ハイパー核の構造とαクラスター

関東学院大学・理工学部
山田泰一

Kashikojima, July 7-8, 2013

元場さん、退職おめでとうございます！

1981.4　M1：元場さんとの初対面（池田研究室、坂東さんと一緒）

80年代の初め:　日本におけるハイパー核研究のスタート
軽いp殻ハイパー核の構造研究（元場・坂東・池田）

$\alpha+x+\Lambda$ model: $x=p,n,d,t,{}^{3}H,{}^{3}He$

sd殻やほかの核はどうか？
山田・池田・元場・坂東　の　スタート

${}^{21}_{\Lambda}Ne(\alpha+{}^{16}O+\Lambda)$, ${}^{13}_{\Lambda}C(3\alpha+\Lambda)$,
${}^{9}_{\Lambda}Be=(\alpha+\alpha+\Lambda)+(\alpha+\alpha^{*}+\Lambda)$
${}^{9}Be(K-,\pi-)$, $(\pi+,K+)$, (stopped $K-,\pi-$)

山田のD論 (1986.3)

・・

90年代　肥山・上村・元場・山田・山本　の　スタート

・・

・・

2012.4　船木さん、肥山研

Hyper-THSR w.f. を用いたハイパー核の構造研究
<u>船木</u>・山田・肥山・池田

軽いハイパー核の構造研究（クラスター模型）

Glue-like role of Λ

Shrinkage of core nucleus

➡ Reduction of B(E2)

Observed in ${}^{7}_{\Lambda}$Li

Tanida et al., PRL86 (2001)

精密4体計算：Hiyama et al. PRC59(1999)

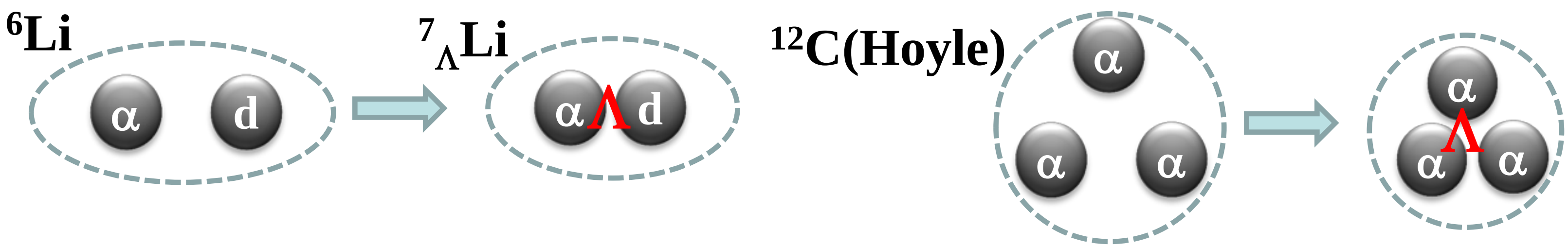
Light p-shell: α+x+Λ model: x=p,n,d,t,^{3}H,^{3}He

Motoba, Bando, Ikeda, PTP70(1983), 71(1984)

${}^{13}_{\Lambda}$C(3α+Λ), ${}^{21}_{\Lambda}$Ne(α+^{16}O+Λ) 山田のD論の一部

${}^{20}_{\Lambda}$Ne(α+^{15}O+Λ) 1980年後半までの研究

特筆すべき発見



Shrinkage = コア核の密度変化（monopole） 外場：ΛN 相互作用

微視的クラスター模型： 成功を収めた大きな理由

1．殻模型的状態とクラスター状態を同時に記述 ⇔ 密度変化

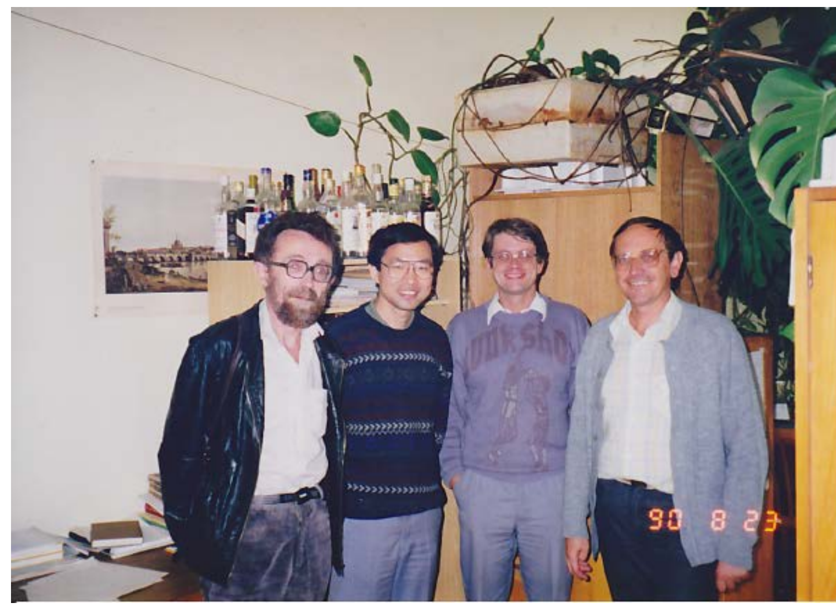
2．Λ粒子を加える前の芯核の波動関数：信頼のおけるもの

クラスター状態とモノポールの密接な関係

元場さん、退職おめでとうございます！

1988.9 .10 プラハ

1990.8 .23 プラハ Rez INP
2国間:代表者　元場さん



1994 バンクーバー

1. クラスター構造とモノポール励起:　代表例　16O

2. Hyper-THSR 波動関数を用いたハイパー核の構造研究

クラスターガス的状態:　12C, 16O　(11B, 13C)

$^{13}_{\Lambda}C$　　船木・山田・肥山・池田

Cluster structures and isoscalar monopole excitations in light nuclei

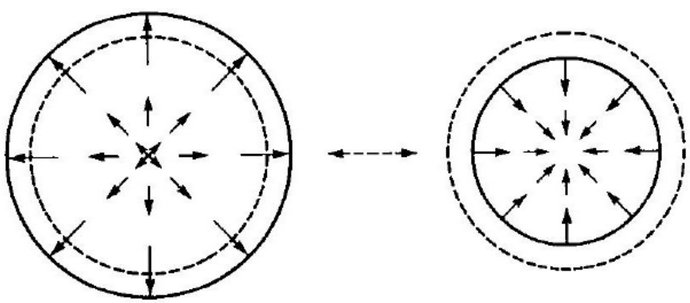
- **Isoscalar Monopole Mode**

  ↔ density fluctuation

<u>Typical example</u>

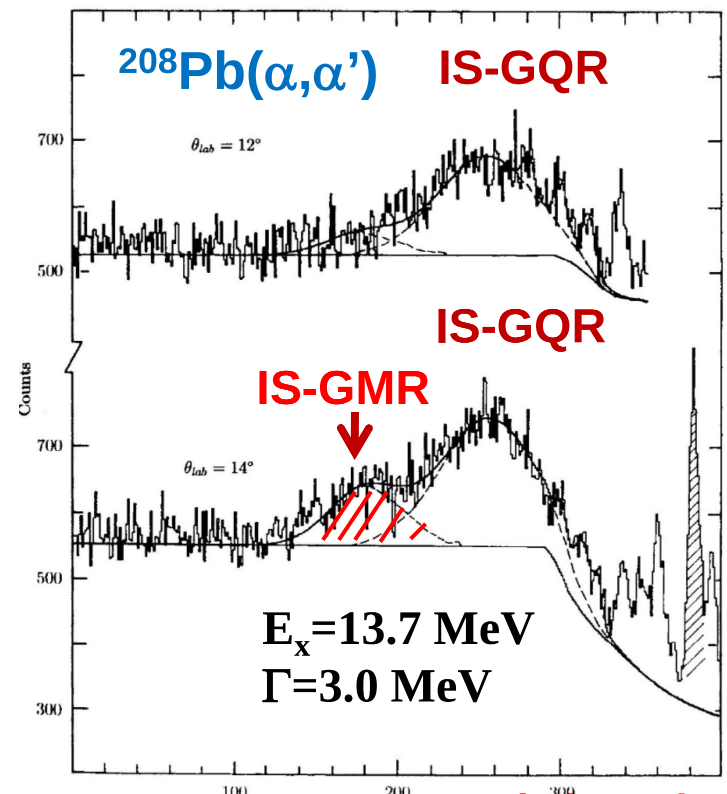
IS-GMR (heavy, medium-heavy nuclei)

exhaust EWSR ~ 100%



breathing mode

$$E_x \simeq 80/A^{-1/3} \quad [\mathrm{MeV}]$$



RPA cal.

Harakeh et al., PRL(1977)

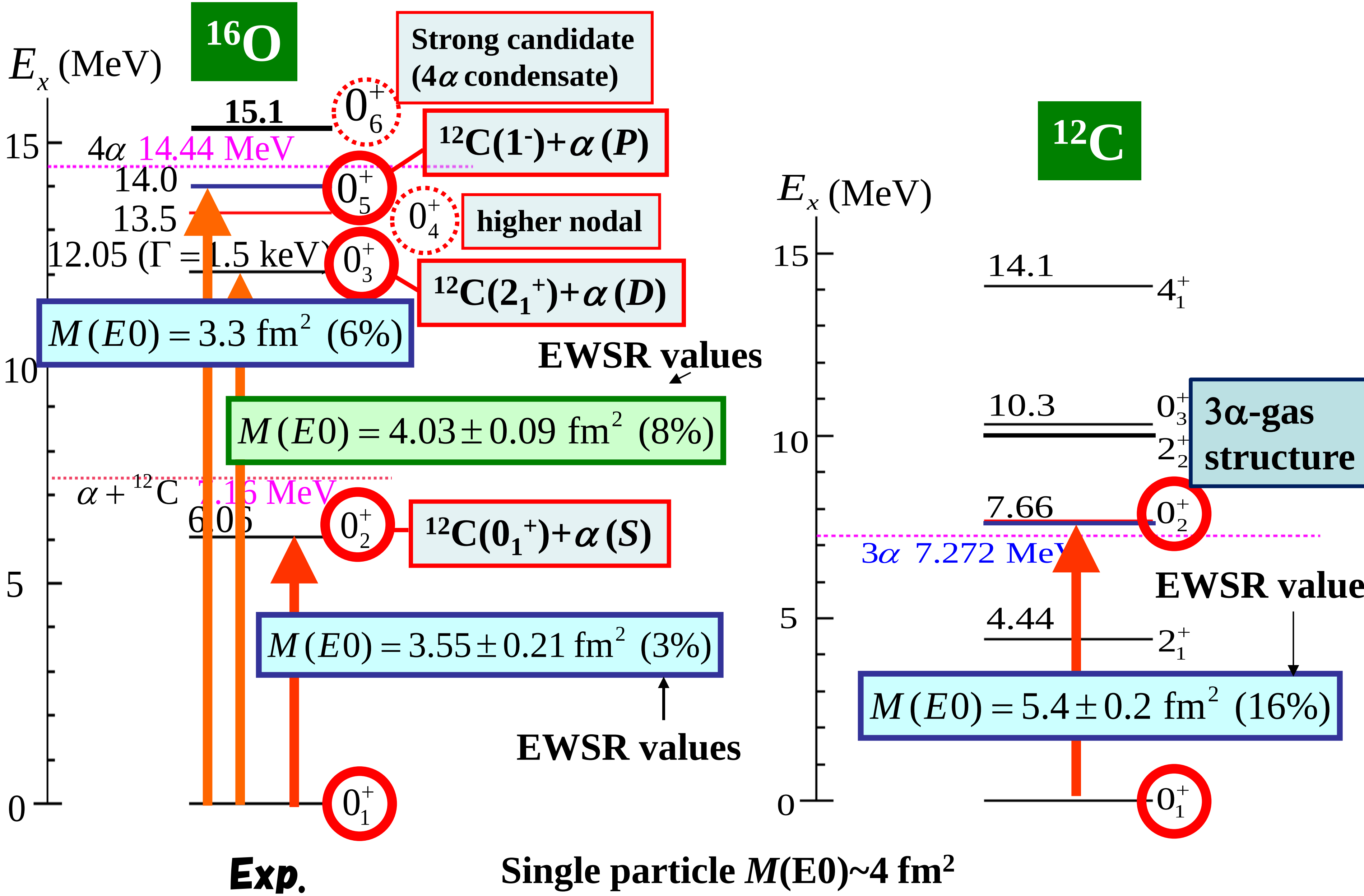
- **Light Nuclei (*p*-, *sd*-shell,,,)**

  Isoscalar monopole strengths are fragmented.

  Monopole strengths to cluster states: ~ 20% of EWSR

  For example, ^{16}O, ^{12}C, ^{11}B, ^{13}C, ^{24}Mg,…

Monopole strengths *M(E0)* in ^{16}O and ^{12}C



Single particle M(E0)~4 fm^2

Single-particle IS-monopole strength

$$M(\mathrm{E0}) \sim \langle u_\mathrm{f} \,|\, r^2 \,|\, u_\mathrm{i} \rangle \sim (3/5) \times R^2 = 4.4\ \mathrm{fm}^2$$

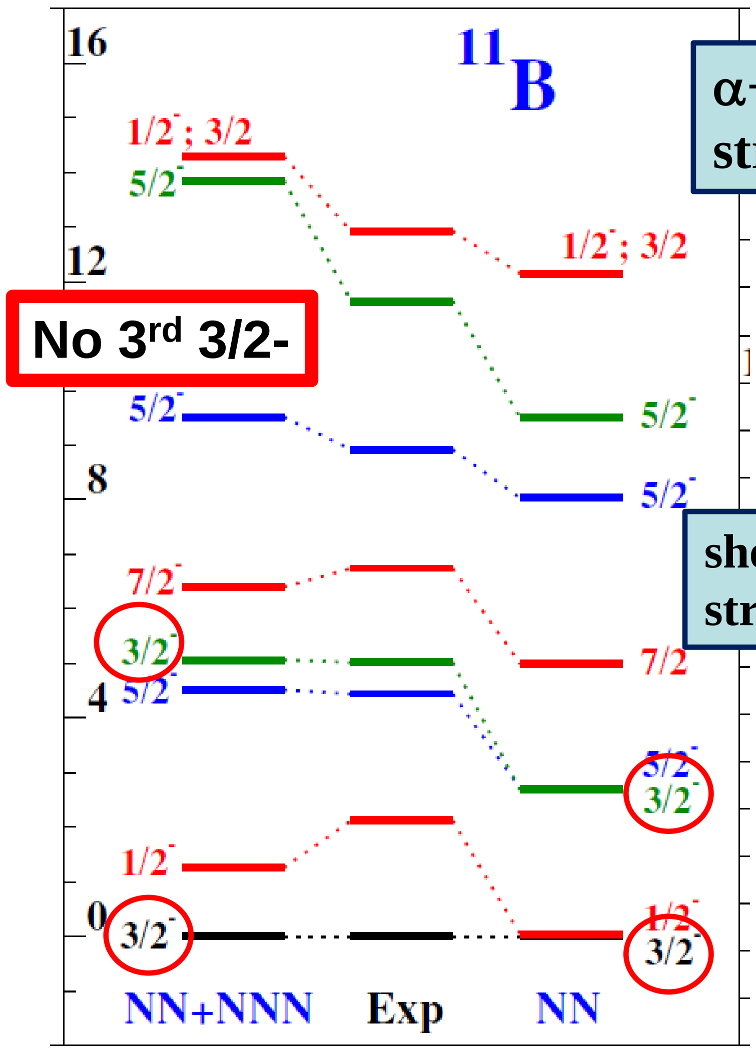
(using R = nuclear radius = 2.7 fm)

Uniform-density approximation for $u_f(r)$ and $u_i(r)$

$$u(r) = (3/R^3)^{1/2} \quad \text{for} \quad 0 \le r \le R$$
$$u(r) = 0 \quad \text{for} \quad R < r$$

No-core shell model

Navratil et al., JPG36(2009)

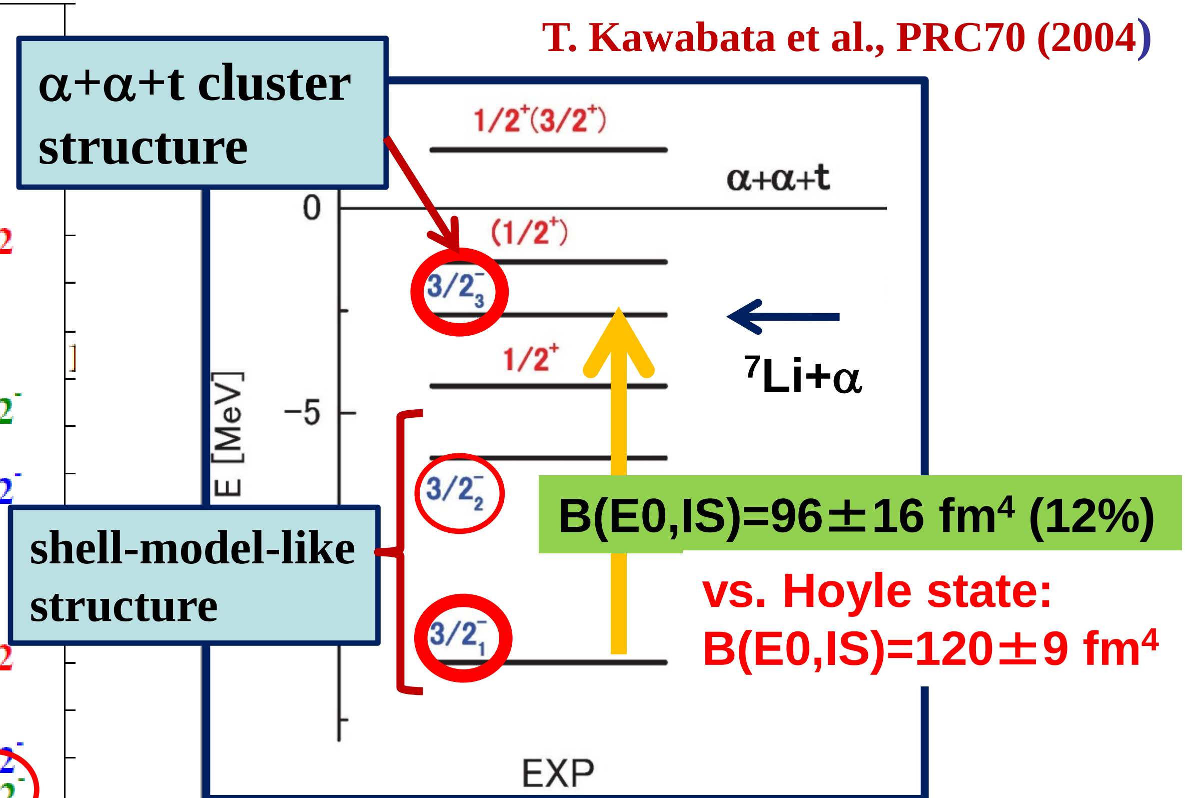


^{11}B

^{11}B energy levels (T=1/2)

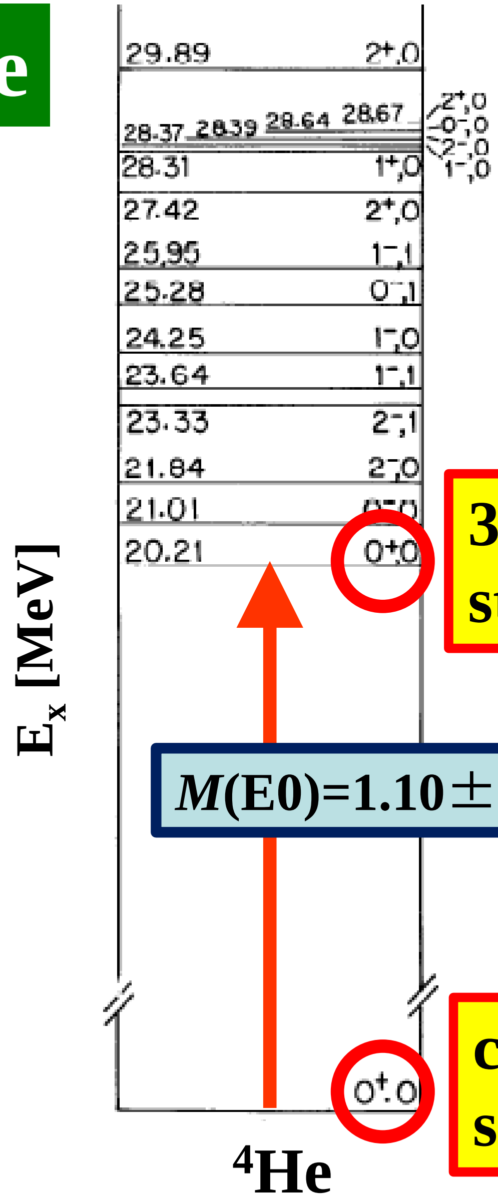
Experimentally, three 3/2- states

T. Kawabata et al., PRC70 (2004)



J_i	J_f	B(E0,IS) EXP [fm^4]	% of EWSR
g.s	2nd 3/2-	< 9	< 1%
	3rd 3/2-	96±16	12%

^{4}He

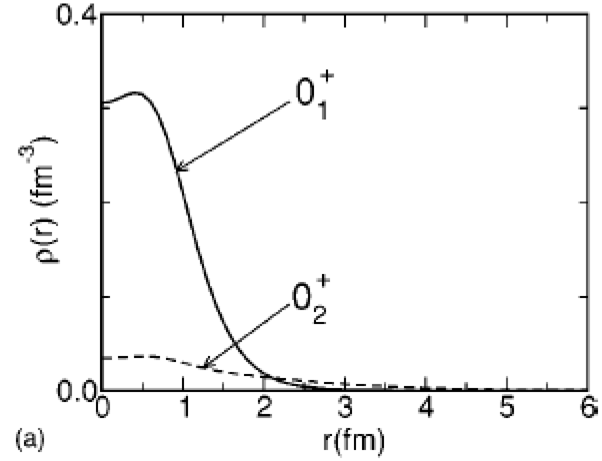


3*N*+*N* cluster structure

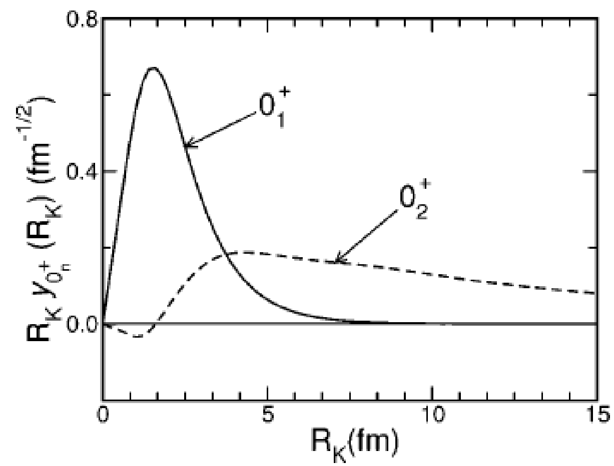
M(E0)=1.10±0.16 fm^2 (11%)

closed shell structure

Density distribution



Overlap amp. between ^{4}He and ^{3}He



Hiyama, Gibson, Kamimura, PRC70 (2004)
Furutani et al., PTP60 (1978)

- **Light Nuclei (*p*-, *sd*-shell,,,)**

  Isoscalar monopole strengths are fragmented.
  Monopole strengths to cluster states: ~ 20% of EWSR
  For example, ^{16}O, ^{12}C,^{11}B,^{13}C,^{24}Mg,…

- **<u>Questions:</u>**

  (1) Why cluster states are excited rather strongly from shell-model-like ground states?

  (2) What kind of features exist in monopole excitations?

- **Light Nuclei (*p*-, *sd*-shell,,,)**

  Isoscalar monopole strengths are fragmented.

  Monopole strengths to cluster states: ~ 20% of EWSR

  For example, ^{16}O, ^{12}C, ^{11}B, ^{13}C, ^{24}Mg,…

- **<u>Present study</u>:**

  IS-monopole strength fun. of $^{16}O(\alpha,\alpha')$

IS Monopole Strength Function of ^{16}O
Exp. vs Cal.

$^{16}O(\alpha,\alpha')$

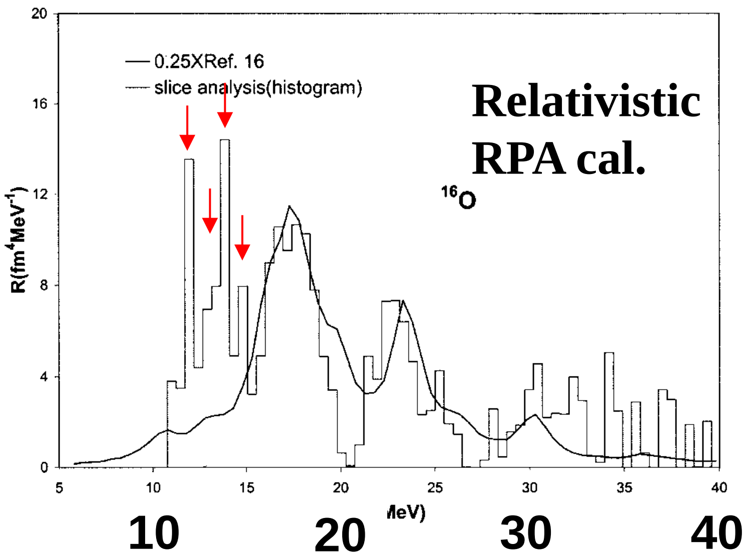


FIG. 7. The histogram is the ... ength con-
verted to monopole response fun... Ex [MeV} ... shows the
monopole response function from Ref. [10] multiplied by 0.25 and
shifted by 4.2 MeV.

Exp. condition: E_x > 10 MeV

Exp: histogram
Lui et al., PRC 64 (2001)

discrete peaks at Ex≤15 MeV
three bumps at 18, 23, 30 MeV

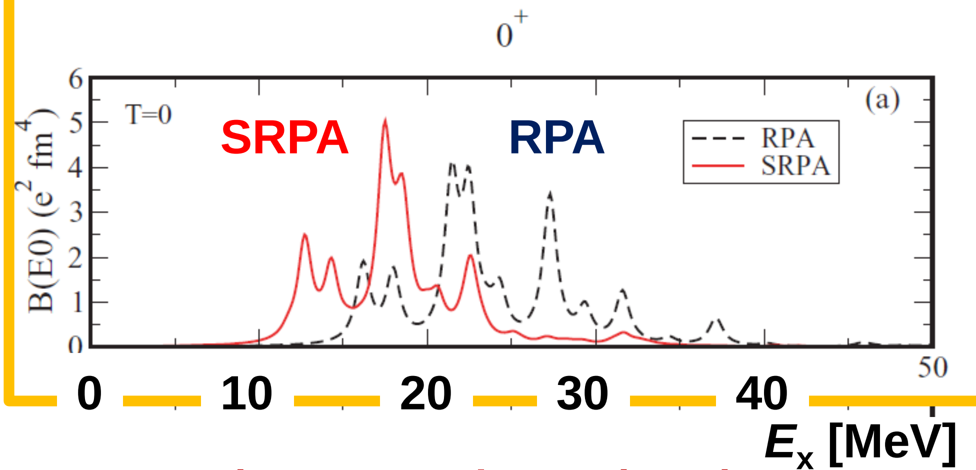
Cal: real line
Relativistic RPA
Ma et al., PRC 55 (1997)
Multiplied by 0.25
Shifted by 4.2 MeV

Not well reproduced by RRPA cal.

Non-rel. RPA calculations

Experiment

SRPA (+RPA) calculation



Gambacurta et al., PRC(2010)

$^{16}O(\alpha,\alpha')$

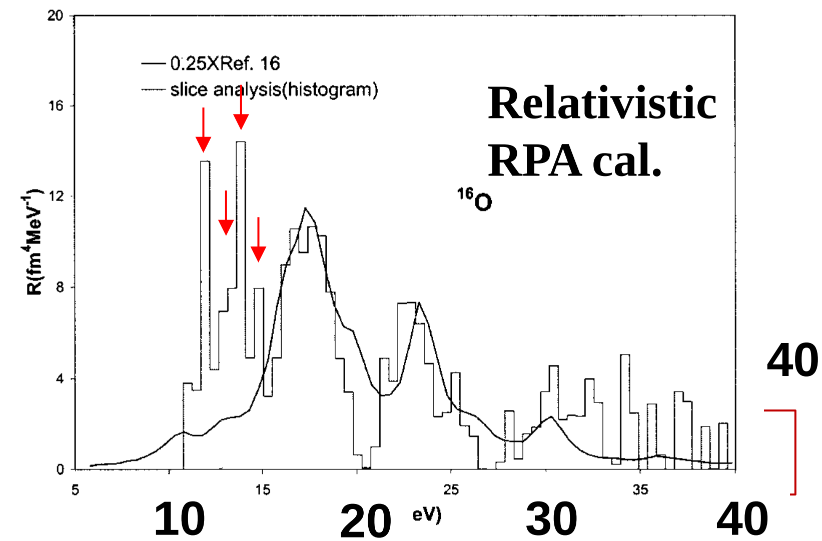


FIG. 7. The histogram is the experimental $E0$ strength converted to monopole response function. The black line shows the mo… shi…

Exp. condition: E_x > 10 MeV

(1) Gross structure at higher energy region (Ex > 18 MeV) , i.e. 3-bump structure, is reproduced by SRPA calculation.

