Microscopic understanding of nuclear saturation properties and EoS and three-nucleon force

M. Kohno, Kyushu Dental College

- Microscopic understanding of nuclear saturation and independent particle model
- Contributions of 3NF of Ch-EFT
  - G-matrix calculations including effective two-body interactions from three-nucleon force
- Hyperons in neutron matter
  - Hyperon s.p. potentials predicted by recent YN interactions
  - Estimation of the contribution of possible ΛNN force

saturation → considering nuclear matter

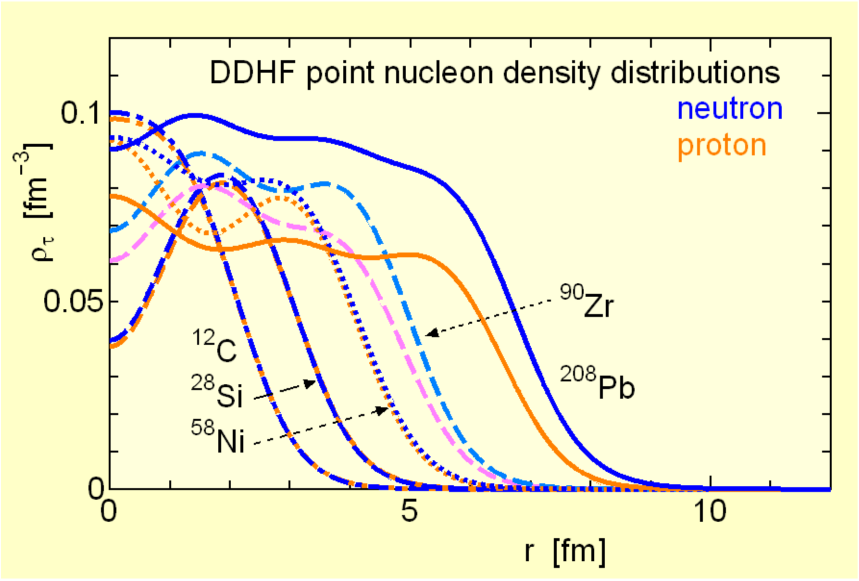
- Weizsacker-Bethe mass formula

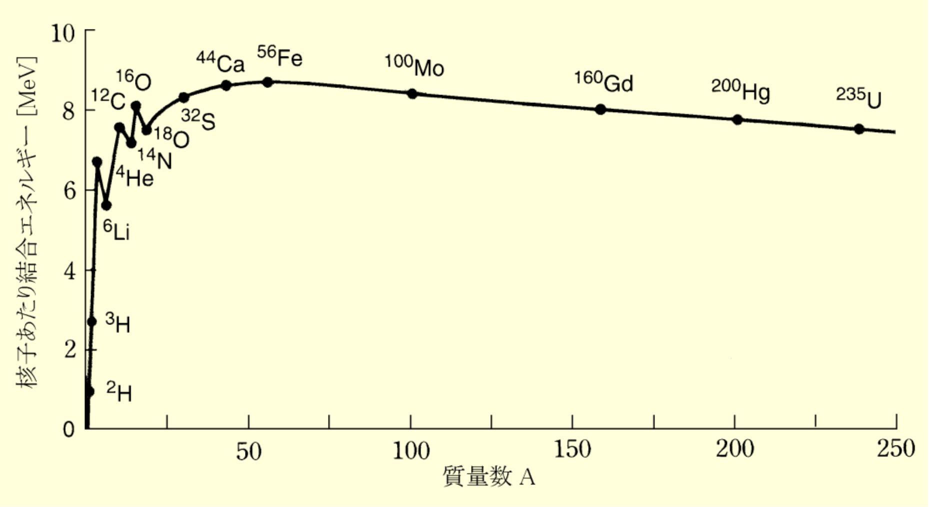
$$M(Z,N)-Zm_p-Nm_n=-a_vA+a_sA^{2/3}+a_c\frac{Z^2}{A^{1/3}}+a_i\frac{(N-Z)^2}{A}+\delta(Z,N)$$

