Four-body Faddeev-Yakubovsky equations using two-cluster RGM kernels
-- Applications to 4*N*, 4*d*’ and 4α systems --

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2. 二体 RGM kernel を用いた三体・四体 Faddeev-Yakubovsky 方程式
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4. *α* 粒子の Faddeev-Yakubovsky 方程式
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2013.7.7 hyper 核物理の発展と今後の展望

1. 導入

クォーク模型バリオン間相互作用 fss2 による少数バリオン系の包括的理解をめざす

(3 体バリオン系)

- ^{3}H (n+n+p), $^{3}_{\Lambda}$H (n+p+Λ) の結合エネルギー
- 3 核子系の散乱観測量 (nd, pd 弾性散乱, 3 体崩壊反応) … 福川

(3 体クラスター系)

- ^{12}C (3α), ^{9}Be (n+2α), $^{9}_{\Lambda}$Be (Λ+2α), ^{6}He (2n+α), $^{6}_{\Lambda\Lambda}$He (2Λ+α) 系の束縛状態計算　αα, nα, Λα RGM kernel

枠組み: 2 体クラスター RGM kernel を用いた 3 体, 4 体クラスター Faddeev-Yakubovsky方程式

Renormalized RGM kernel の利用 → G-行列計算 → α でfolding → Λ-α 有効相互作用を構成

多クラスター Faddeev-Yakubovsky方程式の満たすべき要件

1. 変分法 (h.o. basis, SVM, Gauss 展開法，等) と (同一のinput で) 同じ結果を与える。
2. 現象論的2 体クラスター間ポテンシャルではなく，構成粒子間の 2 体力から出発して RGM kernel を作る → Pauli forbidden state *u* は “クラスター相対運動に対する直交条件” として自然に出る。… 2 体RGM kernel を用いた対直交条件型 (堀内型) OCM
3. 例えば2α, 3α, 4α と通して議論できる。Induced 3-body force (3 クラスターにまたがる反対称化の効果) や 2 クラスター間力の off-shell変換の効果 (エネルギー依存性を除去したことによる $1/\sqrt{N}$ の効果, 等) を議論できる。… 核力における $V_{low\text{-}k}$ や SRG 変換に対するヒントを与える。
4. 2体クラスター間にパウリ禁止状態があるときの　Faddeev redundant component が適切に処理できて、方程式が実際解けること。3体は簡単だが、4体以上では自明でない。…)

Four-cluster Faddeev-Yakubovsky formalism using two-cluster RGM kernels

Removal of the energy dependence by the renormalized RGM

Matsumura, Orabi, Suzuki, Fujiwara, Baye, Descouvemont, Theeten

3-cluster semi-microscopic calculations using 2-cluster non-local RGM kernels:

Phys. Lett. B659 (2008) 160; Phys. Rev. C76, 054003 (2007)

N=1-K

$$[\varepsilon - H_0 - V_{\rm RGM}(\varepsilon)]\,\chi = 0 \quad \text{with } V_{\rm RGM}(\varepsilon) = V_{\rm D} + G + \varepsilon K$$

$$\Rightarrow \Lambda[\varepsilon - H_0 - V_{\rm RGM}]\,\Lambda\psi = 0 \quad \text{with } V_{\rm RGM} = V_{\rm D} + G + W$$

εK method

$(\varepsilon = E - E_{\rm int})$

$$W = \Lambda[\frac{1}{\sqrt{N}}(H_0 + V_{\rm D} + G)\frac{1}{\sqrt{N}} - (H_0 + V_{\rm D} + G)]\Lambda$$