(2) Discrete peaks at $E_x \leq 15$ MeV are not reproduced well.
In particular, the transition to 2nd 0+ state (E_x=6.1 MeV) is not seen in SRPA (+RPA) calculation.

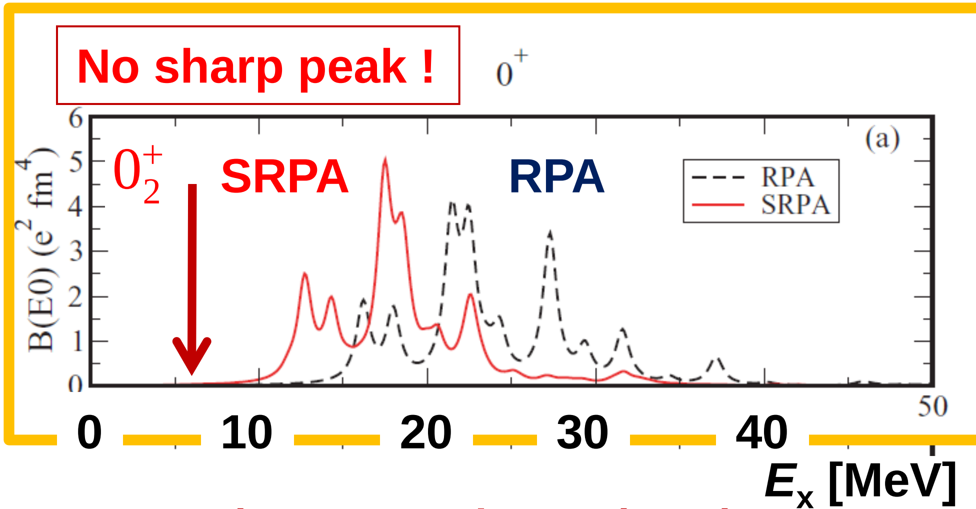
		Experiment				4α OCM		
	Ex [MeV]	R [fm]	M(E0) [fm^2]	Γ [MeV]		R [fm]	M(E0) [fm^2]	Γ [MeV]
0^+_1	0.00	2.71				2.7		
0^+_2	6.05		3.55			3.0	3.9	
0^+_3	12.1		4.03			3.1	2.4	
0^+_4	13.6		no data	0.6		4.0	2.4	0.60
0^+_5	14.0		3.3	0.185		3.1	2.6	0.20
0^+_6	15.1		no data	0.166		5.6	1.0	0.14

over 15%
of total EWSR

20%
of total EWSR

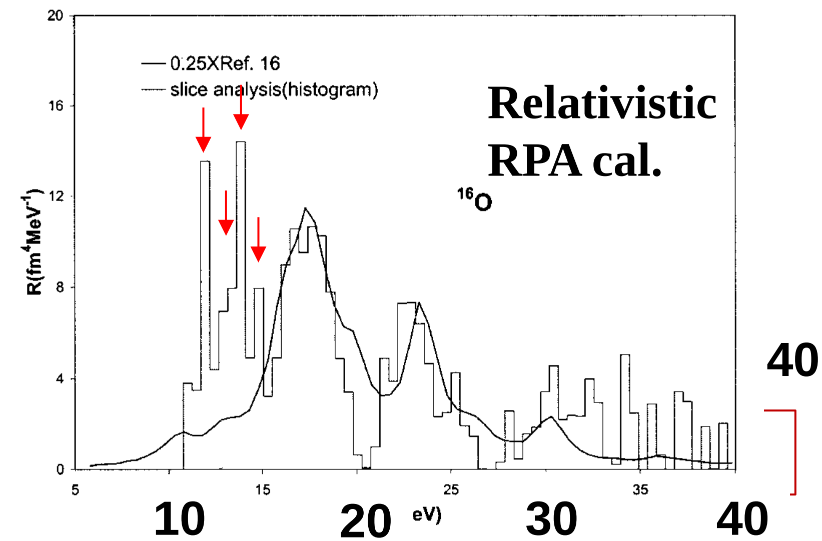
Experiment

SRPA (+RPA) calculation



Gambacurta et al., PRC(2010)

^{16}O(α,α')



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In particular, the transition to 2nd 0+ state (E_x=6.1 MeV) is not seen in SRPA (+RPA) calculation.

Purposes of my talk

- **What kind of states contribute to the discrete peaks?**

- **Recently, 4α OCM（直交条件模型）succeeded in describing the structure of the lowest six 0+ states up to 4α threshold (E_x≈15 MeV).**

Funaki et al., PRL101(2008)

- **We will study the IS monopole strength function with the 4α OCM.**

OCM: Orthogonality Condition Model

Cluster-model analyses of ^{16}O

- **α+^{12}C OCM**

  Y. Suzuki, PTP55 (1976), 1751

- **α+^{12}C GCM**

  M. Libert-Heinemann, D. Bay et al., NPA339 (1980)

- **4α THSR wf** Not include α + ^{12}C configuration.

  Tohsaki, Horiuchi, Schuck, Roepke, PRL87 (2001)
  Funaki, Yamada et al., PRC82(2010)

- **4α OCM** 4α-gas, α + ^{12}C, shell-model-like configurations

  Funaki, Yamada et al., PRL101 (2008)

  Reproduction of lowest six 0+ states up to 4α threshold (15MeV)

$^{16}O = \alpha + ^{12}C$ cluster model

Y. Suzuki, PTP55 (1976), 1751

Even-parity

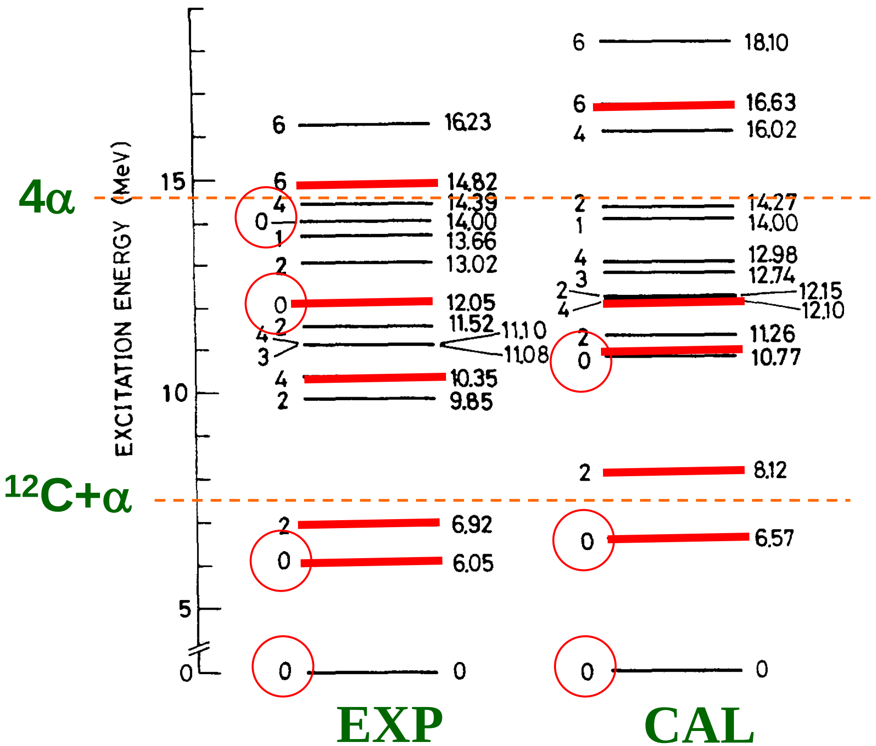


Fig. 2 (a). Energy levels of ^{16}O for the even-parity states [Ref. 30)].

Odd-parity

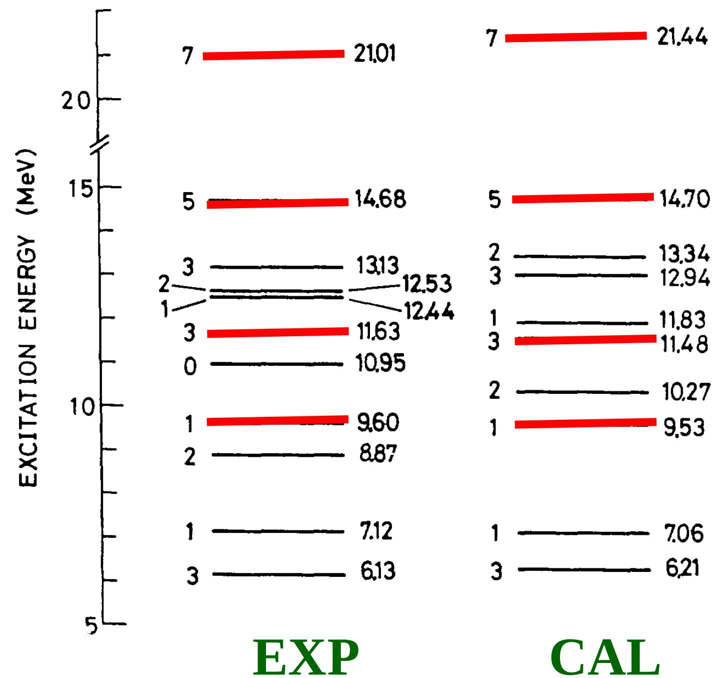
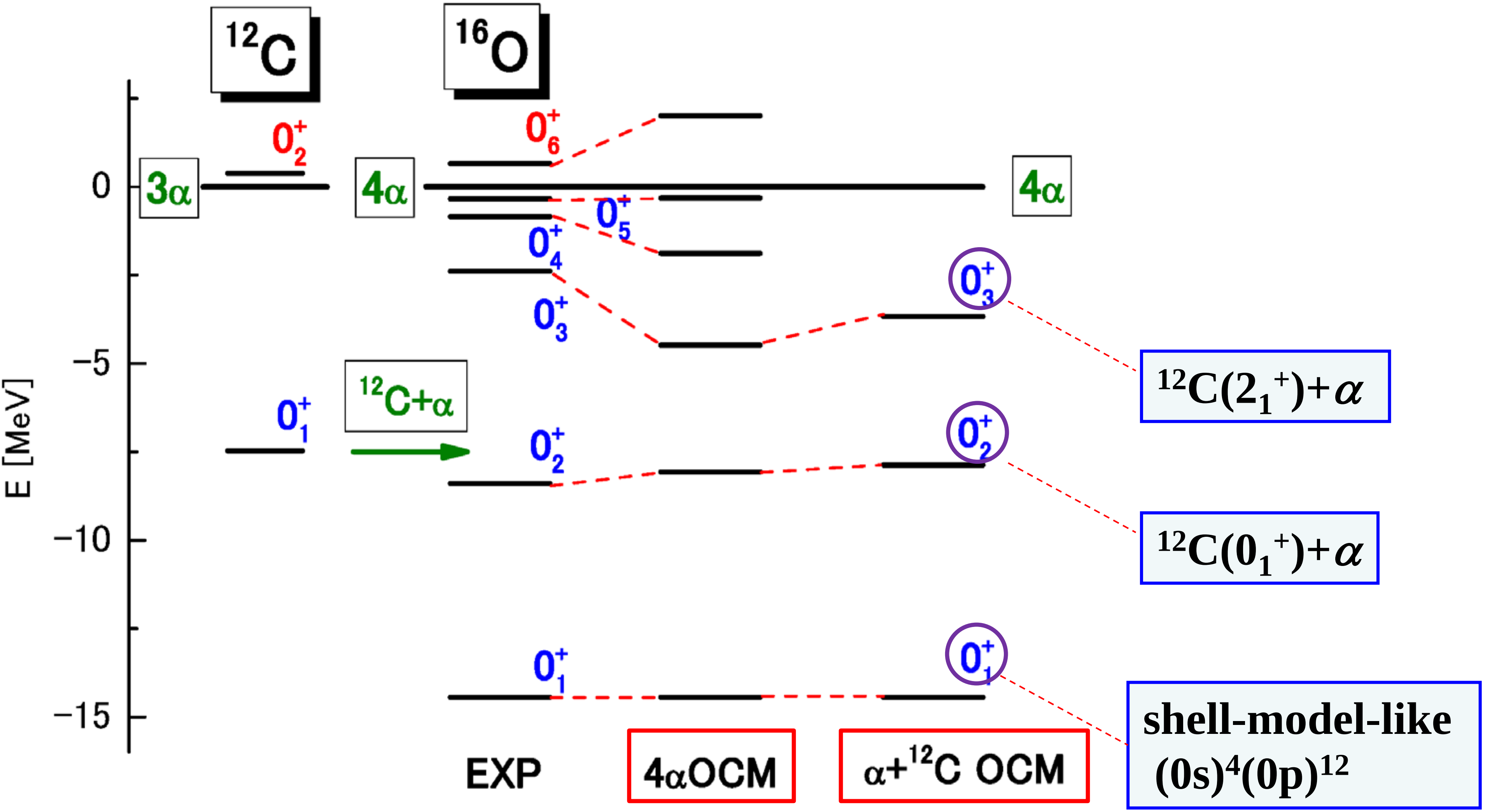


Fig. 2 (b). Energy levels of ^{16}O for the odd-parity states [Ref. 30)].

^{12}C+α : molecular states



Funaki et al., PRL101 (2008)

Suzuki, PTP55 (1976)

		Experimental data				4α OCM		
	E_x [MeV]	R [fm]	M(E0) [fm^2]	Γ [MeV]		R [fm]	M(E0) [fm^2]	Γ [MeV]
0^+_1	0.00	2.71				2.7		
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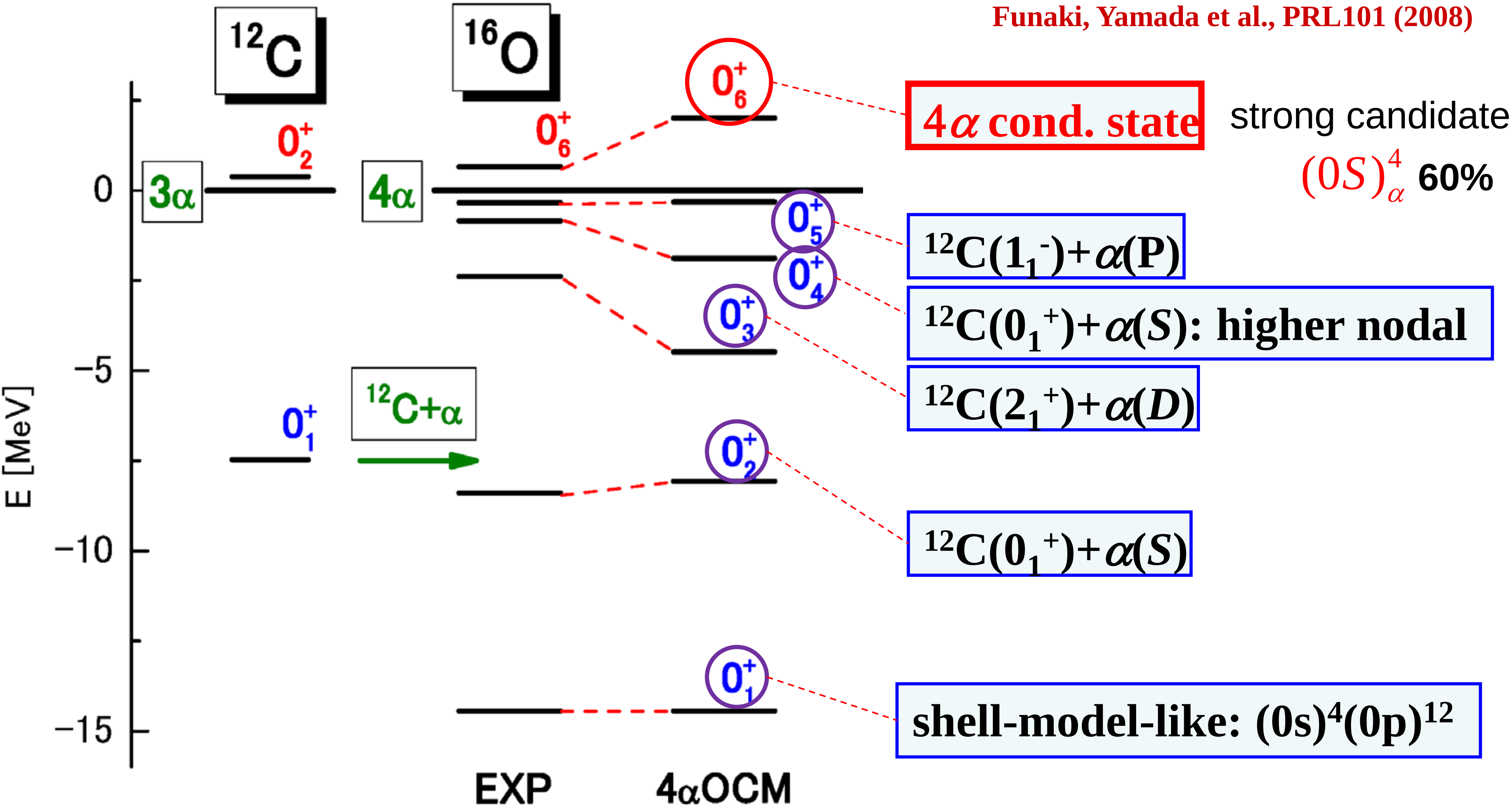
over 15%
of total EWSR

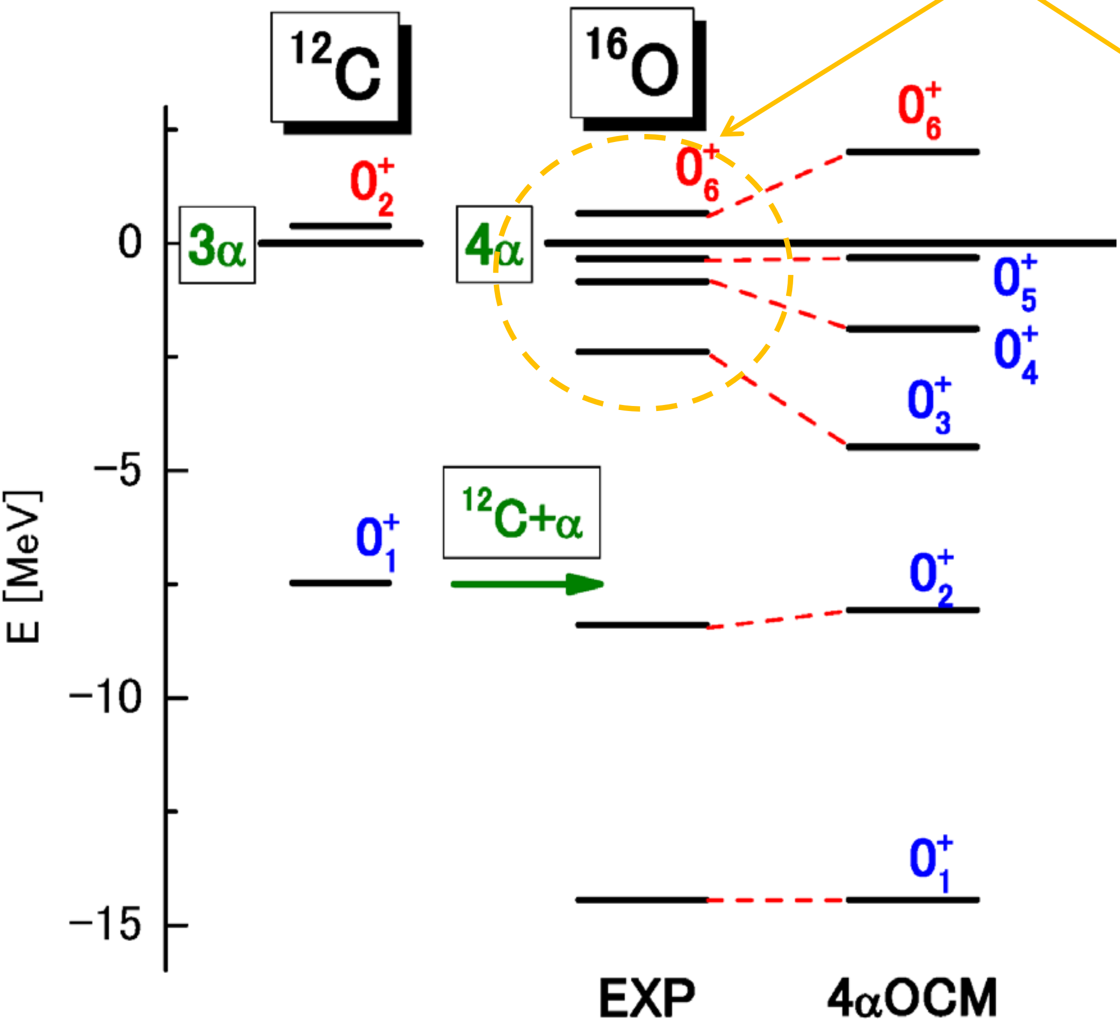
20%
of total EWSR

4α OCM calculation

Funaki, Yamada et al., PRL101 (2008)



Good correspondence in energy



^{16}O(α,α')

Relativistic RPA cal.

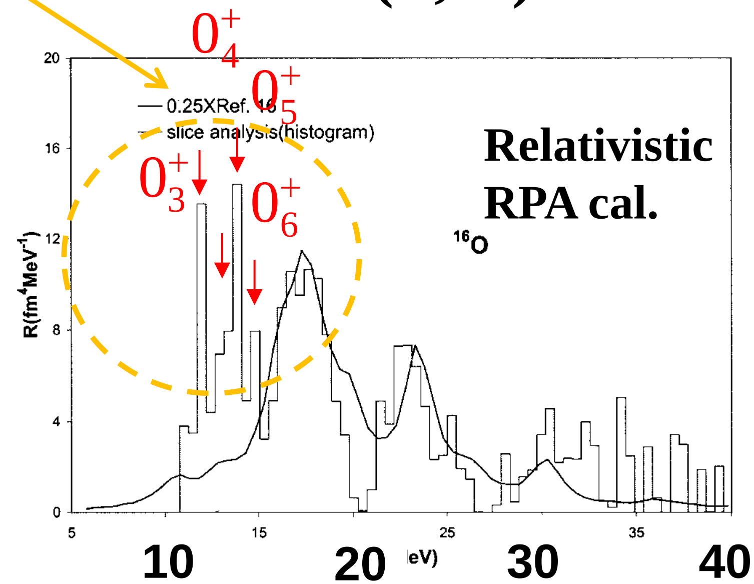


FIG. 7. The histogram is the experimental E0 strength con...

Exp. condition: E_x > 10 MeV

shifted by 4.2 MeV.

Exp: histogram
Lui et al., PRC 64 (2001)
Cal: real line
Ma et al., PRC 55 (1997)
Multiplied by 0.25
Shifted by 4.2 MeV

IS monopole strength function $S(E)$ within 4α OCM framework

Monopole Strength Function with 4α OCM

$$S(E) = \sum_n \delta(E - E_n) |\langle 0_n^+ | \mathcal{O} | 0_1^+ \rangle|^2, \quad \mathcal{O} = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2$$

$$R(E) = \langle 0_1^+ | \frac{\mathcal{O}^\dagger \mathcal{O}}{E - H + i\epsilon} | 0_1^+ \rangle,$$

$\left| \mathbf{0}_{\mathbf{n}}^{+} \right\rangle$: **resonance state with** $E_{\rm n}$ **-** $i\Gamma_{\rm n}/2$

$$S(E) = -\frac{1}{\pi} \mathrm{Im}[R(E)]$$
$$= \frac{1}{\pi} \sum_n \frac{\Gamma_n / 2}{(E - E_n)^2 + (\Gamma_n / 2)^2} \left| M(0_n^+ - 0_1^+) \right|^2$$

$M(0_n^+ - 0_1^+) = \langle 0_n^+ | \mathcal{O} | 0_1^+ \rangle$ **: calculated by 4α OCM**

$$\Gamma_{\rm n} = \sqrt{\Gamma_{\rm n}(\mathrm{OCM})^2 + (\mathrm{exp.\ resolution})^2}$$

50 keV

$\mathbf{E_n}$ **: experimental energy of *n*-th 0+ state**

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over 15%
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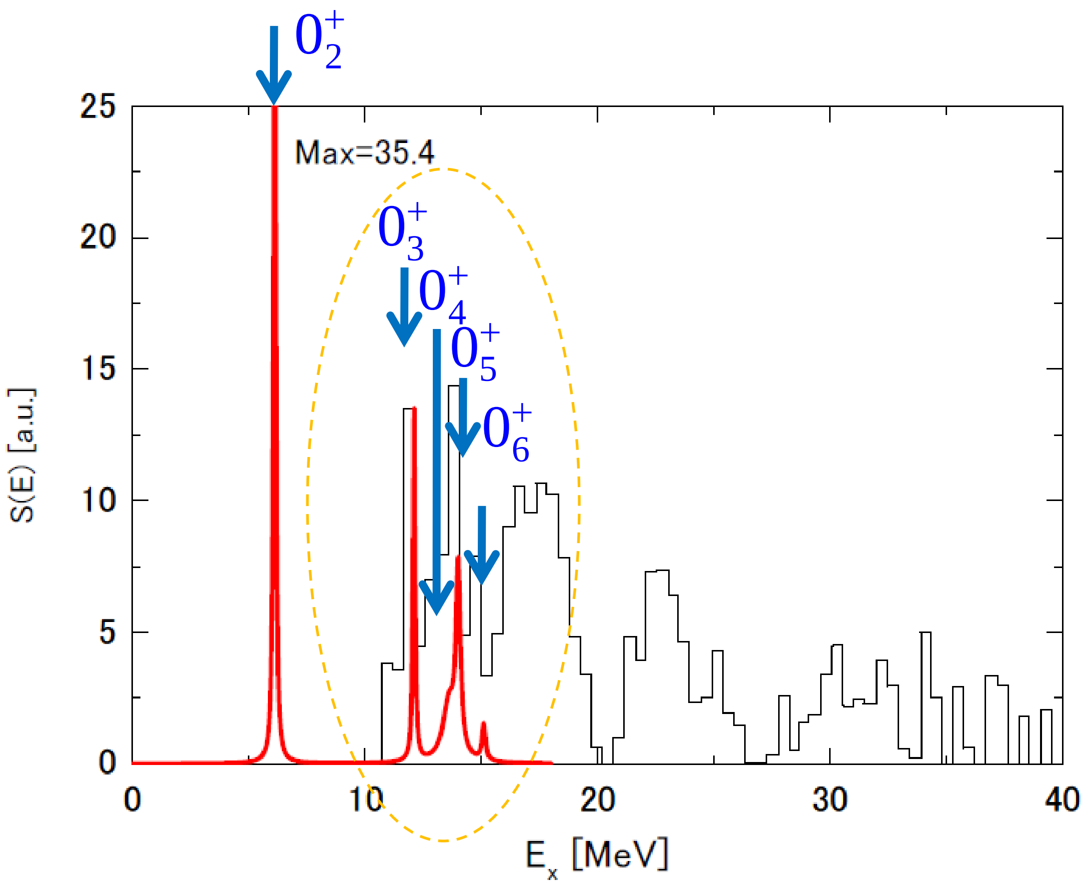
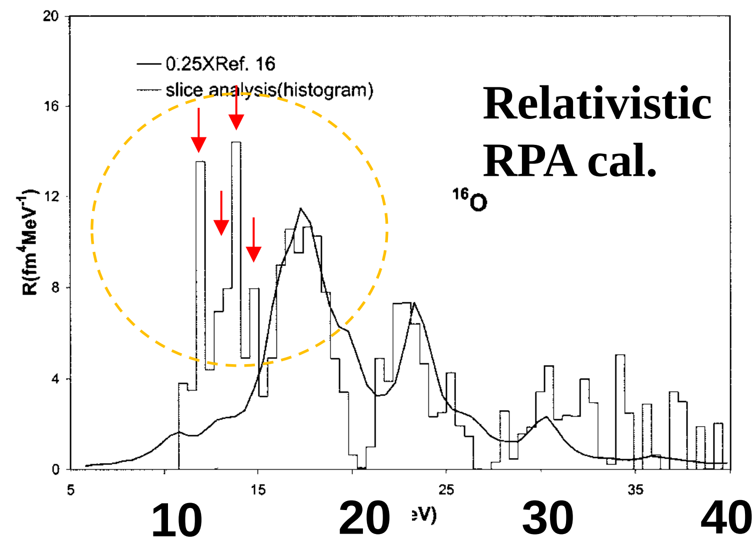
Exp. vs. Cal.