- B.E./A in nuclear matter

  a_v = 15～17 MeV

- central density

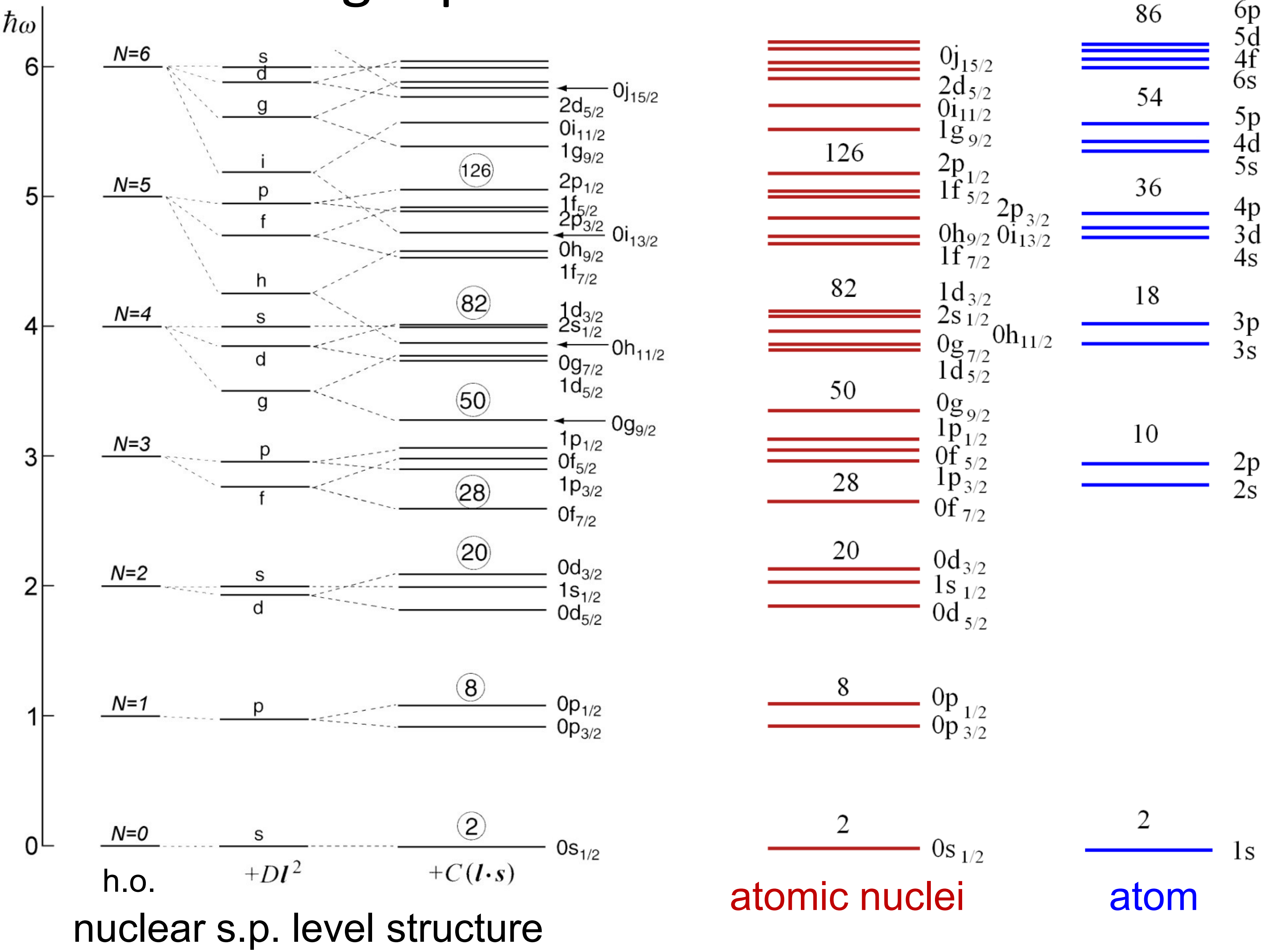




nuclear matter density

$$\rho = 0.17\ \mathrm{fm}^{-3}$$

single-particle level structure

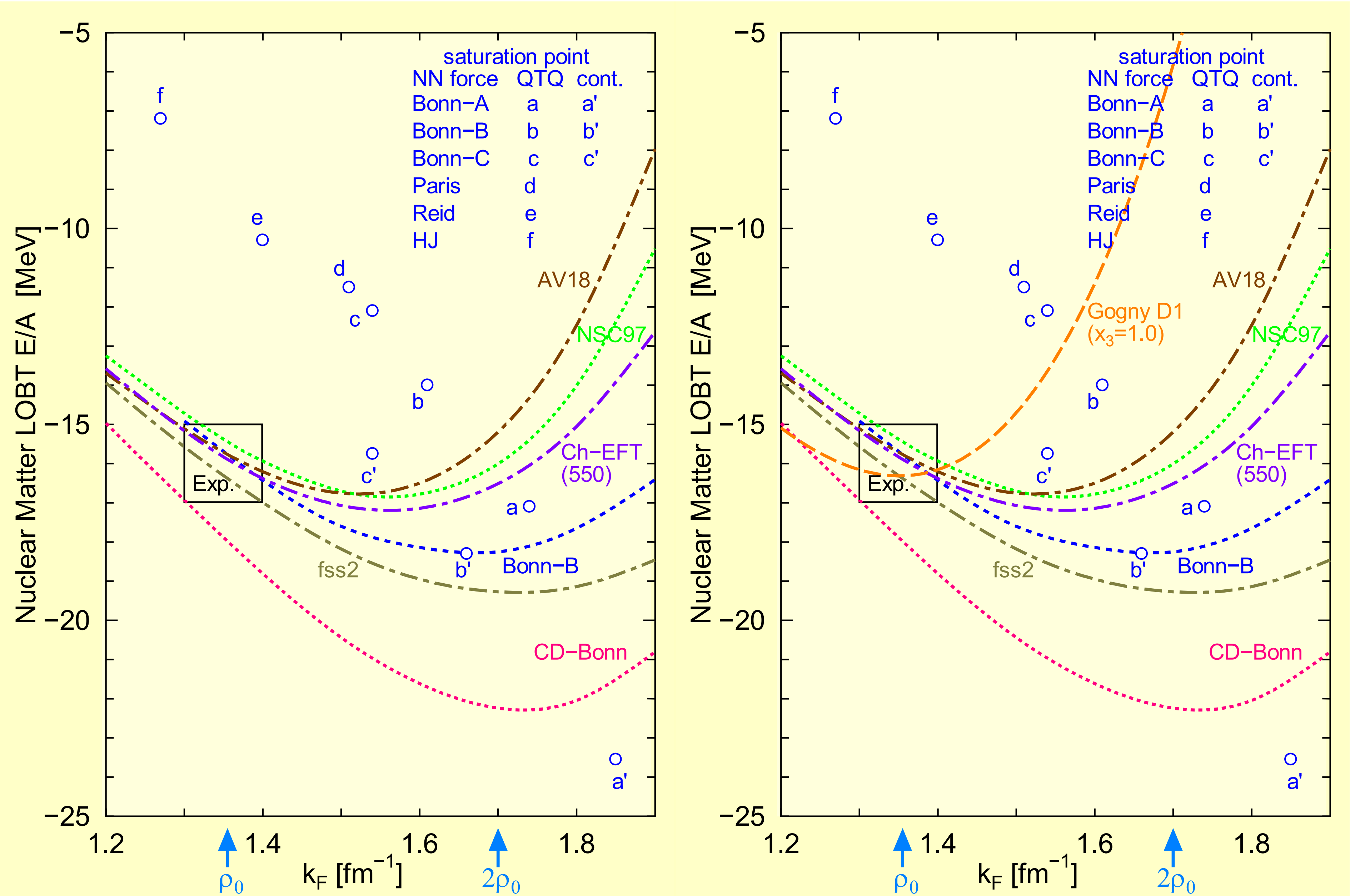


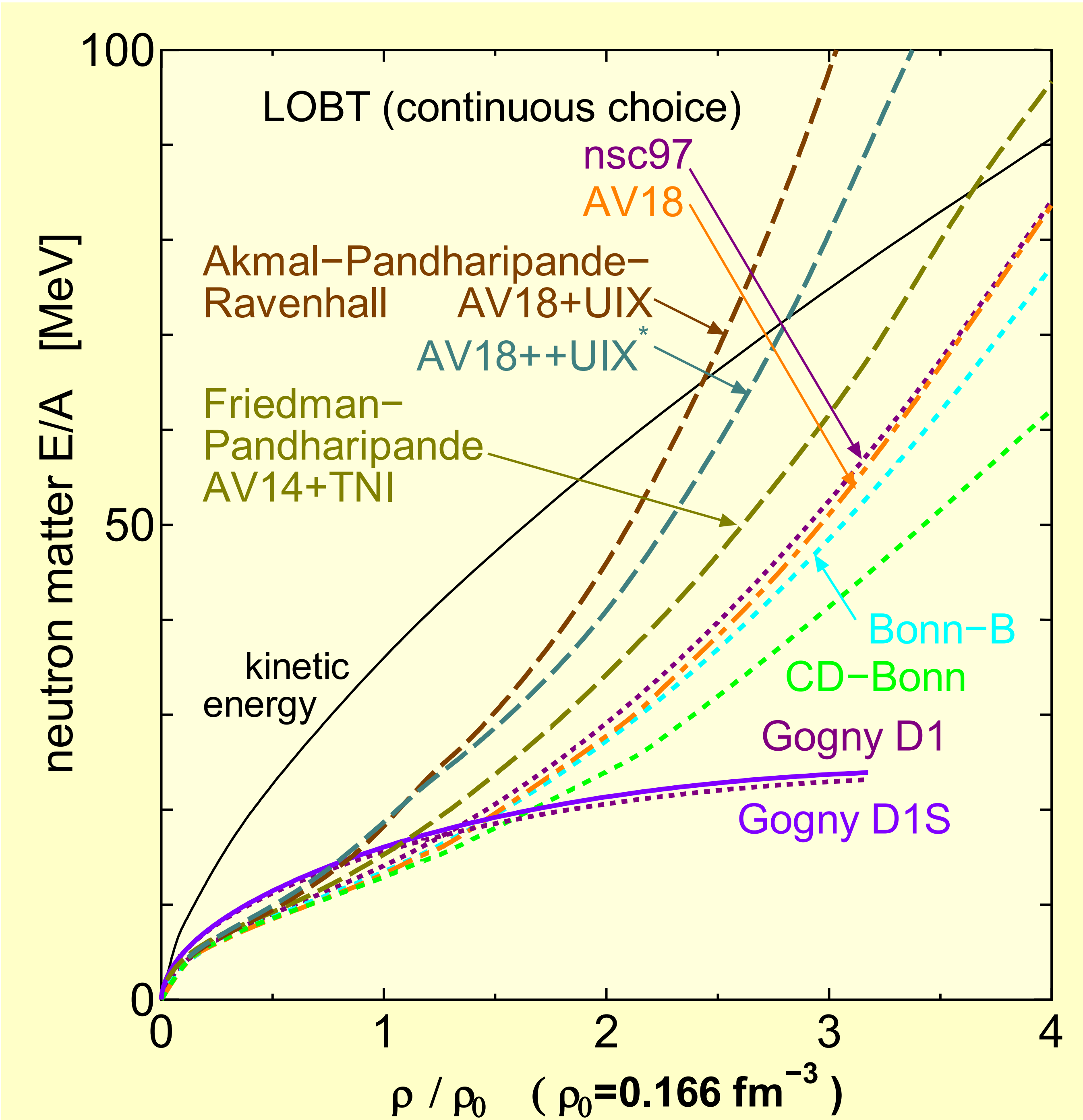
- Long history of microscopic understanding of nuclei : Brueckner as a standard framework.
  - G-matrix: G can be treated in the perturbation, after taking care of the high-momentum singularity with the medium effects. The concept of the healing distance.
  - $G(\omega) = v + v \frac{Q}{\omega - (t + U + t + U)} G(\omega)$
  - $U(k) = \sum_{k'} \langle kk' | G(\omega = e_k + e_{k'}) | kk' \rangle, \quad e_k = t(k) + U(k)$
  - U_N is determined self-consistently（inclusion of many-body correlations）
- G-matrix as effective interactions for describing nuclei (shell model, DDHF, ⋯⋯).
  - Reflecting the failure of reproducing nuclear saturation properties, phenomenological adjustments are necessary.（e.g., density dependent repulsive interaction is introduced in DDHF calculations.)

Microscopic understanding of bulk properties of nuclei

- Saturation properties and independent-particle model.
  - Saturation properties and spin-orbit strength: not understood quantitatively with the realistic NN interactions, which have high-momentum (short-range) singularities.
- Recent progresses in (effective) interaction theory.
  - Systematic description of 2N+3N forces in the Ch-EFT and development in the lattice QCD description.
  - $V_{\text{low k}}$, CCM, no-core shell model, ab-initio calculations, ...
- 3NF is essential to quantitatively describe saturation properties and the strength of the spin-orbit field.
- Estimate the effects of 3NF using the Ch-EFT 3NF.

LOBT saturation curves in symmetric nuclear matter





Friedman and Pandharipande, Nucl. Phys. A361, 501 (1981)

Akmal, Pandharipande, and Ravenhall, Phys. Rev. C58, 1804 (1998)

Gogny and Padjen, Nucl. Phys. A293, 365 (1977)

Decharge and Gogny, Phys. Rev. C21, 1568 (1980)

AV18: Wiringa, Stoks, and Schiavilla, Phys. Rev. C51, 38 (1995)

CD-Bonn: Machleidt, Phys. Rev. C63, 024001 (2001)

nsc97: Rijken, Stoks, and Yamamoto, Phys. Rev. C59, 21 (1999)

Bonn-B: Adv. Nucl. Phys. 19, 189 (1989)

spin-orbit force

- Essential for single-particle shell-structure
  - magic number
  - Strong spin-orbit field is not explained by pion exchange or relativistic effect
  - vector mesons, but not sufficient quantitatively
- Contributions from second order tensor effect and/or three-nucleon force with isobar Δ excitation have been considered, but not settled:
  - J. Fujita and H. Miyazawa, P.T.P. 17, 366 (1957)
  - K. Ohta, T. Terasawa and M. Tohyama, P. R. C22, 2233 (1980)
  - K. Ando and H. Bando, P.T.P. 66, 227 (1981)
  - N. Kaiser, P.R. C70, 034307 (2004)

spin-orbit field

- Thomas type one-body field： $U_{ls}^{0}\frac{1}{r}\frac{d\rho(r)}{dr}\boldsymbol{l}\cdot\boldsymbol{\sigma}$
- Relation to effective two-body LS int.（Scheerbaum, N.P. A257,77(1976)）

Define $B_S = -\frac{2\pi}{q}\int V_{ls}^{\,^3O}(r)j_1(qr)r^3dr$ ($q \cong 0.7\ \mathrm{fm}^{-1}$), then

$U_{ls}^{0} = \frac{3}{4}B_S$: Realistic NN forces predict $B_S \cong 90$ MeV· fm^5.

