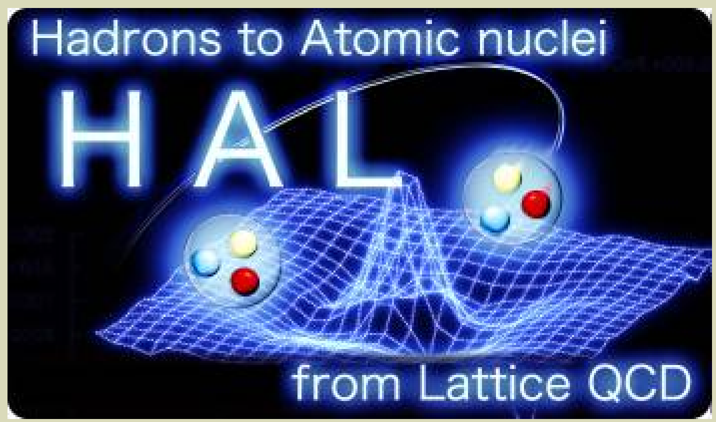
Hyperonic Interactions from Lattice QCD and toward an Application to Few-Body Problems

H. Nemura[1],
for HAL QCD Collaboration
S. Aoki[2], B. Charron[3], T. Doi[4],
T. Hatsuda[4], Y. Ikeda[4], T. Inoue[5], N. Ishii[1],
K. Murano[2], K. Sasaki[1], and M. Yamada[1],



[1]*Center for Computational Science, University of Tsukuba, Japan*
[2]*Yukawa Institute for Theoretical Physics, Kyoto University, Japan*
[3]*Department of Physics, University of Tokyo, Japan*
[4]*Theoretical Research Division, Nishina Center RIKEN, Japan*
[5]*College of Bioresouce Science, Nihon University, Japan*
[6]*Strangeness Nuclear Physics, Nishina Center RIKEN, Japan*

Plan of research

QCD

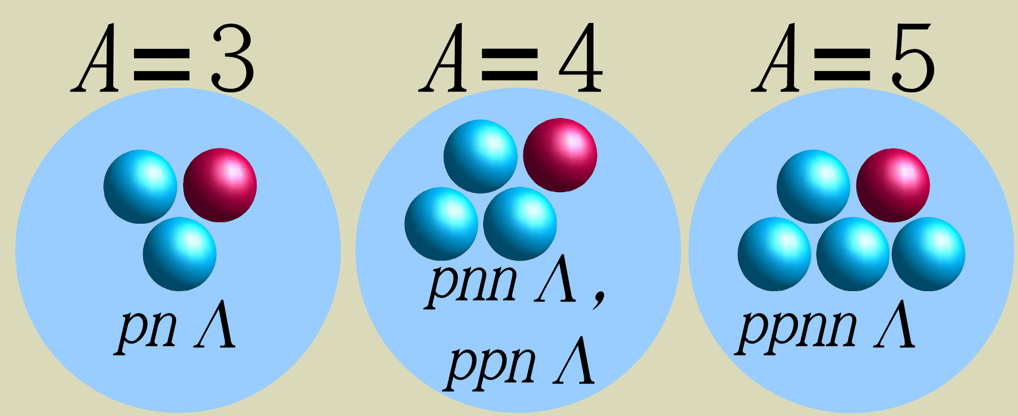
Baryon interaction

J-PARC
hyperon-nucleon (YN)
scattering

Structure and reaction of (hyper)nuclei

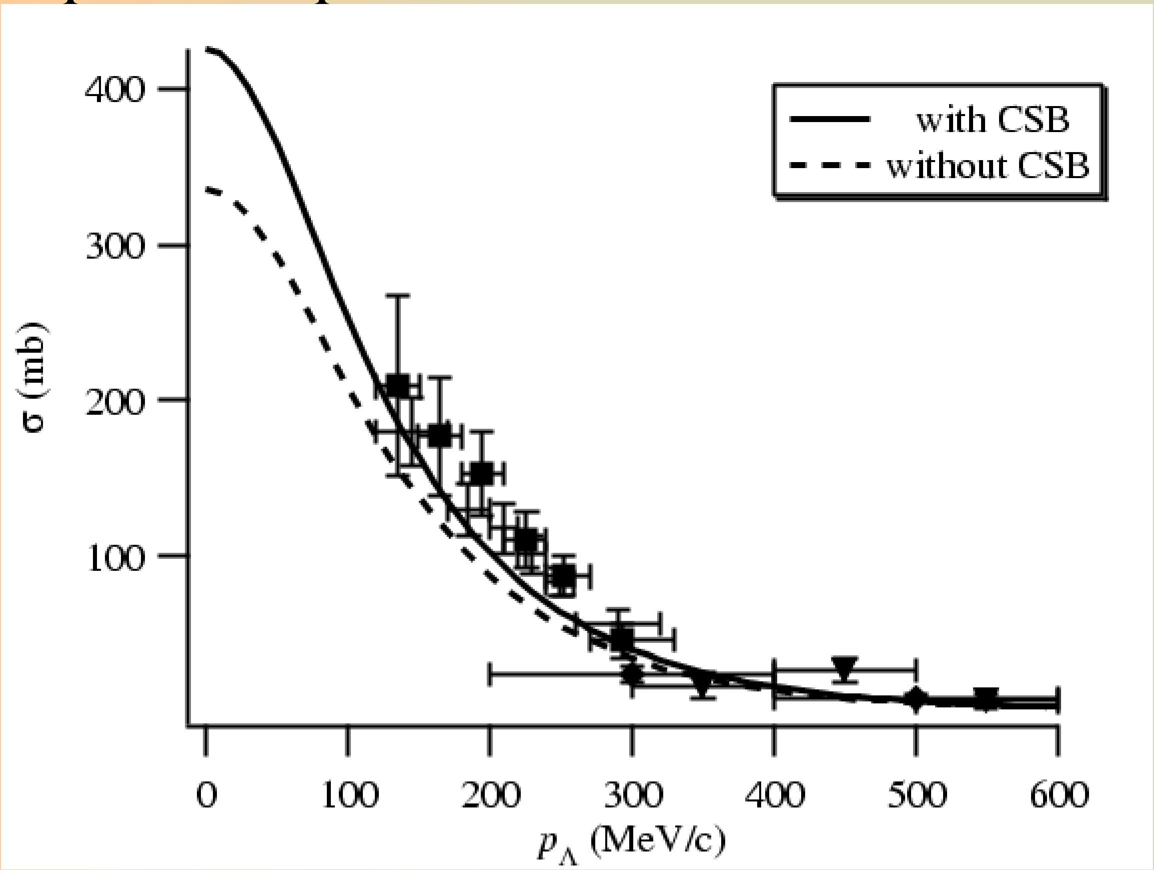
Equation of State (EoS) of nuclear matter

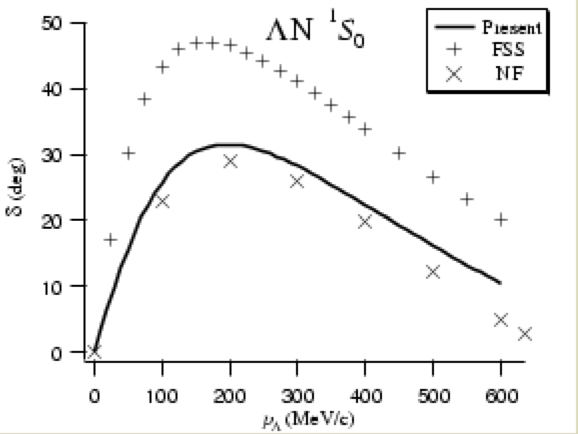
Neutron star and supernova

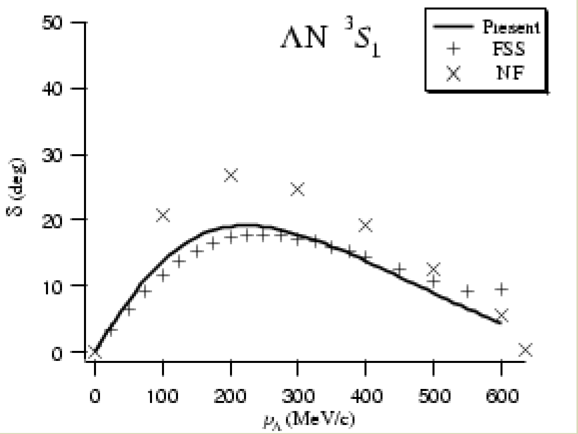


Experimental data for ΛN interaction:

- Only total cross section.
- No phase shift analysis is available.
- Spin-dependence is unclear







Outline

- Introduction
- Formulation --- potential (central + tensor)
- Numerical results:
  - $N\Lambda$ force (V_C + V_T)
  - $N\Sigma$ (I=3/2) force (V_C + V_T)
- Recent improvement for Vc and VT
- Stochastic variational calculation of 4He with using a lattice potential
- Summary and outlook

Introduction:

- Study of hyperon-nucleon (*YN*) and hyperon-hyperon (*YY*) interactions is one of the important subjects in the nuclear physics.
  - Structure of the neutron-star core,
    - Hyperon mixing, softning of EOS, inevitable strong repulsive force,
  - H-dibaryon problem,
    - To be, or not to be,
- The project at J-PARC:
  - Explore the multistrange world,
- However, the phenomenological description of *YN* and *YY* interactions has large uncertainties, which is in sharp contrast to the nice description of phenomenological *NN* potential.

The purposes of this work

- *NY* forces from lattice QCD

- Spin dependence
- Potential (central + tensor)

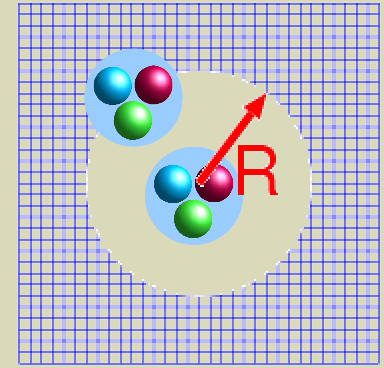
- Numerical calculation:
  - Full lattice QCD by using N_F=2+1 PACS-CS full QCD gauge configurations with the spatial lattice volume (2.86 fm)^3

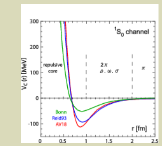
Formulation

i) basic procedure:

asymptotic region

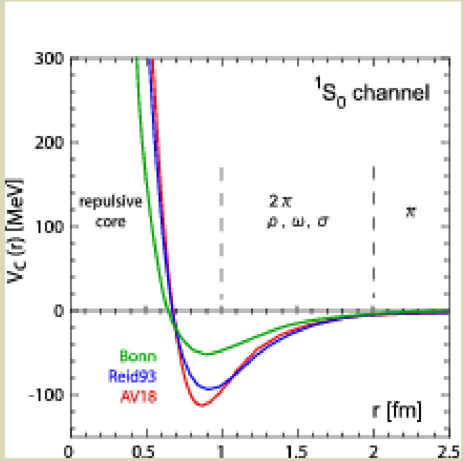
--> phase shift

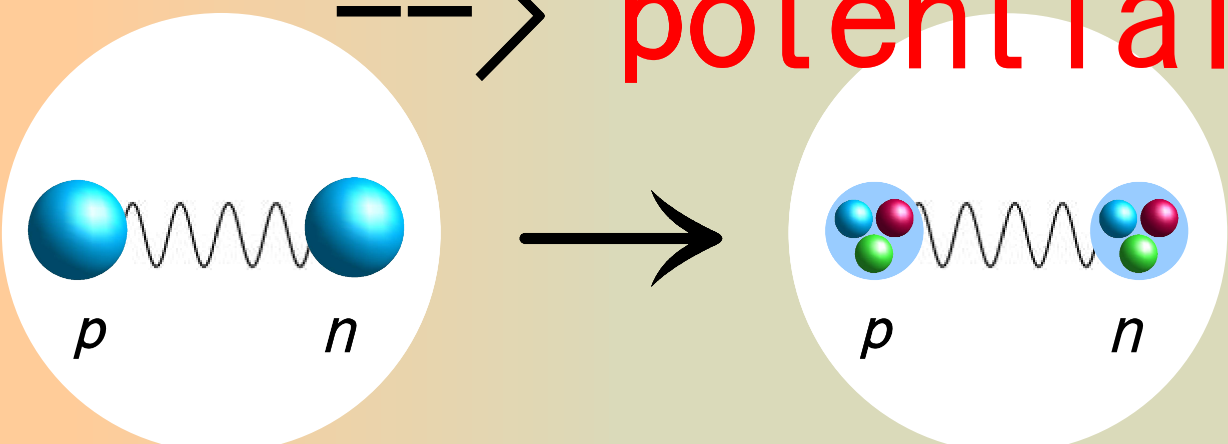




ii) advanced (HAL's) procedure: interacting region

--> potential





Formulation

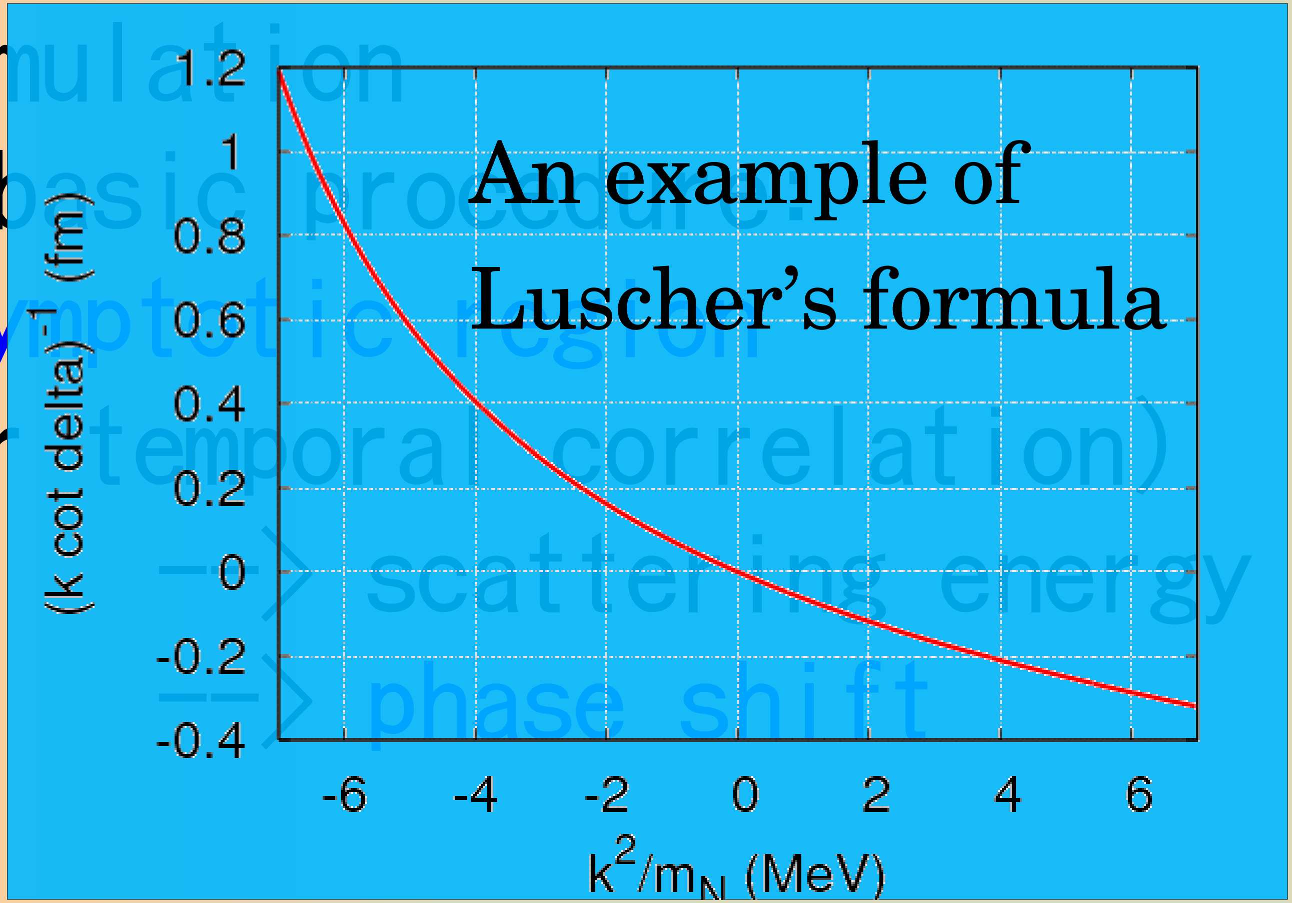
i) basic procedure:

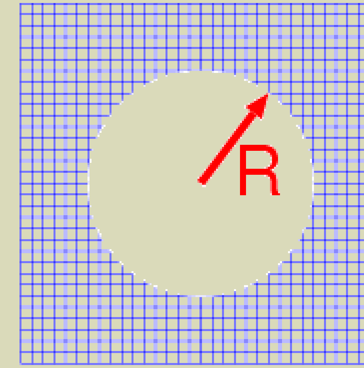
asymptotic region

(or temporal correlation)

--> scattering energy

--> phase shift





$$E = \frac{k^2}{2\mu}$$

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

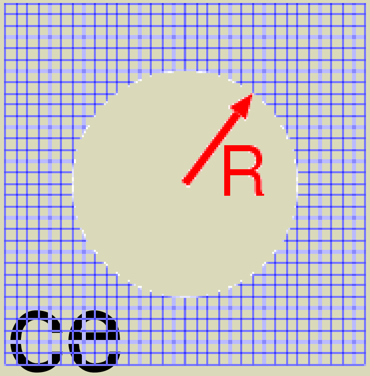
$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in Z^3} \frac{1}{(n^2 - q^2)^s} \qquad \Re\, s > \frac{3}{2}$$

Luscher, NPB354, 531 (1991).

Aoki, et al., PRD71, 094504 (2005).

HAL formulation

ii) advanced procedure:



make better use of the lattice output ! (wave function)

interacting region

--> potential

Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., arXiv:0805.2462[hep-ph].

NOTE:

> Potential is not a direct experimental observable.

> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

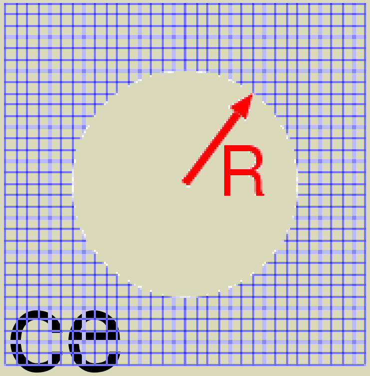
....

HAL formulation

ii) advanced procedure:

make better use of the lattice

output ! (wave function)



interacting region

--> potential

Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., arXiv:0805.2462[hep-ph].

> Phase shift
> Nuclear many-body problems

A recipe for NΛ potential:

- .

- The equal time BS wave function with angular momentum (J,M) on the lattice,

$$\phi^{(JM)}_{\alpha\beta}(\vec{r}) = \sum_{\vec{x}} \left\langle 0 \left| p_\alpha(\vec{r}+\vec{x})\,\Lambda_\beta(\vec{x}) \right| p\Lambda\ ;k\ ,JM \right\rangle$$

$$p_\alpha(x) = \varepsilon_{abc}(u_a(x)\,C\gamma_5\,d_b(x))\,u_{c\alpha}(x)\ ,$$

$$\Lambda_\alpha(x) = \varepsilon_{abc}\left\{(d_a C\gamma_5 s_b)\,u_{c\alpha} + (s_a C\gamma_5 u_b)\,d_{c\alpha} - 2(u_a C\gamma_5 d_b)\,s_{c\alpha}\right\}$$

- The 4-point $N\Lambda$ correlator on the lattice,

$$F^{(JM)}_{\alpha\beta}(\vec{x},\vec{y},t-t_0) = \left\langle 0 \left| p_\alpha(\vec{x},t)\,\Lambda_\beta(\vec{y},t)\,\overline{\Theta^{(JM)}_{p\Lambda}}(t_0) \right| 0 \right\rangle$$

$$= \sum_n A^{(JM)}_n \left\langle 0 \left| p_\alpha(\vec{x})\,\Lambda_\beta(\vec{y}) \right| E_n \right\rangle e^{-E_n(t-t_0)}$$

$\overline{\Theta^{(JM)}_{p\Lambda}}(t_0)$ wall source at $t = t_0$

An improved recipe for lattice potential:

- cf. Ishii (HAL QCD), PLB712 (2012) 437.

- Take account of the temporal correlation as well as the spatial correlation of the NBS amplitude in terms of the R-correlator:

$$R(t,\vec{r}) = \frac{C_{YN}(t,\vec{r})}{C_Y(t)C_N(t)}$$

$$R(t+\Delta t,\vec{r}) = \mathrm{e}^{-\Delta t H} R(t,\vec{r})$$
$$= (1-\Delta t H) R(t,\vec{r})$$

- Time-dependent effective Schroedinger eq.:

$$-\frac{\partial}{\partial t} R(t,\vec{r}) = H R(t,\vec{r})$$

An improved recipe for NY potential:

- cf. Ishii (HAL QCD), PLB712 (2012) 437.

- Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu}\nabla^2 R(t,\vec{r})+\int d^3 r' \, U(\vec{r},\vec{r}')R(t,\vec{r}')=-\frac{\partial}{\partial t}R(t,\vec{r})$$

$$\rightarrow \frac{k^2}{2\mu}R(t,\vec{r})$$

$$U(\vec{r},\vec{r}')=V_{NY}(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

- A general expression of the potential:

$$V_{NY}=V_0(r)+V_\sigma(r)(\vec{\sigma}_N\cdot\vec{\sigma}_Y)$$
$$+V_T(r)S_{12}+V_{LS}(r)(\vec{L}\cdot\vec{S}_+)$$
$$+V_{ALS}(r)(\vec{L}\cdot\vec{S}_-)+O(\nabla^2)$$

A recipe for NΛ potential:

- cf. Ishii (HAL QCD), PLB712 (2012) 437.

- Effective central potential is obtained from the effective Schroedinger equation.

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2+V(r)\right)R(t,\vec{r})=-\frac{\partial}{\partial t}R(t,\vec{r})$$

➔

$$V(r)=\frac{-\frac{\partial}{\partial t}R(t,\vec{r})}{R(t,\vec{r})}+\frac{\hbar^2}{2\mu}\frac{\nabla^2 R(t,\vec{r})}{R(t,\vec{r})}$$

A recipe for NY potential: (contd.)