Extra non-local kernel

$$V^{new}_{\rm RGM}(\omega) = [\omega - H_0 - \Lambda(\omega - H_0)\Lambda] + \Lambda V_{\rm RGM}\Lambda$$

$$\equiv V(\omega) + v \quad \text{with } \omega \text{ (parameter)} \neq \varepsilon$$

$$\mathcal{A}\{\phi_{\rm int} u\} = 0 \Leftrightarrow Ku = \lambda u \ (\lambda = 1)$$

$$\Lambda = 1 - |u\rangle\langle u|$$

$$T(\varepsilon,\omega) = V^{new}_{RGM}(\omega) + V^{new}_{RGM}(\omega) G_0^{(+)}(\varepsilon) T(\varepsilon,\omega)$$

$$T(\varepsilon,\omega) = \tilde{T}(\varepsilon) + (\varepsilon - H_0)|u\rangle \frac{1}{\varepsilon - \omega}\langle u|(\varepsilon - H_0)$$

$$\langle u|[1 + G_0^{(+)}(\varepsilon)\tilde{T}(\varepsilon)] = [1 + \tilde{T}(\varepsilon)G_0^{(+)}(\varepsilon)]|u\rangle = 0$$

RGM *T*-matrix **: Prog. Theor. Phys. 107 (2002) 745; 993**

(also obtained by Kukulin's method)

4α case

$\Sigma_{i<j}\, |u_{i,j}\rangle \langle u_{i,j}|\psi_\lambda\rangle = \lambda|\psi_\lambda\rangle$ in $|\psi_\lambda\rangle \in [4]$

Projection operator onto the (pairwise) Pauli-allowed state

$\lambda = 0$: パウリ許容 $\mathscr{P} = \Sigma\, |\psi_\lambda\rangle \langle\psi_\lambda|$

$\lambda > 0$: パウリ禁止 $|\psi_\lambda\rangle = (1/\lambda)\, \Sigma_{i<j}\, |u_{i,j}\rangle \langle u_{i,j}|\psi_\lambda\rangle$

$$\mathscr{P}[E - H_0 - \sum_{i<j} (V_{\mathrm{RGM}})_{i,j}]\mathscr{P}\Psi = 0$$

: 4-cluster OCM using V_{RGM}

$$\Updownarrow$$

$$\psi = G_0(E)\tilde{T}(E - h_{\bar{0}})P[(1 + P_{(34)})\psi + \varphi]$$

$$\varphi = G_0(E)\tilde{T}(E - h_{\bar{0}})\tilde{P}[(1 + P_{(34)})\psi + \varphi]$$

: 4-cluster Faddeev-Yakubovsky equation using RGM *T*-matrix

where

$$P = P_{(12)}P_{(23)} + P_{(13)}P_{(23)}, \quad \tilde{P} = P_{(13)}P_{(24)}$$

Total wave function

$$\mathscr{P}\Psi = \Psi = (1 + P)\{[1 + P_{(34)}(1 + P)]\psi + (1 + \tilde{P})\varphi\}$$

$$\Rightarrow \forall \langle u_{i,j} \,|\, \Psi\rangle = 0$$

Cf. **Non [4]-symmetric trivial solutions in the 4α system are removable.**

(Faddeev redundant components)

B_8B_8 interactions by fss2

Y. F., C. Nakamaoto, Y. Suzuki, M. Kohno
PRC64 (2001) 054001
PRC65 (2002) 014001

A natural and accurate description of *NN*, *YN*, *YY* interactions in terms of (*3q*)-(*3q*) RGM

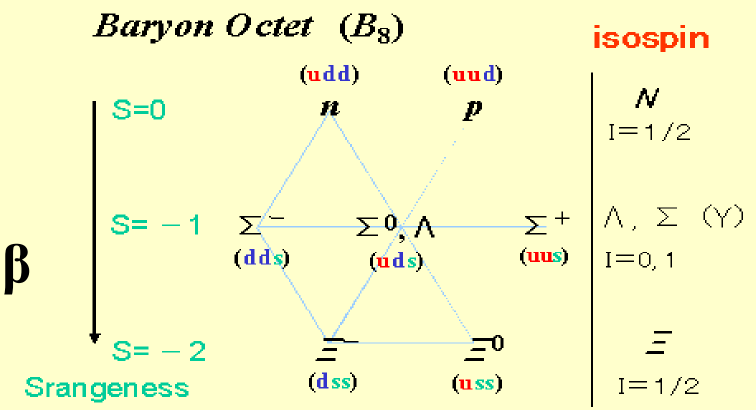
Number of parameters: less than 20

- Short-range repulsion and *LS* by quarks
- Medium-attraction and long-rang tensor by S, PS and V meson exchange potentials (fss2) (*Cf*. FSS without V)

