Lambda-N and Sigma-N interactions from 2+1 lattice QCD with almost realistic masses

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for HAL QCD Collaboration

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\textsuperscript{5}University of Tours,  \textsuperscript{6}Osaka University,  
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Outline

- Introduction
  - Brief introduction of HAL QCD method
  - Importance of LN–SN tensor force for hypernuclei
- Effective block algorithm for various baryon-baryon channels,
  [arXiv:1510.00903 (hep-lat)]
- Preliminary results of LN–SN potentials at nearly physical point
  - LN–SN(I=1/2), central and tensor potentials
  - SN(I=3/2), central and tensor potentials
- Summary
Plan of research

QCD

Baryon interaction

Structure and reaction of (hyper)nuclei

Equation of State (EoS) of nuclear matter

Neutron star and supernova

J-PARC, JLab, GSI, MAMI, ...
YN scattering, hypernuclei

$A = 3$
$A = 4$
$A = 5$

$pn \Lambda$
$pnn \Lambda$, $ppn \Lambda$
$ppnn \Lambda$
What is realistic picture of hypernuclei?

\[ B(\text{total}) = B(\Lambda^4\text{He}) + B(\Lambda^5\text{He}) \]

A conventional picture:
\[
B(\text{total}) = B(\Lambda^4\text{He}) + B(\Lambda^5\text{He})
\]
\[
= 28 + 3 \text{ MeV.}
\]

A (probably realistic) picture:
\[
B(\text{total}) = (B(\Lambda^4\text{He}) - \Delta E_c) + (B(\Lambda^5\text{He}) + \Delta E_c)
\]
\[
= ?? + ??? \text{ MeV.}
\]
Comparison between $d=p+n$ and core+Y

![Comparison between $d=p+n$ and core+Y](image)

<table>
<thead>
<tr>
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<th>$T_S$ (MeV)</th>
<th>$T_D$ (MeV)</th>
<th>$V_{NN}$(central) (MeV)</th>
<th>$V_{NN}$(tensor) (MeV)</th>
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<td>−9.22</td>
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</tbody>
</table>

Rearrangement effect of $^5\Lambda$He


\[ H = \sum_{i=1}^{A} \left( m_i c^2 + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{i<j}^{A-1} v^{(NN)}_{ij} + \sum_{i=1}^{A-1} v^{(NY)}_{iY} = H_{\text{core}} + H_{\text{Y-core}}, \]

\[ H_{\text{core}} = \sum_{i=1}^{A-1} \frac{p_i^2}{2m_N} - \frac{\left( \sum_{i=1}^{A-1} p_i \right)^2}{2(A-1)m_N} + \sum_{i<j}^{A-1} v^{(NN)}_{ij} = T_{\text{core}} + V_{NN}. \]
What is realistic picture of hypernuclei?

\[ B(\text{total}) = B(4\text{He}) + B(^5\text{He}) \]

A conventional picture:
\[
B(\text{total}) = B(4\text{He}) + B(^5\text{He})
= 28 + 3 \text{ MeV.}
\]

A (probably realistic) picture:
\[
B(\text{total}) = (B(4\text{He}) - \Delta E_c) + (B(^5\text{He}) + \Delta E_c)
= 24 + 7 \text{ MeV.}
\]
Lattice QCD calculation
Multi-hadron on lattice

i) basic procedure:
   asymptotic region
   \[\rightarrow\] phase shift

ii) HAL’s procedure:
   interacting region
   \[\rightarrow\] potential
Formulation

Lattice QCD simulation

\[ L = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q \]

\[ \langle O(\bar{q}, q, U) \rangle = \int dU d\bar{q} dq e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \]
\[ = \int dU \det D(U) e^{-S_U(U)} O(D^{-1}(U)) \]
\[ = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i)) \]

\[ \langle \langle t \rangle \rangle \]
Formulation

Lattice QCD simulation

\[ L = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A^a_\mu) q - m \bar{q} q \]