- For $J = 1$, ϕ comprises *S*-wave and *D*-wave,

$$| \phi \rangle = | \phi_S \rangle + | \phi_D \rangle$$

where,

$$| \phi_S \rangle = \mathcal{P} | \phi \rangle = (1/24) \sum_{\mathcal{R} \in O} \mathcal{R} | \phi \rangle$$

$$| \phi_D \rangle = Q | \phi \rangle = (1 - \mathcal{P}) | \phi \rangle$$

- Therefore, we have 2-component Schrödinger eq.

S-wave:

$$\mathcal{P} (T + V_C + V_T S_{12}) |\phi\rangle = - \partial / \partial t \, \mathcal{P} |\phi\rangle$$

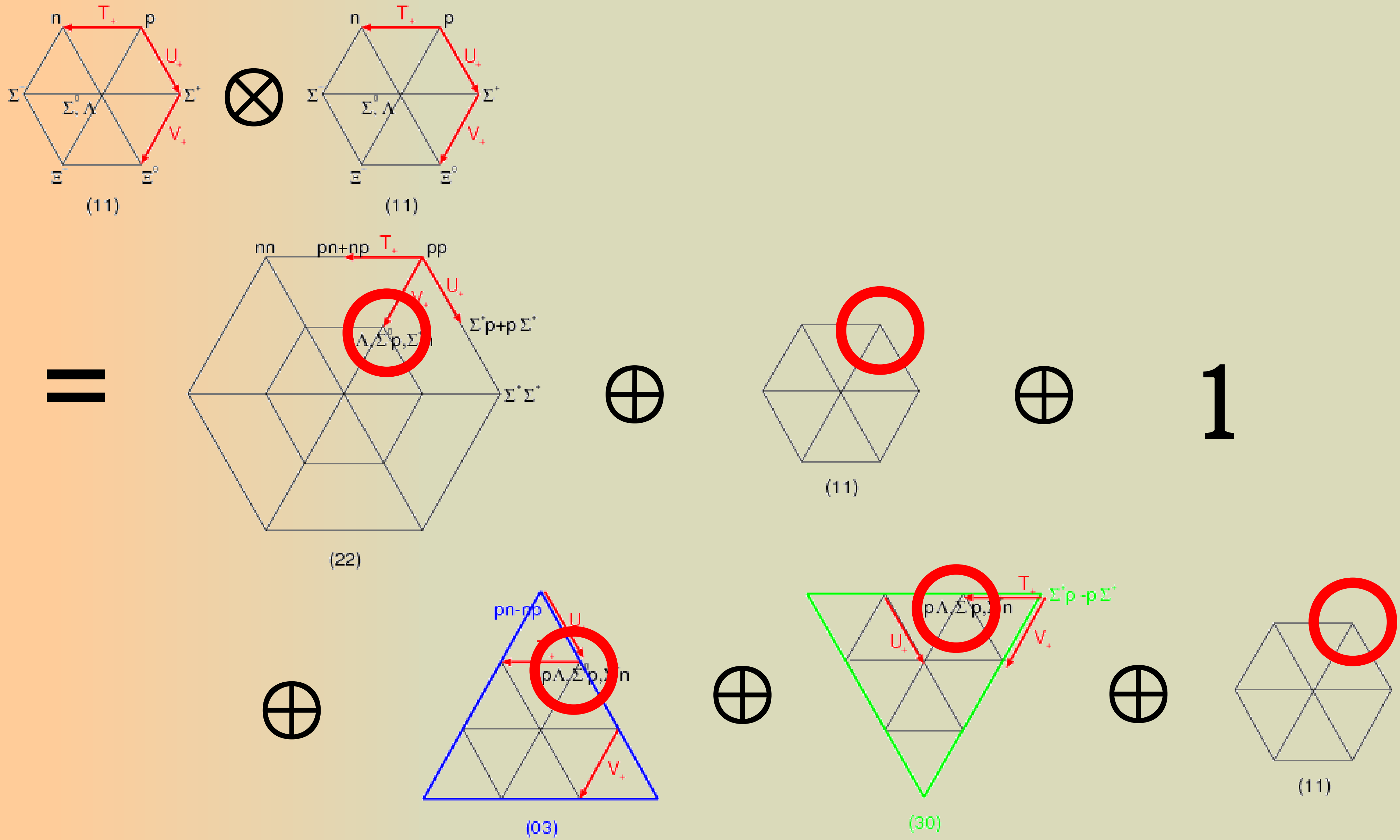
D-wave:

$$Q (T + V_C + V_T S_{12}) |\phi\rangle = - \partial / \partial t \, Q |\phi\rangle$$

- Obtain the $V_C(r)$ and the $V_T(r)$ simultaneously.

Numerical results

$$8 \otimes 8 = 27 \oplus 8_s \oplus 1 \oplus \overline{10} \oplus 10 \oplus 8_a$$



	1S0	3S1-3D1
ΛN	{27}+{8s}	{8a}+{10*}
ΣN(I=1/2)	{27}+{8s}	{8a}+{10*}
ΣN(I=3/2)	{27}	{10}

$$\left\{(\Sigma N)_{(I,I_z)=(\frac{1}{2},\frac{1}{2})} - 3p\Lambda\right\}_{\text{symmetric}} \in \{\mathbf{27}\}. \qquad \left\{(\Sigma N)_{(I,I_z)=(\frac{1}{2},\frac{1}{2})} - p\Lambda\right\}_{\text{antisymmetric}} \in \{\overline{\mathbf{10}}\}.$$

$$\left\{3\,(\Sigma N)_{(I,I_z)=(\frac{1}{2},\frac{1}{2})} + p\Lambda\right\}_{\text{symmetric}} \in \{\mathbf{8}_s\}. \qquad \left\{(\Sigma N)_{(I,I_z)=(\frac{1}{2},\frac{1}{2})} + p\Lambda\right\}_{\text{antisymmetric}} \in \{\mathbf{8}_a\}.$$

$$\left\{(\Sigma N)_{(I,I_z)=(\frac{3}{2},\frac{1}{2})}\right\}_{\text{symmetric}} \in \{\mathbf{27}\}. \qquad \left\{(\Sigma N)_{(I,I_z)=(\frac{3}{2},\frac{1}{2})}\right\}_{\text{antisymmetric}} \in \{\mathbf{10}\}.$$

Full QCD calculations by using N_F=2+1 PACS-CS gauge configurations:

- S. Aoki, et al., (PACS-CS Collaboration), PRD**79**, 034503 (2009), arXiv:0807.1661 [hep-lat].
- Iwasaki gauge action at β=1.90 on $32^3 \times 64$ lattice
- O(a) improved Wilson quark action
- $1/a$ = 2.17 GeV (a = 0.0907 fm)