- effective δ-type LS force: $iW(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\overleftarrow{\nabla}\times\delta(\boldsymbol{r})\overrightarrow{\nabla}$

Hartree-Fock LS mean field ➡ $\frac{1}{2}W\frac{1}{r}\frac{d(\rho(r)+\rho_\tau(r))}{dr}\boldsymbol{l}\cdot\boldsymbol{\sigma}$

When $\rho_p(r) = \rho_n(r) = \frac{1}{2}\rho(r)$, $\frac{3}{4}W\frac{1}{r}\frac{d\rho(r)}{dr}\boldsymbol{l}\cdot\boldsymbol{\sigma}$. Thus, $W \Leftrightarrow B_S$.

- In standard DDHF calculations（Skyrme int., Gogny int. , and others）$W_{ph} = 120\sim130$ MeV· fm^5.

Evaluation of B_S using momentum space G-matrices

- $B_S = -\frac{2\pi}{q}\int V_{ls}^{^3O}(r) j_1(qr) r^3 dr$, using momentum space G-matrices:

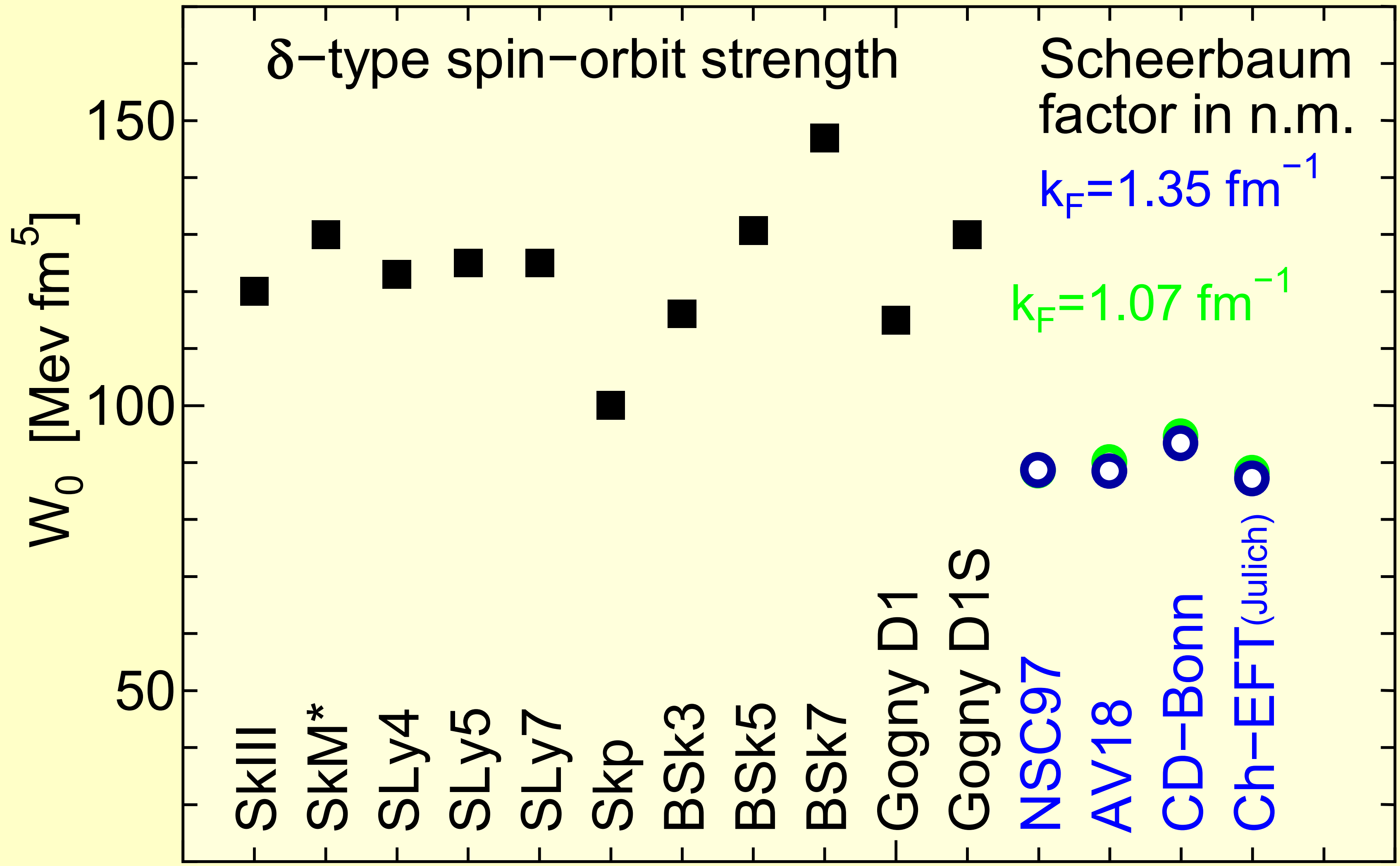
$$B_S = \frac{1}{k_F^3}\sum_{JT}(2J+1)(2T+1)\int_0^{q_{max}} dq\, W(\bar{q}, q)$$

$$\times \{(J+2)G_{J+1}^{JT}(q) + G_J^{JT}(q) - (J-1)G_{J-1}^{JT}(q)\}$$

$$q_{max} = \frac{1}{2}(k_F + \bar{q}), \quad W(\bar{q}, q) = \begin{cases} \theta(k_F - \bar{q}) \text{ for } 0 \le q \le \frac{|k_F - \bar{q}|}{2} \\ \frac{k_F^2 - (\bar{q} - 2q)^2}{8\bar{q}q} \text{ for } \frac{|k_F - \bar{q}|}{2} \le q \le \frac{k_F + \bar{q}}{2} \end{cases}$$

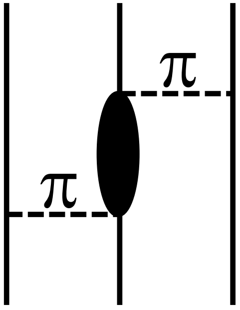
- Sheerbaum factor $B_S\ (= W)$ calculated in nuclear matter with modern NN potentials (AV18, Nijmegen, CD-Bonn, and Julich N^3LO) is around $\sim 90\ \mathrm{MeV \cdot fm^5}$, which is about $\frac{3}{4}$ of the phenomenological strength.

phenomenological W_0 and Scheerbaum factor



Three-nucleon force

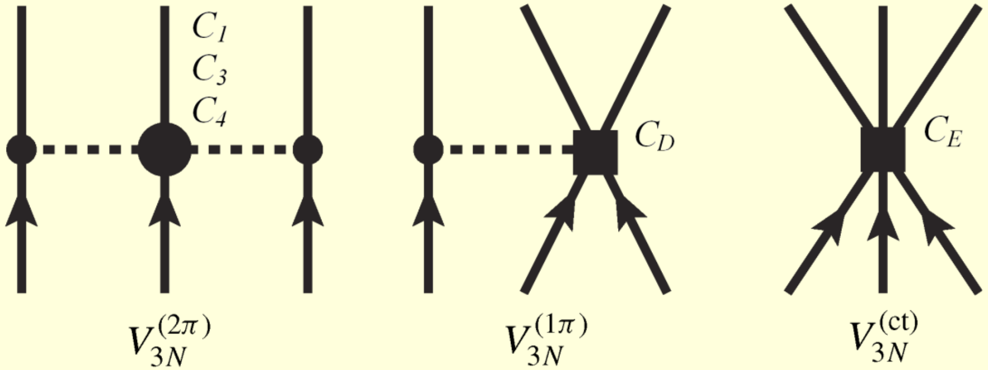
- Importance
  - Quantitative understanding of saturation properties, EoS.
  - Ab-initio calculations（MC, NCSM, CCM, ...）for few body systems（$A \lesssim 12$）
  - Nucleon-nucleus and nucleus-nucleus reactions.
- Process in which the isobar Δ_{33} (anti-nucleon) excited.
  - A. Klein, Phys. Rev. 90 (1953) 1101. 他
  - Fujita-Miyazawa, P.T.P. 17 (1957) 360.



- In recent years, systematic and quantitative treatment becomes possible in the Ch-EFT framework.
- Making reduced two-nucleon interactions from the 3NF, and carry out G-matrix calculations.

（It is very hard to treat directly 3NF in nuclear matter.）

3NF in Ch-EFT



$$V_{3N}^{(2\pi)} = \sum_{i\neq j\neq k} \frac{g_A^2}{8f_\pi^4} \frac{\vec{\sigma}_i \cdot \vec{q}_i \vec{\sigma}_j \cdot \vec{q}_j}{(\vec{q}_i^2 + m_\pi^2)(\vec{q}_j^2 + m_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta ,$$