Y. F. ,C. Nakamoto, Y. Suzuki, PRC54 (1996) 2180

Model Hamiltonian

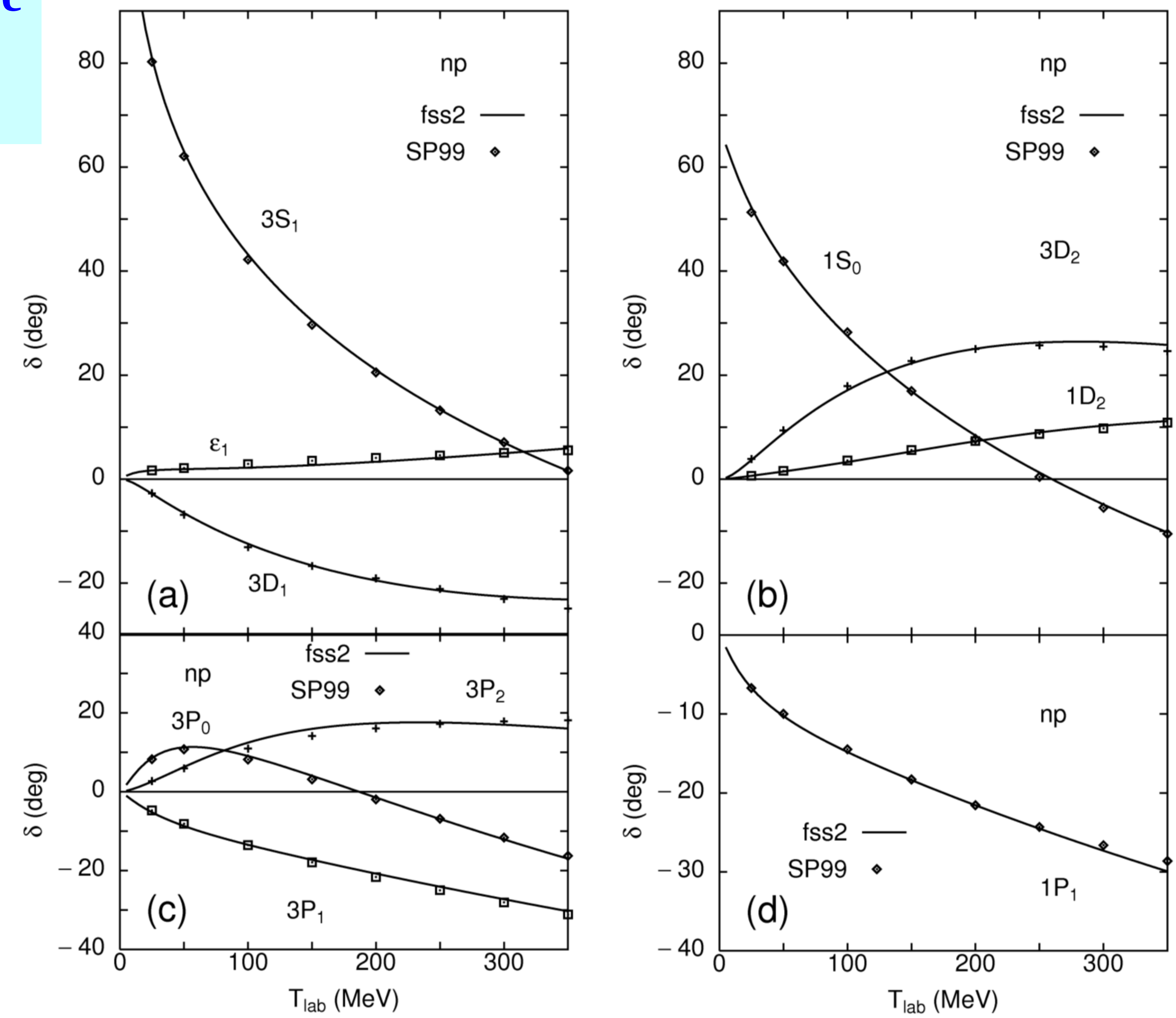
$$H = \sum_{i=1}^{6} (m_i + p_i^2/2m_i) + \sum_{i<j}^{6} (U_{ij}^{\mathrm{Conf}} + U_{ij}^{\mathrm{FB}} + \sum_\beta U_{ij}^{\mathrm{S}\beta} + \sum_\beta U_{ij}^{\mathrm{PS}\beta} + \sum_\beta U_{ij}^{\mathrm{V}\beta})$$