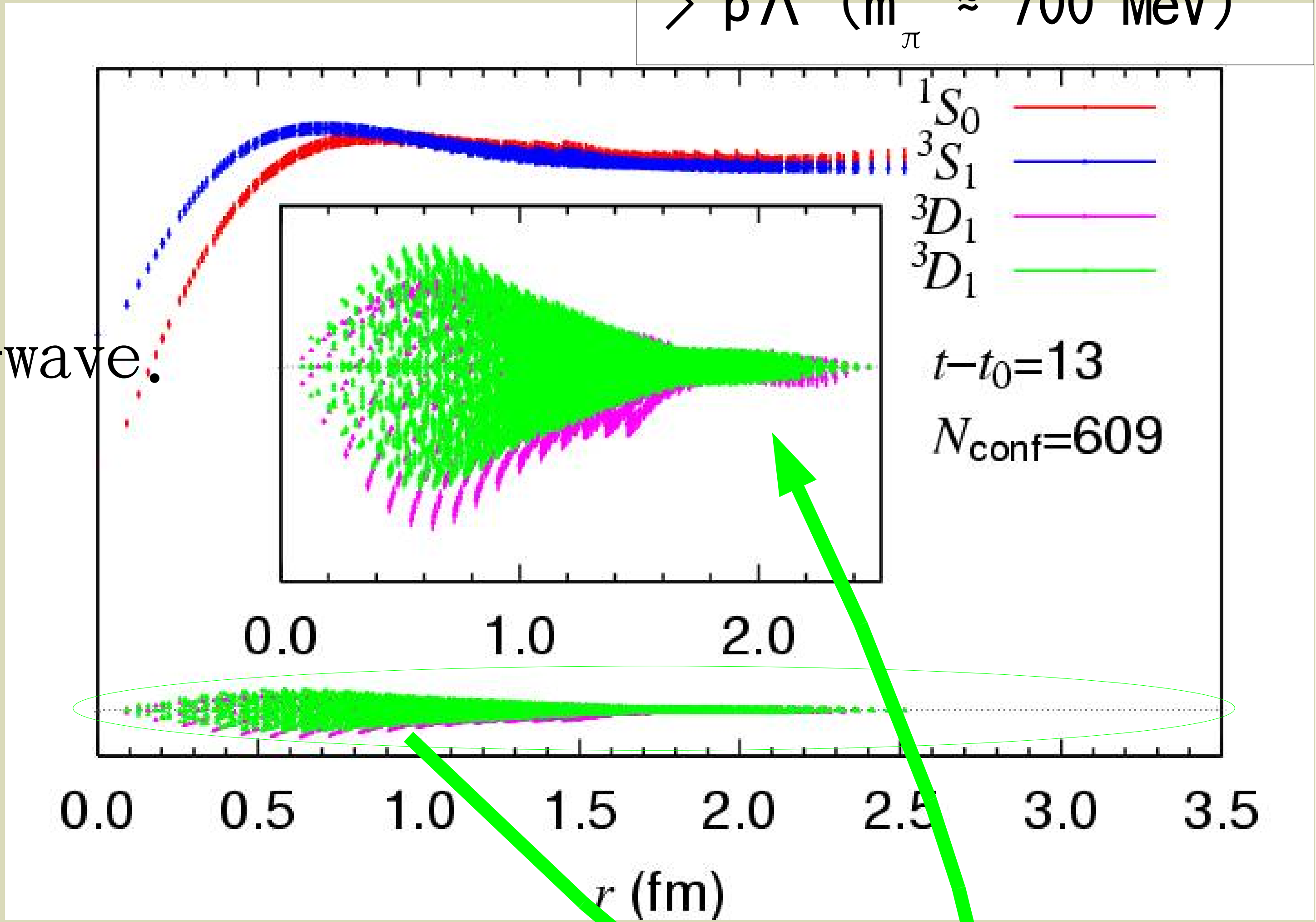
$(\kappa_{ud})_{N_{\rm conf}}$	m_π	m_ρ	m_K	m_{K^*}	m_N	m_Λ	m_Σ	m_Ξ
2+1 flavor QCD by PACS-CS with κ_s = 0.13640 @present calc (Dirichlet BC along T)								
$(0.13700)_{609}$	700.0(4)	1108(3)	785.8(3)	1159(2)	1573(4)	1632(4)	1650(5)	1700(4)
Exp.	135	770	494	892	940	1116	1190	1320

ΛN potential

Results --- wave function

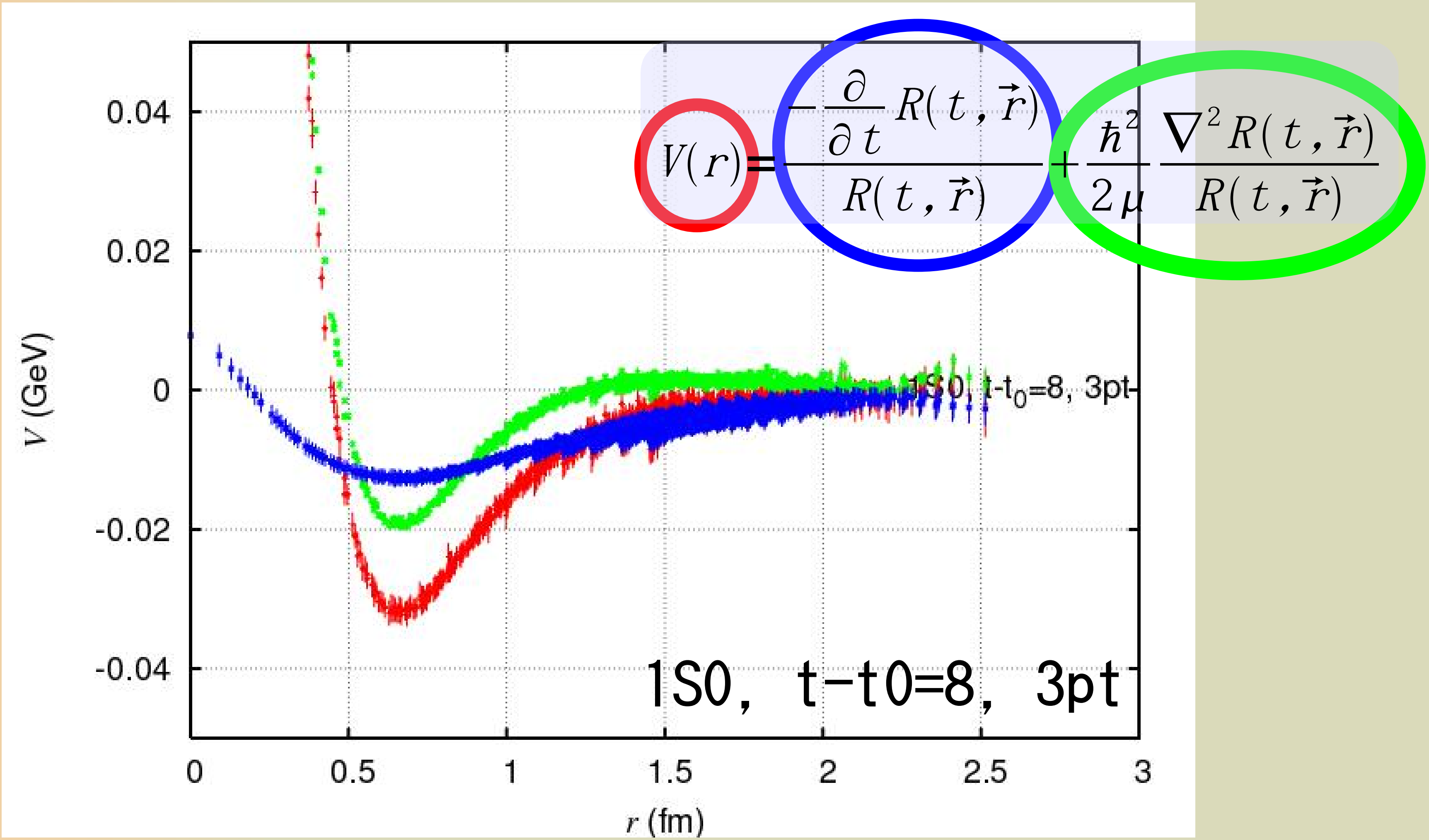
- $J = 0$:
  - S–wave.
- $J = 1$:
  - S– and D–wave.

