Proposed Novel Route to Reach a P-wave Superfluid Fermi Gas

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Introduction
- ultracold Fermi gas and neutron star
- A new proposal to reach a p-wave superfluid Fermi gas
  - parity mixing effect and p-wave pair amplitude
  - time-evolution of p-wave superfluid state
- Summary
Current possible approach by human beings (21st century)

Equation of state (EoS)

internal structure + TOV eq. (Tolman-Oppenheimer-Volkoff equation)

“Mass-radius (MR)” relation

Observable! Observable!

experimentalists on the earth

Theorists on the earth
Our strategy: Application of ultracold Fermi gas system as a quantum simulator for neutron star

Fermi atoms ($^6$Li, $^{40}$K) are trapped in a magnetic/optical potential, and are cooled down to $<$O($\mu$K), where quantum effects are important. In this gas system, one can tune the strength of an interaction between atoms by using a Feshbach resonance.

$$N = 10^5 \sim 10^8$$
Phase diagram of an ultracold Fermi gas

- Normal Fermi gas
- Fluctuating preformed pair
- Bose gas
- Superfluid phase

\[ T \sim 0 \]
\[ p_F r_e \sim 0 \]
\[ p_F r_e \lesssim 3 \]
\[ r_e = 2.7 \text{ fm} \]

**Pairing interaction**

- Effective range \( r_e \)

**Neutron star**

**Cold Fermi gas**
cold Fermi gas physics meets neutron star physics!

Cold Fermi gas EOS  Neutron star EOS

Exp.: $^6\text{Li}$ (Horikoshi)

pairing interaction

$E/E_{FG}$

$T \approx 0$

$k_F a_s$^{-1}

$E/N$ [MeV]

$a_s = -18.5 \text{ fm}$

our result ($r_c = 2.7 \text{ fm}$)

strong-coupling theory

Inclusion of pairing fluctuations beyond MF level

Horikoshi, Koashi, Gonokami, Tajima, Ohashi, arXiv: 1612.04026
cold Fermi gas physics meets neutron star physics!

Cold Fermi gas EOS  Neutron star EOS

Exp.: $^6$Li (Horikoshi)  

$T \approx 0$

pairing interaction

Horikoshi, Koashi, Gonokami, Tajima, Ohashi, arXiv: 1612.04026

$p$-wave SF?
Next strategy: approach to deeper inside neutron star using a p-wave superfluid Fermi gas

cold Fermi gas + p-wave SF theory difference between cold Fermi gas and neutron star physics

Neutron star EoS

NO EXPERIMENTAL DATA!
A tunable p-wave pairing interaction has been realized in an ultracold Fermi gas. However, this interaction, which is necessary to form p-wave Cooper-pairs, destroys the system, before the p-wave state grows enough!

\[ \frac{\pi}{2} \leq \frac{T}{T_c} \leq \pi \]

p-wave interacting Fermi gas

\[ ^{23}\text{Na} \]

\[ N_{\text{cond}} (10^5) \]

\[ \text{Time (ms)} \]

\[ \sim 100 \text{ ms} \gg \text{lifetime } \sim 5-20 \text{ ms} \]
A tunable p-wave pairing interaction has been realized in an ultracold Fermi gas.

However,

this interaction, which is necessary to form p-wave Cooper-pairs, destroys the system, before the p-wave state grows enough!
The purpose of this talk

We theoretically propose a novel idea to reach a $p$-wave superfluid Fermi gas. Using this proposal, one may be able to overcome the long-standing difficulty that all the cold Fermi gas experiments aiming to realize this unconventional Fermi superfluid.

\[
\Delta(r, r') = U(r - r') \langle \Psi_\sigma(r) \Psi_\sigma'(r') \rangle
\]

Pairing interaction

Cooper-pair amplitude

$p$-wave interacting Fermi gas

\[
\Delta_{p\text{-wave}} = U_{p\text{-wave}} \times 0 = 0 \\
\Delta_{p\text{-wave}} = U_{p\text{-wave}} \times \langle \Psi_\sigma \Psi_\sigma' \rangle \\
\Delta_{p\text{-wave}} = U_{p\text{-wave}} \times \langle \Psi_\sigma \Psi_\sigma' \rangle
\]
The purpose of this talk

We theoretically propose a novel idea to reach a $p$-wave superfluid Fermi gas. Using this proposal, one may be able to overcome the long-standing difficulty that all the cold Fermi gas experiments aiming to realize this unconventional Fermi superfluid.