$$\langle \phi(3q)\phi(3q) | E-H | A\{\phi(3q)\phi(3q)\chi(\mathbf{r})\}\rangle = 0$$

QMPACK homepage http://qmpack.homelinux.com/~qmpack/php

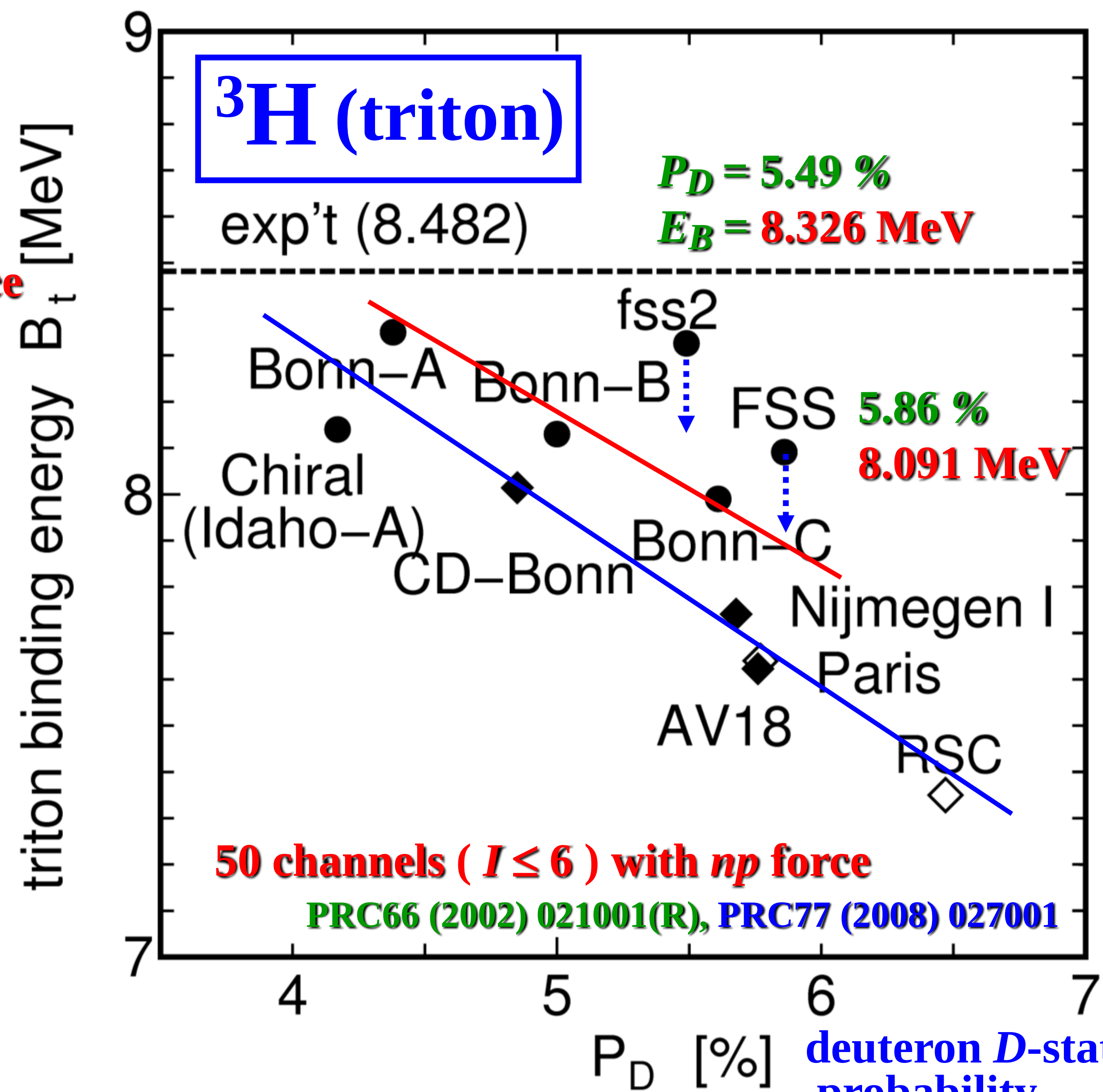
NN phase shifts by fss2



I ≤ 2

effect of charge dependence

~190 keV



deuteron *D*-state probability

Effect of 3-body force ~ 350 keV ?

almost half of 0.5 – 1 MeV for the meson exchange models

4 体同種 Fermion 粒子系の Faddeev-Yakubovsky 方程式

$$\psi = G_0 t P[(1-P_{(34)})\psi + \varphi]$$

$$\varphi = G_0 t \tilde{P}[(1-P_{(34)})\psi + \varphi]$$

with $\quad t = V^{RGM} + V^{RGM} G_0 t,$

$$P = P_{(12)}P_{(23)} + P_{(13)}P_{(23)}, \quad \tilde{P} = P_{(13)}P_{(24)}$$