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta}\left[-4c_1 m_\pi^2 + 2c_3 \vec{q}_i \cdot \vec{q}_j\right] + c_4 \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \cdot (\vec{q}_i \times \vec{q}_j).$$

$$\vec{q}_i = \vec{p}_i{}' - \vec{p}_i$$

$$V_{3N}^{(1\pi)} = -\sum_{i\neq j\neq k} \frac{g_A c_D}{8f_\pi^4 \Lambda_\chi} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{\vec{q}_j^2 + m_\pi^2} \vec{\sigma}_i \cdot \vec{q}_j \vec{\tau}_i \cdot \vec{\tau}_j ,$$

$$V_{3N}^{(\mathrm{ct})} = \sum_{i\neq j\neq k} \frac{c_E}{2f_\pi^4 \Lambda_\chi} \vec{\tau}_i \cdot \vec{\tau}_j ,$$

c_1, c_3, c_4: fixed

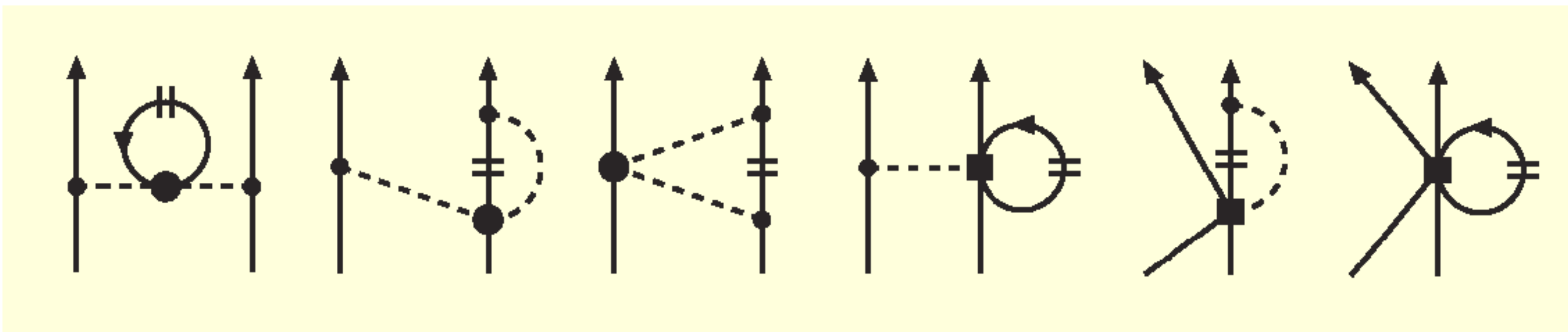
c_D, c_E: adjustable

$$\Lambda_\chi = 700\,\mathrm{MeV},\, m_\pi = 138.04\,\mathrm{MeV}$$

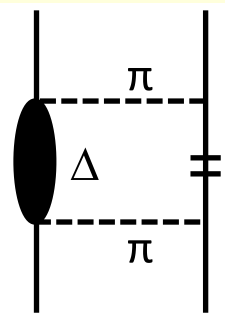
Reduced two-nucleon force ($\frac{1}{3}$ is needed for energy)

$$\langle ab|v_{12(3)}|cd\rangle_A \equiv \frac{1}{3}\sum_h \langle abh|v_{123}|cdh\rangle_A$$

Diagrams by Weise group



The factor $\frac{1}{3}$ is, in most cases, missing in the literature.



This Pauli blocking term gives large repulsive *contrib*ution.

G-matrix calculations, after making partial wave decomposition

Reduced NN force and partial wave decomposition

- Example: c_1 term of the Ch-EFT 3NF

$$-\frac{c_1 g_A^2 m_\pi^2}{f_\pi^4}\left\{\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1)(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2)}{(\boldsymbol{q}_1^2+m_\pi^2)(\boldsymbol{q}_2^2+m_\pi^2)}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1)(\boldsymbol{\sigma}_3\cdot\boldsymbol{q}_3)}{(\boldsymbol{q}_1^2+m_\pi^2)(\boldsymbol{q}_3^2+m_\pi^2)}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)+\frac{(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2)(\boldsymbol{\sigma}_3\cdot\boldsymbol{q}_3)}{(\boldsymbol{q}_2^2+m_\pi^2)(\boldsymbol{q}_3^2+m_\pi^2)}(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\right\}$$

- Sum over the third nucleon $(|\boldsymbol{k}_3|\leq k_F,\sigma_3,\tau_3)$ in nuclear matter. Note the factor of $\frac{1}{3}$.

$$-\frac{c_1 g_A^2 m_\pi^2}{f_\pi^4}\ \frac{1}{3}\sum_{\boldsymbol{k}_3,\sigma_3,\tau_3}\langle \boldsymbol{k}_1'\sigma_1'\tau_1',\boldsymbol{k}_2'\sigma_2'\tau_2',\boldsymbol{k}_3\sigma_3\tau_3|\left\{\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1)(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2)}{(\boldsymbol{q}_1^2+m_\pi^2)(\boldsymbol{q}_2^2+m_\pi^2)}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1)(\boldsymbol{\sigma}_3\cdot\boldsymbol{q}_3)}{(\boldsymbol{q}_1^2+m_\pi^2)(\boldsymbol{q}_3^2+m_\pi^2)}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)+\frac{(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2)(\boldsymbol{\sigma}_3\cdot\boldsymbol{q}_3)}{(\boldsymbol{q}_2^2+m_\pi^2)(\boldsymbol{q}_3^2+m_\pi^2)}(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\right\}$$

$$|[\boldsymbol{k}_1\sigma_1\tau_1,\boldsymbol{k}_2\sigma_2\tau_2]_a,\boldsymbol{k}_3\sigma_3\tau_3+[\boldsymbol{k}_2\sigma_2\tau_2]_a,\boldsymbol{k}_3\sigma_3\tau_3,[\boldsymbol{k}_1\sigma_1\tau_1]_a+\boldsymbol{k}_3\sigma_3\tau_3,[\boldsymbol{k}_1\sigma_1\tau_1,\boldsymbol{k}_2\sigma_2\tau_2]_a\rangle$$

- Carry out the summation

effective two-body LS component ($c_1$term)

$$\frac{c_1 g_A^2 m_\pi^2}{f_\pi^4} \iiint_{|\boldsymbol{k}_3| \le k_F} d\boldsymbol{k}_3 \times \frac{i(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (-\boldsymbol{k}_1' \times \boldsymbol{k}_1 + (\boldsymbol{k}_1' - \boldsymbol{k}_1) \times \boldsymbol{k}_3)}{((\boldsymbol{k}_1' - \boldsymbol{k}_3)^2 + m_\pi^2)((\boldsymbol{k}_1 - \boldsymbol{k}_3)^2 + m_\pi^2)}.$$

partial wave decomposition

$$-\delta_{S1} \frac{c_1 g_A^2 m_\pi^2}{f_\pi^4} \frac{\ell(\ell+1) + 2 - J(J+1)}{2\ell+1} \left\{ Q_{W,0}^{\ell-1}(k_1', k_1) - Q_{W,0}^{\ell+1}(k_1', k_1) - W_{\ell s,0}^{\ell}(k_1', k_1) \right\}$$

functions

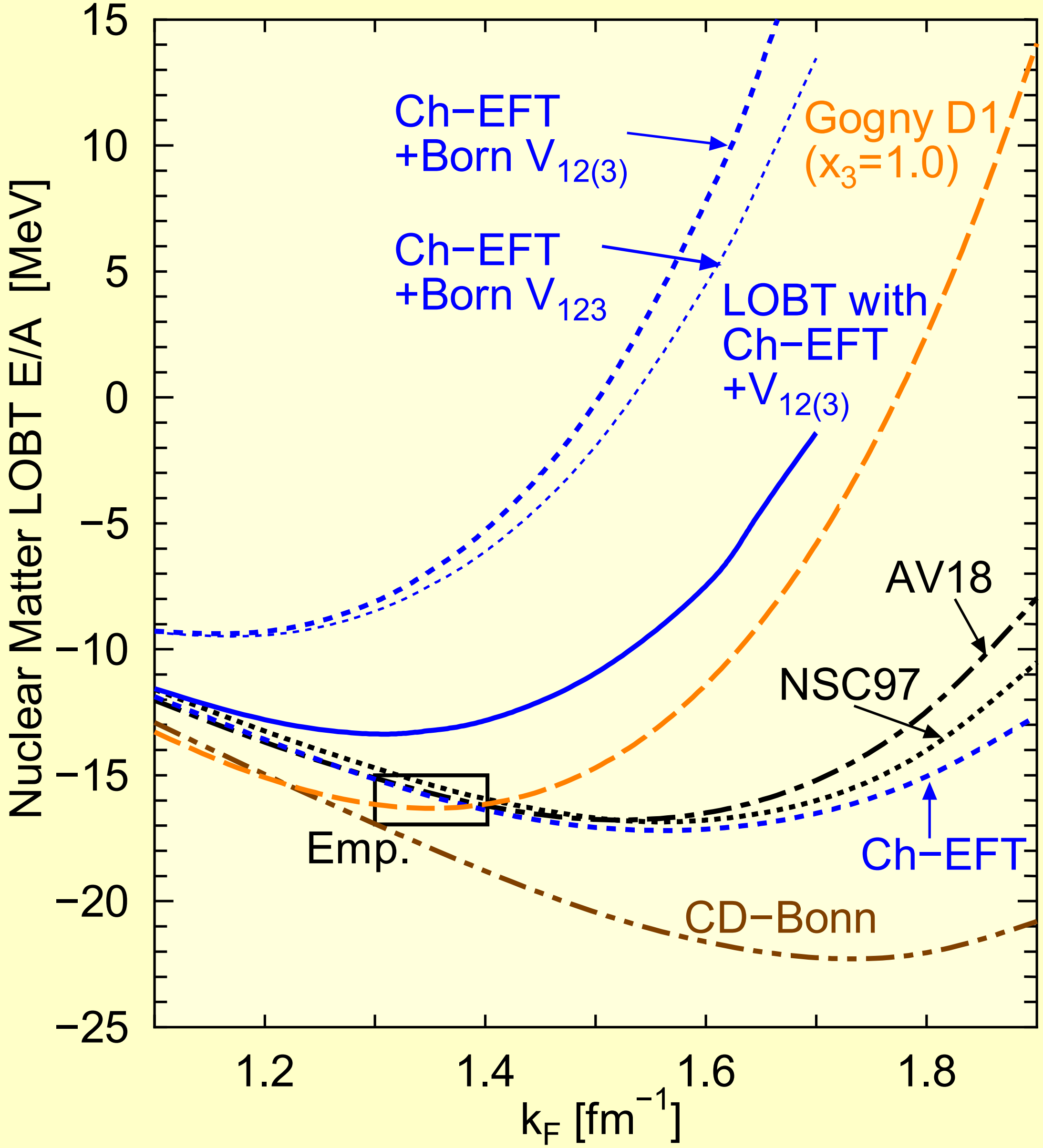
$$Q_{W,0}^{\ell}(k_1', k_1) \equiv \frac{1}{(2\pi)^2} \frac{1}{2k_1' k_1} \int_0^{k_F} dk_3 Q_\ell(x') Q_\ell(x),$$

$$W_{\ell s,0}^{\ell}(k_1', k_1) \equiv \frac{1}{(2\pi)^2} \frac{1}{2k_1' k_1} \int_0^{k_F} dk_3 k_3 \times \{k_1' Q_\ell(x)(Q_{\ell-1}(x') - Q_{\ell+1}(x')) + k_1 Q_\ell(x')(Q_{\ell-1}(x) - Q_{\ell+1}(x))\},$$