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\[ = \int dU \ det \ D(U) e^{-S_U(U)} O(D^{-1}(U)) \]
\[ = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i)) \]

\[ \langle \langle \langle \text{pn} (t) \text{pn} (t_0) \rangle \rangle \rangle \]
Multi-hadron on lattice
Lattice QCD simulation

\[ L = -\frac{1}{4} G_{\mu \nu}^a G^{a \mu \nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q \]

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\[ = \int dU \det D(U) e^{-S_U(U)} O(D^{-1}(U)) \]
\[ = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i)) \]

\[ \langle p\Lambda(\mathbf{t}) p\Lambda(\mathbf{t}_0) \rangle \]
Multi-hadron on lattice
i) basic procedure:

asymptotic region
(or temporal correlation)

$\rightarrow$ scattering energy

$\rightarrow$ phase shift

\[ E = \frac{k^2}{2\mu} \]

\[ k \cot \delta_0(k) = \frac{2}{\sqrt{\pi L}} Z_{00}(1 ; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2) \]

\[ Z_{00}(1 ; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\mathbf{n} \in \mathbb{Z}^3} \frac{1}{(n^2 - q^2)^s} \quad \mathbb{R} s > \frac{3}{2} \]

An example of Luscher’s formula

\[ E = \frac{k^2}{2\mu} \]

\[ k \cot \delta_0 (k) = \frac{2}{\sqrt{\pi} L} Z_{00} \left( 1 ; \left( k L / (2\pi) \right)^2 \right) = \frac{1}{a_0} + O(k^2) \]

\[ Z_{00} (1 ; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{n \in \mathbb{Z}^3} \frac{1}{(n^2 - q^2)^s} \]

\[ \Re s \geq \frac{3}{2} \]

Multi-hadron on lattice
Lattice QCD simulation

\[ L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A^a_\mu) q - m \bar{q} q \]

\[ \langle O(\bar{q}, q, U) \rangle = \int dU \int d\bar{q} dq e^{-S(\bar{q}, q, U)} O(\bar{q}, q) \]

\[ = \int dU \det D(U) e^{-S_U(U)} O(D^{-1}(U)) \]

\[ F^{(JM)}_{\alpha\beta}(\vec{r}, t - t_0) \]

\[ \rightarrow \langle p\Lambda(\vec{r}, t) \rangle \]

Calculate the scattering state
Multi-hadron on lattice

ii) HAL’s procedure: make better use of the lattice output! (wave function) interacting region

$\rightarrow$ potential


NOTE:

$\triangleright$ Potential is not a direct experimental observable.
$\triangleright$ Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)
Multi-hadron on lattice

ii) HAL’s procedure: 
make better use of the lattice output! (wave function)

interacting region

→ potential

→ Phase shift

→ Nuclear many-body problems

The potential is obtained at moderately large imaginary time; no single state saturation is required.

\[
R^{(J,M)}_{\alpha\beta}(\vec{r}, t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X} + \vec{r}, t) B_{2,\beta}(\vec{X}, t) \overline{J}^{(J,M)}_{B_3B_4}(t_0) \right| 0 \right\rangle / \exp\{-m_{B_1} + m_{B_2}\}(t-t_0),
\]

\[
= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X} + \vec{r}, 0) B_{2,\beta}(\vec{X}, 0) \right| E_n \right\rangle e^{-(E_n - m_{B_1} + m_{B_2})(t-t_0)}
+ O(e^{-(E_{th} - m_{B_1} + m_{B_2})(t-t_0)}),
\]