IS monopole S(E) with 4α OCM



^{16}O(α,α')



shifted by 4.2 MeV.

Exp. condition: E_x > 10 MeV

It is likely to exist discrete peaks on a small bump at E_x < 15MeV
This small bump may come from the contribution from continuum states of α+^{12}C

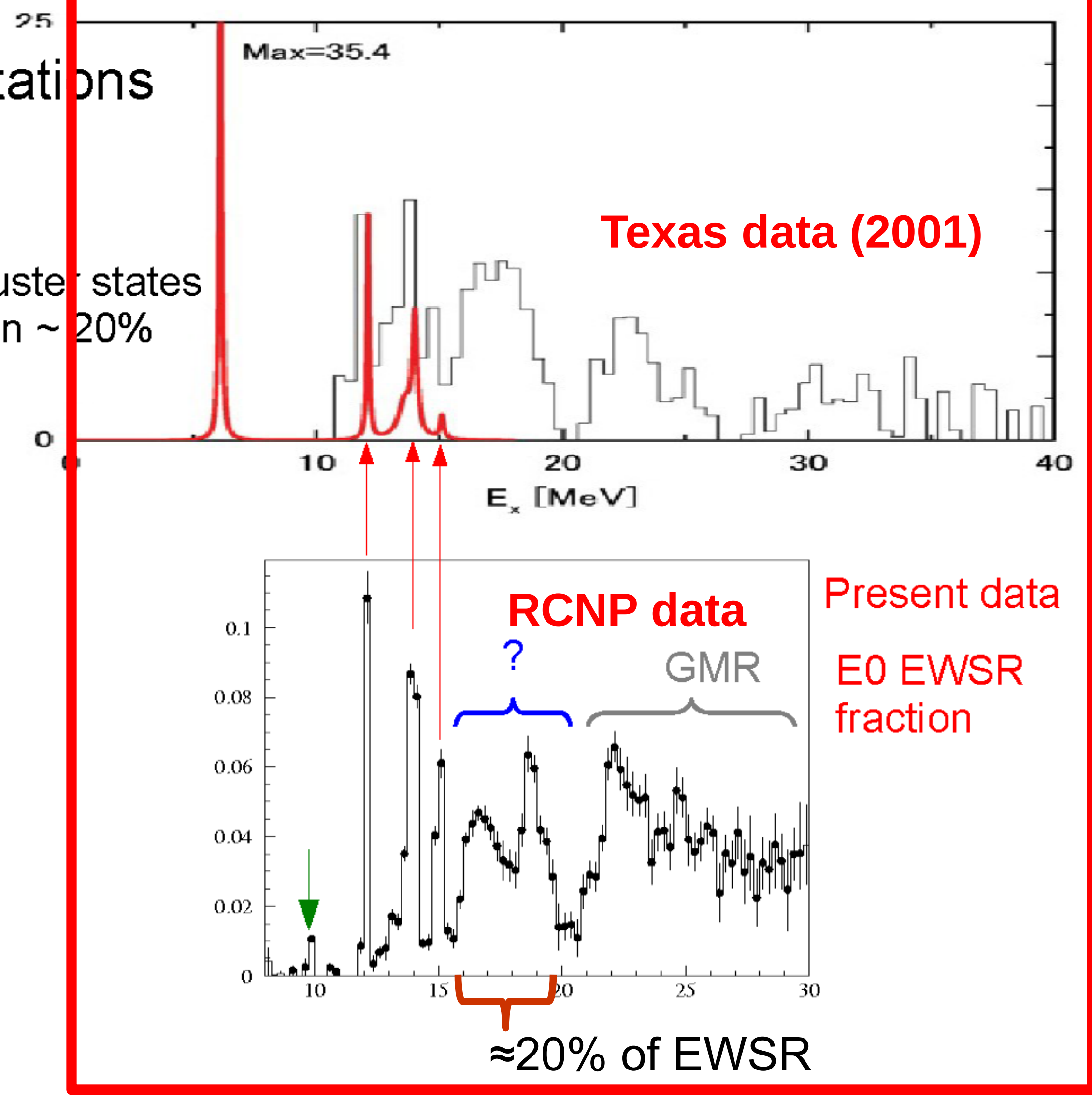
Isoscalar E0 strength in ^{16}O

- Isoscalar monopole excitations
  T. Yamada et al,
  Phys. Rev.C 85 (2012) 034315
  - Monopole excitations to α-cluster states
    $E_x \leqq 16$ MeV, EWSR fraction ~ 20%

- E_α = 386MeV@RCNP

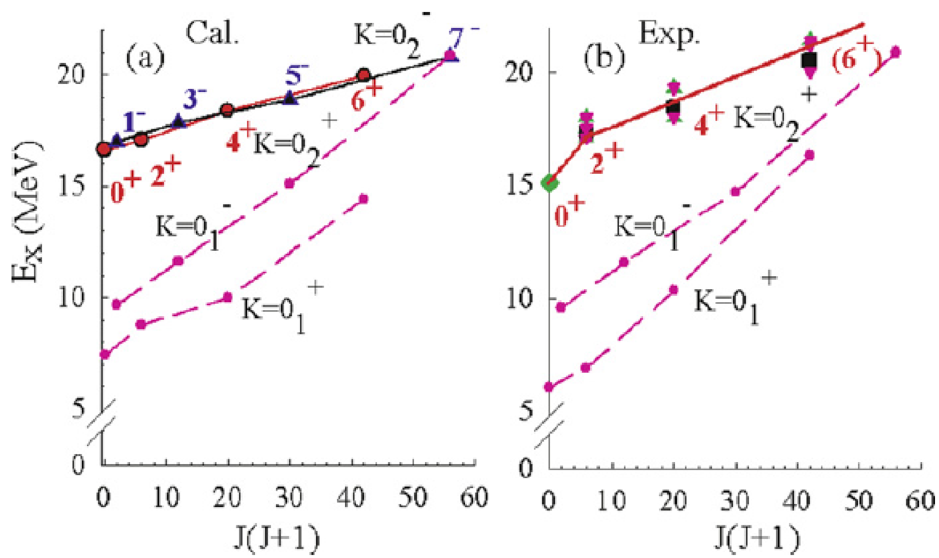
new analysis of $^{16}O(\alpha,\alpha')$

Itoh (Tohoku)'s talk,
RCNP workshop, 19 July 2012



Hoyle+α states and Linear Chain States

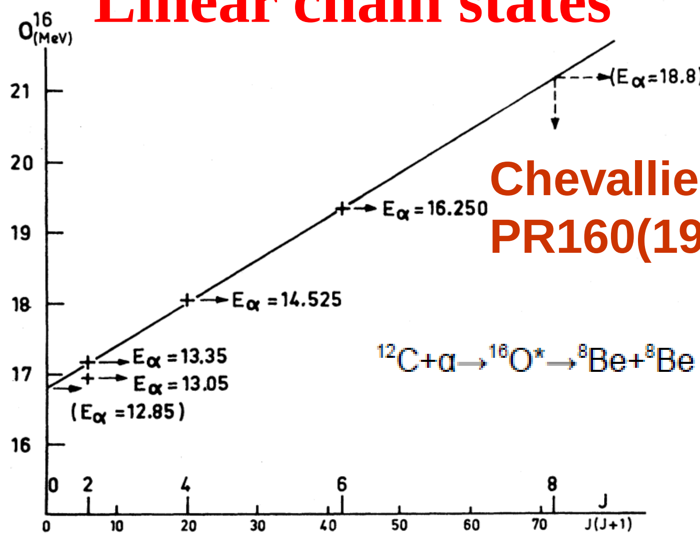
Hoyle+α cluster



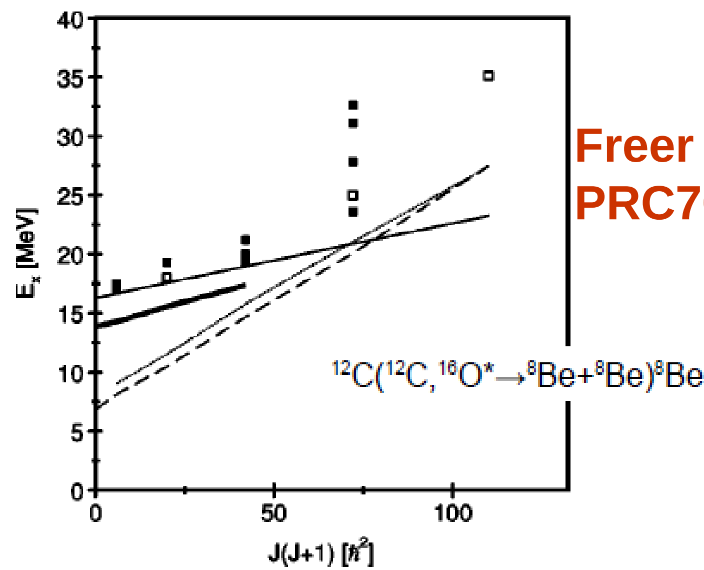
Ohkubo et al., PLB684(2010)

Funaki et al., 4α OCM

Linear chain states



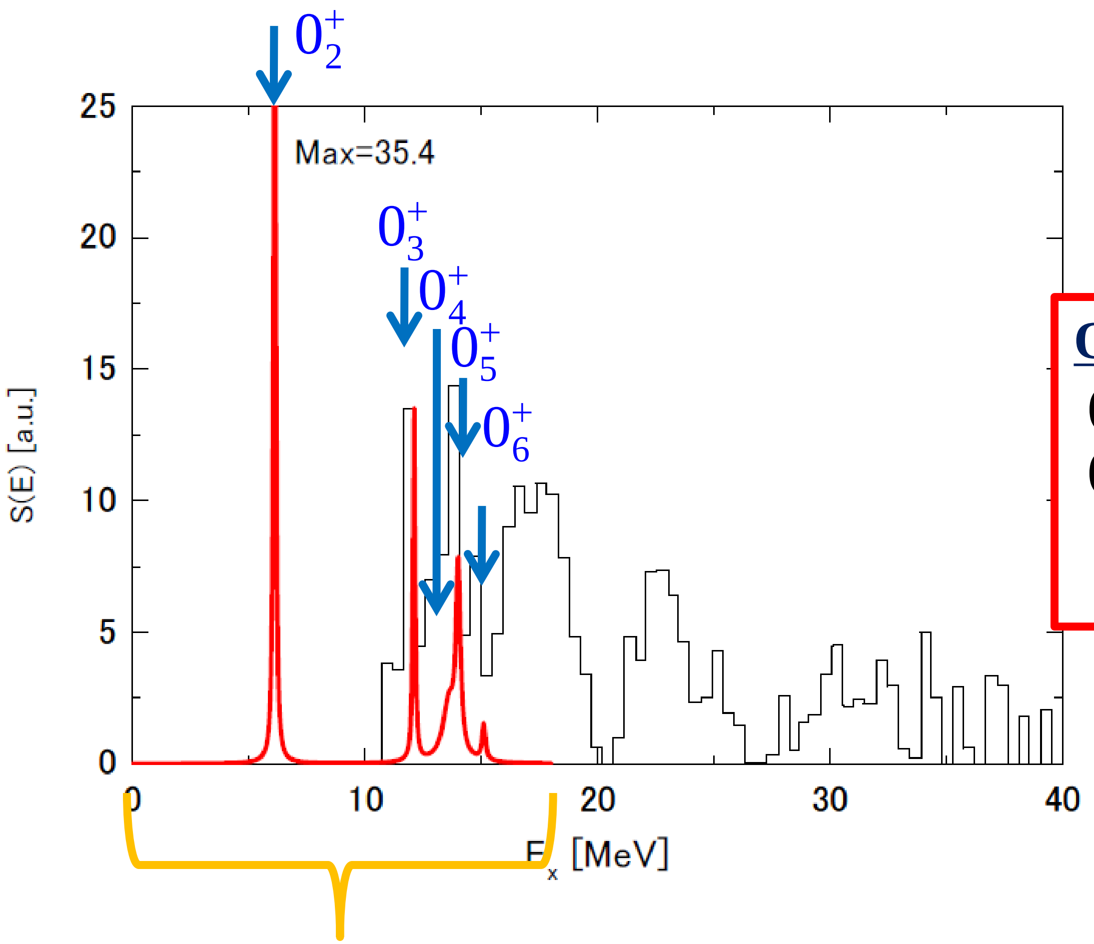
Chevallier et al., PR160(1967)



Freer et al., PRC70(2004)

Exp. vs. Cal.

IS monopole S(E) with 4α OCM



Two features in IS monopole excitations

Origin: dual nature of G.S. of ^{16}O

(1) α-clustering degree of freedom

(2) mean-field-type one

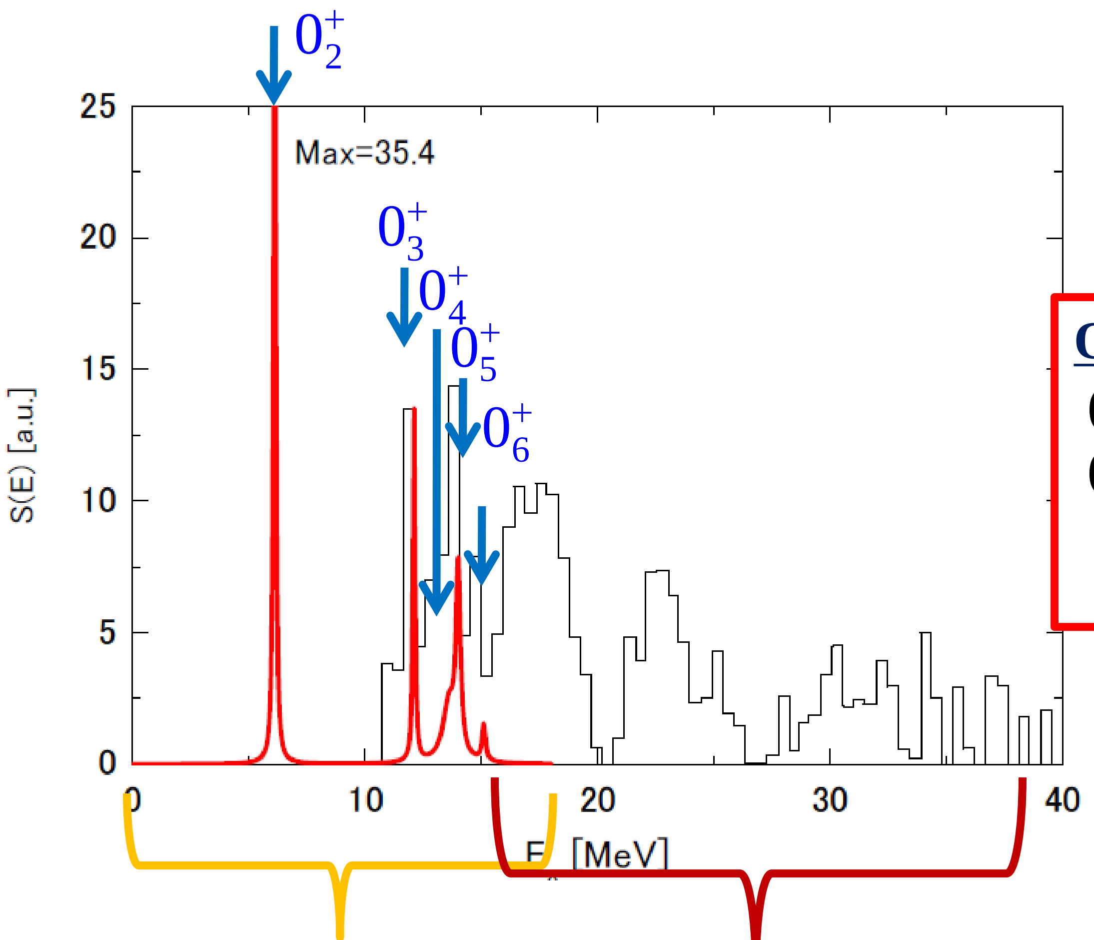
$(0s)^4(0p)^{12}$: SU(3)(00)= ^{12}C+α :

Bayman-Bohr theorem

Excitation to cluster states (α-cluster type)

Exp. vs. Cal.

IS monopole S(E) with 4α OCM



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Bayman-Bohr theorem

Excitation to cluster states (α-cluster type)

Monopole excitation of mean-field type (RPA)

Dual nature of ground state of ^{16}O

mean-field character and α-clustering character

Ground state of ^{16}O

(λ,μ)

Dominance of doubly-closed-shell structure: $(0s)^4(0p)^{12}$ **= SU(3)(0,0)**

Cluster-model calculations: 4α OCM, 4α THSR, α+^{12}C OCM,...
Mean-field calculations : RPA, QRPA, RRPA,......

Supported by no-core shell model calculations:
Dytrych et al., PRL98 (2007)

Bayman & Bohr, NPA9 (1958/59)

Bayman-Bohr theorem : **SU(3)[f](λμ) is equivalent to "a cluster-model wf"**

Doubly-closed-shell w.f, $(0s)^4(0p)^{12}$, is mathematically equivalent to a single α-cluster w.f.

This means that the ground state w.f. of ^{16}O originally has an α-clustering degree of freedom together with mean-filed-type degree of free dom.

We call dual nature of g.s.

Bayman-Bohr theorem

Nucl. Phys. 9, 596 (1958/1959)

c.o.m. w.f. of ^{16}O

$$\phi_{\rm cm}(\mathbf{R}_{\rm cm}) = \left(\frac{32\nu}{\pi}\right)^{3/4} \exp(-16\nu \mathbf{R}_{\rm cm}^2)$$

$$\frac{1}{\sqrt{16!}} \det\left|(0s)^4(0p)^{12}\right| \times \left[\phi_{\rm cm}(\mathbf{R}_{\rm cm})\right]^{-1} \quad \text{: closed shell}$$

$$= N_0 \sqrt{\frac{12!4!}{16!}} A\left\{\left[u_{40}(\xi_3, 3\nu)\phi_{L=0}(^{12}\mathrm{C})\right]_{J=0} \phi(\alpha)\right\}$$

relative wf (S-wave)

$$= N_2 \sqrt{\frac{12!4!}{16!}} A\left\{\left[u_{42}(\xi_3, 3\nu)\phi_{L=2}(^{12}\mathrm{C})\right]_{J=0} \phi(\alpha)\right\}$$

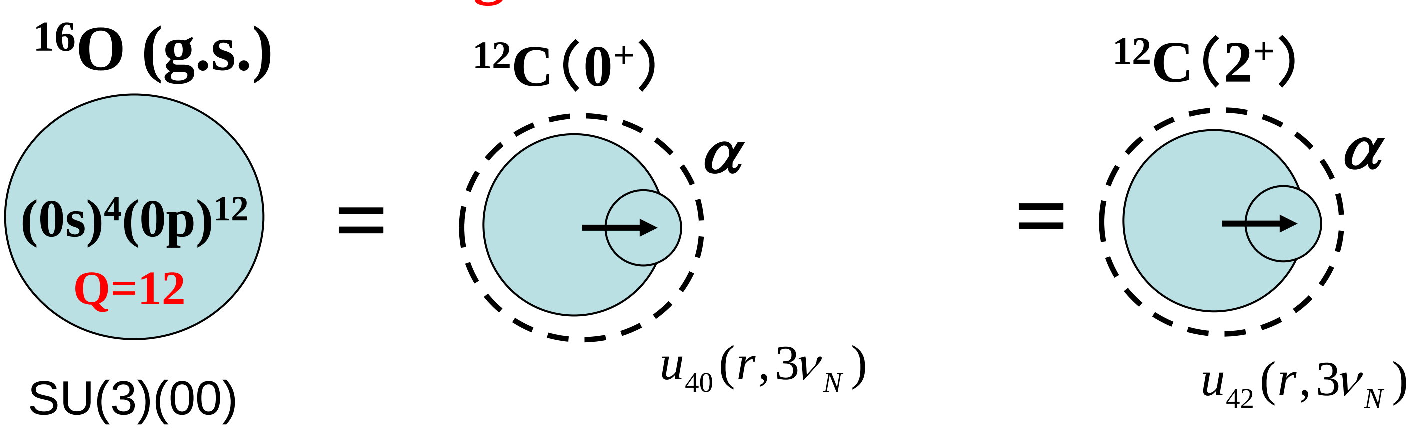
relative wf (D-wave)

} **α-degree of freedom**

➔G.S. has mean-field-type and α-cluster degrees of freedom.

We call dual nature of g.s.

0^+_1



Bayman-Bohr theorem

Nucl. Phys. 9, 596 (1958/1959)

c.o.m. w.f. of ^{16}O

$$\phi_{\rm cm}(\mathbf{R}_{\rm cm}) = \left(\frac{32\nu}{\pi}\right)^{3/4} \exp(-16\nu \mathbf{R}_{\rm cm}^2)$$

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} α-degree of freedom

➔G.S. has mean-field-type and α-cluster degrees of freedom.

Excitation of mean-field-type degree of freedom in g.s
➔ 1p1h states (3-bump structure)

Excitation of α-cluster degree of freedom in g.s
➔ α+^{12}C cluster states: 2nd 0+, 3rd 0+

IS monopole operator

$$\mathcal{O} = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2 = \underbrace{\sum_{i=1}^{4} (\mathbf{r}_i - \mathbf{R}_\alpha)^2 + \sum_{i=5}^{16} (\mathbf{r}_i - \mathbf{R}_{12\mathrm{C}})^2}_{\text{internal parts}} + \underbrace{3(\mathbf{R}_\alpha - \mathbf{R}_{12\mathrm{C}})^2}_{\text{relative part}}$$

Bayman-Bohr theorem

Nucl. Phys. 9, 596 (1958/1959)

c.o.m. w.f. of ^{16}O

$$\phi_{\rm cm}(\mathbf{R}_{\rm cm}) = \left(\frac{32\nu}{\pi}\right)^{3/4} \exp(-16\nu \mathbf{R}_{\rm cm}^2)$$

$$\frac{1}{\sqrt{16!}} \det\left|(0s)^4(0p)^{12}\right| \times [\phi_{\rm cm}(\mathbf{R}_{\rm cm})]^{-1}$$: closed shell

$$= N_0 \sqrt{\frac{12!4!}{16!}} A\left\{\left[u_{40}(\xi_3, 3\nu)\phi_{L=0}(^{12}\mathrm{C})\right]_{J=0} \phi(\alpha)\right\}$$

relative wf (S-wave)

$$= N_2 \sqrt{\frac{12!4!}{16!}} A\left\{\left[u_{42}(\xi_3, 3\nu)\phi_{L=2}(^{12}\mathrm{C})\right]_{J=0} \phi(\alpha)\right\}$$

relative wf (D-wave)

α-degree of freedom

➔G.S. has mean-field-type and α-cluster degrees of freedom.

IS monopole operator

$$\mathcal{O} = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2$$

$$\begin{aligned}\mathcal{O}\,|(0s)^4(0p)^{12}\rangle = \sum |1p1h\rangle \quad &= \quad c_1|(1s_{\frac{1}{2}})(0s_{\frac{1}{2}})^{-1}\rangle + c_2|(1p_{\frac{3}{2}})(0p_{\frac{3}{2}})^{-1}\rangle \\ &+ \quad c_3|(1p_{\frac{1}{2}})(0p_{\frac{1}{2}})^{-1}\rangle\end{aligned}$$

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Nucl. Phys. 9, 596 (1958/1959)

c.o.m. w.f. of ^{16}O

$$\phi_{\rm cm}(\mathbf{R}_{\rm cm}) = \left(\frac{32\nu}{\pi}\right)^{3/4} \exp(-16\nu \mathbf{R}_{\rm cm}^2)$$

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relative wf (S-wave)

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➔G.S. has mean-field-type and *α*-cluster degrees of freedom.

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$$\mathcal{O} = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2$$

$$\mathcal{O}\,|(0s)^4(0p)^{12}\rangle = \sum |1p1h\rangle = c_1|(1s_{\frac{1}{2}})(0s_{\frac{1}{2}})^{-1}\rangle + c_2|(1p_{\frac{3}{2}})(0p_{\frac{3}{2}})^{-1}\rangle + c_3|(1p_{\frac{1}{2}})(0p_{\frac{1}{2}})^{-1}\rangle$$

Excitation of mean-field-type degree of freedom in g.s

➔ 1p1h states are produced (3-bump structure)

Bayman-Bohr theorem

Nucl. Phys. 9, 596 (1958/1959)

$$\frac{1}{\sqrt{16!}}\det\left|(0s)^4(0p)^{12}\right|\times\left[\phi_{\rm cm}(\mathbf{R}_{\rm cm})\right]^{-1}$$ **: closed shell**

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$$\phi_{\rm cm}(\mathbf{R}_{\rm cm})=\left(\frac{32\nu}{\pi}\right)^{3/4}\exp(-16\nu\mathbf{R}_{\rm cm}^2)$$

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$$=N_2\sqrt{\frac{12!4!}{16!}}A\left\{\left[\underline{u_{42}(\xi_3,3\nu)}\phi_{L=2}(^{12}\mathrm{C})\right]_{J=0}\phi(\alpha)\right\}$$

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➔ 1p1h states (3-bump structure)

Excitation of *α*-cluster degree of freedom in g.s
➔ α+^{12}C cluster states: 2nd 0+, 3rd 0+

IS monopole operator

$$\mathcal{O}=\sum_{i=1}^{16}(\boldsymbol{r}_i-\boldsymbol{R}_{\rm cm})^2=\underbrace{\sum_{i=1}^{4}(\mathbf{r}_i-\mathbf{R}_\alpha)^2+\sum_{i=5}^{16}(\mathbf{r}_i-\mathbf{R}_{12\mathrm{C}})^2}_{\text{internal parts}}+\underbrace{3(\mathbf{R}_\alpha-\mathbf{R}_{12\mathrm{C}})^2}_{\text{relative part}}$$

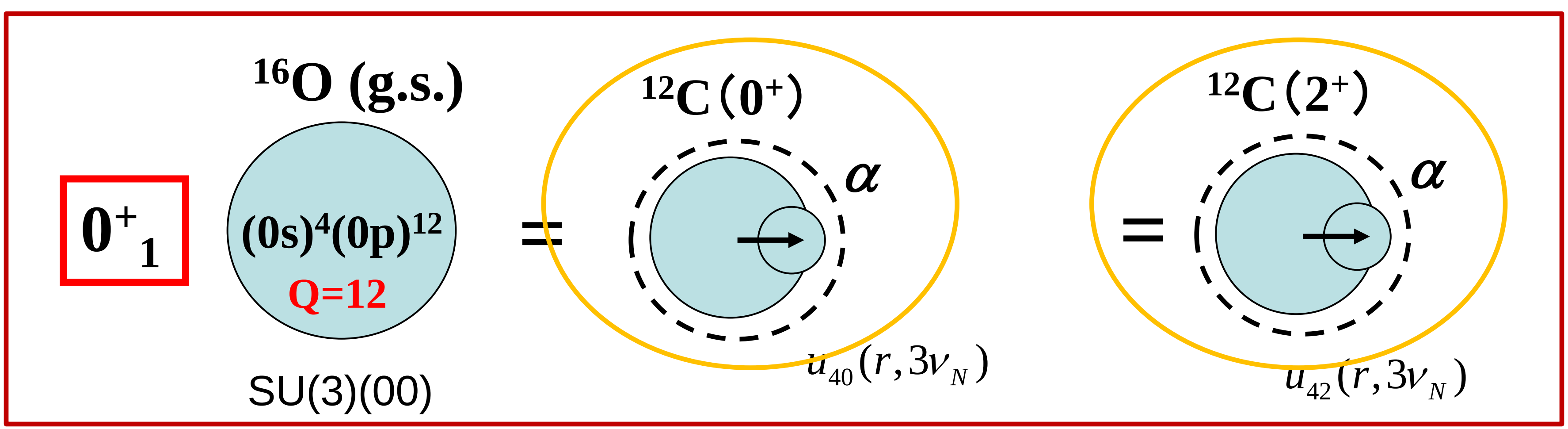
Monopole transitions: $0^+_1 - 0^+_2$, $0^+_1 - 0^+_3$

Monopole operator

$$\mathcal{O} = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\mathrm{cm}})^2$$
$$= \mathcal{O}(\alpha) + \mathcal{O}(^{12}\mathrm{C}) + 3\xi_3{}^2$$

Relative motion is excited.