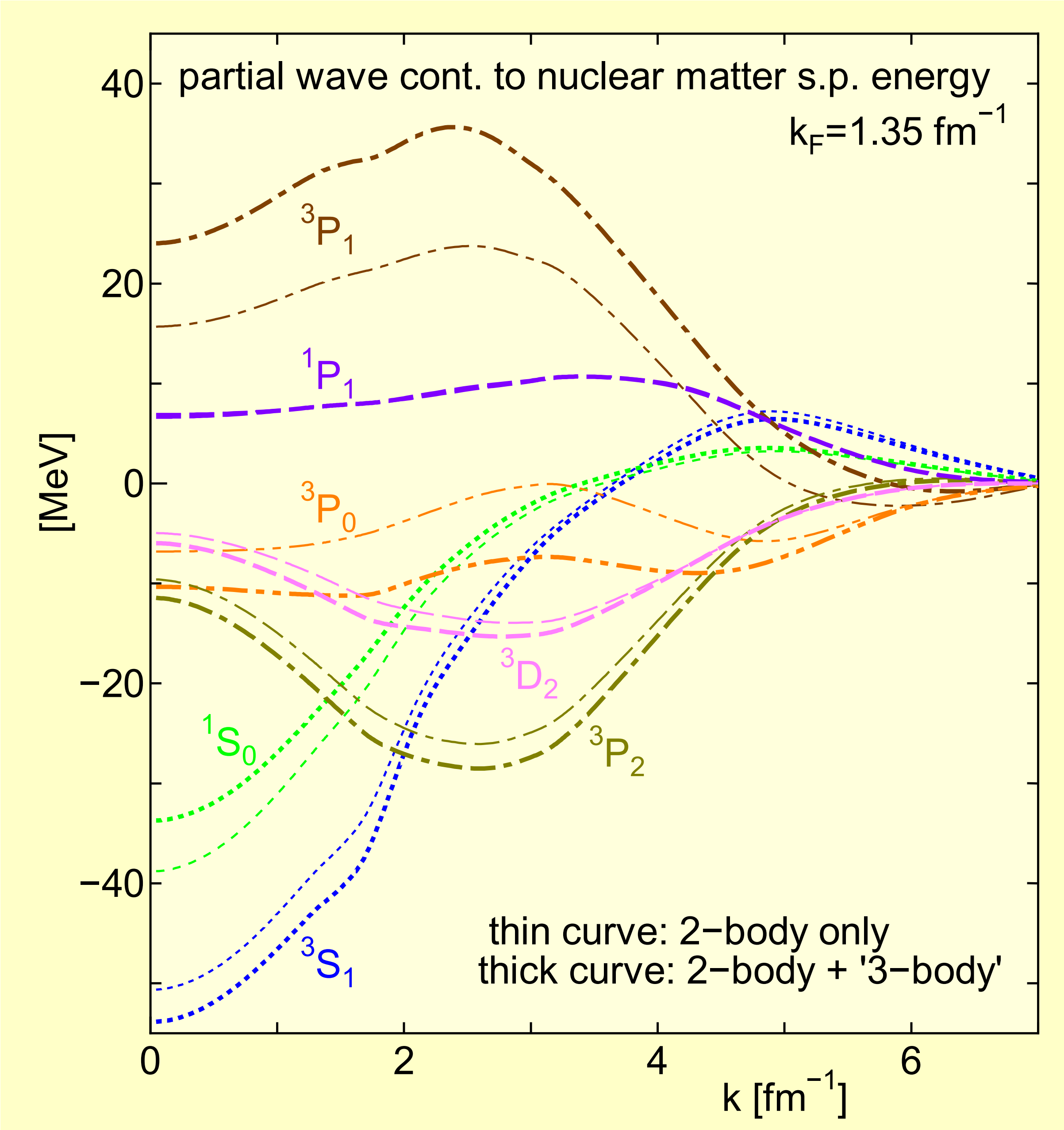
$Q_l(x)$: 2nd Legendre fn.

$$x = \frac{k_1^2 + k_3^2 + m_\pi^2}{2k_1 k_3}$$

LOBT calculations in nuclear matter with $v_{12} + v_{12(3)}$



partial wave contributions to s.p. potential

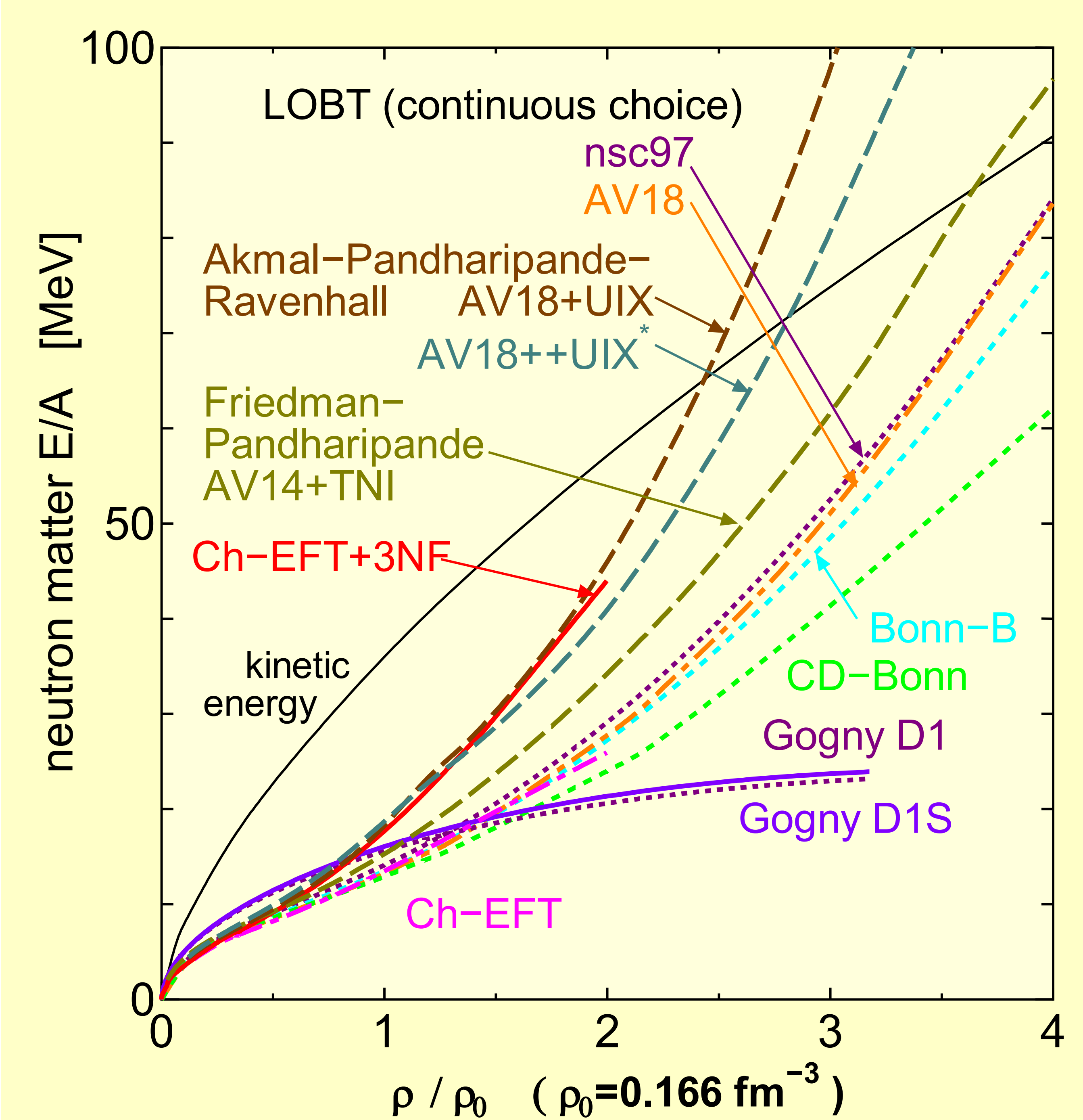


thin curves：
only ChEFT 2NF
thick curves：
2NF+“3NF”

Strong $^3P_1$ repulsion cancels with $^3P_2$ attraction. $^1P_1$ repulsion cancels with $^3D_2$ attraction.

Effects of “3NF”

- Enhancement of $^3S_1$ tensor force gives attraction.
- Repulsion in $^1S_0$.
- Net repulsive contributions from p-waves.
- LS force enhanced.