(4)

where \( E_n (|E_n\rangle) \) is the eigen-energy (eigen-state) of the six-quark system and \( A_n = \sum_{\alpha'\beta'} P^{(JM)}_{\alpha'\beta'} \langle E_n | B_{4,\alpha'} B_{3,\alpha'} | 0 \rangle \). At moderately large \( t - t_0 \) where the inelastic contribution above the pion production \( O(e^{-(E_{th} - 2m_N)(t-t_0)}) = O(e^{-m_\pi(t-t_0)}) \) becomes exiguous, we can construct the non-local potential \( U \) through \( \left( \frac{\nabla^2}{2\mu} - \frac{k^2}{2\mu} \right) R(\vec{r}) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}') \). In lattice QCD calculations

\footnote{The potential is obtained from the NBS wave function at moderately large imaginary time; it would be \( t - t_0 \gg 1/m_\pi \sim 1.4 \text{ fm} \) even for the physical pion mass. Furthermore, no single state saturation between the ground state and the first excited states, \( t - t_0 \gg (\Delta E)^{-1} = ((2\pi)^2/(2\mu L^2))^{-1}, \) is required for the present HAL QCD method[20], which becomes \( ((2\pi)^2/(2\mu L^2))^{-1} \approx 4.6 \text{ fm} \) if we consider \( L \sim 6 \text{ fm} \) and \( m_N \approx 1 \text{ GeV} \). In Ref. [14], the validity of the velocity expansion of the NN potential has been examined in quenched lattice QCD simulations at \( m_\pi \approx 530 \text{ MeV} \) and \( L \approx 4.4 \text{ fm} \).}
The potential is obtained at moderately large imaginary time; no single state saturation is required.

\[ R_{\alpha \beta}^{(J,M)}(\vec{r}, t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X} + \vec{r}, t) B_{2,\beta}(\vec{X}, t) \mathcal{J}_{B_3 B_4}^{(J,M)}(t_0) \right| 0 \right\rangle / \exp\{-m_{B_1} + m_{B_2}(t-t_0)\}, \]

\[ = \sum_{\text{n}} A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X} + \vec{r}, 0) B_{2,\beta}(\vec{X}, 0) \left| E_n \right\rangle e^{-(E_n - m_{B_1} - m_{B_2})(t-t_0)} \right. \]

\[ + O(e^{-(E_{\text{th}} - m_{B_1} - m_{B_2})(t-t_0)}), \quad (4) \]

where \( E_n \) (\( |E_n\rangle \)) is the eigen-energy (eigen-state) of the six-quark system and \( A_n = \sum_{\alpha' \beta'} P^{(J,M)}_{\alpha' \beta' \alpha \beta} \langle E_n | B_{4,\beta'} B_{3,\alpha} | 0 \rangle \). At moderately large \( t-t_0 \) where the inelastic contribution above the pion production \( O(e^{-(E_{\text{th}} - 2m_N)(t-t_0)}) = O(e^{-m_{\pi}(t-t_0)}) \) becomes exiguous, we can construct the non-local potential \( U \) through

\[ \left( \frac{k^2}{2\mu} \right) R(\vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}') R(\vec{r}'). \]

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\[ = \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X} + \vec{r}, 0) B_{2,\beta}(\vec{X}, 0) \left| E_n \right| \right\rangle e^{-(E_n - m_{B_1} - m_{B_2})(t-t_0)} \]

\[ + O(e^{-(E_{th} - m_{B_1} - m_{B_2})(t-t_0)}), \tag{4} \]

where \( E_n \) is the eigen-energy (eigen-state) of the six-quark system and \( A_n = \sum_{\alpha'\beta'} P^{(J,M)}_{\alpha'\beta'} \left\langle E_n \left| B_{4,\alpha'}B_{3,\beta'} \right| 0 \right\rangle \). At moderately large \( t-t_0 \) where the inelastic contribution above the pion production \( O(e^{-(E_{th} - 2m_N)(t-t_0)}) = O(e^{-m_\pi(t-t_0)}) \) becomes negligible, we can construct the non-local potential \( U \) through \( \left( \frac{\nabla^2}{2\mu} - \frac{k^2}{2\mu} \right) R(\vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}') R(\vec{r}') \). In lattice QCD calculations

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An improved recipe for NY potential:

- Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

\[- \frac{1}{2 \mu} \nabla^2 R(t, \vec{r}) + \int d^3 r' U(\vec{r}, \vec{r}') R(t, \vec{r}') = -\frac{\partial}{\partial t} R(t, \vec{r}) \to \frac{k^2}{2 \mu} R(t, \vec{r})\]

- A general expression of the potential:

\[ V_{NY} = V_0(r) + V_{\sigma}(r)(\vec{\sigma}_N \cdot \vec{\sigma}_Y) + V_T(r) S_{12} + V_{LS}(r)(\vec{L} \cdot \vec{S}_+) + V_{ALS}(r)(\vec{L} \cdot \vec{S}_-) + O(\nabla^2) \]
Determination of baryon–baryon potentials at nearly physical point
**Effective block algorithm for various baryon–baryon correlators**

HN, CPC207,91(2016), arXiv:1510.00903(hep-lat)

Numerical cost (# of iterative operations) in this algorithm:

\[ 1 + N_c^2 + N_c^2 N_\alpha^2 + N_c^2 N_\alpha^2 + N_c^2 N_\alpha + N_c^2 N_\alpha = 370 \]

In an intermediate step:

\[ (N_c^! N_\alpha^B) \times N_u^! N_d^! N_s^! \times 2^{N_\Lambda + N_{\Sigma^-} - B} = 3456 \]

In a naïve approach:

\[ (N_c^! N_\alpha^B)^2B \times N_u^! N_d^! N_s^! = 3,981,312 \]
Generalization to the various baryon–baryon channels strangeness $S=0$ to $-4$ systems

\[ \langle pn\bar{p}n \rangle, \]
\[ \langle p\Lambda\bar{p}\Lambda \rangle, \langle p\Sigma^+\bar{n}n \rangle, \langle p\Lambda\Sigma^0\bar{p} \rangle, \]
\[ \langle \Sigma^+n\bar{p}\Lambda \rangle, \langle \Sigma^+n\Sigma^+\bar{n} \rangle, \langle \Sigma^+n\Sigma^0\bar{p} \rangle, \]
\[ \langle \Sigma^0\bar{p}p\Lambda \rangle, \langle \Sigma^0p\Sigma^+\bar{n} \rangle, \langle \Sigma^0p\Sigma^0\bar{p} \rangle, \]
\[ \langle \Lambda\Lambda\Lambda\Lambda \rangle, \langle \Lambda\Lambda\pi\Xi^- \rangle, \langle \Lambda\Lambda\bar{n}\Xi^0 \rangle, \langle \Lambda\Lambda\Sigma^+\Sigma^- \rangle, \langle \Lambda\Lambda\Sigma^0\Xi^0 \rangle, \]
\[ \langle p\Xi\Lambda\Lambda \rangle, \langle p\Xi\bar{p}\Xi^- \rangle, \langle p\Xi\bar{n}\Xi^0 \rangle, \langle p\Xi\bar{\Sigma}+\Sigma^- \rangle, \langle p\Xi\Sigma^0\Xi^0 \rangle, \langle p\Xi\Sigma^0\Sigma^0 \rangle, \]
\[ \langle n\Xi^0\Lambda\Lambda \rangle, \langle n\Xi^0\bar{p}\Xi^- \rangle, \langle n\Xi^0\bar{n}\Xi^0 \rangle, \langle n\Xi^0\Sigma^+\Sigma^- \rangle, \langle n\Xi^0\Sigma^0\Xi^0 \rangle, \langle n\Xi^0\Sigma^0\Sigma^0 \rangle, \]
\[ \langle \Sigma^+\Sigma^-\Lambda\Lambda \rangle, \langle \Sigma^+\Sigma^-\bar{p}\Xi^- \rangle, \langle \Sigma^+\Sigma^-\bar{n}\Xi^0 \rangle, \langle \Sigma^+\Sigma^-\Sigma^+\Sigma^- \rangle, \langle \Sigma^+\Sigma^-\Sigma^0\Xi^0 \rangle, \]
\[ \langle \Sigma^0\Sigma^0\Lambda\Lambda \rangle, \langle \Sigma^0\Sigma^0\bar{p}\Xi^- \rangle, \langle \Sigma^0\Sigma^0\bar{n}\Xi^0 \rangle, \langle \Sigma^0\Sigma^0\Sigma^+\Sigma^- \rangle, \langle \Sigma^0\Sigma^0\Sigma^0\Xi^0 \rangle, \]
\[ \langle \Sigma^0\Lambda\bar{p}\Xi^- \rangle, \langle \Sigma^0\Lambda\bar{n}\Xi^0 \rangle, \langle \Sigma^0\Lambda\Sigma^+\Sigma^- \rangle, \]
\[ \langle \Xi\Lambda\Xi^-\Lambda \rangle, \langle \Xi\Lambda\Sigma^-\Xi^0 \rangle, \langle \Xi\Lambda\Sigma^0\Xi^- \rangle, \]
\[ \langle \Sigma^-\Xi^0\Xi^-\Lambda \rangle, \langle \Sigma^-\Xi^0\Sigma^-\Xi^0 \rangle, \langle \Sigma^-\Xi^0\Sigma^0\Xi^- \rangle, \]
\[ \langle \Sigma^0\Xi^-\Xi^-\Lambda \rangle, \langle \Sigma^0\Xi^-\Sigma^-\Xi^0 \rangle, \langle \Sigma^0\Xi^-\Sigma^0\Xi^- \rangle, \]
\[ \langle \Xi^-\Xi^0\Xi^-\Xi^0 \rangle. \]