Yamada et al., PTP120 (2008)



Monopole transitions: $0^+_1 - 0^+_2$, $0^+_1 - 0^+_3$

Monopole operator

$$\mathcal{O} = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2$$

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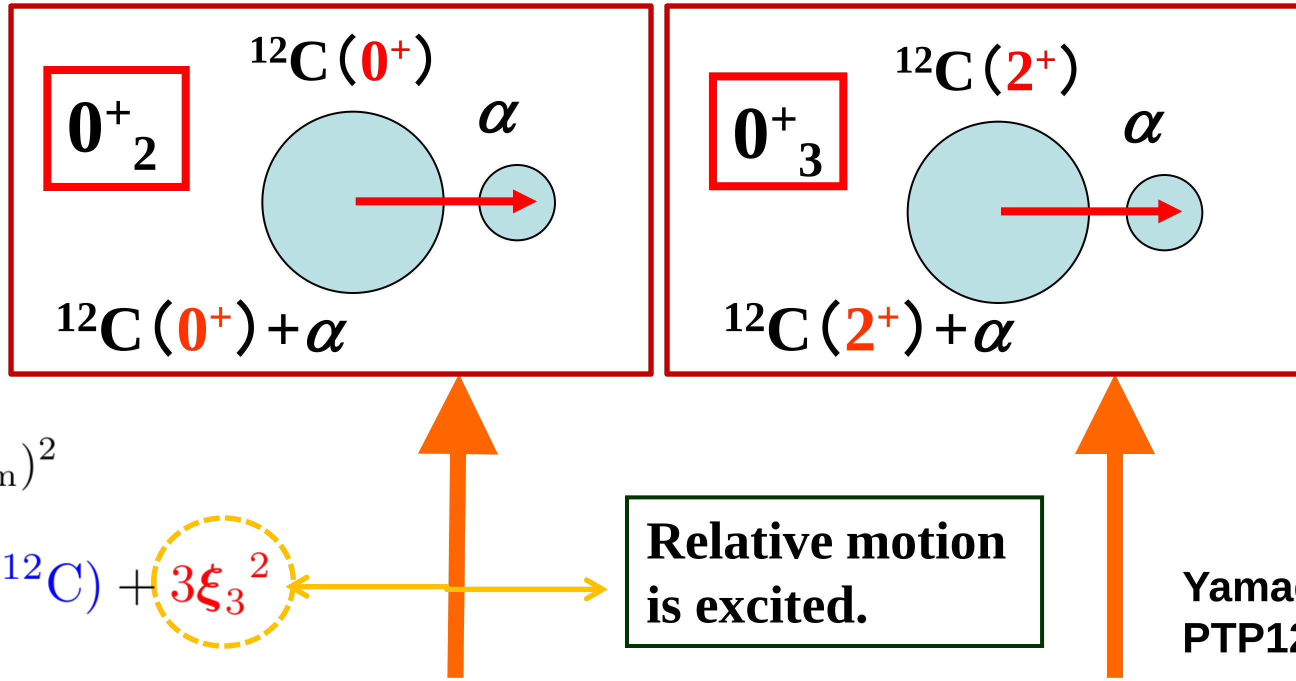
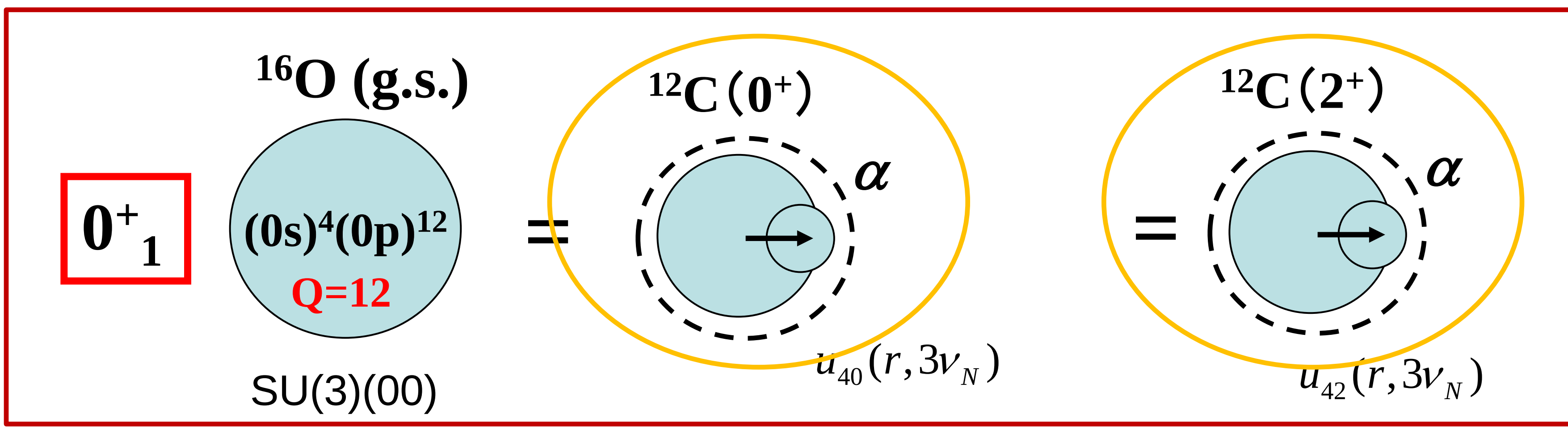
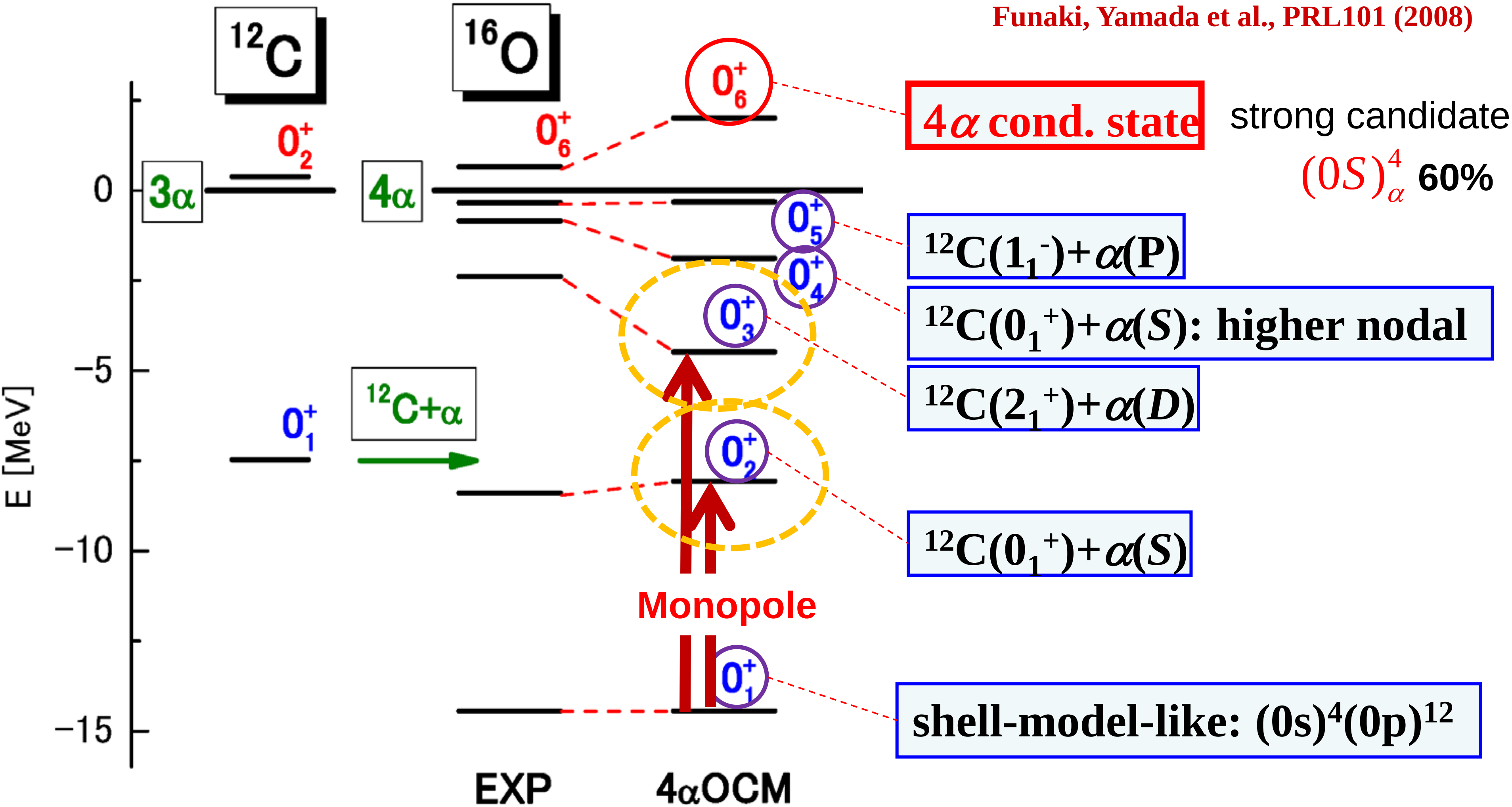


Yamada et al., PTP120 (2008)



4α OCM calculation

Funaki, Yamada et al., PRL101 (2008)



		Experiment				4α OCM		
	Ex [MeV]	R [fm]	M(E0) [fm^2]	Γ [MeV]		R [fm]	M(E0) [fm^2]	Γ [MeV]
0^+_1	0.00	2.71				2.7		
0^+_2	6.05		3.55			3.0	3.9	
0^+_3	12.1		4.03			3.1	2.4	
0^+_4	13.6		no data	0.6		4.0	2.4	0.60
0^+_5	14.0		3.3	0.185		3.1	2.6	0.20
0^+_6	15.1		no data	0.166		5.6	1.0	0.14

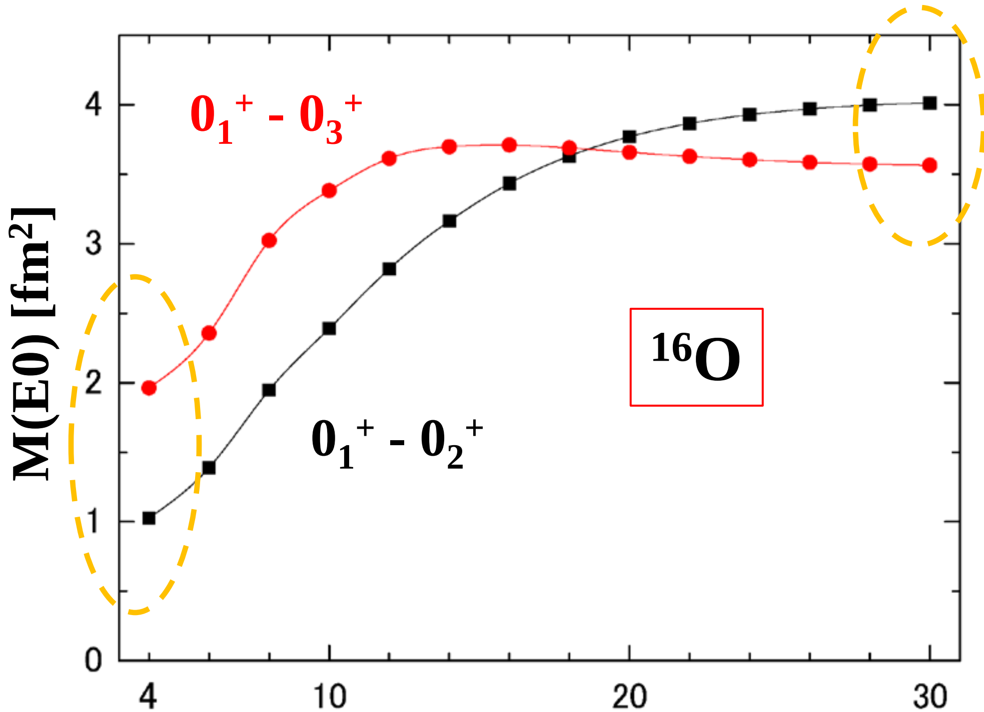
over 15%
of total EWSR

20%
of total EWSR

Importance of α-type ground-state correlation in monopole strengths

<u>$^{16}O = \alpha + ^{12}C$ OCM analysis</u>

Monopole strengths: M(E0)



H.O. quanta（Model space of ^{16}O g.s. with $\alpha + ^{12}C$ OCM)

2nd 0+ Exp. : 4.03±0.09 fm²
3rd 0+ Exp. : 3.55±0.21 fm²

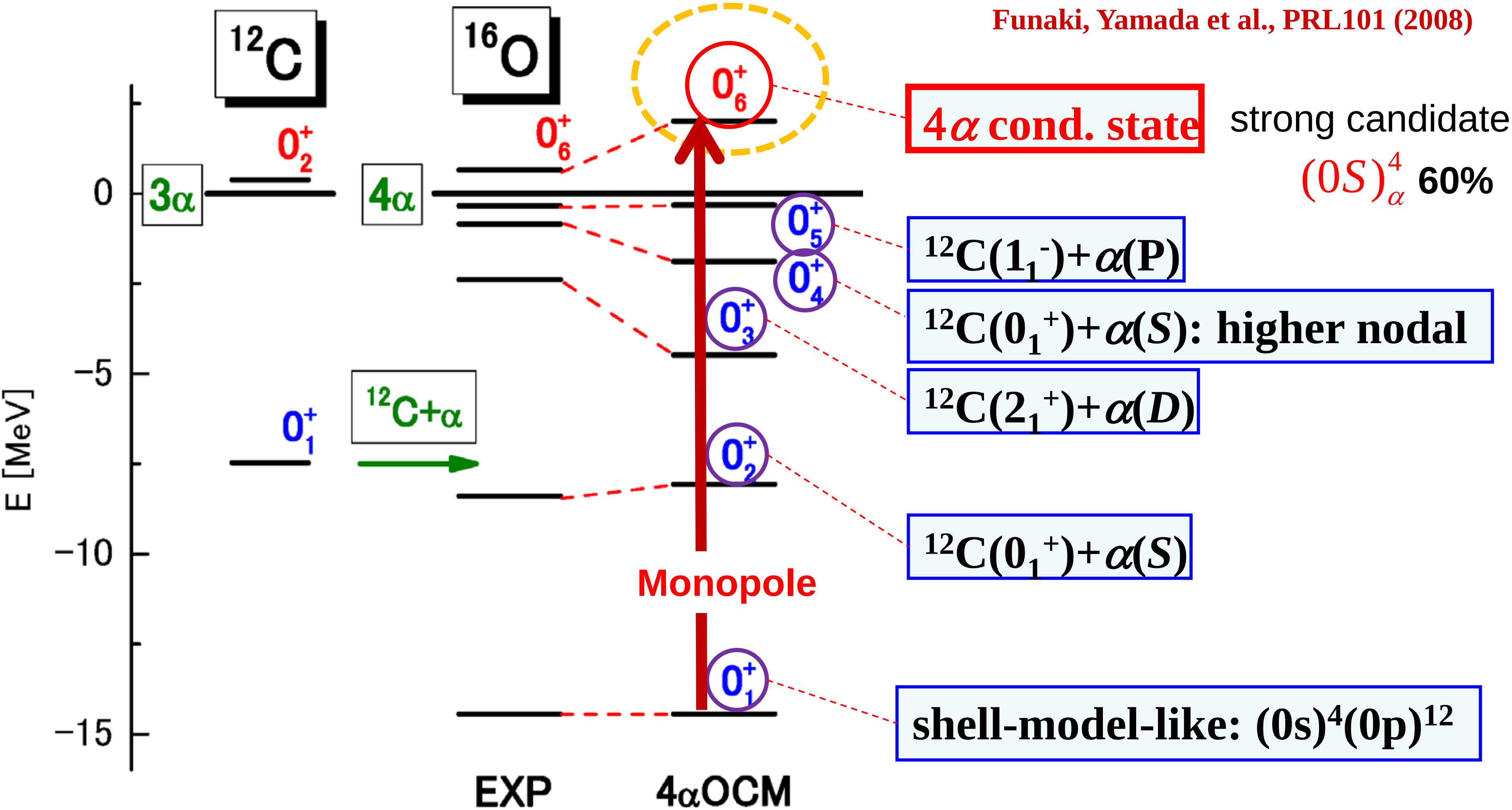
Importance of G.S. correlation
(α clustering in g.s.)

<u>**SU(3) (0,0) limit**</u>

Yamada et al., PTP120(2008)

4α OCM calculation

Funaki, Yamada et al., PRL101 (2008)



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over 15%
of total EWSR

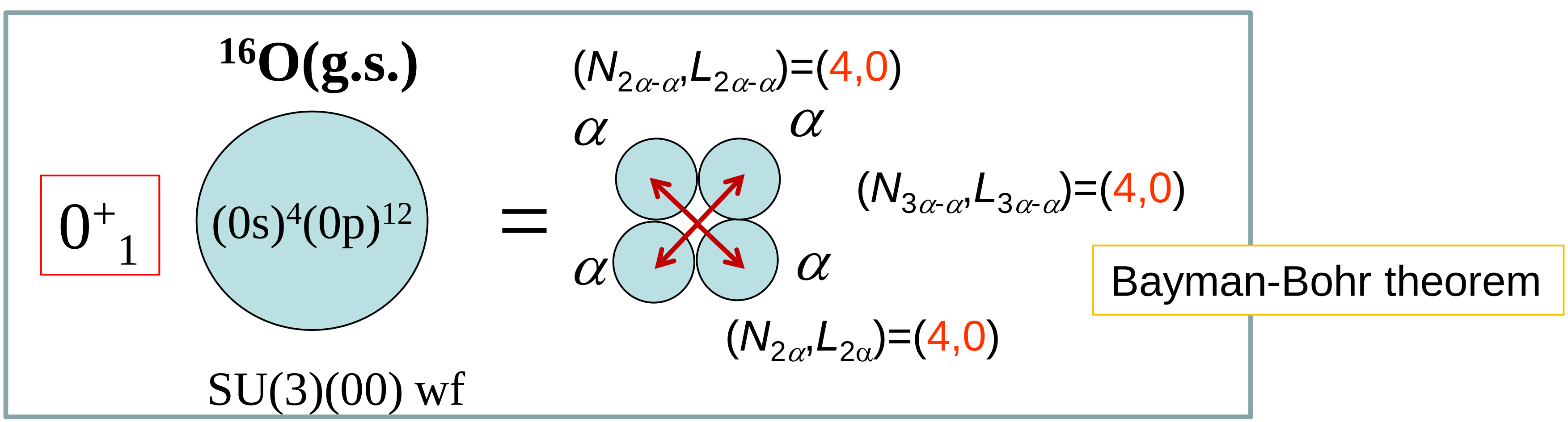
20%
of total EWSR

Bayman-Bohr theorem

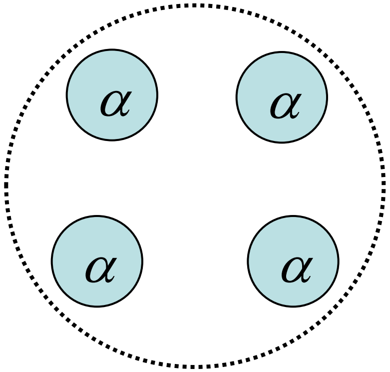
$$\frac{1}{\sqrt{16!}}\det|(0s)^4(0p)^{12}| \times [\phi_{\rm cm}(\boldsymbol{R}_{\rm cm})]^{-1}$$ **: closed shell**

$$= \hat{N}_0\sqrt{\frac{4!4!4!4!}{16!}}\mathcal{A}\Big\{ \Big[u_{40}(\boldsymbol{\xi}_3, 3\nu)\Big[u_{40}(\boldsymbol{\xi}_2, \frac{8}{3}\nu)u_{40}(\boldsymbol{\xi}_1, 2\nu)\Big]_{L=0}\Big]_{J=0}$$
$$\times\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4)\Big\}$$ **4α-cluster wf**

➔ G.S. has a 4α-cluster degree of freedom.



0^+_6 state ≈ 4α gas-like or $\alpha+{}^{12}$C(Hoyle)



4α -gas-like

M(E0)=1.0 fm^2 by 4α OCM

Monopole transition

$$\mathcal{O} = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2$$

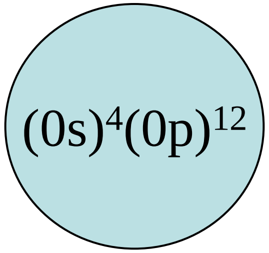
$$= \sum_{k=1}^{4}\sum_{i=1}^{4} (\mathbf{r}_{i+4(k-1)} - \mathbf{R}_{\alpha_k})^2 + \sum_{k=1}^{4} 4(\mathbf{R}_{\alpha_k} - \mathbf{R}_{\rm cm})^2$$

internal part **relative part**

coherent excitation

0^+_1

^{16}O(g.s.)



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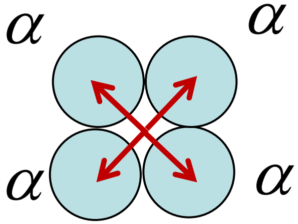


$(N_{2\alpha\text{-}\alpha}, L_{2\alpha\text{-}\alpha})=(4,0)$

$(N_{3\alpha\text{-}\alpha}, L_{3\alpha\text{-}\alpha})=(4,0)$

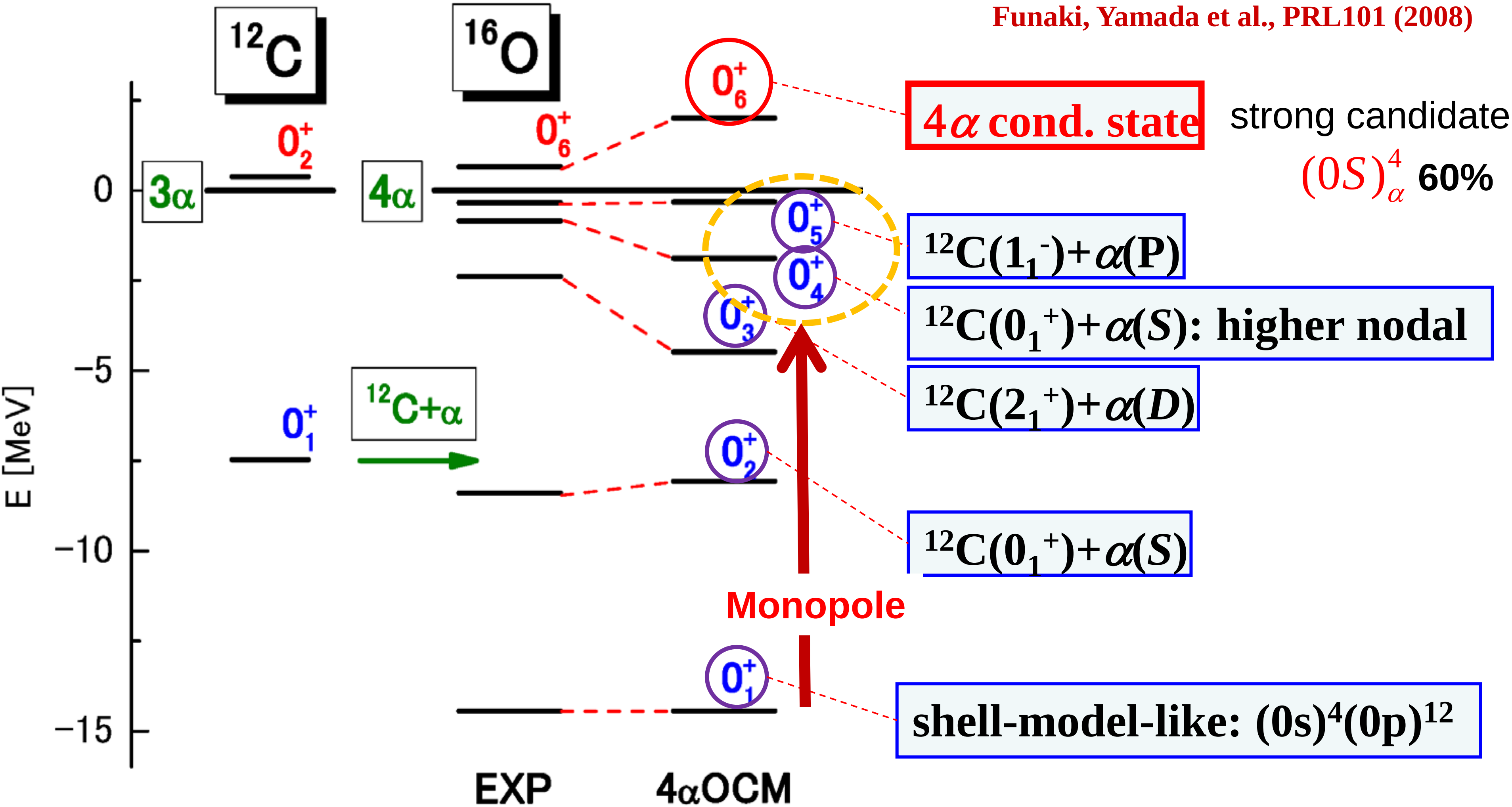
$(N_{2\alpha}, L_{2\alpha})=(4,0)$

Bayman-Bohr theorem

SU(3)(00) wf

4α OCM calculation

Funaki, Yamada et al., PRL101 (2008)



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over 15%
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20%
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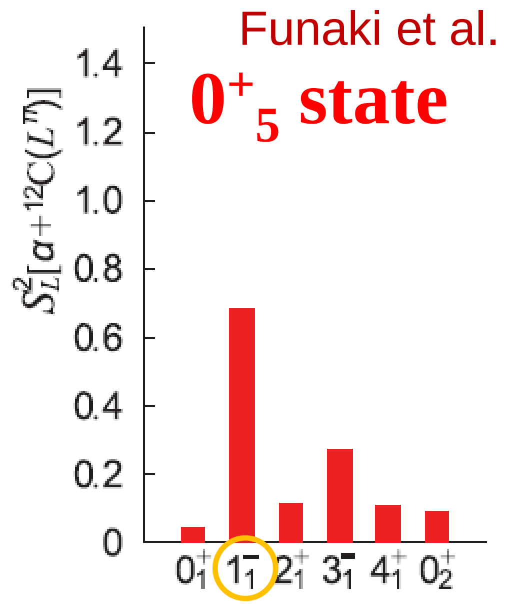
Monopole excitation to 0^+_5 and 0^+_4 state

S^2-factors of $\alpha+^{12}C(L^\pi)$ channels

0^+_5 state: $^{12}C(1_1^-)+\alpha(P)$ main configuration



Bayman-Bohr theorem:

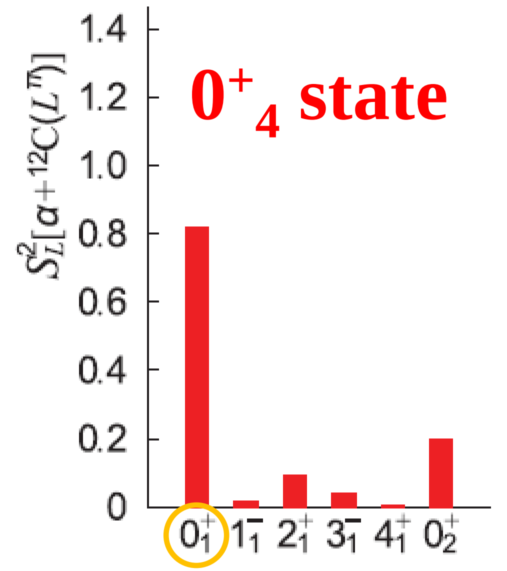
$(0s)^4(0p)^{12}$ has no configuration of $^{12}C(1-)+\alpha$

Why this state is excited?

Coupling with $^{12}C(0+,2+)+\alpha$ and $^{12}C(Hoyle)+\alpha$

Coherent contribution from these configurations

0^+_4 state: $^{12}C(0_1^+)+\alpha(S)$: higher nodal



Coherent contributions from

$^{12}C(0+,2+)+\alpha$ and $^{12}C(Hoyle)+\alpha$ configurations

Dual nature of the ground states in ^{16}O common to light nuclei

^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, ….

For example, ^{20}Ne g.s.