Friedman and Pandharipande, Nucl. Phys. A361, 501 (1981)

Akmal, Pandharipande, and Ravenhall, Phys. Rev. C58, 1804 (1998)

Gogny and Padjen, Nucl. Phys. A293, 365 (1977)

Decharge and Gogny, Phys. Rev. C21, 1568 (1980)

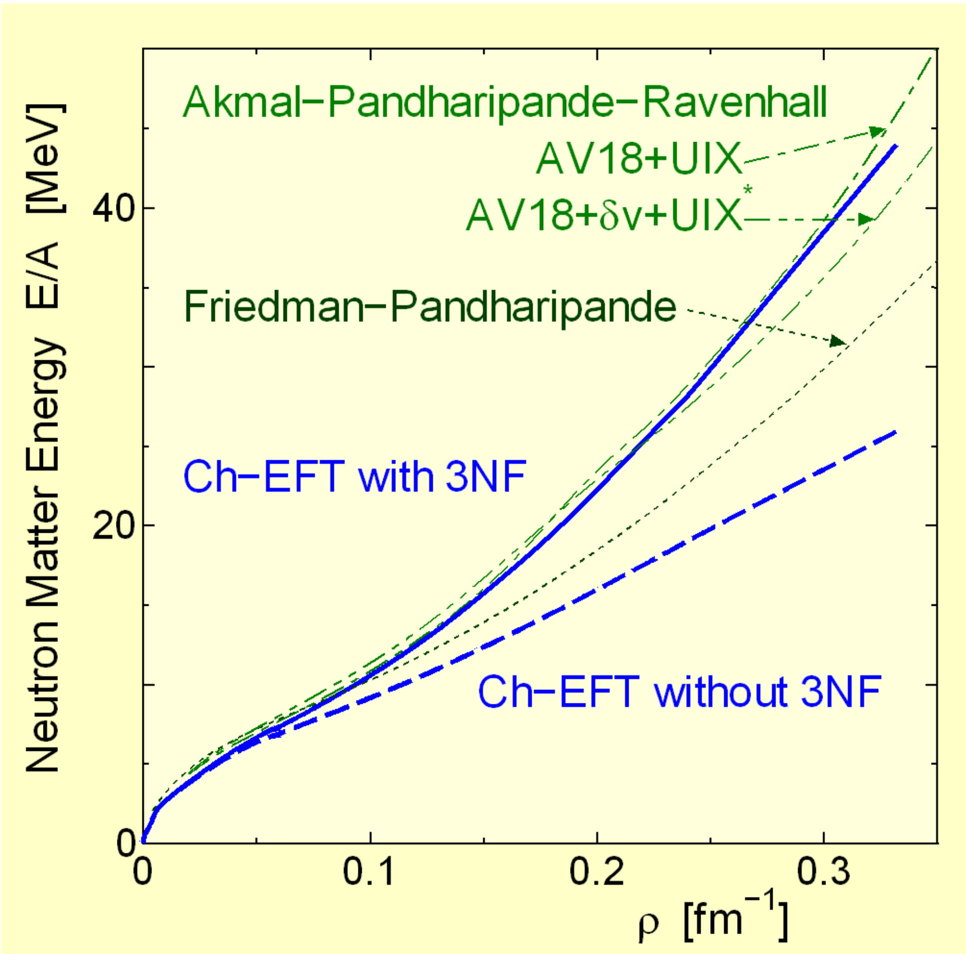
AV18: Wiringa, Stoks, and Schiavilla, Phys. Rev. C51, 38 (1995)

CD-Bonn: Machleidt, Phys. Rev. C63, 024001 (2001)

nsc97: Rijken, Stoks, and Yamamoto, Phys. Rev. C59, 21 (1999)

Bonn-B: Adv. Nucl. Phys. 19, 189 (1989)

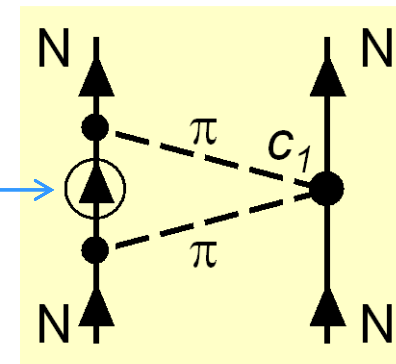
LOBT E/A with including the 3NF in neutron matter agrees with the GMC energy by the Illinois group.



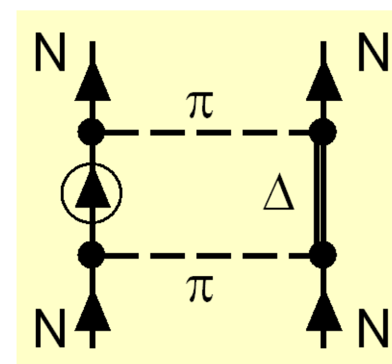
(no (little) contribution from c_E (c_D) term in neutron matter.)

Most repulsive contribution comes from Pauli blocking

hole state



Traditional view



Friedman and Pandharipande,Nucl. Phys. A361, 501 (1981)

Akmal, Pandharipande, and Ravenhall, Phys. Rev. C58, 1804 (1998)

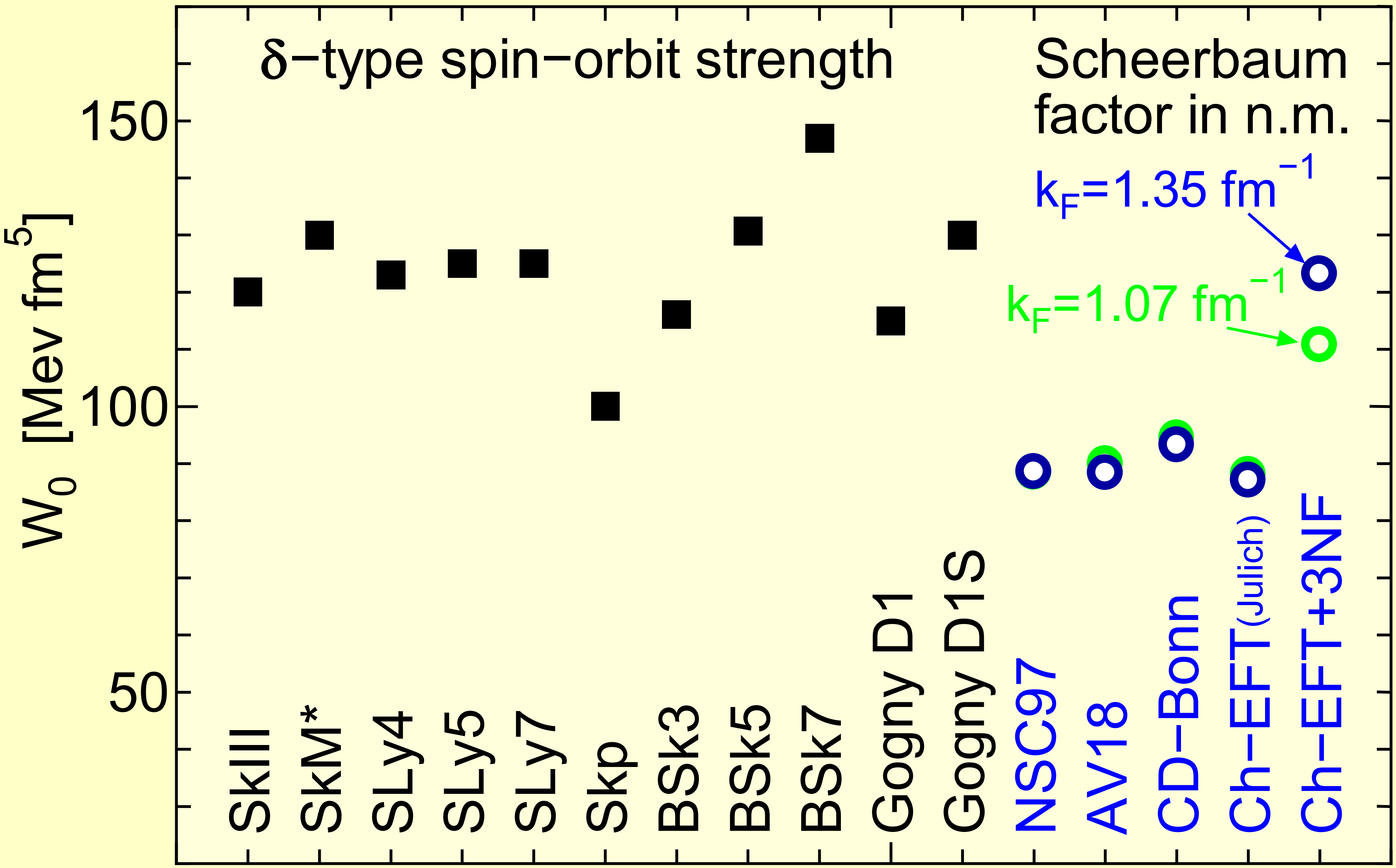
Scheerbaum factor B_S including effective 2NF from 3NF

（LS components come only from c_1 and c_3 terms.）

$k_F = 1.35\ \mathrm{fm}^{-1}$	AV18	NSC97	CD-Bonn	N^3LO	N^3LO+3NF
$B_S(T=0)$	2.0	1.9	3.1	2.5	7.0
$B_S(T=1)$	86.4	86.7	90.2	84.6	116.2
B_S	88.4	88.6	93.3	87.1	123.2
$k_F = 1.07\ \mathrm{fm}^{-1}$	AV18	NSC97	CD-Bonn	N^3LO	N^3LO+3NF
$B_S(T=0)$	1.4	1.3	2.3	1.6	4.1
$B_S(T=1)$	88.1	88.7	92.2	86.5	106.7
B_S	89.5	90.0	94.5	88.1	110.8

Cf. $W = B_S \cong 120\ \mathrm{MeV \cdot fm^5}$ in Skyrme HF

Phenomenological W_0 and Scheerbaum factor



Implication for the spin-orbit field in the drip line region. Calculations in pure neutron matter.