Make better use of the computing resources!

HN, CPC 207, 91(2016) [arXiv:1510.00903[hep-lat]],
(See also arXiv:1604.08346)
Almost physical point lattice QCD calculation using $N_f=2+1$ clover fermion + Iwasaki gauge action

- APE-Stout smearing ($\rho=0.1$, $n_{\text{stout}}=6$)
- Non-perturbatively $O(a)$ improved Wilson Clover action at $\beta=1.82$ on $96^3 \times 96$ lattice
- $1/a = 2.3$ GeV ($a = 0.085$ fm)
- Volume: $96^4 \rightarrow (8\text{fm})^4$
- $m_\pi = 145 \text{MeV}$, $m_K = 525 \text{MeV}$
- DDHMC(ud) and UVPHMC(s) with preconditioning

- NBS wf is measured using wall quark source with Coulomb gauge fixing, spatial PBD and temporal DBC; #stat=207 configs x 4 rotation x Nsrc
  (Nsrc=4 → 20 → 52 → 96 (2015FY+))
LN-SN potentials at nearly physical point

The methodology for coupled-channel $V$ is based on:
Ishii, et al., JPS meeting, March (2016).

#stat: (this/scheduled in FY2015+) < 0.05 (=>$0.2) \rightarrow 0.54

$$\Lambda N - \Sigma N \ (I=1/2)$$

\begin{align*}
V_c( ^1 S_0 ) & \quad V_c( ^3 S_1 - ^3 D_1 ) \quad V_T( ^3 S_1 - ^3 D_1 ) \\
\Sigma N \ (I=3/2) & \\
V_c( ^1 S_0 ) & \quad V_c( ^3 S_1 - ^3 D_1 ) \quad V_T( ^3 S_1 - ^3 D_1 )
\end{align*}$$
LN-SN potentials at nearly physical point

The methodology for coupled-channel $V$ is based on:
- Ishii, et al., JPS meeting, March (2016).