Dual nature of the ground states in ^{12}C and ^{16}O : common to light nuclei

^{20}Ne

^{20}Ne, ^{24}Mg, ^{32}S, ..., ^{44}Ti,....
^{11}B, ^{13}C,....

$$\Phi_J(^{20}\mathrm{Ne}) = |(0s)^4(0p)^{12}(1s0d)^4 : SU(3)(80), J\rangle_{\mathrm{internal}}$$ **: SU(3) wf**

$$= N_J\sqrt{\frac{4!16!}{20!}}\mathcal{A}\left\{\underline{u_{8J}(\boldsymbol{r}_{\alpha-^{16}\mathrm{O}})}\phi(\alpha)\phi(^{16}\mathrm{O})\right\}$$ **: cluster wf**

relative wf (J-wave) α+16O

Activation of mean-field degree of freedom

➔ $K^\pi = 2^-$ band : $5p1h$ state

Activation of *α*-cluster degree of freedom

➔ ***α*+^{16}O cluster states of $K^\pi = 0^+_4, 0^-$ bands**

$K^\pi = 0^+_4$ band : higher nodal states

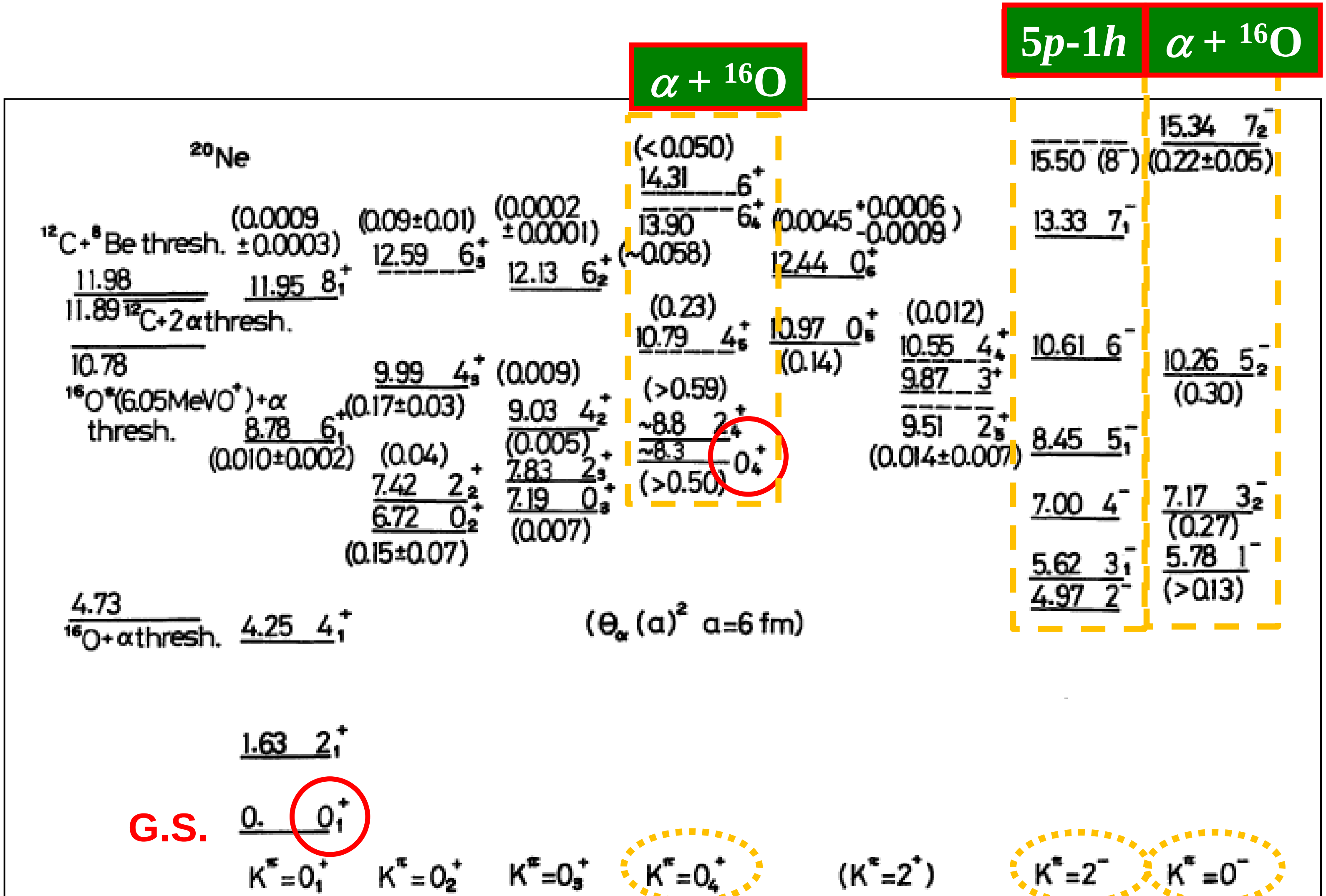
α+^{16}O comp. = 80% for low spins: Q=10 quanta

$K^\pi = 0^-$ band : parity – doublet states

Almost pure α+^{16}O structures for low spins: Q=9 quanta

Observed levels of ^{20}Ne

α+^{16}O model, 2α+^{12}C model
AMD (Kimura)



$$\Phi_J(^{20}\mathrm{Ne}) = |(0s)^4(0p)^{12}(1s0d)^4 : SU(3)(80), J\rangle_{\mathrm{internal}}$$

$$= N_J \mathcal{A}\left\{u_{8J}(\boldsymbol{r}_{\alpha-(\alpha+^{12}\mathrm{C})})\underline{\phi(\alpha)}[u_{4L}(\boldsymbol{r}_{\alpha-^{12}\mathrm{C}})\underline{\phi(\alpha)}\phi_L(\underline{^{12}\mathrm{C}})]_0\right\}$$

Dual nature of the ground states in ^{12}C and ^{16}O : common to light nuclei

^{20}Ne

^{20}Ne, ^{24}Mg, ^{32}S, ….., ^{44}Ti,….
^{11}B, ^{13}C,….

$$\Phi_J(^{20}\mathrm{Ne}) = |(0s)^4(0p)^{12}(1s0d)^4 : SU(3)(80), J\rangle_{\mathrm{internal}}$$

: SU(3) wf

$$= N_J\sqrt{\frac{4!16!}{20!}}\mathcal{A}\left\{\underline{u_{8J}(\boldsymbol{r}_{\alpha-^{16}\mathrm{O}})}\phi(\alpha)\phi(^{16}\mathrm{O})\right\}$$

relative wf (J-wave)

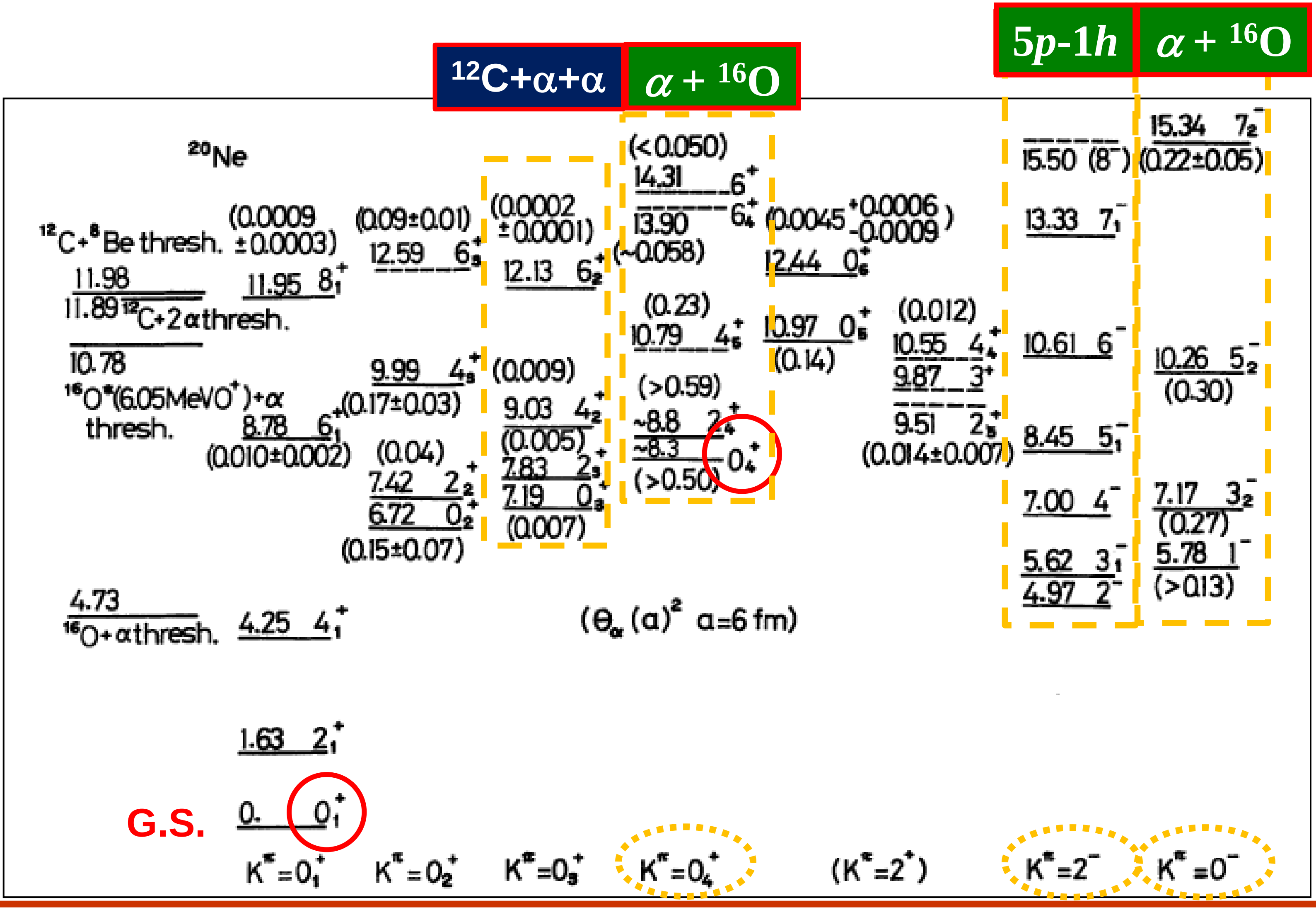
: cluster wf
α+16O

$$= N_J\mathcal{A}\left\{u_{8J}(\boldsymbol{r}_{\alpha-(\alpha+^{12}\mathrm{C})})\phi(\alpha)[u_{4L}(\boldsymbol{r}_{\alpha-^{12}\mathrm{C}})\phi(\alpha)\phi_L(^{12}\mathrm{C})]_0\right\}$$

: cluster wf
α+α+12C

Observed levels of ^{20}Ne

α+^{16}Omodel, 2α+^{12}C model
AMD (Kimura)

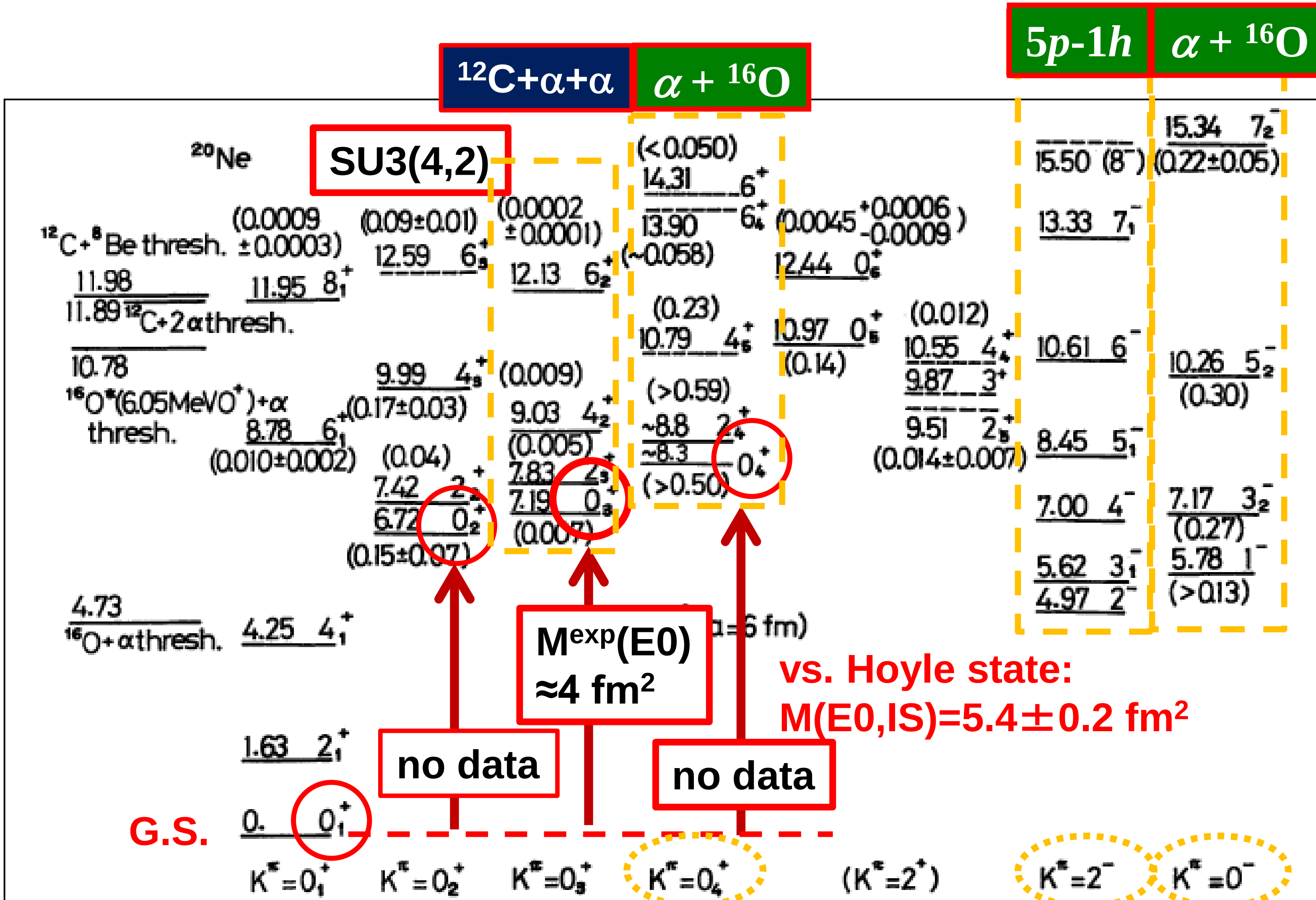


$$\Phi_J(^{20}\mathrm{Ne}) = |(0s)^4(0p)^{12}(1s0d)^4 : SU(3)(80), J\rangle_{\mathrm{internal}}$$

$$= N_J\mathcal{A}\{u_{8J}(\boldsymbol{r}_{\alpha-(\alpha+^{12}\mathrm{C})})\underline{\phi(\alpha)}[u_{4L}(\boldsymbol{r}_{\alpha-^{12}\mathrm{C}})\underline{\phi(\alpha)}\phi_L\underline{(^{12}\mathrm{C})}]_0\}$$

Observed levels of ^{20}Ne

α+^{16}O model, 2α+^{12}C model
AMD (Kimura)



$$\Phi_J(^{20}\mathrm{Ne}) = |(0s)^4(0p)^{12}(1s0d)^4 : SU(3)(80), J\rangle_{\mathrm{internal}}$$

$$= N_J \mathcal{A}\left\{u_{8J}(\boldsymbol{r}_{\alpha-(\alpha+^{12}\mathrm{C})})\underline{\phi(\alpha)}[u_{4L}(\boldsymbol{r}_{\alpha-^{12}\mathrm{C}})\underline{\phi(\alpha)}\phi_L(\underline{^{12}\mathrm{C}})]_0\right\}$$

Isoscalar monopole transition in neutron-rich nuclei

Useful to search for cluster states

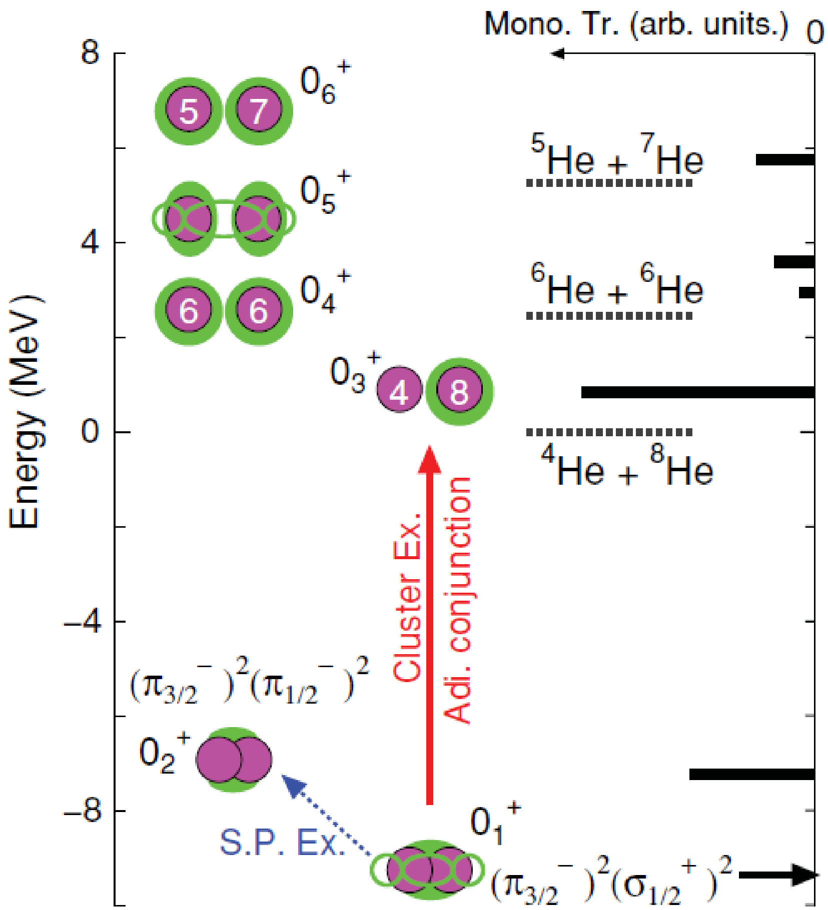
For example, ^{12}Be

Isoscalar monopole transition in neutron-rich nuclei

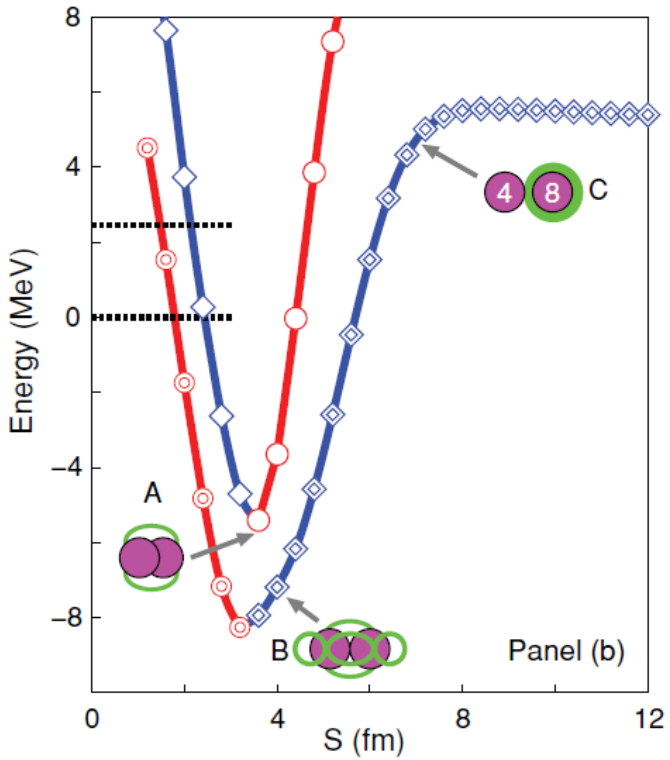
^{12}Be

α+α+4*n* description
based on Generalized two-center cluster model

M. Ito, Phys. Rev. C 83, (2011)



Adiabatic potential



1. クラスター構造とモノポール励起:　代表例　16O

2. Hyper-THSR 波動関数を用いたハイパー核の構造研究

クラスターガス的状態:　12C, 16O　(11B, 13C)

$^{13}{}_{\Lambda}C$　　船木・山田・肥山・池田

Structure study of ${}^{13}_{\Lambda}C$ with Hyper - THSR wf

Funaki, Yamada, Hiyama, Ikeda

The details will be given at the next JPS meeting by Funaki-san

Introduction

- **Cluster picture as well as mean-field picture is important viewpoint to understand structure of light nuclei.**
- **Structure of light nuclei**

  Cluster states + Shell-model-like states

  Microscopic cluster models, AMD,....

  ^{8}Be=2α, ^{12}C=3α , ^{16}O=^{12}C+α / 4α, …

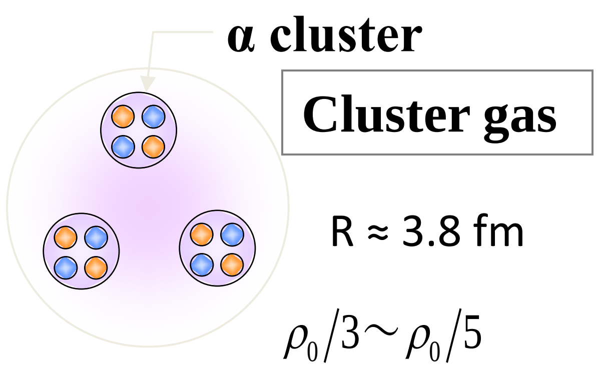
- **α-condensate-like states in 4*n* nuclei:**

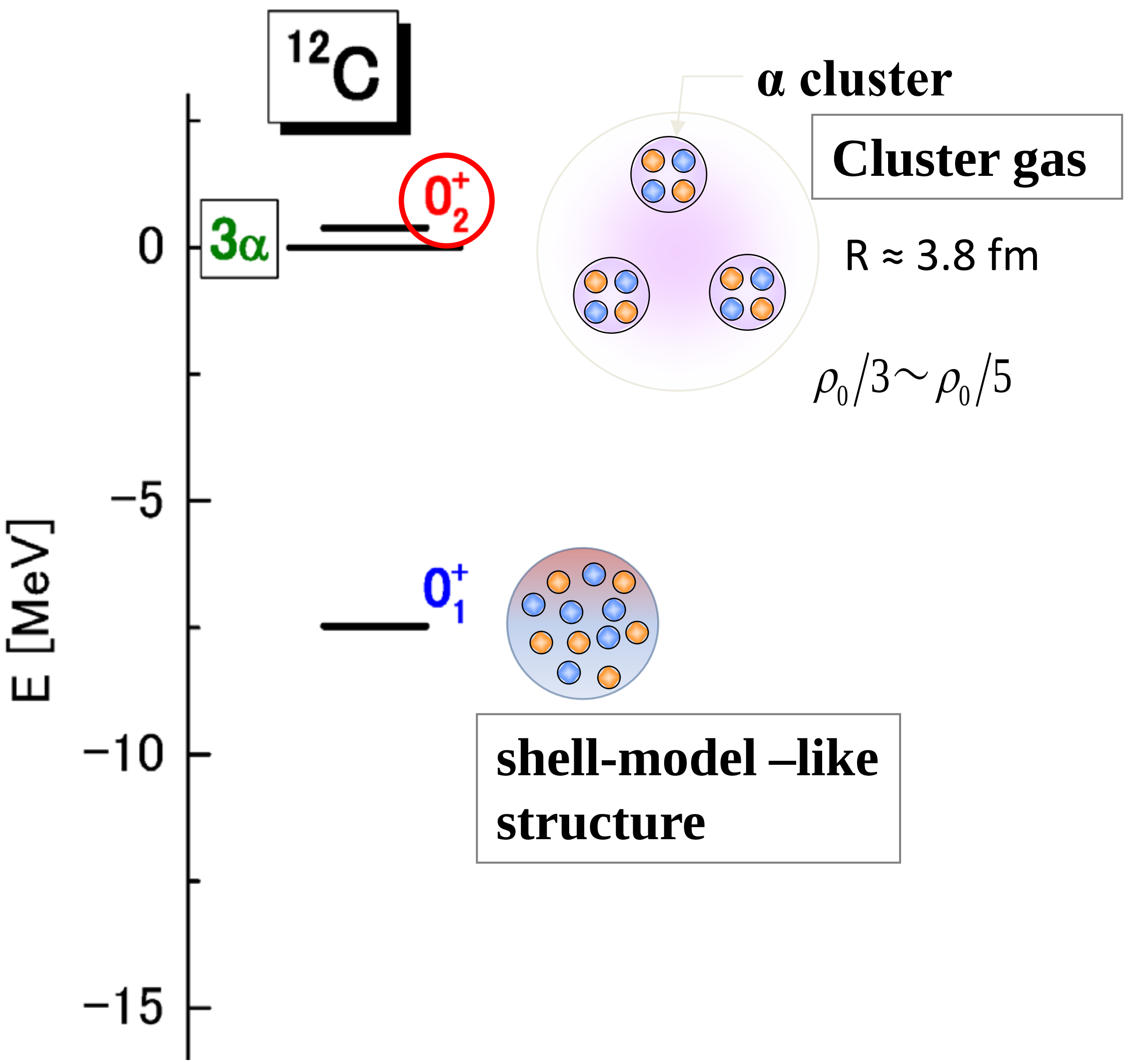
  α-gas-like state described by a product state of α-particles, all with their c.o.m. in (0*S*) orbit.

  Typical state:

  ^{12}C: Hoyle state (2^{nd} 0^{+})

  Tohsaki, Horiuchi, Schuck, Roepke, Phy. Rev. Lett.87 (2001)
  Funaki et al., PRC (2003)
  Yamada et al., EPJA (2005), Matsumura et al.,NPA(2004)

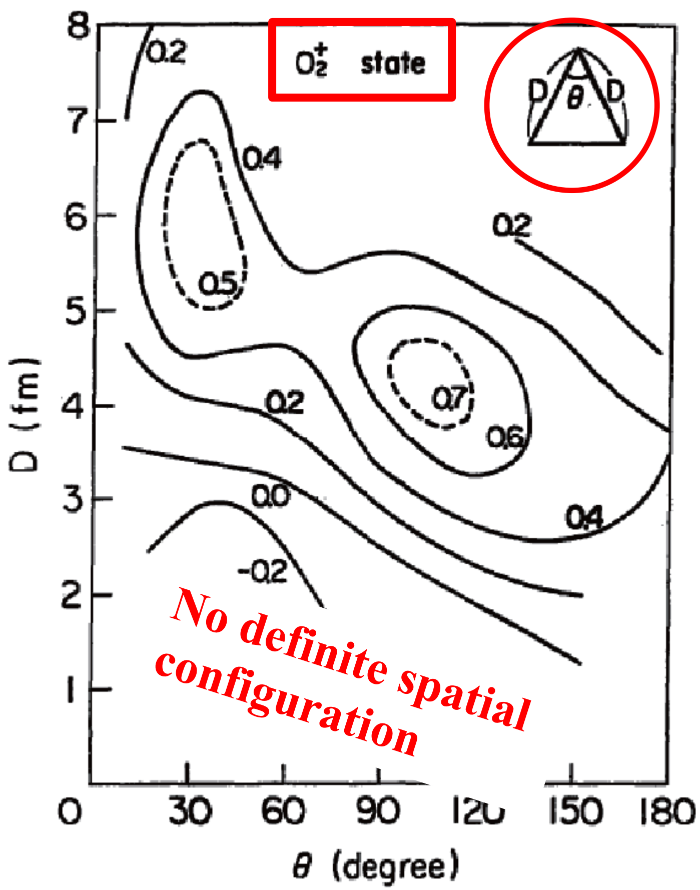




12C
3α
0
−5
−10
−15
E [MeV]
0+2
0+1
α cluster
Cluster gas
R ≈ 3.8 fm
ρ0/3∼ρ0/5
shell-model –like structure

Overlap 3α GCM

$$\langle \text{Brink wf} \,|\, \text{exact } 0_2^+ \text{ state} \rangle$$



Uegaki et al, PTP57(1977)

α-gas-like nature of Hoyle state

Horiuchi, PTP51(1974): 3α OCM

Uegaki et al, PTP57(1977): 3α GCM

Fukushima &Kamimura, (1977): 3α RGM

The 0^+_2 state has a distinct clustering and has **no definite spatial configuration**.

Chernykh, Feldmeir et al., PRL98(2007)

UCOM + FMD, 3α RGM

About **55 components** of the Brink-type wave functions are needed to reproduce the full RGM solution for the Hoyle state.