> pΛ ($m_\pi \approx 700$ MeV)



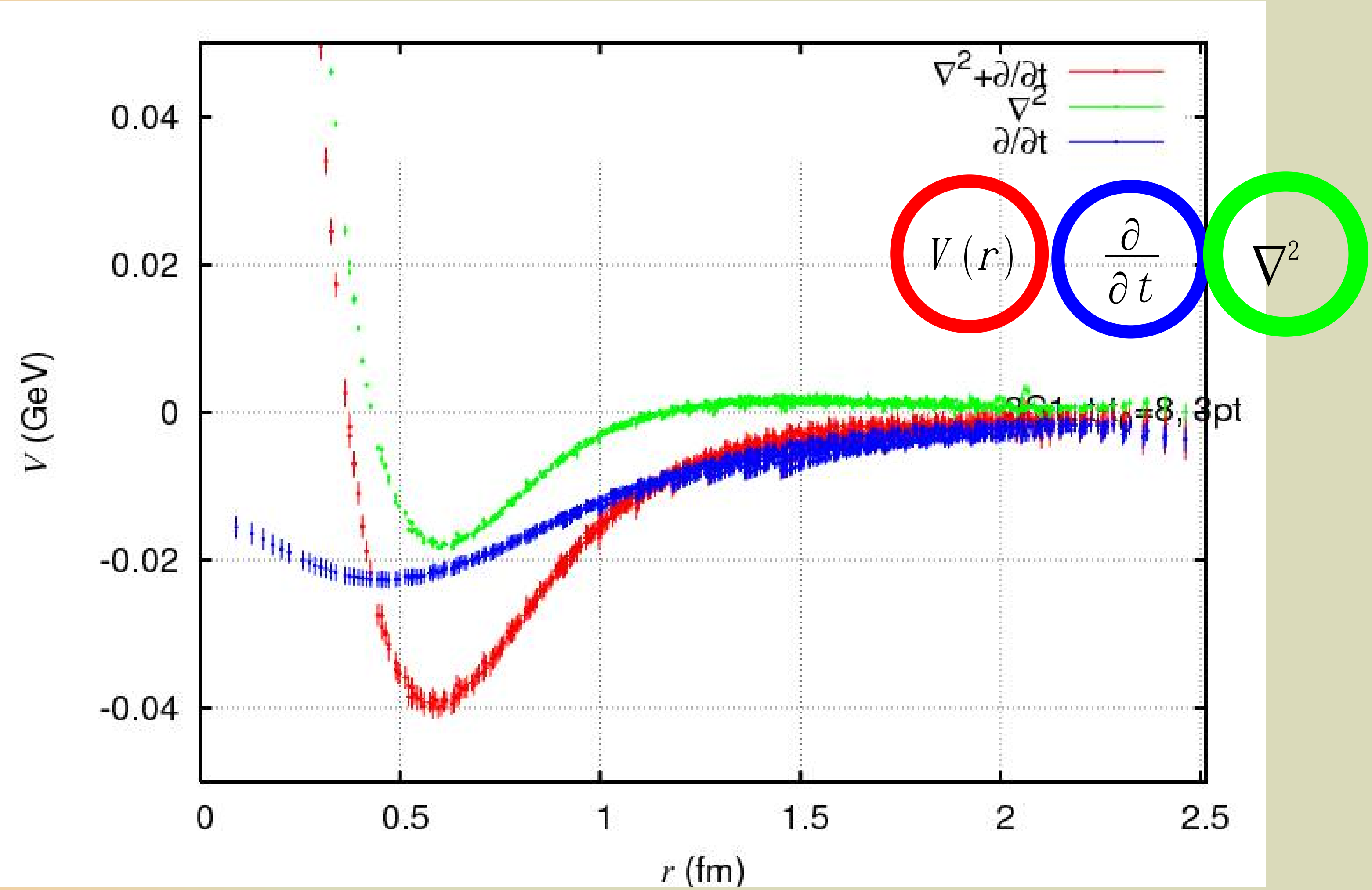
$$\phi_D(\vec{r}) = f(r)\left[Y_2(\hat{r})\times\chi_1\right]_{J=1,\,M}$$

$V_C(\Lambda N;\ 1S0)$



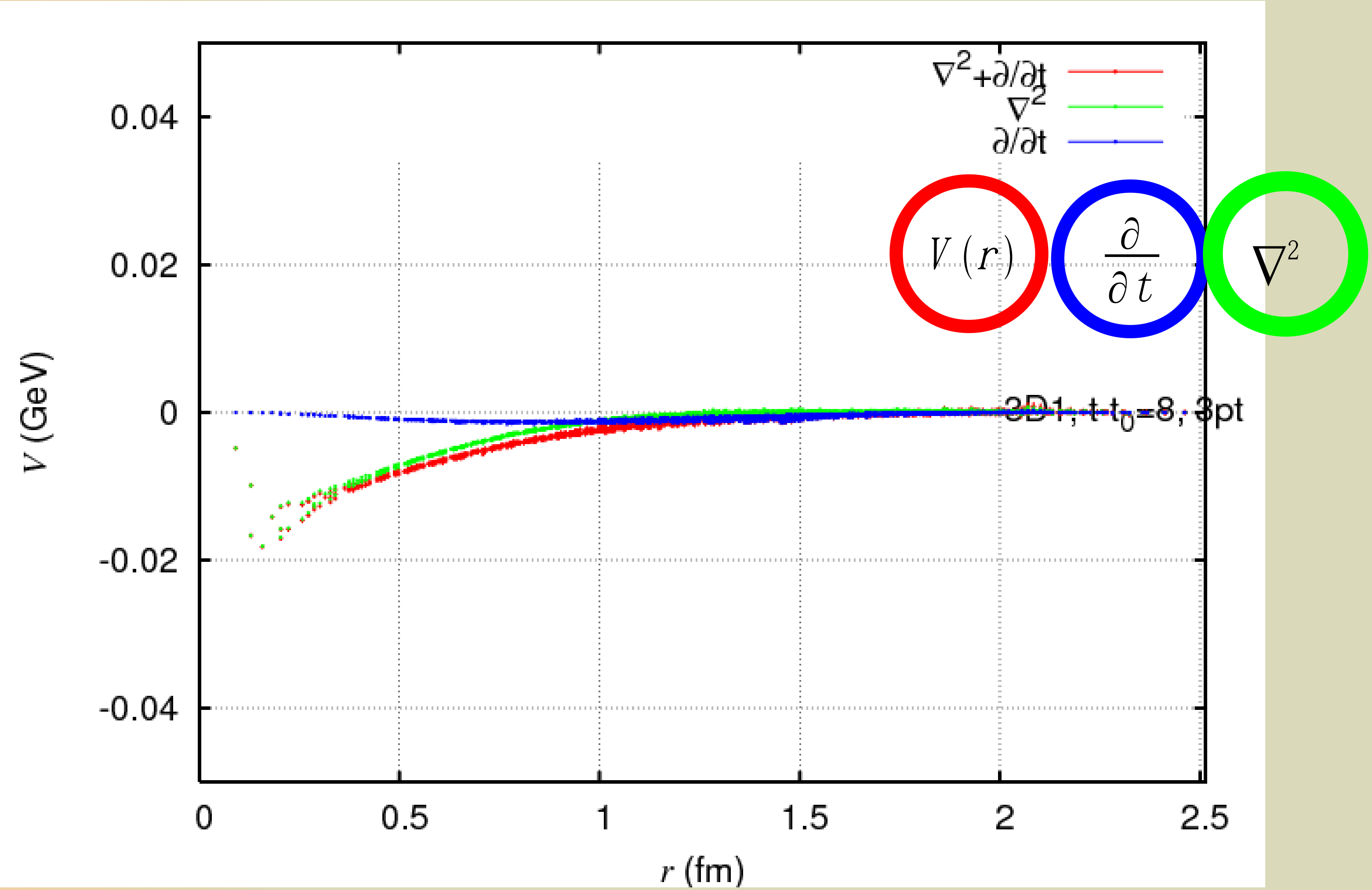
- {27}+{8s}
- Similar to NN (1S0)
- Sizable contribution from time-derivative part