Total wave function

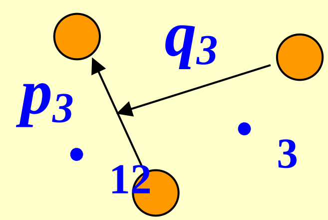
$$\Psi = (1+P)\{[1-P_{(34)}(1+P)]\psi + (1+\tilde{P})\varphi\}$$

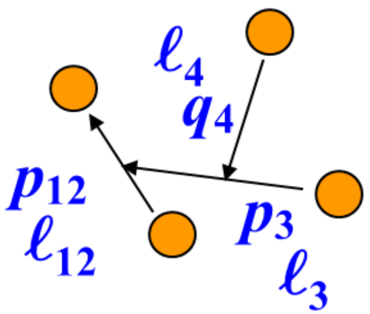
(3 body case)

$$\psi = G_0 t P \psi$$

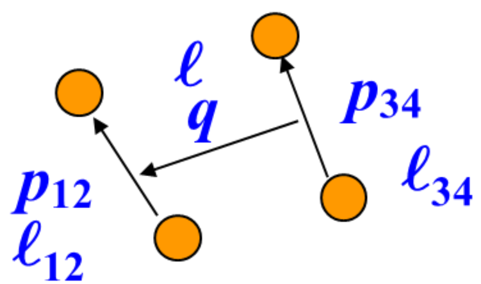
Total wave function

$$\Psi = (1+P)\psi$$





Y-type



H-type

(・ $_{12}s_{12}$)I_{12} ≤ I_{max}= 6

・ $_{12}$+ ・ $_3$+ ・ $_4$, ・ $_{12}$+ ・ $_{34}$+ ・ ≤ (・

by A. Nogga

- $(3q)$-$(3q)$ folded cut-off Coulomb with R_{cou} = 10 fm
  ⟹ smaller than point Coulomb
- $^1S_0$ charge independence breaking (CIB)

Approximate treatment in the isospin basis:

H. Witala et al. Phys. Rev. C43, 1619 (1991)

Scatt. length	a_s (fm)	F_{BB}
pp	–17.80	0.9934
nn	–18.0	0.9944
np	–23.76	1

⟸ reduction factor only for $^1S_0$

$$^3\mathrm{H}: \frac{2F_{nn}+1}{3} f_1^S$$

$$^3\mathrm{He}: \frac{2F_{pp}+1}{3} f_1^S + \frac{2}{3}\mathrm{Coulomb}$$

$$^3\mathrm{He}: \frac{F_{pp}+F_{nn}+1}{3} f_1^S + \frac{1}{3}\mathrm{Coulomb}$$

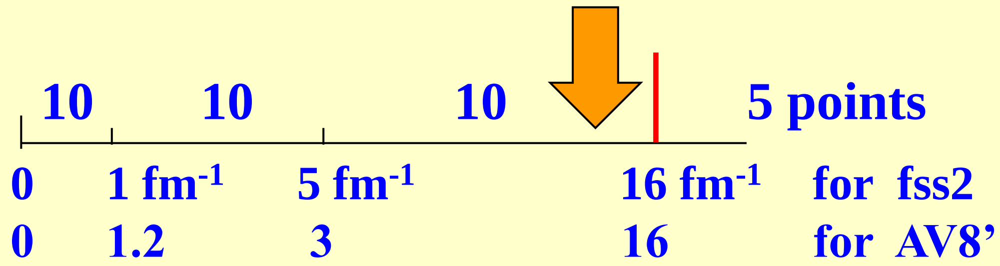
for isospin I=1 pairs

^{3}H , ^{3}He , ^{4}He(α)

	Coul (keV)	CIB (keV)	*E* (MeV)	E^{exp} (MeV)	diff (MeV)
^{3}H	–	182	–8.143	–8.482	0.34
^{3}He	682	208	–7.436	–7.718	0.28
^{4}He	810	538	–26.64	–28.30	1.66

(^{4}He calculation is by *n*=6-6-3)

mesh	精度
n=4-4-2	2 桁
n=6-6-3	3 桁
n=10-10-5	4 桁



Benchmark test

H. Kamada et. al., Phys. Rev. C64, 044001 (2001)

^{4}He	fss2	10-10-5		AV8'	10-10-5 ← 4 桁精度	
• sum max	E (MeV)	KE (MeV)	R_c (fm)	E (MeV)	KE (MeV)	R_c (fm)
2	–24.73	76.29	1.498	–21.46	83.10	1.607
4	–26.32	85.46	1.443	–24.88	97.32	1.512
6	–27.76	87.64	1.433	–25.53	100.94	1.493
8	–27.92	88.18	1.430	–25.90	102.38	1.485
10	–27.95	88.31	1.429	–25.95	102.67	1.483
Stochastic Variational Method				–25.92	102.35	1.486

R_c^{exp} (^{4}He)=1.457(4) fm

Deuteron properties

deuteron	fss2	AV8'	(SVM)	AV18
ε_d (MeV)	2.2246	2.2436	2.242	2.2246
P_d (%)	5.49	5.78	5.77	5.76
rms (fm)	1.960	1.961	1.961	1.967
Q_d (fm2)	0.270	0.269		0.270
η	0.0252	0.0252		0.0250
μ_d (μ_0)	0.849	0.847		0.847
KE (MeV)	(17.49)	19.89	19.881	19.814

^{3}H binding energy ~ 350 keV missing in fss2: almost half of 0.5 – 1 MeV for meson exch. models

他の計算との比較

A. Nogga, H. Kamada, W. Glöckle, and B.R. Barrett, Phys. Rev. C65, 054003 (2002)

model	P_d (%)	^{3}H (MeV)	^{3}He (MeV)	^{4}He (MeV)	diff (MeV)
fss2	5.490	–8.143	–7.436	–26.64	1.66
CD-Bonn	4.833	–8.013	–7.288	–26.26	2.04
AV18	5.760	–7.628	–6.917	–24.25	4.02
Nijm I	5.678	–7.741	–7.083	–24.98	3.32
Nijm II	5.652	–7.659	–7.008	–24.56	3.74
Nijm93	5.755	–7.668	–7.014	–24.53	3.77
Exp.		–8.482	–7.718	–28.296	