Tohsaki,Horiuchi,Schuck,Roepke, PRL87(2001)

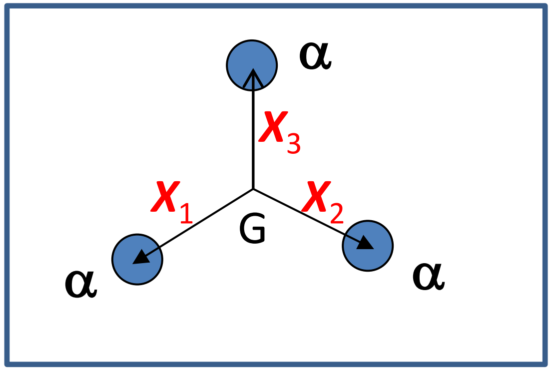
THSR wave function: α-condensate-type cluster wf

1 base THSR : **Funaki et al., PRC67, (2003)**

$$\left| \left\langle \Phi_{3\alpha}^{THSR} \,\middle|\, \mathbf{exact}\ \mathbf{0}_2^+\ \mathbf{state}\ (3\alpha\text{RGM}) \right\rangle \right|^2 \approx 99\%$$

THSR wave function

Tohsaki, Horiuchi, Schuck, Roepke, PRL (2001)



$(0S)_\alpha^3$ B: parameter

$$\Phi_{3\alpha}(B) = \mathcal{A}\left\{\prod_{i=1}^{3}\left[\exp\left(-\frac{2}{B^2}\vec{X}_i^2\right)\phi(\alpha_i)\right]\right\}$$

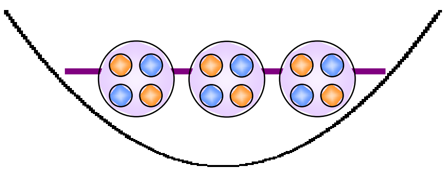
$B \to \infty$ $\Phi_{3\alpha}(B) \to \prod_{i=1}^{3}\left[\exp\left(-\frac{2}{B^2}\vec{X}_i^2\right)\phi(\alpha_i)\right]$

product state

$B \to b$ $\Phi_{3\alpha}(B) \to (0s)^4(0p)^8$

b: nucleon size parameter

Condensed into the lowest orbit



$(0S)_\alpha^3$

Funaki et al., PRC (2003)

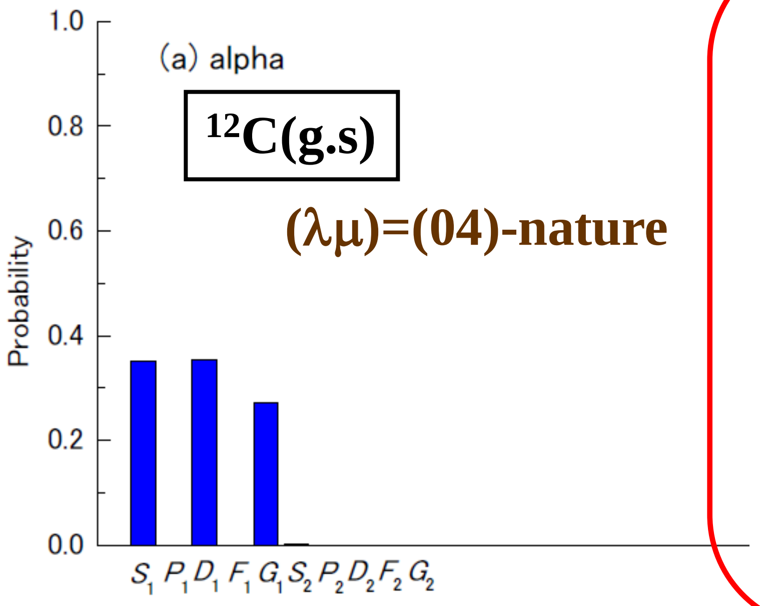
$$\left|\left\langle \Phi_{3\alpha}^{\mathrm{THSR}}(B) \middle| \textbf{exact } \mathbf{0_2^+} \textbf{ state } (3\alpha\mathrm{RGM})\right\rangle\right|^2 \approx 0.9$$

3α RGM: Kamimura & Fukushima (1978)

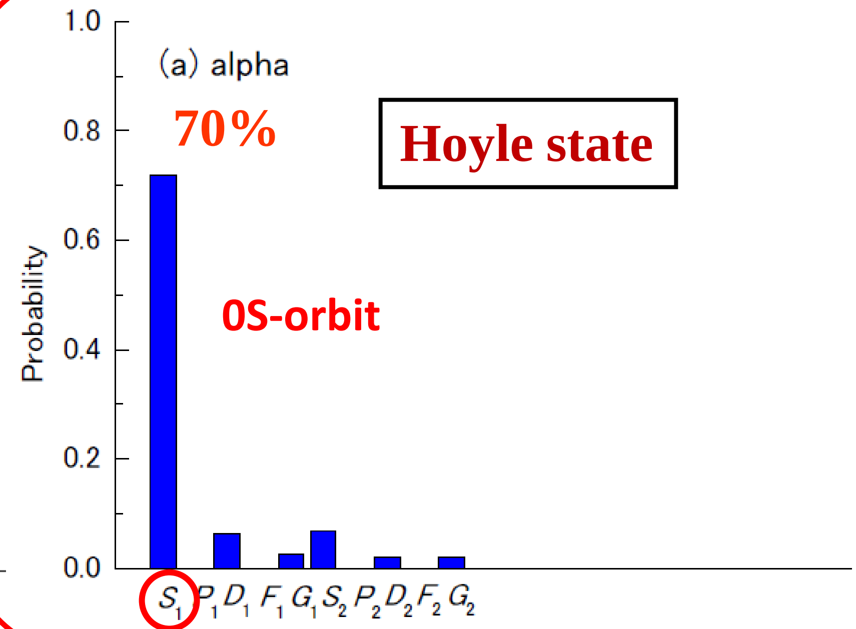
Deformation (B_x,B_y,B_z) ➔ 0.999

$R \approx 3.8$ fm: alpha-gas-like structure

Occupation probabilities of α-orbits in ^{12}C



SU(3) nature: confirmed by no-core shell model , Dytrych et al., PRL98 (2007)



Yamada & Schuck EPJA26 (2005) with 3α OCM wf
Matsumura & Suzuki, NPA739 (2004)
Funaki et al., PRC (2010) with 3α THSR wf

Single cluster density matrix: $\rho(\mathbf{r},\mathbf{r}')$

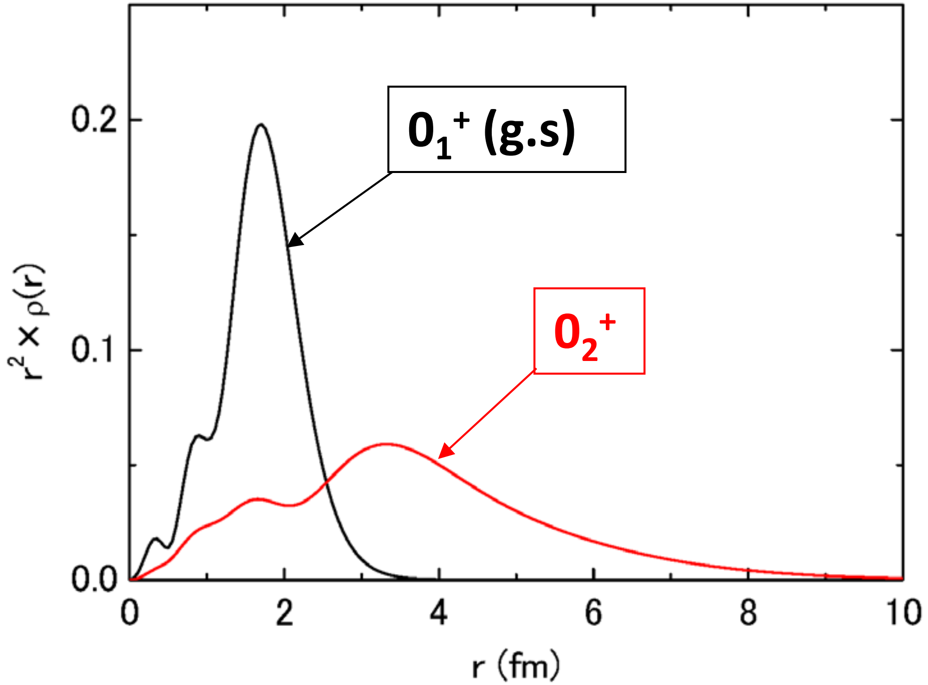
$$\int d\mathbf{r}'\rho_\alpha(\mathbf{r},\mathbf{r}')\varphi_a(\mathbf{r}') = \lambda_\alpha\varphi_\alpha(\mathbf{r}),$$

$\varphi(\mathbf{r})$: **single-cluster orbital w.f.**

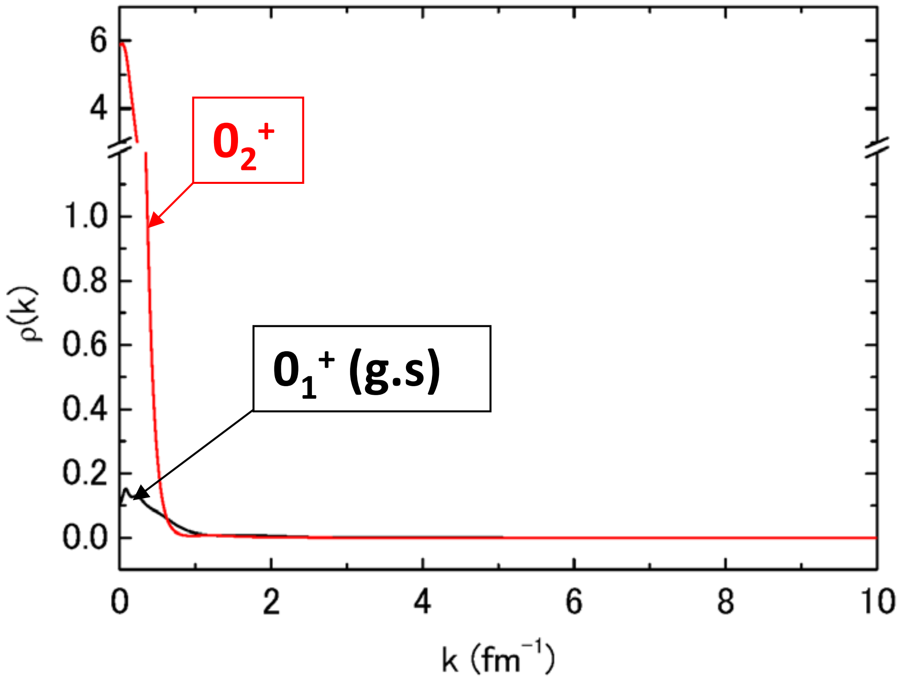
λ : **occupation probability**

α-density distribution and α-momentum distribution in ¹²C

Density × r^2



Momentum distribution

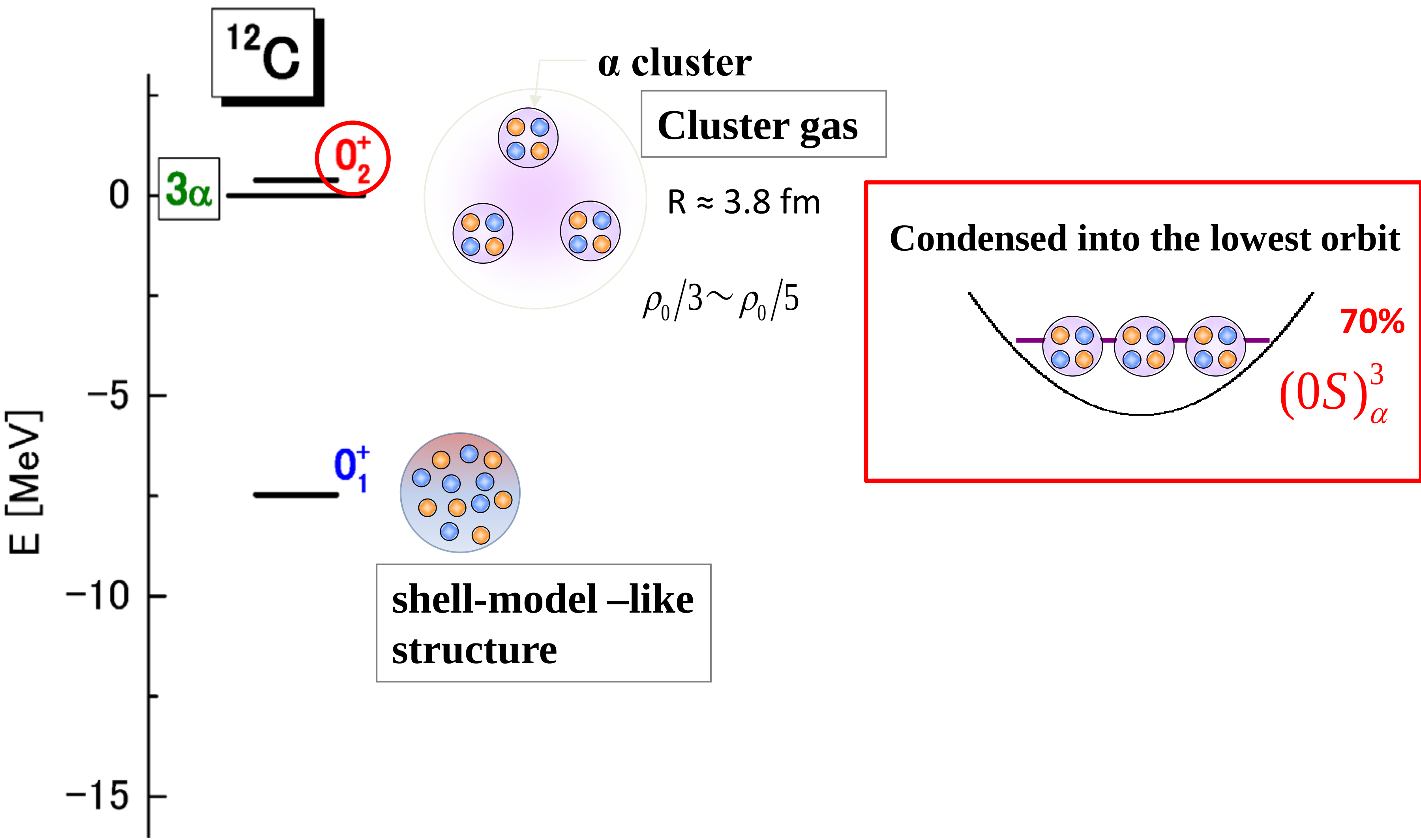


Compact (0_1^+) vs. Dilute (0_2^+)

0_2^+ state: δ function-like

Similar to dilute atomic cond.

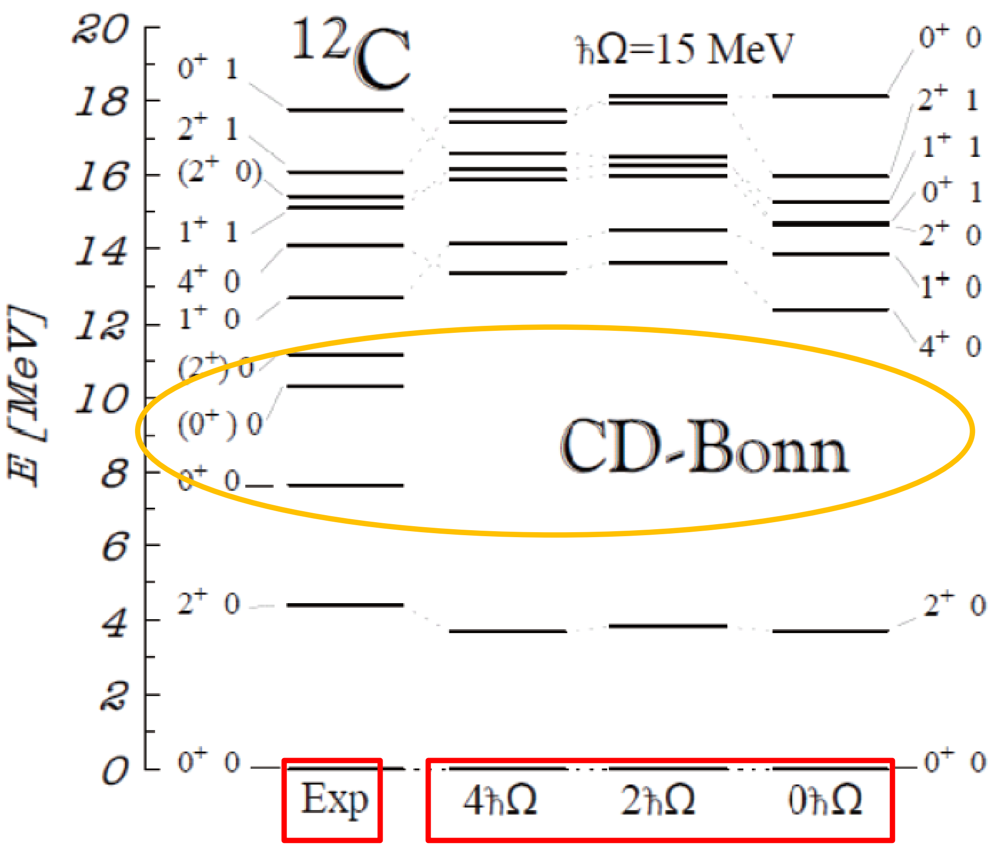
Yamada & Schuck EPJA26 (2005) with 3α OCM wf

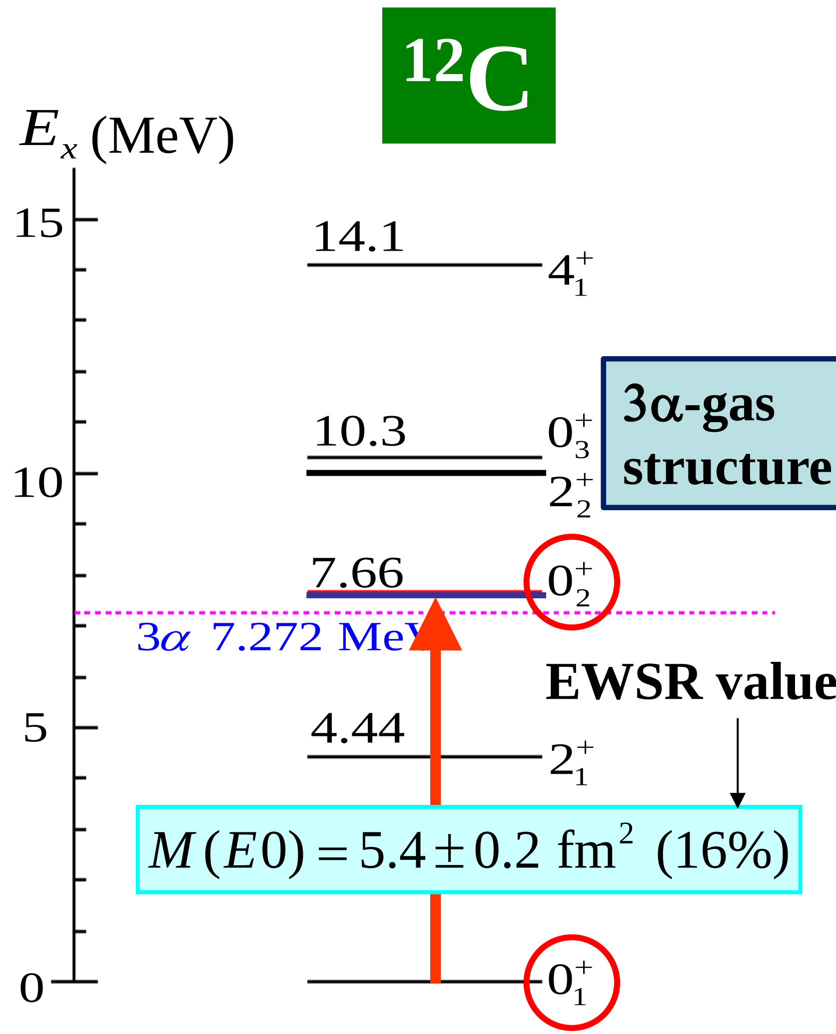


12C
E [MeV]
0
−5
−10
−15
3α
0_2^+
0_1^+
α cluster
Cluster gas
R ≈ 3.8 fm
$\rho_0/3 \sim \rho_0/5$
shell-model –like structure
Condensed into the lowest orbit
70%
$(0S)_\alpha^3$

Monopole strengths *M(E0)* in ^{12}C

No-core shell model

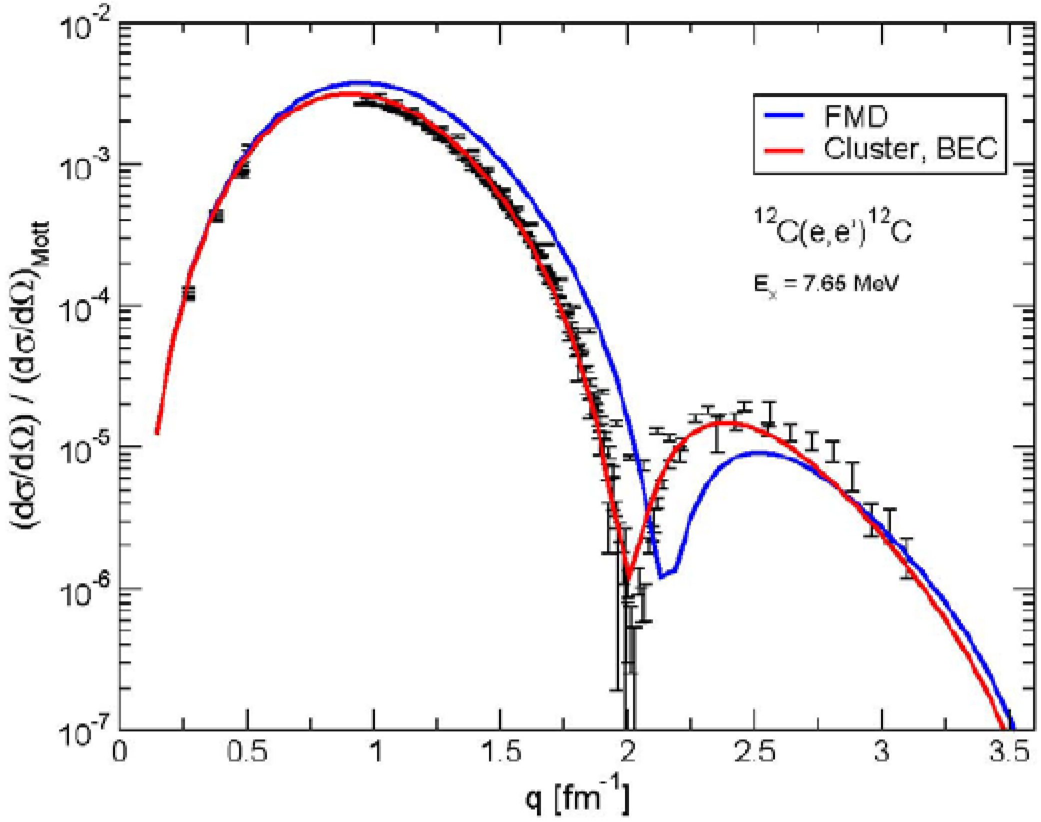




P. Navr’atil et al., PRC62 (2000)

Single particle M(E0)~5.4 fm^2

$^{12}C(e,e')^{12}C$(Hoyle)



blue : FMD : Chernykh et al., PRL98(2007)

read : Cluster (3α RGM): Kamimura, NPA351(1981)

BEC (3α condensate: THSR): Funaki et al., PRC67(2003)

Hyper-THSR w.f.

THSR: α-condensate-type w.f.

Core part

THSR w.f. **+ Λ**

full microscopic w.f.

^{12}C	structure	1 base THSR
1st 0+	shell-model-like	94 %
2nd 0+	3α-gas-like	99.9%
3rd 0+	linear-chain-like	99 %

core核状態を ほとんど 1基底 で記述！

8Be=2α, 12C=3α, 20Ne=α+16O

興味の焦点

1. Λ粒子が加わることにより、12Cの2nd 0+における α凝縮的様相は生き残るか？

2. linear-chain-likeな構造はΛ粒子が付くことにより、安定化するか？

   ⇔ glue-like role

軽いハイパー核の構造研究(クラスター模型)

Glue-like role of Λ

Shrinkage of core nucleus

➡ **Reduction of B(E2)**

Observed in $^{7}_{\Lambda}$Li

Tanida et al., PRL86 (2001)

精密4体計算: Hiyama et al. PRC59(1999)

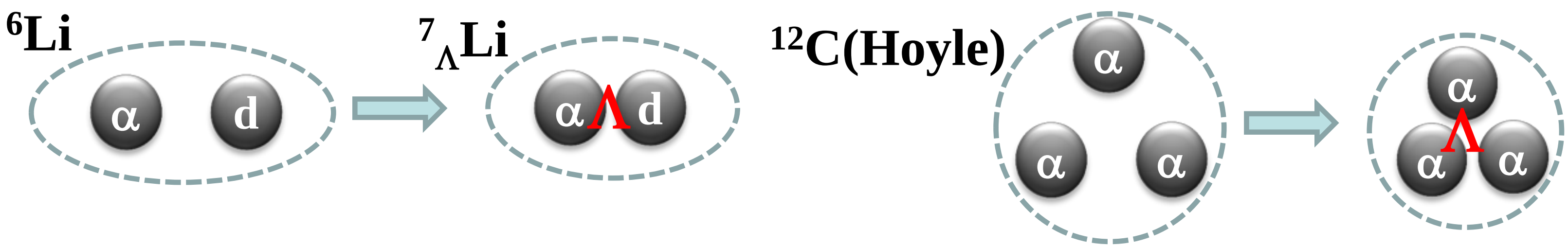
Light p-shell: α+x+Λ model: x=p,n,d,t,^{3}H,^{3}He

Motoba, Bando, Ikeda, PTP70(1983), 71(1984)

$^{13}_{\Lambda}$C(3α+Λ), $^{21}_{\Lambda}$Ne(α+^{16}O+Λ)　山田のD論の一部

$^{20}_{\Lambda}$Ne(α+^{15}O+Λ)　1980年後半までの研究

特筆すべき発見



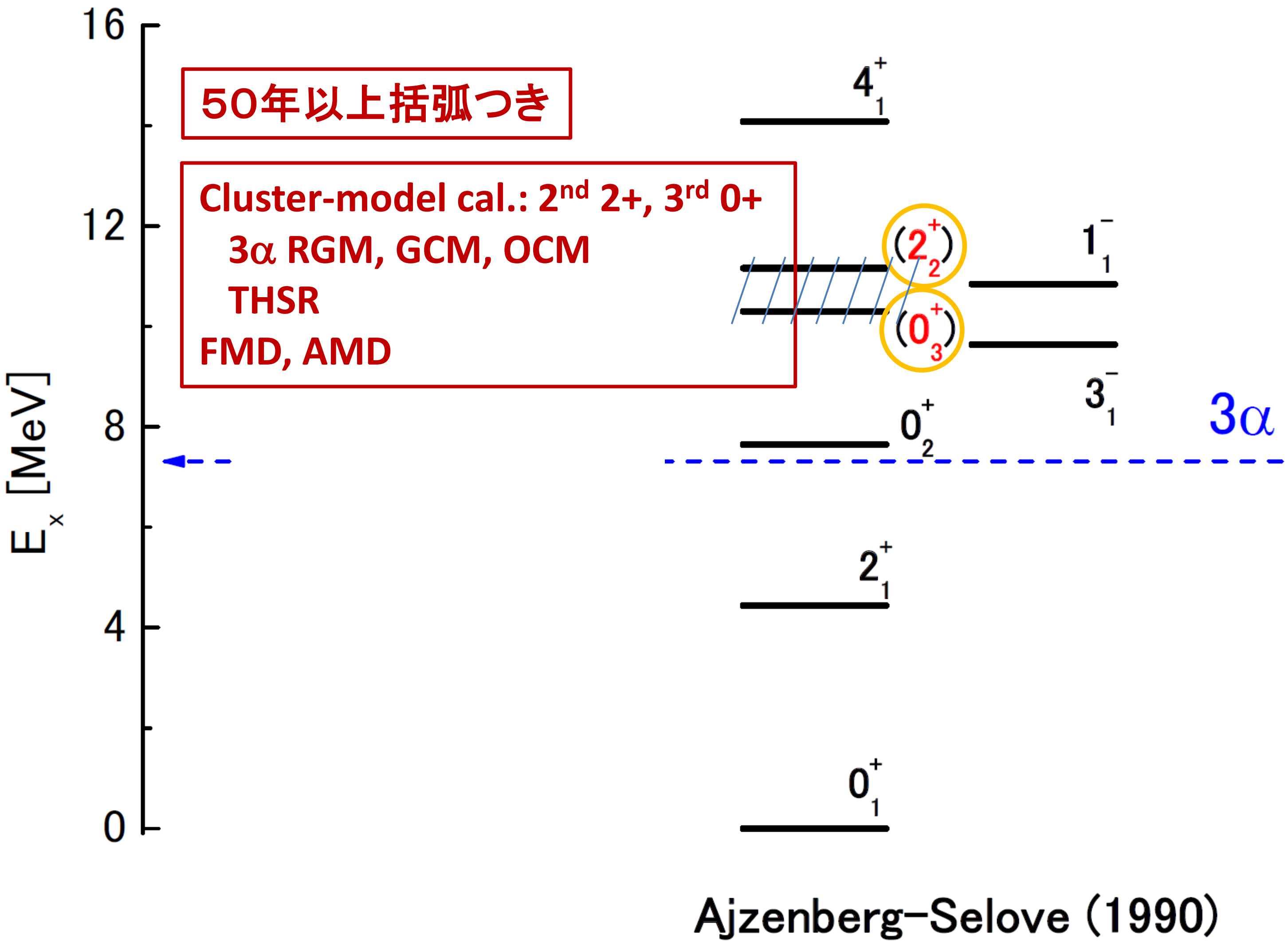
Shrinkage = コア核の密度変化(monopole)　外場: ΛN 相互作用

微視的クラスター模型: 成功を収めた大きな理由

1. 殻模型的状態とクラスター状態を同時に記述 ⇔ 密度変化
2. Λ粒子を加える前の芯核の波動関数:信頼のおけるもの

クラスター状態とモノポールの密接な関係

Energy levels of ^{12}C (Exp.)