$V_c(\Lambda N;\ 3S1\text{-}3D1)$



- {10*}+{8a}
- Sizable attractive contribution from time-derivative part

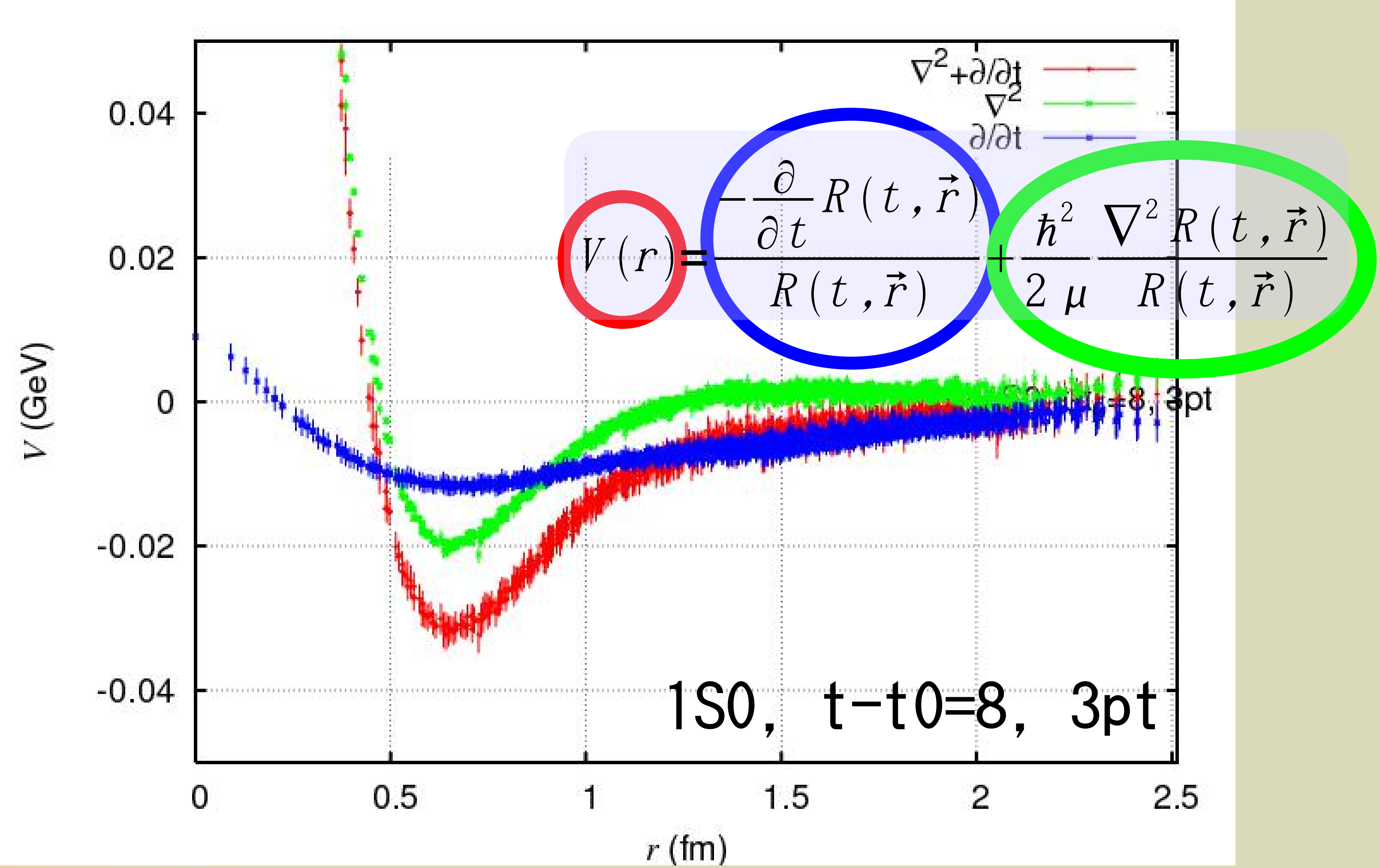
$V_T(\Lambda N;\ 3S1\text{-}3D1)$



- Weaker tensor force than NN
- Small contribution from time-derivative part

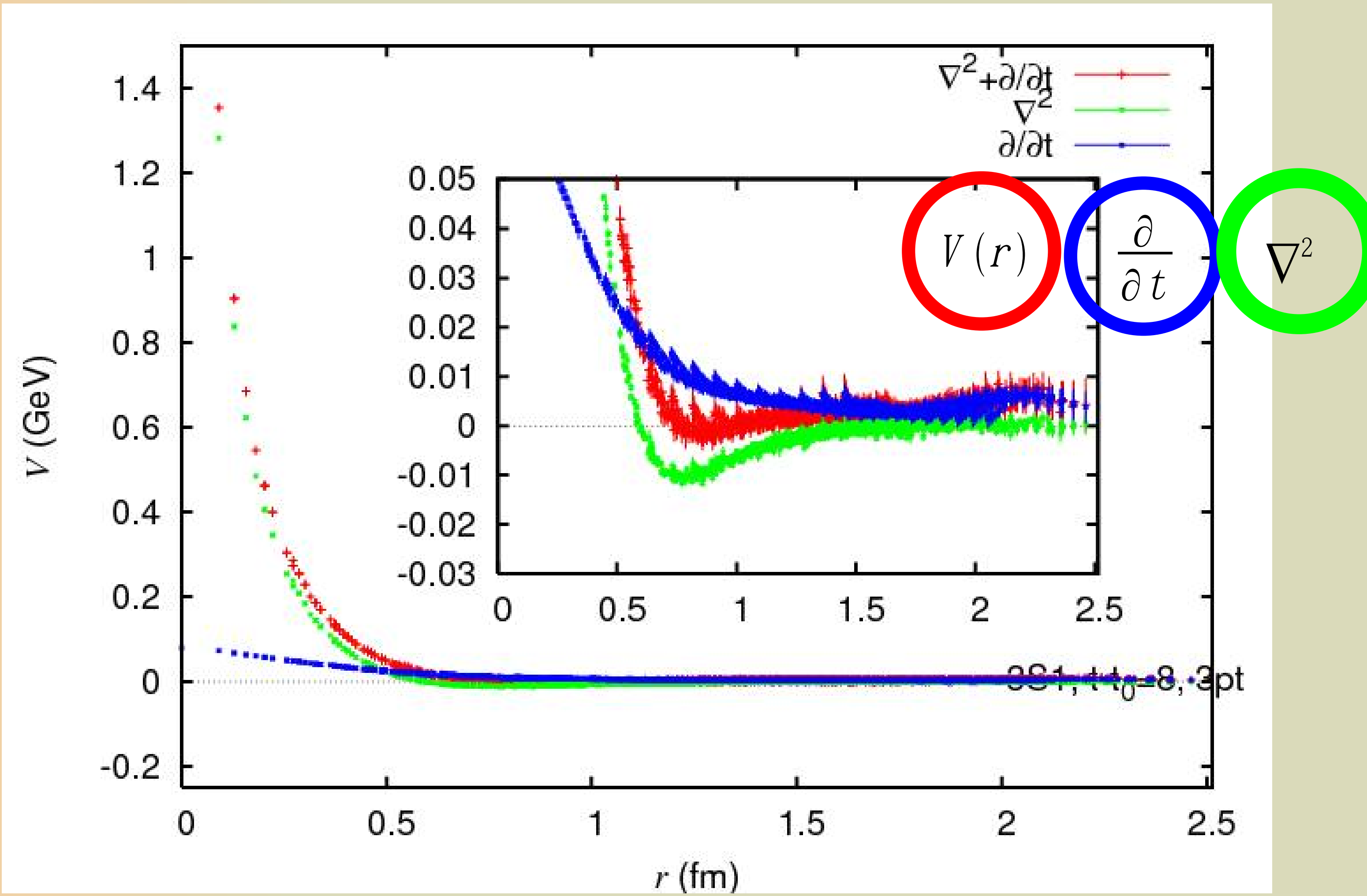
$\Sigma N(I=3/2)$ potential

$V_C(\Sigma N(I=3/2);\ 1S0)$



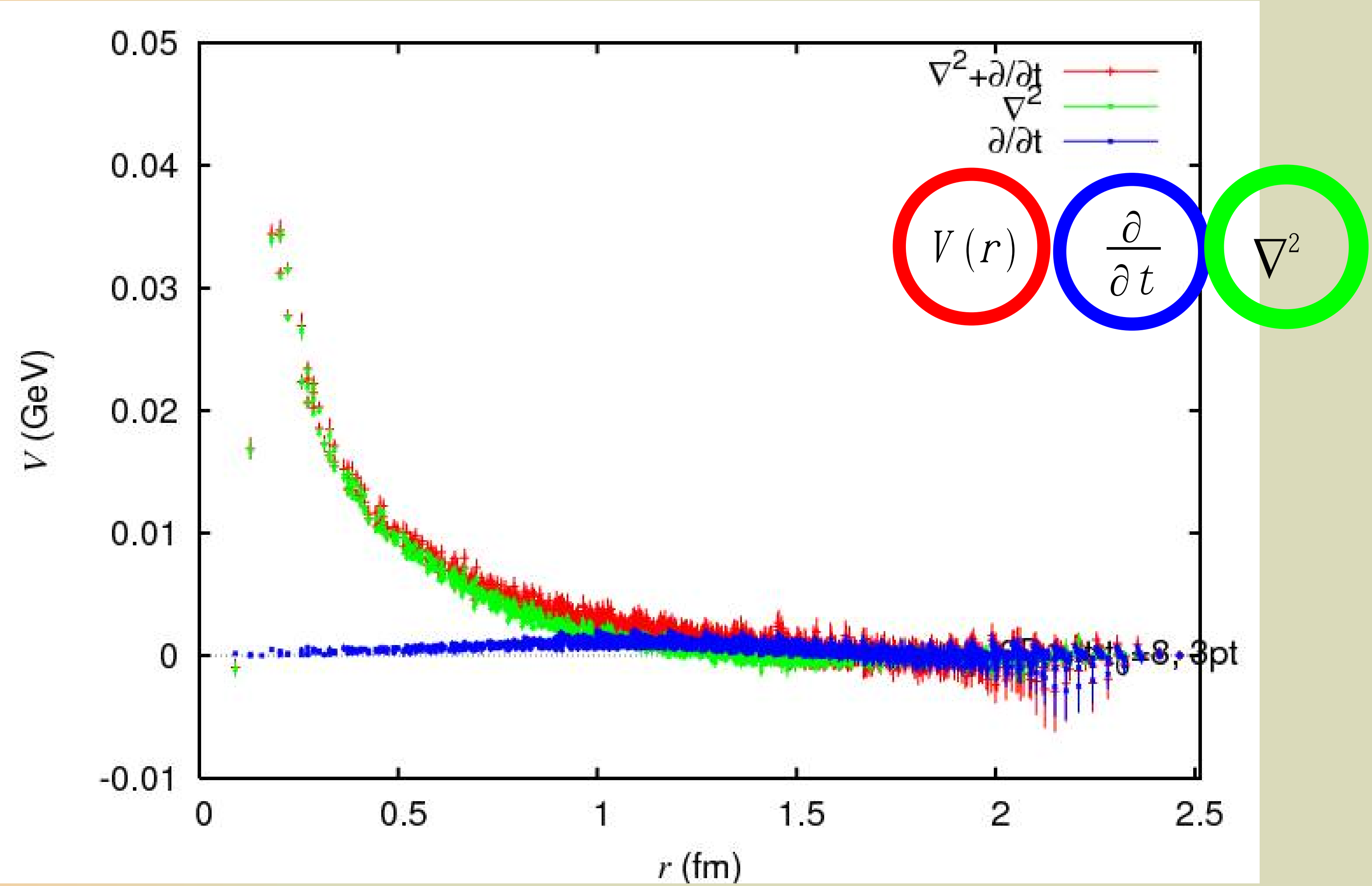
- {27}
- Similar to NN (1S0) (as well as Lambda-N (1S0))
- Sizable contribution from time-derivative part

$V_C(\Sigma N(I=3/2);\ 3S1-3D1)$



- {10}
- Repulsive potential (consistent with quark model)
- sizable repulsive contribution from time-derivative part

$V_T(\Sigma N(I=3/2);\ 3S1-3D1)$



- Weak tensor force
- Small contribution from time-derivative part

Scattering phase shifts

Proton-Lambda scattering (preliminary)

Parametrized potential → Phase shift

Summary:

- The lattice QCD study for Lambda-Nucleon and Sigma-nucleon(I=3/2) interactions.
- pΛ:
  - Central, tensor. For full QCD
  - Time-derivative terms enhance the attractive force.
  - Qualitatively similar to well-known nuclear forces.
    - Repulsive at short distance.
    - Attractive well at medium to long distance.
- NΣ(I=3/2):
  - Central, tensor. For full QCD
  - The 1S0 potential is similar to Lambda-N potential
  - The 3S1 potential is repulsive

Outlook:

- Quark mass dependence.
- Scattering lengths.
    - spin-dependence.
    - Comparison with the hypernuclear data.
- Coupled-channel potential.

- Application to nuclear physics (few-body systems)