0.5 – 1 MeV 3 – 4 MeV missing in MEP's

Ours: – 28.0 MeV + 0.8 MeV (Coulomb) + 0.5 MeV (CIB) = – 26.7 MeV
1.7 MeV missing → almost half of standard MEP's

Faddeev redundant components

1) Y-type 座標における 3体部分系の redundant component : $(1+P)|uf\rangle=0$

2) 2体 - 2体の H-type 座標における core exchange type の redundant component : $(1+\tilde{P})|\widetilde{uu}\rangle=0$

3) genuine 4体系の redundant component :

$$\lambda\,|uF\rangle = P[(1+P_{(34)})uF+uG]+|uf\rangle\langle uf\,|(1+P_{(34)})uF+uG\rangle$$

$$\lambda\,|uG\rangle = \tilde{P}[(1+P_{(34)})uF+uG]+|\widetilde{uu}\rangle\langle\widetilde{uu}\,|(1+P_{(34)})uF+uG\rangle$$

$\Rightarrow$ **we can prove** with $\lambda=-1$

$$\langle uf\,|uF\rangle=0\ ,\ \langle uf\,|(1+P_{(34)})uF+uG\rangle=0$$

$$\langle\widetilde{uu}\,|uG\rangle=0\ ,\ \langle\widetilde{uu}\,|(1+P_{(34)})uF+uG\rangle=0$$

and $\Psi^{\lambda}=(1+P)\{[1-P_{(34)}(1+P)]uF+(1+\tilde{P})uG\}=0$ **trivial solution**

4) modified Faddeev-Yakubovsky equation :

$$\psi = G_0 \tilde{t} P[(1+P_{(34)})\psi + \varphi] - |uf\rangle\langle uf|(1+P_{(34)})\psi + \varphi\rangle$$

$$- |uF\rangle\langle(1+P_{(34)})uF + uG|(1+P_{(34)})\psi + \varphi\rangle$$

$$\varphi = G_0 \tilde{t} \tilde{P}[(1+P_{(34)})\psi + \varphi] - |u\tilde{u}\rangle\langle u\tilde{u}|(1+P_{(34)})\psi + \varphi\rangle$$

$$- |uG\rangle\langle(1+P_{(34)})uF + uG|(1+P_{(34)})\psi + \varphi\rangle$$

we can prove $\langle u|\Psi\rangle = 0$ and

$$\langle uf|\psi\rangle = 0 \;,\; \langle uf|(1+P_{(34)})\psi + \varphi\rangle = 0$$

$$\langle u\tilde{u}|\varphi\rangle = 0 \;,\; \langle u\tilde{u}|(1+P_{(34)})\psi + \varphi\rangle = 0$$

for identical 4-boson systems

4*d*’ energy and rms radius

$v = v_0\, e^{-\kappa r^2}(1+P_r)/2$ (pure Serber)

$\nu = 0.12$ fm^{-2}, $\kappa = 0.46$ fm^{-2}, $v_0 = -$ 153 MeV

Faddeev-Yakubovsky (6-6-3 mesh)

• su m max	$E_{4d'}$ (MeV)	KE (MeV)	*Rc* (fm)	rms (fm)
0	–0.100	5.160	14.79	14.90
2	–0.099	5.033	14.79	14.89
4	–0.768	23.67	5.362	5.645
6	–6.872	81.98	1.891	2.589
8	–7.012	83.53	1.875	2.577
10	–7.088	83.41	1.879	2.580
12	–7.089	83.41	1.879	2.580

h.o. variation (total quanta N_{tot})