Superfluid order parameter

$$\Delta(r, r') = U(r - r') \langle \Psi_\sigma(r) \Psi_{\sigma'}(r') \rangle$$

- **Pairing interaction**
- **Cooper-pair amplitude**

NO $p$-wave interaction

1. $U_{p\text{-wave}} = 0$

\[ \langle \Psi_\sigma(r) \Psi_{\sigma'}(r') \rangle = 0 \]

2. $U_{p\text{-wave}} > 0$

\[ \langle \Psi_\sigma(r) \Psi_{\sigma'}(r') \rangle \neq 0 \]

$\Delta_{p\text{-wave}} = U_{p\text{-wave}} \times \langle \Psi_\sigma \Psi_{\sigma'} \rangle \neq 0$

$5 \sim 20$ ms

$\Delta_{p\text{-wave}} > 0$
STEP 1
How to produce p-wave amplitude without using p-wave interaction

\[ \langle \Psi_\sigma(r) \Psi_\sigma'(r') \rangle \]

\[ U_{p\text{-wave}} = 0 \]
Parity mixing effect caused by a *synthetic* spin-orbit coupling

Cold Fermi gas physics can now introduce an *antisymmetric* spin-orbit coupling to the system by using an artificial gauge field technique.

\[ \mathcal{E}_p = \hbar k \cdot \mathbf{r} \]

\[ H_{\text{spin-orbit}} = \lambda p_z \sigma_x \]

Parity is broken!

Pairing symmetry:
- Spin-singlet × even parity
- Spin-triplet × odd parity

Parity-mixing occurs!
S-wave superfluid Fermi gas with a *synthetic* spin-orbit coupling

- **Parity-broken s-wave superfluid Fermi gas**

\[
H = \sum_p \left( c_{p\uparrow}^\dagger, c_{p\downarrow}^\dagger \right) \left( \begin{pmatrix} \varepsilon_p - \mu & \lambda p_z \\ \lambda p_z & \varepsilon_p - \mu \end{pmatrix} \right) \left( \begin{pmatrix} c_{p\uparrow} \\ c_{p\downarrow} \end{pmatrix} \right) - U_{s\text{-wave}} \sum_{p, p', q} c_{p+q/2\uparrow}^\dagger c_{-p+q/2\downarrow}^\dagger c_{-p'+q/2\downarrow} c_{p'+q/2\uparrow}
\]

**Spin-orbit coupling**

**S-wave pairing interaction**

- **BCS-Leggett strong-coupling theory at \( T=0 \)**

- **s-wave superfluid order parameter**

\[
\Delta_{s\text{-wave}} = -U_{s\text{-wave}} \sum_p \langle c_{p\uparrow} c_{-p\downarrow} \rangle \neq 0 \quad \text{s-wave superfluid state}
\]

- **p-wave Cooper-pair amplitude**

\[
\langle c_{p\uparrow} c_{-p\uparrow} \rangle = -\langle c_{p\downarrow} c_{-p\downarrow} \rangle = \text{sgn}(p_z) \frac{\Delta_{s\text{-wave}}}{4} \sum_p \left[ \frac{1}{E_p^+} - \frac{1}{E_p^-} \right] \neq 0
\]