#stat: (this/scheduled in FY2015+) $< 0.05 \Rightarrow 0.2 \quad 0.54$

\[
\Lambda N - \sum N \quad (I = 1/2)
\]
\[
\begin{align*}
V_C (^1S_0) & & V_C (^3S_1 - ^3D_1) & & V_T (^3S_1 - ^3D_1) \\
\sum N \quad (I = 3/2)
\end{align*}
\]
\[
\begin{align*}
V_C (^1S_0) & & V_C (^3S_1 - ^3D_1) & & V_T (^3S_1 - ^3D_1)
\end{align*}
\]
Effective mass plot of the single baryon’s correlation function.

Potentials obtained at $t-t_0 = 5$ to $12$ will be shown.
The eigenvalues of the normalization kernel in eq. (3.3) for $S=-1$ two-baryon (BB) system

<table>
<thead>
<tr>
<th>$I$</th>
<th>$J$</th>
<th>BB</th>
<th>Eigenvalues (uncoupled)</th>
<th>Eigenvalues (coupled)</th>
</tr>
</thead>
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<tr>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$N\Lambda$</td>
<td>1</td>
<td>0 $\frac{10}{9}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N\Sigma$</td>
<td>$\frac{1}{5}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>$N\Lambda$</td>
<td>1</td>
<td>$\frac{8}{9}$ $\frac{10}{9}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N\Sigma$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\frac{3}{2}$</td>
<td>0</td>
<td>$N\Sigma$</td>
<td>$\frac{10}{9}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{3}{2}$</td>
<td>1</td>
<td>$N\Sigma$</td>
<td>$\frac{7}{9}$</td>
<td></td>
</tr>
</tbody>
</table>

Oka, Shimizu and Yazaki (1987)
Very preliminary result of LN potential at the physical point

\[
\left( \frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} \right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \cdots \tag{8}
\]
Very preliminary result of LN potential at the physical point

\[
\left( \frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} \right) R(\vec{r}, t) = \int d^3 r' U(\vec{r}, \vec{r}') R(\vec{r}, t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \cdot(8)
\]

\[
V_C \left( \frac{1}{S_0} \right)
\]
Very preliminary result of LN potential at the physical point

\[
\left( \frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} \right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \cdot \cdot \cdot (8)
\]
Very preliminary result of LN potential at the physical point

\[
\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \cdot(8)\]

\[
\sum N - N
\]

\[
\Lambda N
\]

\[
\Lambda N
\]
Very preliminary result of LN potential at the physical point

\[
V_T(^{3}S_{1} - ^{3}D_{1}) = \left( \nabla^2 - \frac{\partial}{\partial t} \right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}, t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \cdot(8)
\]
Very preliminary result of LN potential at the physical point

\[
\left( \frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} \right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \cdots \tag{8}
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\]

\[\Sigma N( I = 3/2 )\]

\[V_C( ^3 S_1 \rightarrow ^3 D_1 )\]

\[V_C( ^1 S_0 ) \quad V_T( ^3 S_1 \rightarrow ^3 D_1 )\]
Very preliminary result of LN potential at the physical point

\[
\left( \frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} \right) R(\vec{r}, t) = \int d^3 r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \cdot (8)
\]

\[
\sum N(I = 3/2)
\]

\[
V_c (\begin{pmatrix} 3 S_1 \\ 3 D_1 \end{pmatrix})
\]

\[
V_c (\begin{pmatrix} 1 S_0 \end{pmatrix})
\]

\[
V_T (\begin{pmatrix} 3 S_1 \\ 3 D_1 \end{pmatrix})
\]
Summary

(I-1) Preliminary results of LN-SN potentials at nearly physical point. (Lambda-N, Sigma-N: central, tensor)
Statistics approaching to 0.54 (=present/scheduled)
Signals in spin-triplet are relatively going well smoothly.
We will have to increase still more statistics, particularly for spin-singlet channels
Several interesting features seem to be obtained with more high statistics.

(I-2) Effective hadron block algorithm for the various baron-baryon interaction
Paper published/available:

Future work:
(II-1) Physical quantities including the binding energies of few-body problem of light hypernuclei with the lattice YN potentials