N_{tot} max	$E_{4d'}$ (MeV)	$c_{(00)}$	KE (MeV)	*Rc* (fm)	rms (fm)
6	0.416	1	52.25	2.339	2.932
8	–1.604	0.941	63.44	2.130	2.768
10	–4.485	0.914	67.55	2.116	2.758
12	–5.481	0.879	74.14	2.002	2.671
14	–6.181	0.857	77.34	1.980	2.655
16	–6.466	0.842	80.08	1.938	2.623
18	–6.628	0.836	81.10	1.935	2.621
20	–6.689	0.832	81.85	1.923	2.612
22	–6.726	0.829	81.50	1.948	2.631
:	:	:	:	:	:

- • $_{sum\ max}$ = 6 で大きく変化する :
  [(20)(20)](02)(20):(00) のため
- h.o. basis : convergence is very slow
- $E_{3d'} = -$ 0.417 MeV (N_{tot}=60) : small
- $E_{2d'} =$ 0.05 MeV (N_{tot}=100) $v_0 = -$ (151 ~ 152) MeV で bound

4α energy and rms radius

Volkov No.2 m=0.605, b=1.36 fm

Faddeev-Yakubovsky (4-4-2)

• su m max	$E_{4\alpha}$ (MeV)	KE (MeV)	R_α (fm)	rms (fm)
0	–4.21	15.51	3.67	3.95
2	–4.16	15.20	3.69	3.96
4	–6.53	20.71	3.27	3.57
6	–7.28	23.10	3.07	3.40
8	–11.56	43.08	2.71	3.07
10	–15.82	66.39	2.19	2.62
12	–39.06	142.33	1.57	2.13
14	–39.15	141.80	1.57	2.13

$(\text{rms})_{exp}$= 2.710±0.015 fm

- largely overbound
- • sum max=12 で大きく変化する [(40)(40)](04)(40):(00) のため
- b を大きくとって rms radius を大きくしても overbinding は不変

h.o. variation (red: with Coulomb)

N_{tot}	$E_{4\alpha}$ (MeV)	$c_{(00)}$	KE (MeV)	R_α (fm)	rms (fm)
12	–34.14	1	184.98	1.38	2.00
	–19.99	1	184.98	1.38	2.00
14	–37.04	0.964	160.34	1.48	2.07
	–23.47	0.958	158.60	1.49	2.07
16	–38.27	0.935	150.87	1.53	2.10
	–24.90	0.924	148.05	1.54	2.11
18	–38.76	0.917	145.95	1.55	2.12
	–25.50	0.901	142.43	1.57	2.13
20	–38.96	0.907	143.39	1.57	2.13
	–25.77	0.888	139.37	1.59	2.15

$E_{2\alpha}$= –1.105 (0.252) MeV

$E_{3\alpha}$= –7.391 (–2.307) for N_{tot}=60

Cf. S. Oryu, H. Kamada, H. Sekine, T. Nishino, and H. Sekiguchi, Nucl. Phys. A534 (1991)221

まとめ

クォーク模型バリオン間相互作用 fss2 の *NN* 部分を用いて, α 粒子の結合エネルギーと平均二乗半径を、二体クラスター RGM kernel を用いた四体 Faddeev-Yakubovsky方程式を解くことにより検討した。結果は、クーロンなしで – 28.0 MeV, 平均二乗半径は 1.43 fm と多少小さい。
クーロン力と核力の荷電独立性の破れを考慮すると
$E_\alpha = -26.64$ MeV ~ $-28.0 + 0.8 + 0.5$ *vs.* $E_\alpha^{exp} = -28.296$ MeV
charge rms radius = 1.439 fm vs. exp : 1.457 ± 0.004 fm
不足分 –1.66 MeV は, 標準的な中間子交換ポテンシャルのそれのほぼ半分で, 3 核子系での結果と consistent である。

二体クラスター RGM kernel に Pauli 禁止状態が存在する 4*d*’ や 4α系についても四体クラスターFaddeev-Yakubovsky方程式の有効性が示された。

これからの課題

- ${}^4_\Lambda$H, ${}^4_\Lambda$He の計算 (進行中)
- ${}^4_{\Lambda\Lambda}$He ... Λ*N*-Σ*N* coupling と ΛΛ-Ξ*N*-ΣΣ coupling が同時に現れる