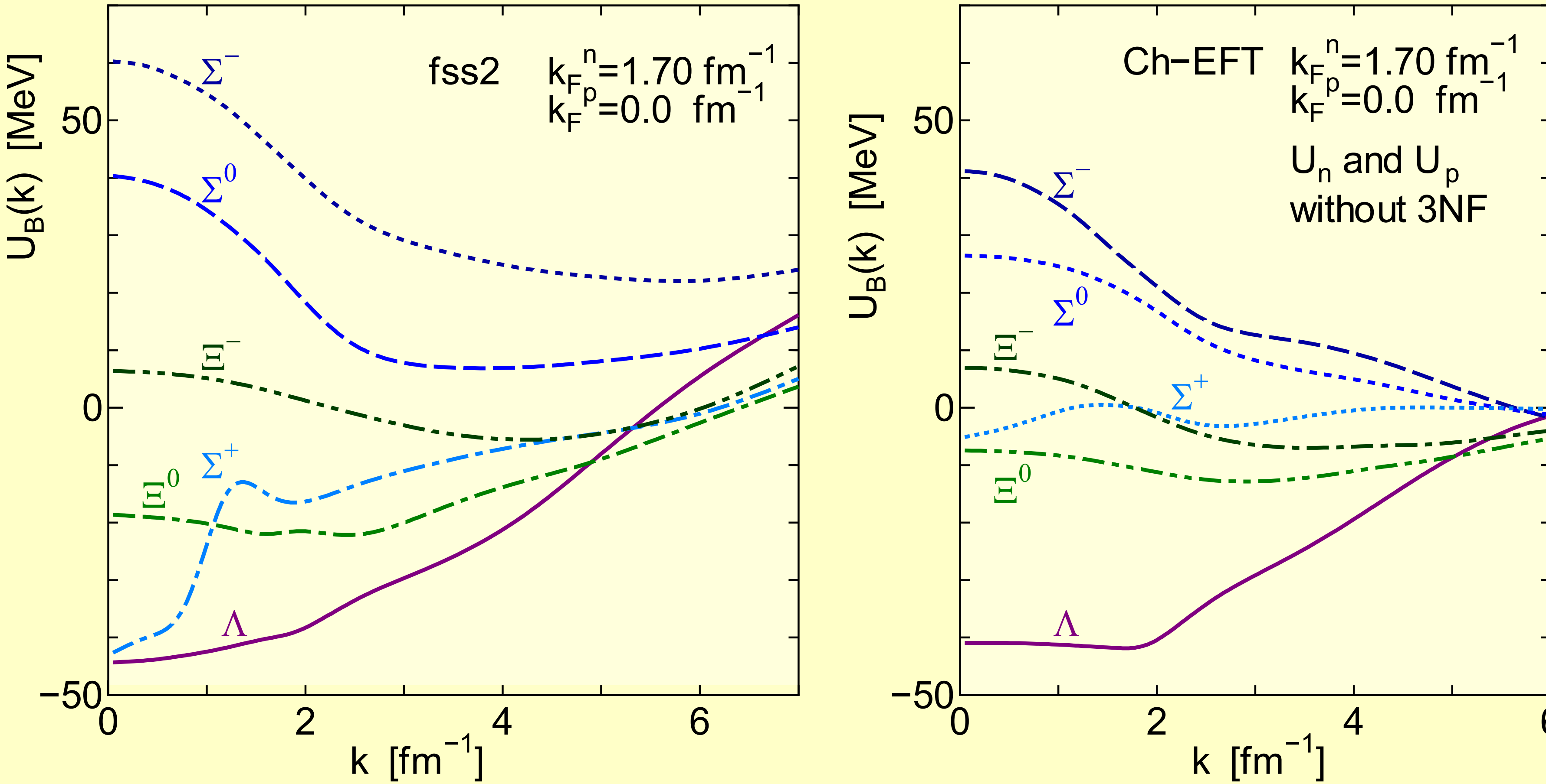
	Symmetric N.M.		Neutron matter	
$k_F = 1.35\ \mathrm{fm}^{-1}$	N3LO	N^3LO+3NF	N3LO	N^3LO+3NF
$B_S(T=0)$	2.5	7.0	-----	-----
$B_S(T=1)$	84.6	116.2	84.7	93.5
B_S	87.1	123.2	84.7	93.5

- Contributions from two-nucleon force change little.
- Contribution form reduced two-nucleon force is about $\frac{1}{3}$. ⇒ Spin-orbit potential in neutron excess nuclei is relatively weak.
  - Influence on the structure and scattering.

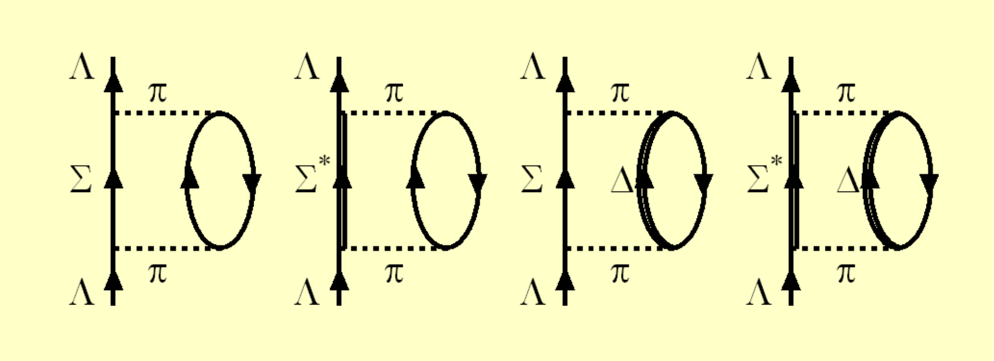
Hyperons in pure neutron matter

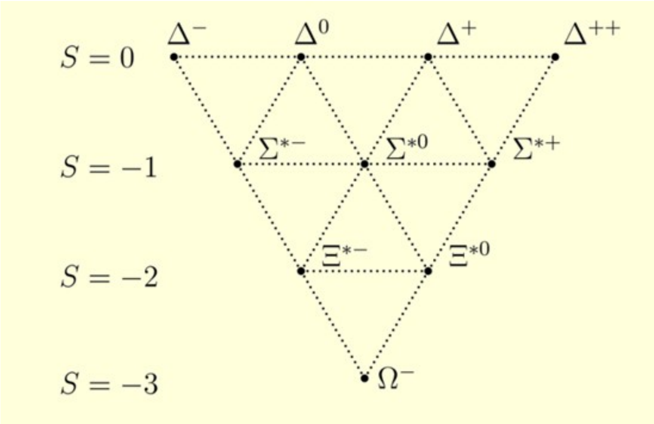
- Hyperons have been expected to emerge in high density neutron star matter to bypass the increase of neutron Fermi energy.
- Report of the measurement of 2 solar-mass neutron star. ➡ Sufficiently hard EoS is needed.
- Hyperons are not welcomed.
- Recent YN potentials（Kyoto-Niigata fss2 and Julich Ch-EFT）predict repulsive Σ^- and Ξ^- single-particle potentials in pure neutron matter.
- Λ s.p. potential is attractive at higher densities.
  - Λ should appear at around $3\rho_0$, as so far expected.
  - There should be repulsive effect. ➡ Consider $\Sigma^*(1385)$.

hyperon s.p. potentials in neutron matter at $k_F^n = 1.7$ fm^{-1} with fss2 and Ch-EFT potentials



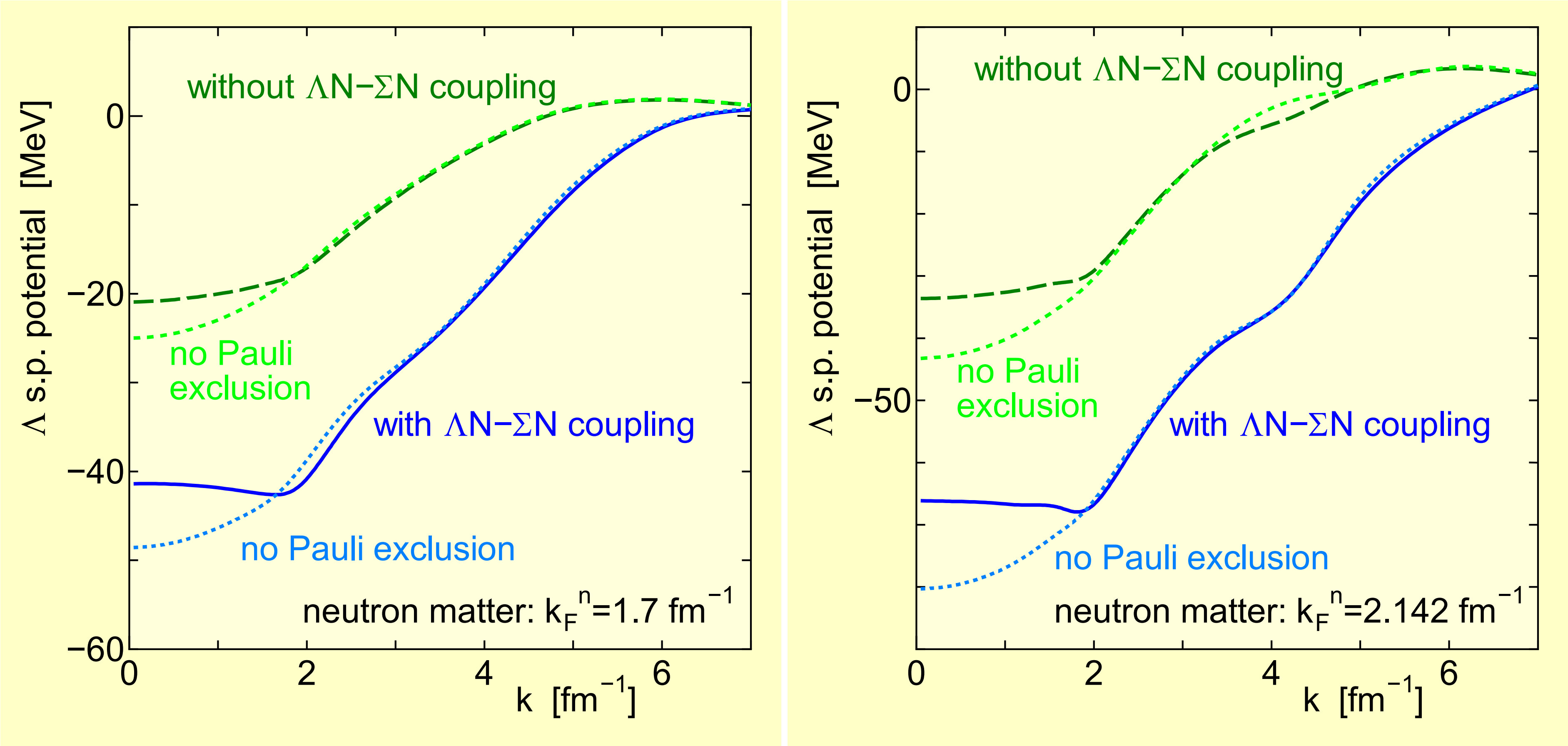
In NN case, the Pauli blocking of the Δ excitation gives repulsive contribution. There is an analogous decouplet baryon in the LN sector: $\Sigma^*(1385)$.