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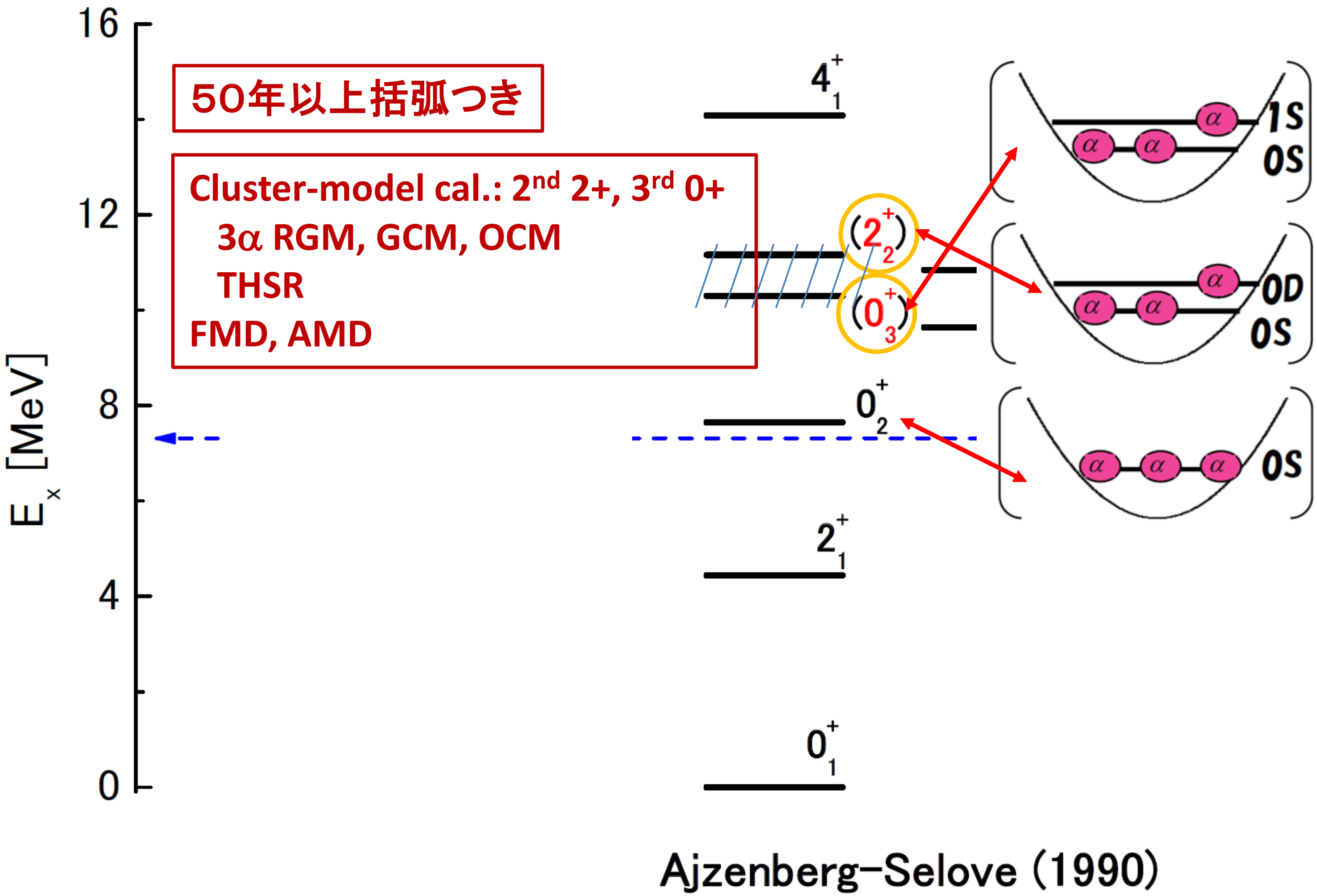
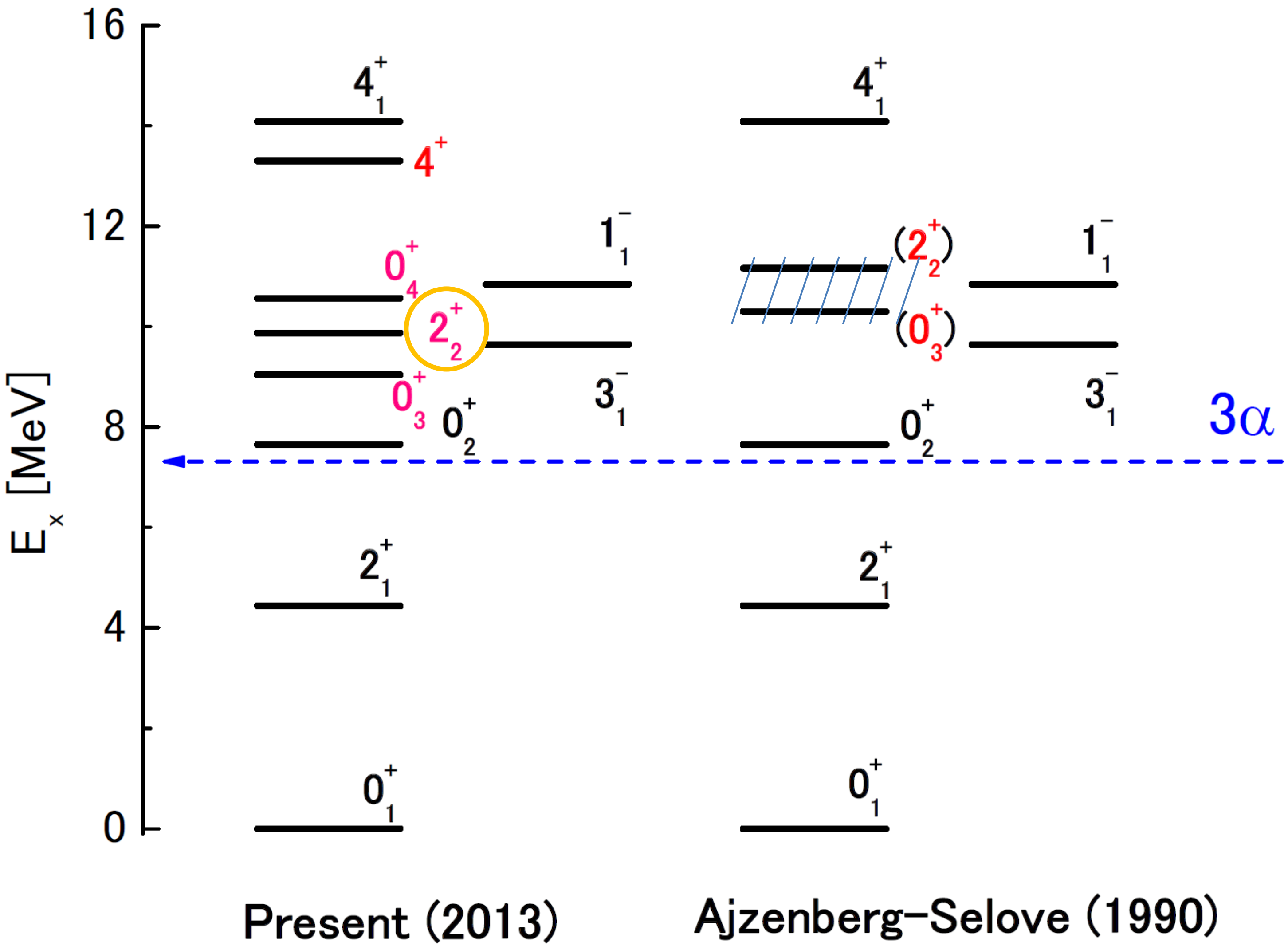
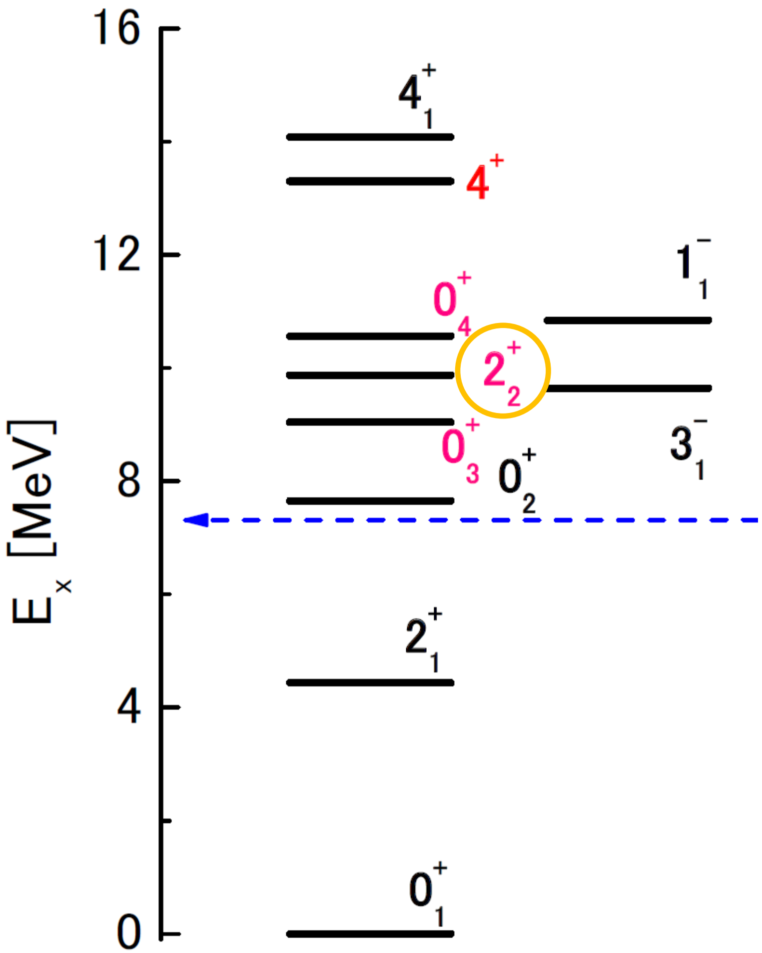


Energy levels of ^{12}C (Exp.)



Energy levels of ^{12}C (Exp.)

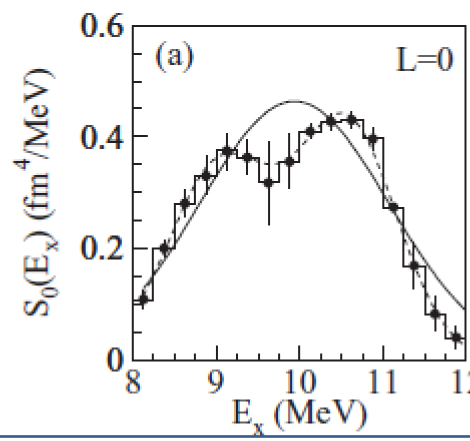


Present (2013)

2_2^+ ^{12}C(α,α') : Itoh et al., NPA738 (2004)
Itoh et al., PRC84 (2011)
^{12}C(p,p') : Freer et al., PRC80 (2009)
Zimmerman et al., PRC84(2011)
^{12}C(γ,α) : Zimmerman et al., PRL110(2013)

実験的に決着

$0_3^+, 0_4^+$ ^{12}C(α,α') : Itoh et al., PRC84 (2011)



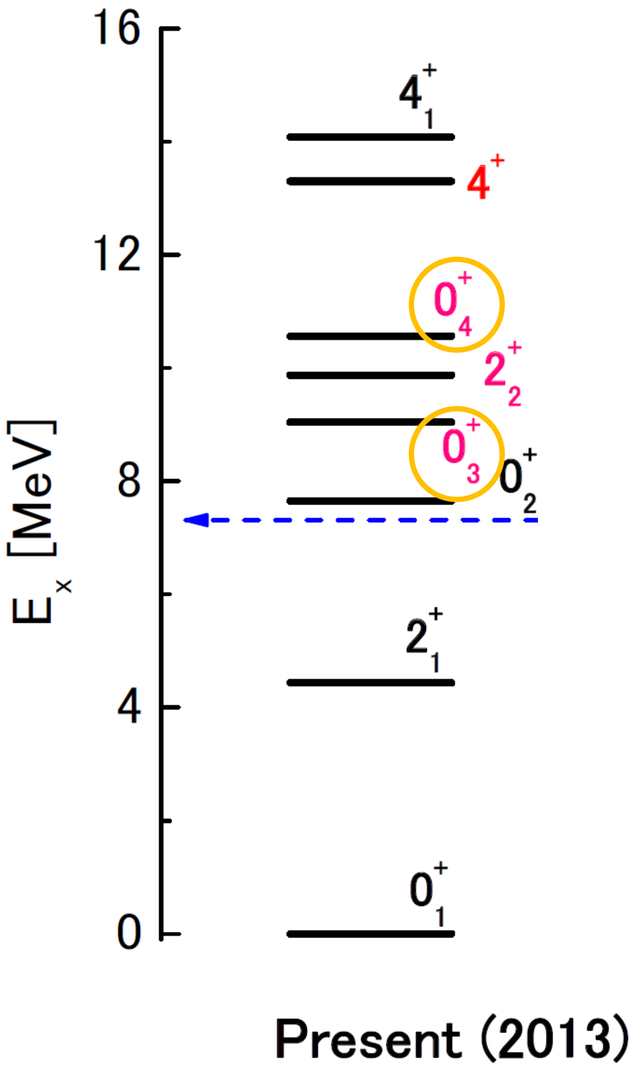
Result of MDA

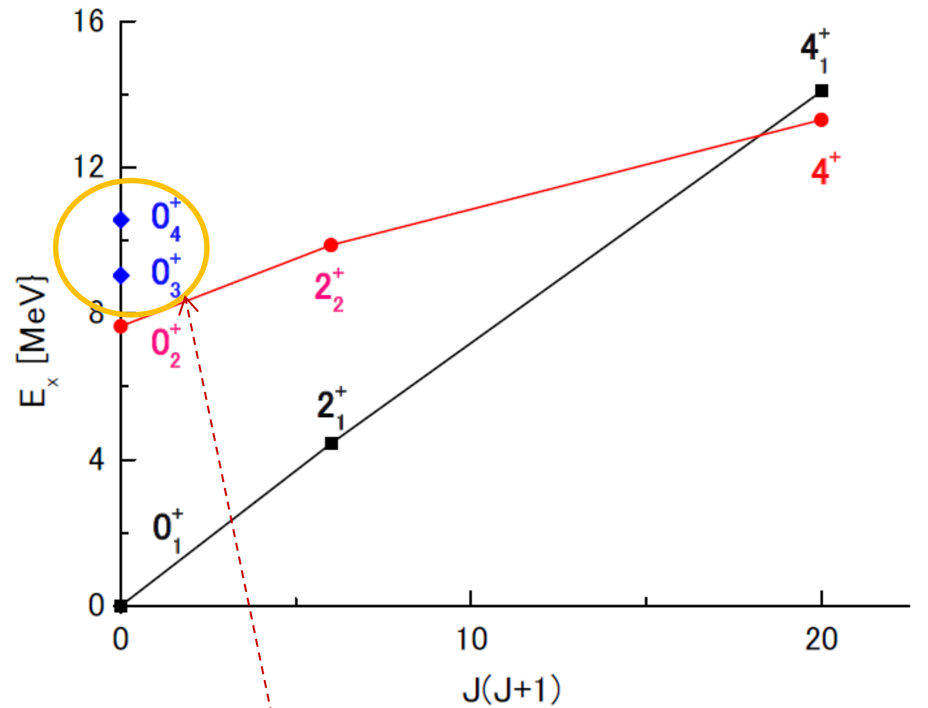
Ex=9.04 MeV
Γ=1.45 MeV

Ex=10.56 MeV
Γ =1.42 MeV

4^+ ^{12}C(α,α+α+α)α, ^{9}Be(α,α+α+α)n
Freer et al., PRC83 (2011)

Energy levels of ^{12}C (Exp.)





- **OCM+CSM: 黒川・加藤**
  - **3rd 0+: family of α cond.**
  - **4th 0+: linear-chain-like**
- **上村 CSM: confirmed**
- **AMD (延與):**
  - **linear-chain-like**

Hyper-THSR w.f.

THSR: α-condensate-type w.f.

Core part

THSR w.f. + Λ

full microscopic w.f.

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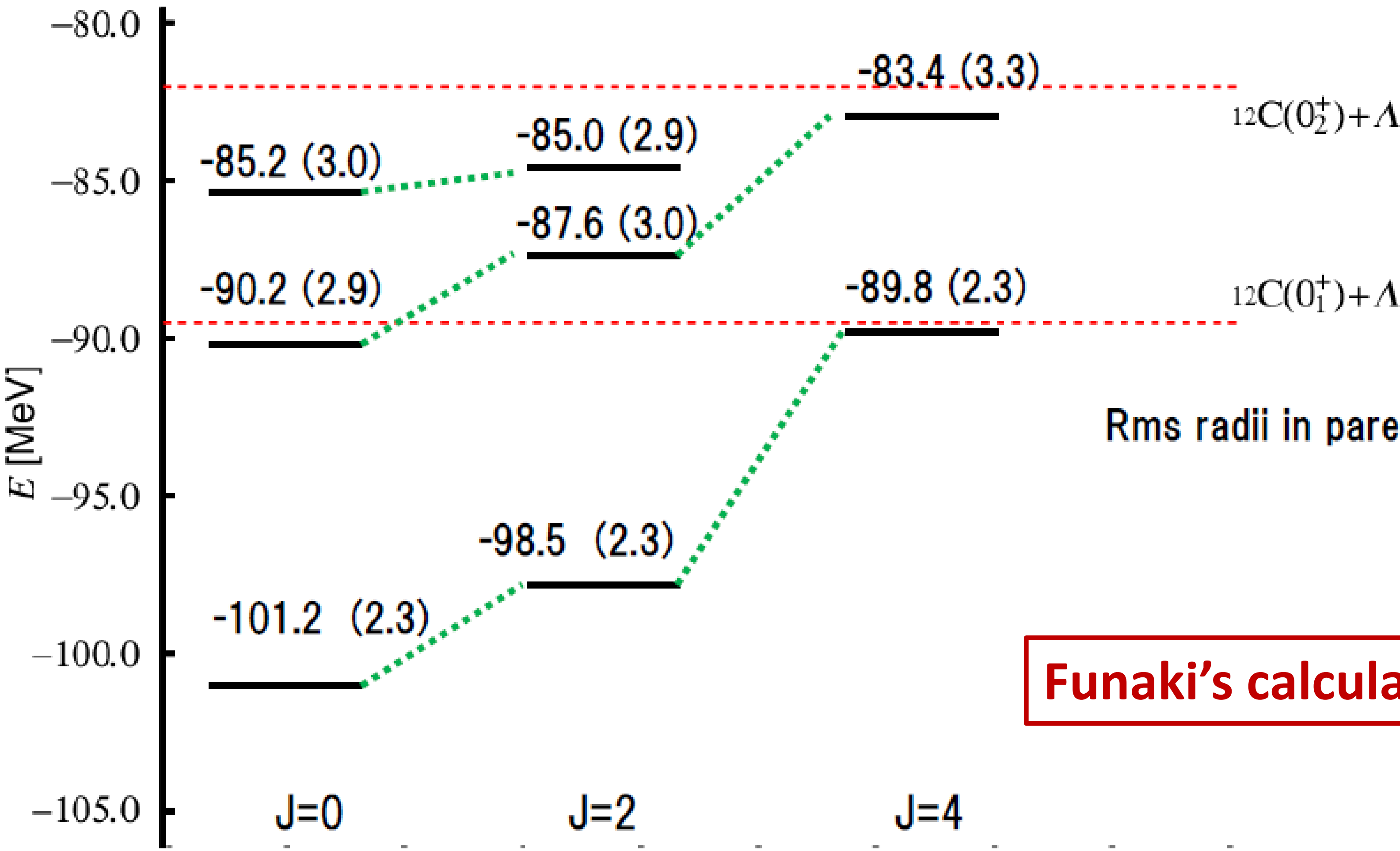
⇔ glue-like role

Energy of $^{13}{}_{\Lambda}C(0^+, 2^+, 4^+)$

YNG (ND) interaction

$$\sum_{B'_\perp, B'_z, \kappa'} \left\langle \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B_\perp, B_z, \kappa) \middle| H - E_\lambda \middle| \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B'_\perp, B'_z, \kappa') \right\rangle f_\lambda(B'_\perp, B'_z, \kappa') = 0$$

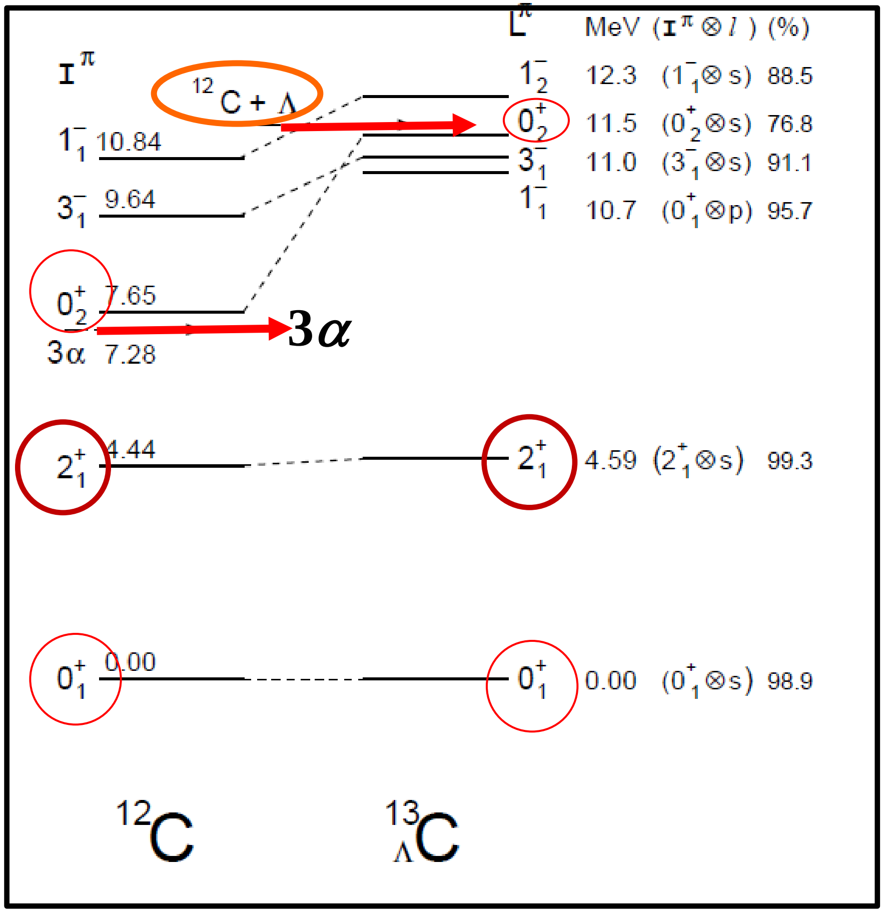


Rms radii in parentheses

Funaki's calculation

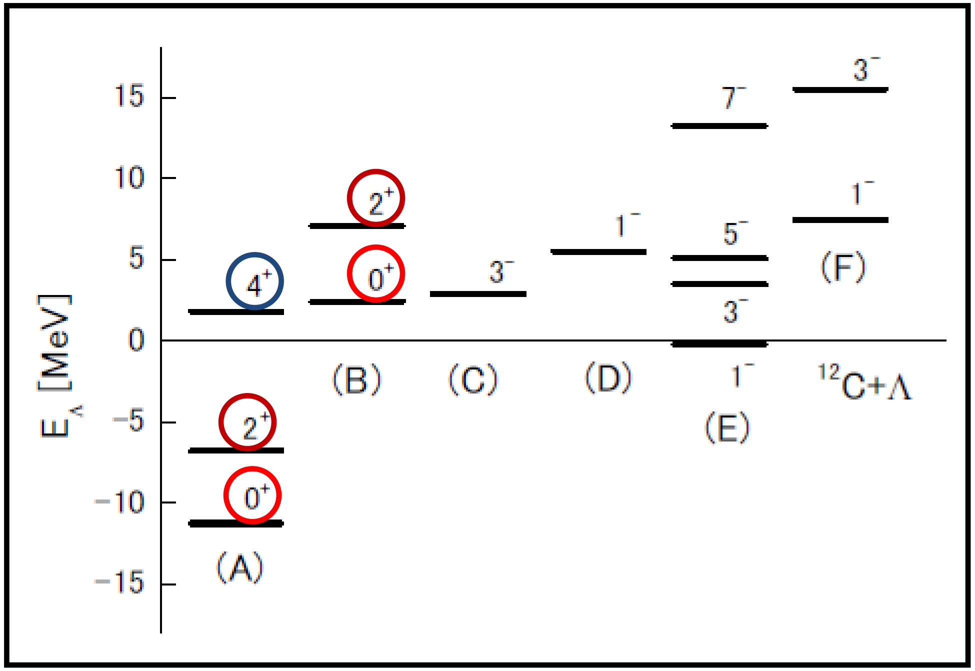
Previous calculations with cluster models

3α+Λ model (Hiyama)



E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, PTP97 (1997); PRL85 (2000)

3α RGM+Λ model (Yamada)

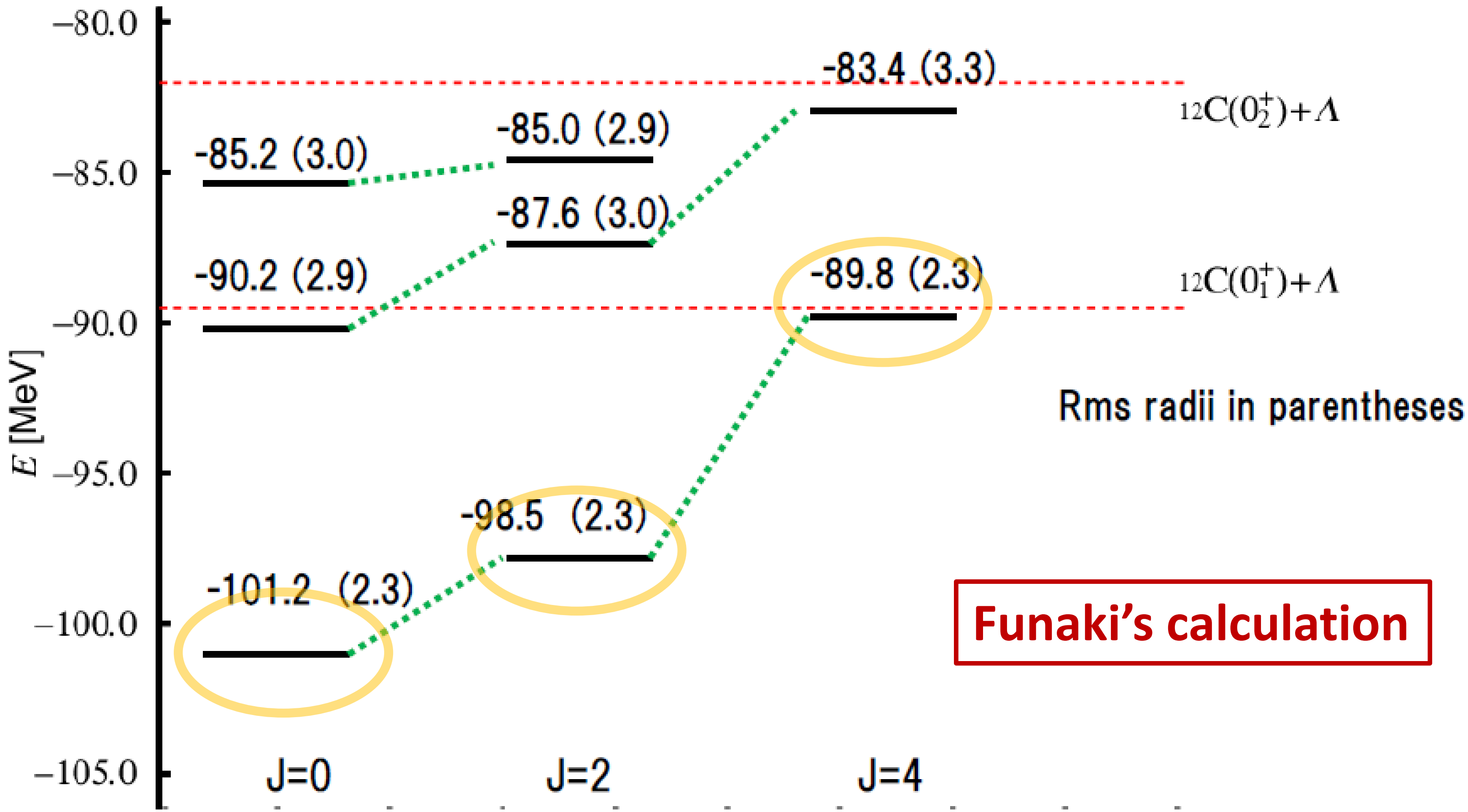


T. Yamada et al., PTP Suppl. 81 (1985)

Energy of $^{13}{}_{\Lambda}C(0^+, 2^+, 4^+)$

YNG (ND) interaction

$$\sum_{B'_\perp, B'_z, \kappa'} \left\langle \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B_\perp, B_z, \kappa) \middle| H - E_\lambda \middle| \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B'_\perp, B'_z, \kappa') \right\rangle f_\lambda(B'_\perp, B'_z, \kappa') = 0$$

