structure!**

The End

Review article:
R. Machleidt and D.R. Entem,
Chiral Effective Field Theory and Nuclear Forces,
Physics Reports 503, 1 (2011).