- The effects of the Σ^* excitation is implicitly included in determining parameters of LN interaction. The Pauli effect has to be taken into account, when the potential is used in nuclear matter.
- ΛN-ΣN coupling is taken into account in G-matrix calculation. The coupling effect can be estimated by switching on and off the coupling and Pauli exclusion.

Contributions of ΛN-ΣN coupling and Pauli exclusion in G-matrix calculations
(using Ch-EFT Baryon-Baryon interaction)

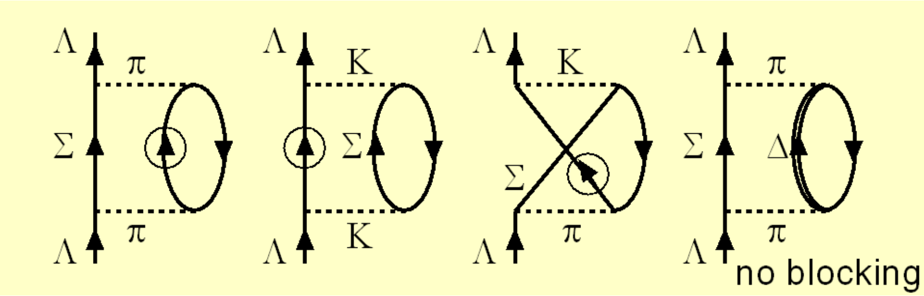


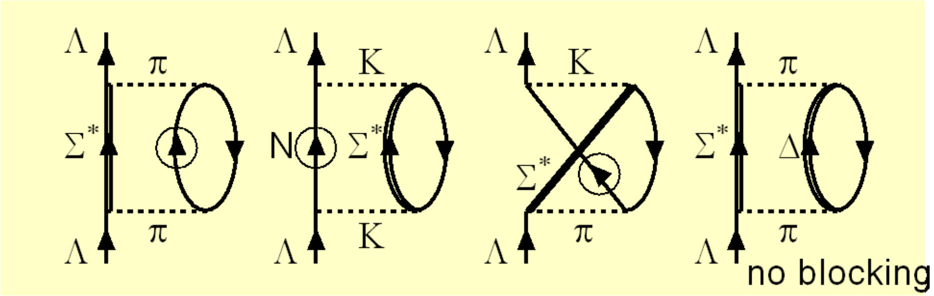
Pauli effect in ΛN-ΣN coupling: 3.1 MeV at $k_F^n = 1.7$ fm^{-1}
4.5 MeV at $k_F^n = 2.142$ fm^{-1}

Estimation of the effect of the Pauli blocking by the 2nd order diagrams. N, Λ, and Σ s.p. potentials are taken into account in the denominator（except for Σ^*）.

coupling constants used (vertex f.f. $e^{-(q/\Lambda)^2}$ with $\Lambda = 0.96$ GeV)

$g_{\pi NN} = 12.677$, $g_{\pi\Lambda\Sigma} = 12.677$, $g_{KN\Lambda} = -11.448$, $g_{KN\Sigma} = 0.7032$, $f_{KN\Sigma^*} = -3.22$, $f_{\pi\Lambda\Sigma^*} = 1.106$



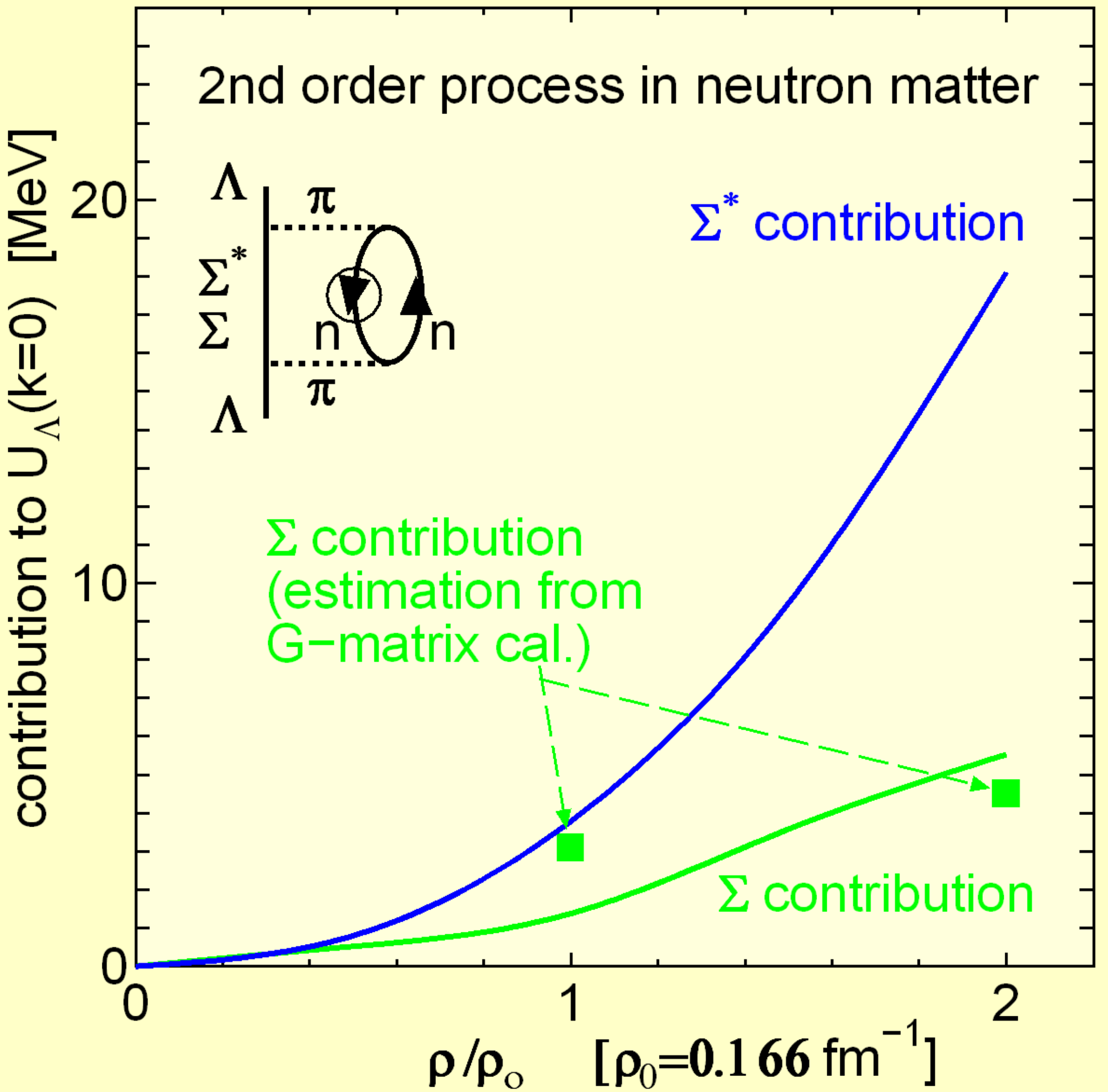


blocking of Σ excitation (MeV)			
$\rho_0/2$	ρ_0	$3\rho_0/2$	$2\rho_0$
+0.52	+1.83	+3.58	+5.53

blocking of Σ^* excitation (MeV)			
$\rho_0/2$	ρ_0	$3\rho_0/2$	$2\rho_0$
+0.80	+3.79	+9.48	+18.11

Included in G-matrix calculations

Not included in G-matrix cal.



2nd order process in neutron matter
Λ
π
Σ*
Σ
n
n
π
Λ
Σ* contribution
Σ contribution
(estimation from
G-matrix cal.)
Σ contribution
contribution to U_Λ(k=0) [MeV]
20
10
0
0
1
2
ρ/ρ_o [ρ_0=0.166 fm^-1]

Summary

- Ch-EFT 3NF (reduced 2N force) gives desirable contributions to the saturation properties and spin-orbit strength of nuclei.
  - The spin-orbit strength and the energy of neutron matter are determined by the low-energy constants which are fixed at the NN level.（The dependence on the form factor is present.)
- It is a future problem to consider the contribution of v_{123} together with many-body correlations.（The framework of the CCM is promising.）
- 3NF should be taken into account in studies of nuclear structures and reactions.
  - There may be effects beyond phenomenological consideration.
- ΛNN interaction (through Σ*) should be considered.