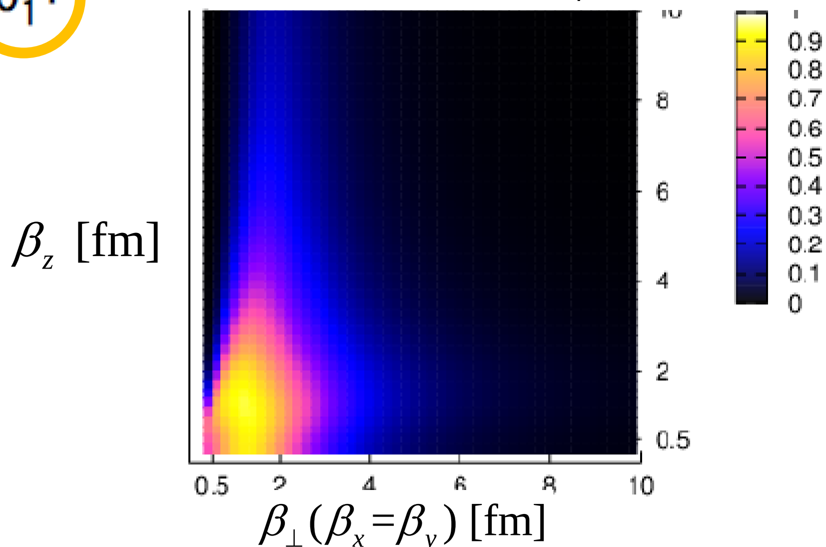


Squared overlap surfaces for 0_1^+, 2_1^+, 4_1^+

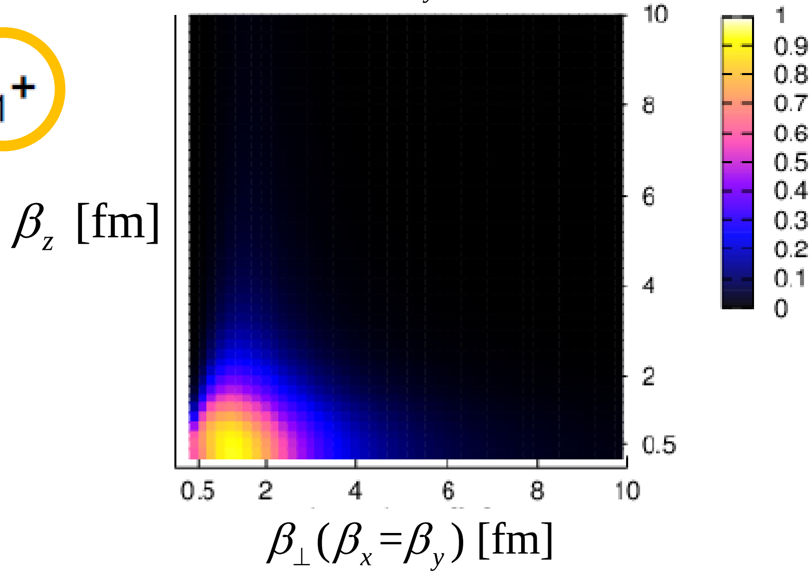
Funaki's calculation

$$O(\beta_\perp, \beta_z, \kappa) = \left| \left\langle \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(\beta_\perp, \beta_z, \kappa) \middle| \text{full Hyper-THSR w.f.} \right\rangle \right|^2$$

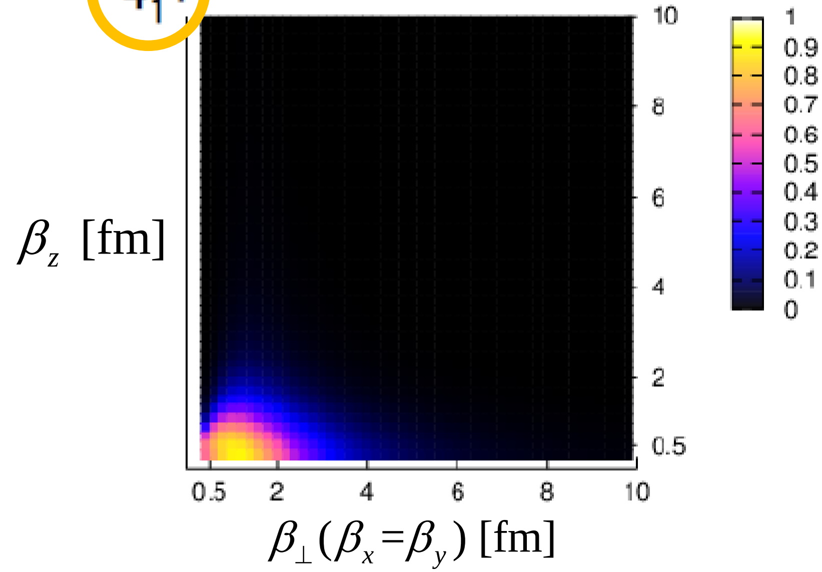
0_1^+



2_1^+



4_1^+

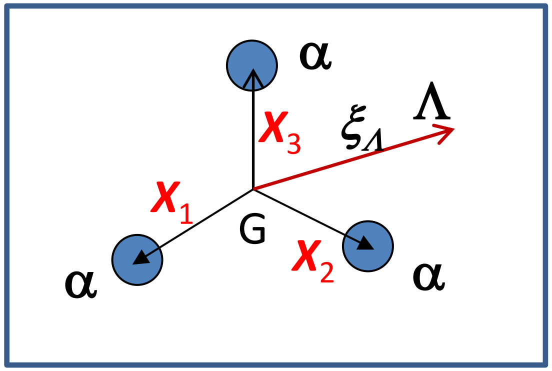


Hyper-THSR w.f.

$$\Phi^{Hyper-THSR}_{[I,\ell]J}(\beta_\perp,\beta_z,\kappa)$$

$$= \hat{P}_I \mathcal{A}\left\{\prod_{i=1}^{3}\left[\exp\left(-\frac{2}{\beta_\perp^2+2b^2}\left(X_i^2+Y_i^2\right)-\frac{2}{\beta_z^2+2b^2}Z_i^2\right)\phi(\alpha_i)\right]\right\}\times\varphi_\ell(\vec{\xi}_\Lambda,\kappa)$$

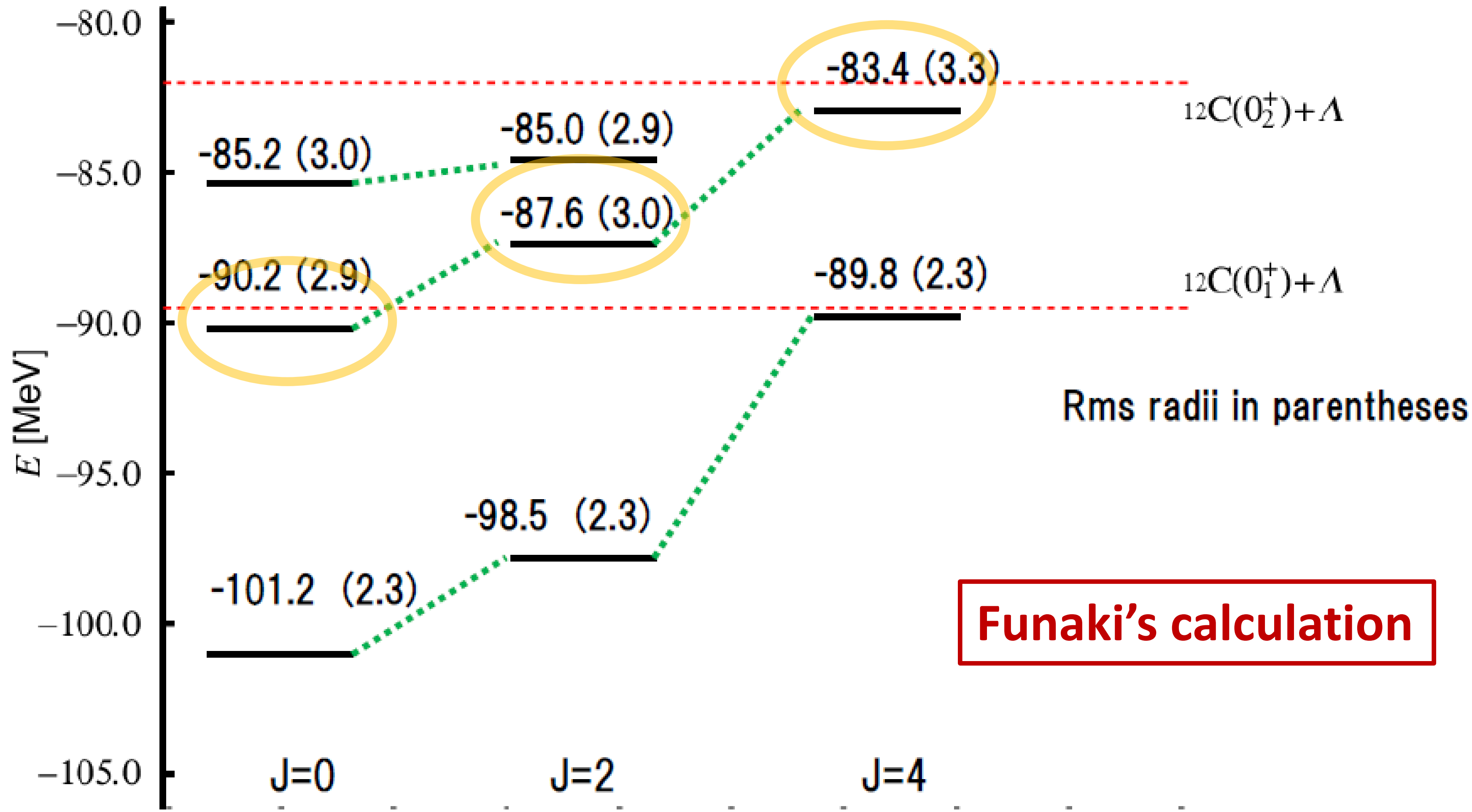
$$\varphi_\ell(\vec{\xi}_\Lambda,\kappa)=N_\ell(\kappa)\xi_\Lambda^\ell\exp\left(-\frac{\xi_\Lambda^2}{\kappa^2}\right)Y_{\ell m}(\hat{\xi}_\Lambda)$$



Energy of $^{13}_{\Lambda}C(0^+, 2^+, 4^+)$

YNG (ND) interaction

$$\sum_{B'_\perp, B'_z, \kappa'} \left\langle \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B_\perp, B_z, \kappa) \middle| H - E_\lambda \middle| \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B'_\perp, B'_z, \kappa') \right\rangle f_\lambda(B'_\perp, B'_z, \kappa') = 0$$

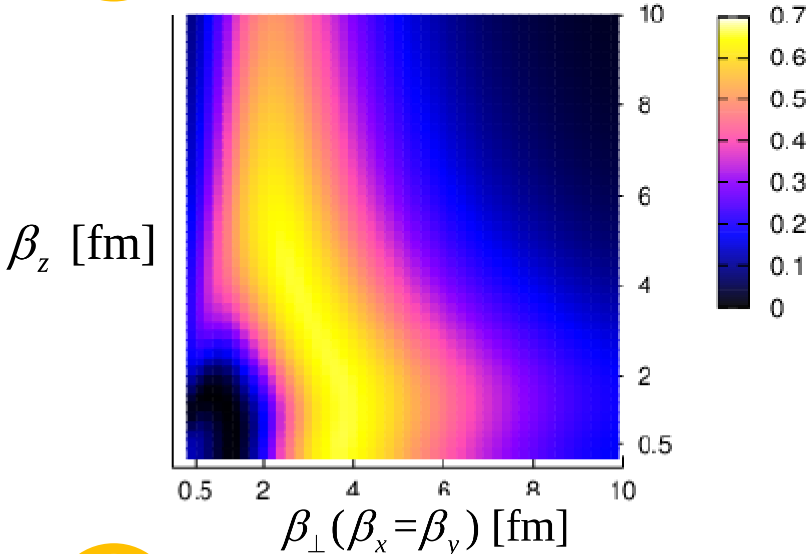


Funaki's calculation

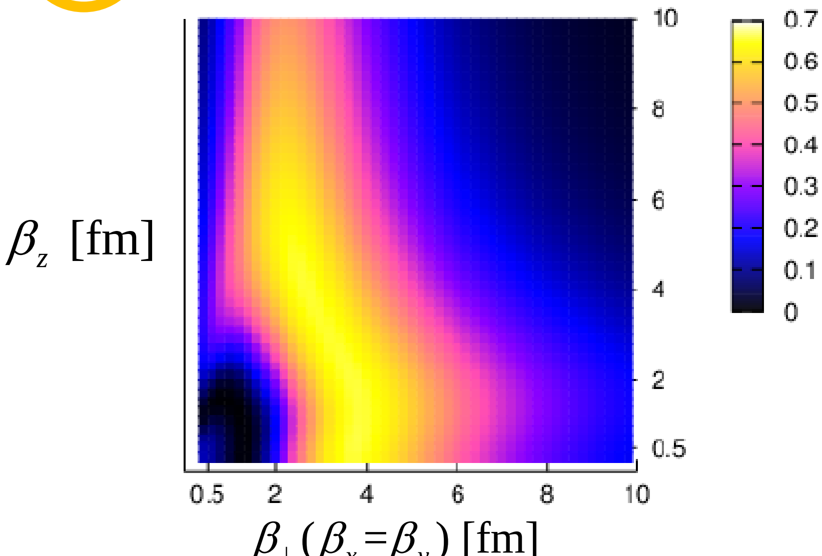
Family of the Hoyle state

Funaki's calculation

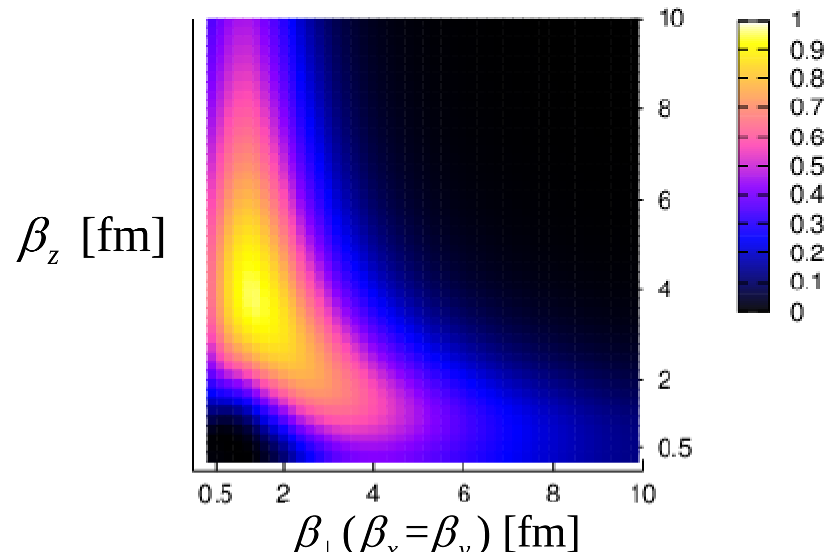
0_2^+



2_2^+



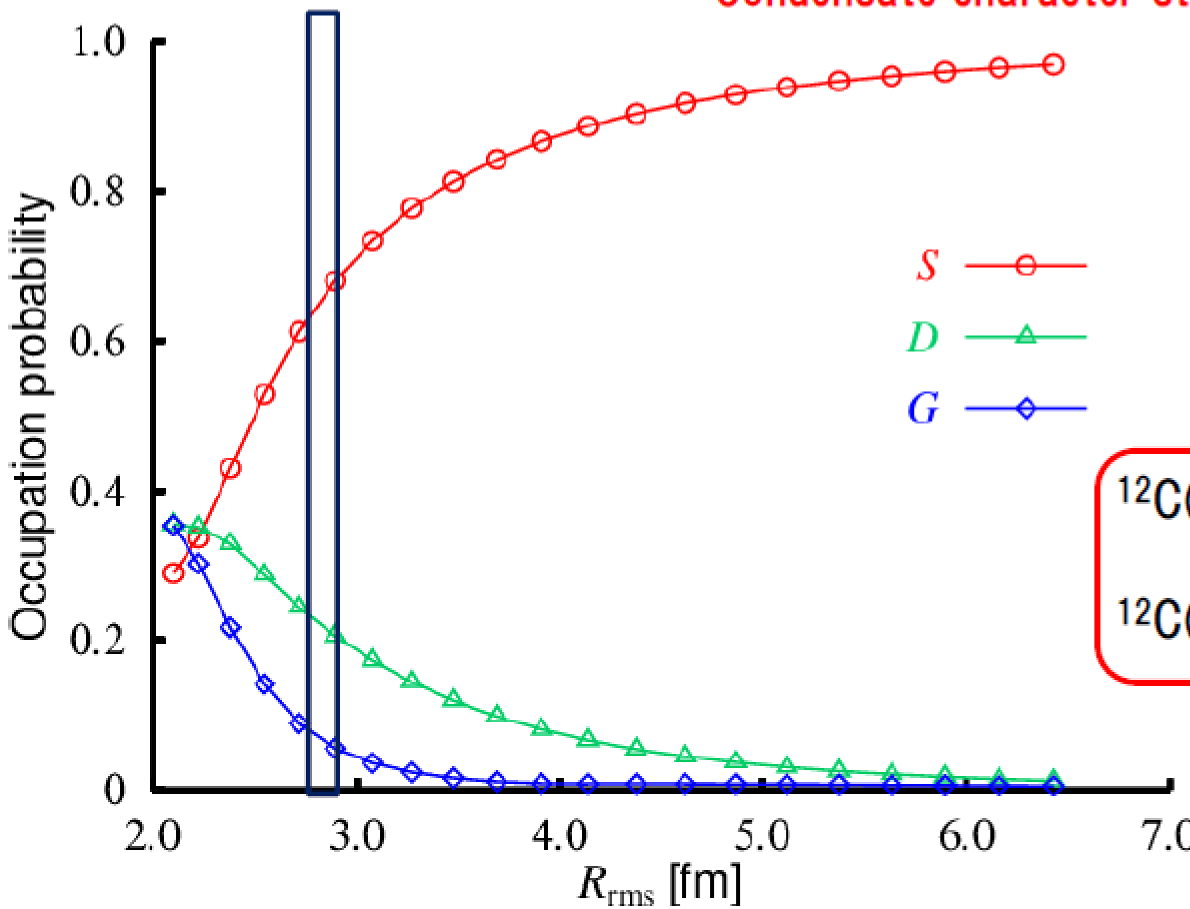
4_2^+



Size dependence of occupation probability (amount of α condensation)

(Hoyle + Λ) R_{rms}=2.9 fm

Condensate character still much survives. ~60 %



$^{12}C(0_1^+)$	2.4 fm
$^{12}C(0_2^+)$	3.8 fm

R_{rms} < 2.5 fm: Alpha's are resolved due to the antisymmetrization.

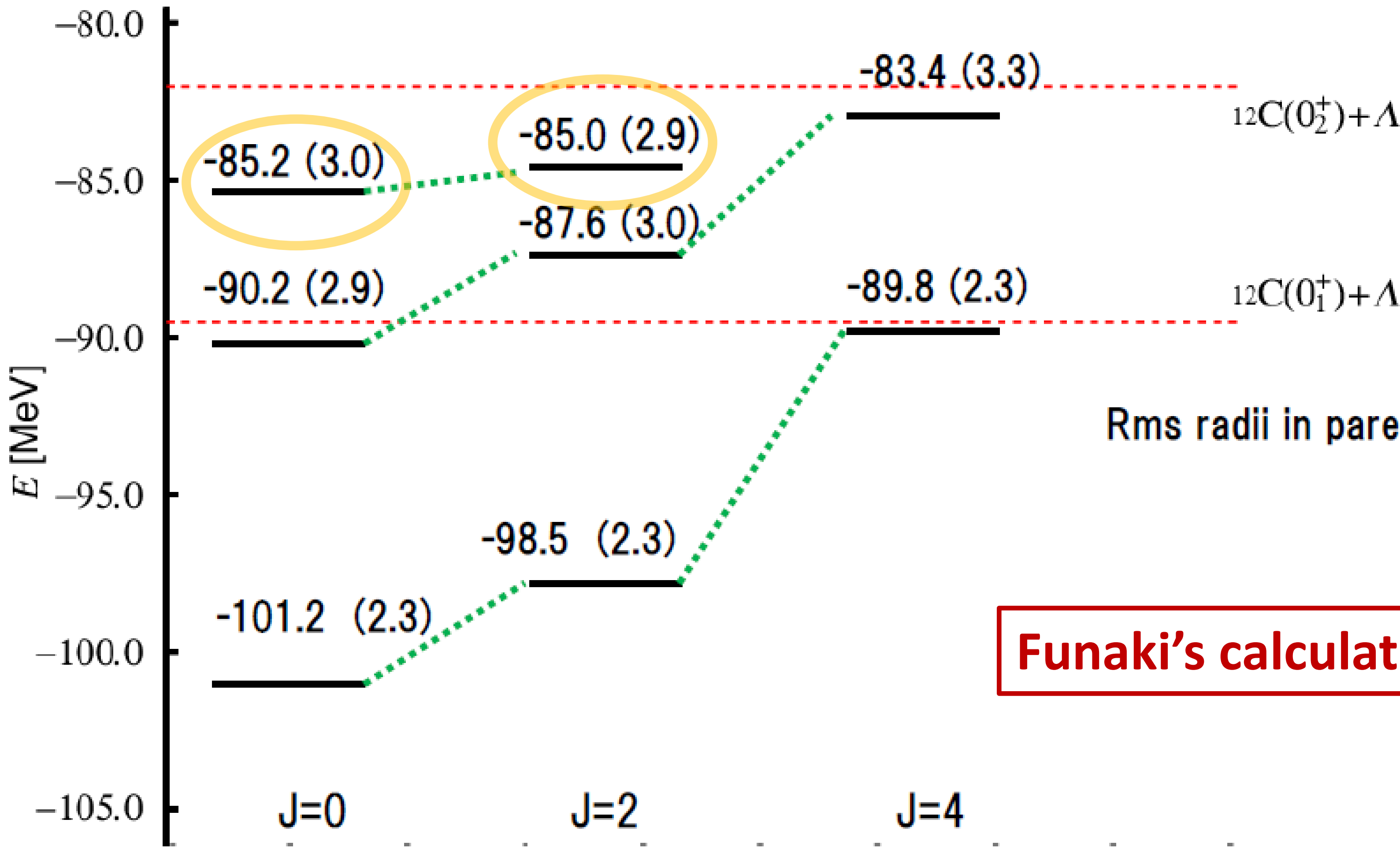
R_{rms} → large: Alpha's occupy a single *S*-orbit only.

Funaki's calculation

Energy of $^{13}{}_{\Lambda}C(0^+, 2^+, 4^+)$

YNG (ND) interaction

$$\sum_{B'_\perp, B'_z, \kappa'} \left\langle \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B_\perp, B_z, \kappa) \middle| H - E_\lambda \middle| \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B'_\perp, B'_z, \kappa') \right\rangle f_\lambda(B'_\perp, B'_z, \kappa') = 0$$



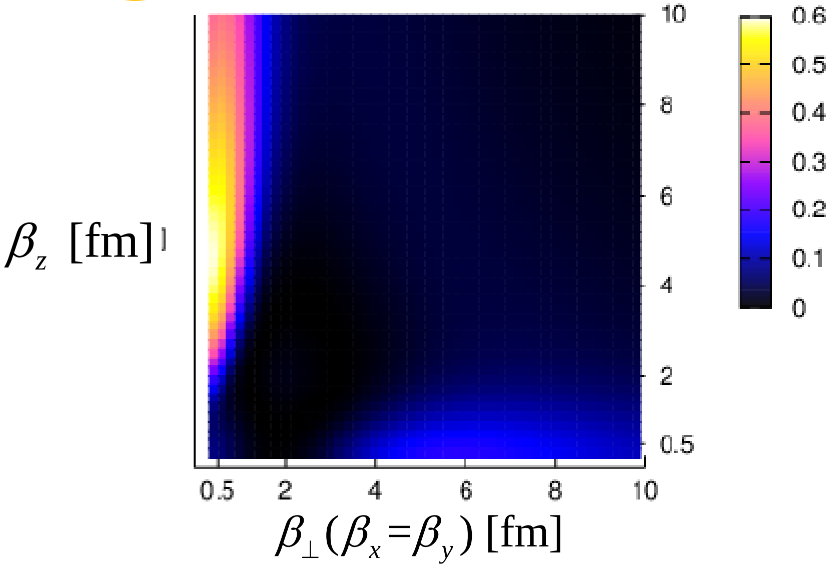
Rms radii in parentheses

Funaki's calculation

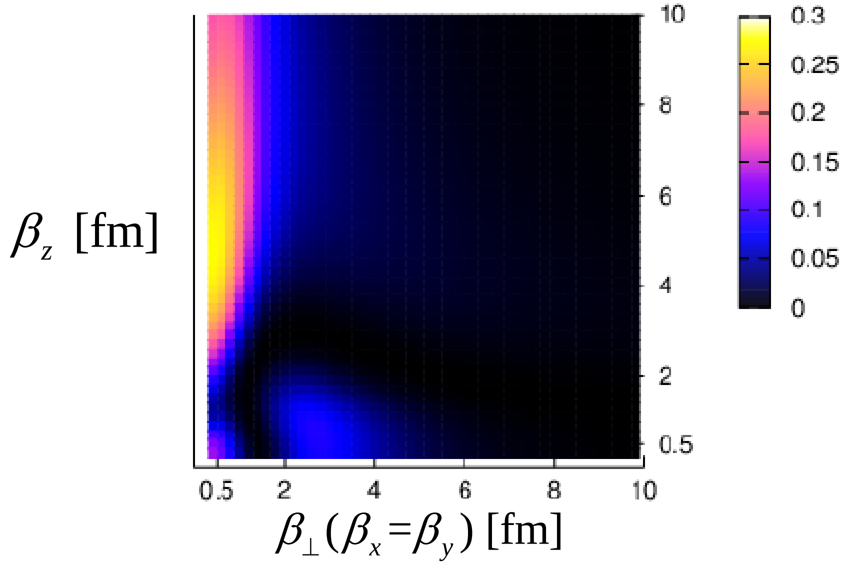
1 dim.-like linear-chain band

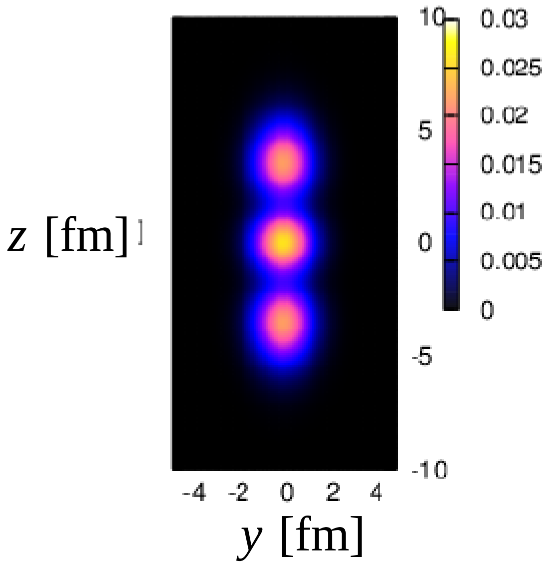
Funaki's calculation

0_3^+



2_3^+





Intrinsic density of 0_3+

Summary (1)

- **IS M(E0) trans. : useful to search for cluster states**

B(E2): nuclear deformation (Rainwater)

They seem to have about 20% of EWSR in low energy region.

- **IS monopole excitations have two features: ^{16}O (typical)**

  (i) α-cluster type: discrete peaks at $E_x \leq 15$ MeV

  (ii) mean-field type: 3-bump structure (18,23,30 MeV)

- **The origin: Dual nature of the ground state of ^{16}O.**

  G.S. has mean-field and α-cluster degrees of freedom

  + α-type g.s. correlation

- **Dual nature is common in light nuclei.**

- **Two features of IS monopole excitations seems to be in general in light nuclei.**

It is interesting to study two features in other 4*n* nuclei, neutron-rich nuclei.

⇒ Importance of Systematic Analyses with IS M(E0) in Light Nuclei

Summary (2)

- **Alpha-gas like states : novel states in light nuclei**

  ^{12}C : Hoyle state, family states of Hoyle

  ^{16}O, ^{11}B, ^{13}C : Hoyle analog states

- **Hyper-THSR w.f. (full microscopic)**

  Promising to describe the structures of light hypernuclei

  Linear-chain states may come down in energy to be stabilized due to glue-like role of Λ