- **p-wave order parameter**

\[
E_p^\pm = \sqrt{(\varepsilon_p - \mu \pm \lambda |p_z|)^2 + \Delta_{s\text{-wave}}^2}
\]
Induced $p$-wave Cooper-pair amplitude in the $s$-wave state

$$
\sum_p \left| \langle c_p \uparrow c_{-p} \uparrow \rangle \right|^2 / N
$$

Yamaguchi, YO, PRA 92, 013615 (2015)
STEP 2

How to reach a p-wave superfluid state, within the lifetime, 5~20ms

\[ U_{p\text{-wave}} > 0 \]

\[ \Delta_{p\text{-wave}} = U_{p\text{-wave}} \times \langle \Psi_\sigma \Psi_{\sigma'} \rangle \neq 0 \]
Tunable Feshbach interaction adjusted by external magnetic field

In $^{40}$K and $^6$Li Fermi gases, we can tune a pairing interaction by adjusting an external magnetic field.

Feshbach resonance

\[
\Delta_{s\text{-wave}} = -U_{s\text{-wave}} \sum_p \langle c_p \uparrow c_{-p} \downarrow \rangle \neq 0
\]

\[
\left\langle c_p \uparrow c_{-p} \uparrow \right\rangle = \sum_{p} |c_p \uparrow c_{-p} \uparrow|^2 / N
\]

BCS \quad BEC
In $^{40}$K and $^6$Li Fermi gases, we can tune a pairing interaction by adjusting an external magnetic field.

**Feshbach resonance**

$$\Delta_{p\text{-wave}}(p) = \sum_p U_{p_{-\text{wave}}}(p, p') \left\langle c_{p \uparrow} c_{-p \uparrow} \right\rangle \neq 0$$

$$\left\langle c_{p \uparrow} c_{-p \uparrow} \right\rangle =$$

**p-wave superfluid Fermi gas**!
Time-dependent Bogoliubov de Gennes (TdBdG) analysis

\[ i \frac{\partial \Psi_p(t)}{\partial t} = \begin{pmatrix} \varepsilon_p & \Delta_{p_z\text{-wave}}(p, t) \\ \Delta^*_{p_z\text{-wave}}(p, t) & -\varepsilon_p \end{pmatrix} \Psi_p(t) \]

\[ U_{s\text{-wave}}, \lambda_{\text{spin-orbit}} \]

Equilibrium s-wave state

\[ \Delta_{s\text{-wave}} = -U_{s\text{-wave}} \sum_p \left\langle c_p^\uparrow c_{-p}^\downarrow \right\rangle \]

\[ \left\langle c_p^\uparrow c_{-p}^\uparrow \right\rangle = \]
Time-dependent Bogoliubov de Gennes (TdBdG) analysis

\[ s\text{-wave} : (k_F a_s)^{-1} = 0 \Rightarrow p_z\text{-wave} : (k_F^3 \nu_p)^{-1} = 0 \]

In our idea, the p-wave order parameter grows much faster than the system lifetime (~5-20ms).
Time-dependent Bogoliubov de Gennes (TdBdG) analysis

\[ s\text{-wave}: (k_F a_s)^{-1} = 0 \Rightarrow p_z \text{-wave}: (k_F^3 v_p)^{-1} = 0 \]

\[ s\text{-wave}: (k_F a_s)^{-1} = -1 \Rightarrow p_z \text{-wave}: (k_F^3 v_p)^{-1} = -6 \]

Non-vanishing \( \Delta_p(t>>0) \) is obtained when the initial momentum distribution is taken to be close to that in the final equilibrium p-wave state.
We have proposed a novel idea to realize a p-wave superfluid Fermi gas. Our approach separately prepares p-wave pair amplitude without relying on any p-wave interaction, but using parity-broken spin-orbit coupling. Thus, it may overcome the current experimental difficulty (short system lifetime (=5~20 ms <<100 ms) by p-wave interaction).

Our idea involves potential importance of cold Fermi gas system for the study of non-equilibrium problems, such as PBF mechanism of neutron-star cooling.