Stochastic variational calculation of 4He with using a lattice potential

- For NN potential, we use Inoue-san’ s SU(3) potential at the lightest quark mass(m_ps = 469 MeV), which has been reported to have a 4N bound state (about 5.1MeV) within a tensor-included effective central potential; NPA881, 28-43 (2011).

Stochastic variational calculation of 4He with using a lattice potential

The wave function of A-body system is described by a linear combination of basis functions as

$$\Psi = \sum_{k=1}^{K} c_k \varphi_k, \qquad \text{with} \qquad \varphi_k = \mathcal{A}\{G(\mathbf{x}; A_k)[\theta_{(LL')_k}(\mathbf{x}; (uu')_k), \chi_{S_k}]_{JM}\eta_{kIM_I}\}, \tag{11}$$

where c_k is the linear variational parameter determined by the variational principle, $\mathcal{A}$ is antisymmetrizer for identical particles. χ_{S_k} (η_{kIM_I}) is the spin (isospin) function of the system. $G(\mathbf{x}; A_k)$ is the correlated Gaussian function which is given by

$$G(\mathbf{x}; A_k) = \exp\left\{-\frac{1}{2}\sum_{i<j}^{A} \alpha_{kij}(\mathbf{r}_i - \mathbf{r}_j)^2\right\} = \exp\left\{-\frac{1}{2}\sum_{i,j=1}^{A-1} A_{kij}\mathbf{x}_i \cdot \mathbf{x}_j\right\}. \tag{12}$$

Stochastic variational calculation of 4He with using a lattice potential

A set of relative coordinates $\{\mathbf{x}_1, \cdots, \mathbf{x}_{A-1}\}$ and the center-of-mass coordinate $\mathbf{x}_A$ are given by a linear transformation of single particle coordinates $\{\mathbf{r}_1, \cdots, \mathbf{r}_A\}$ such as

$$\mathbf{x}_i = \sum_{j=1}^{A} U_{ij}\mathbf{r}_j, \qquad (i = 1, \cdots, A). \tag{13}$$

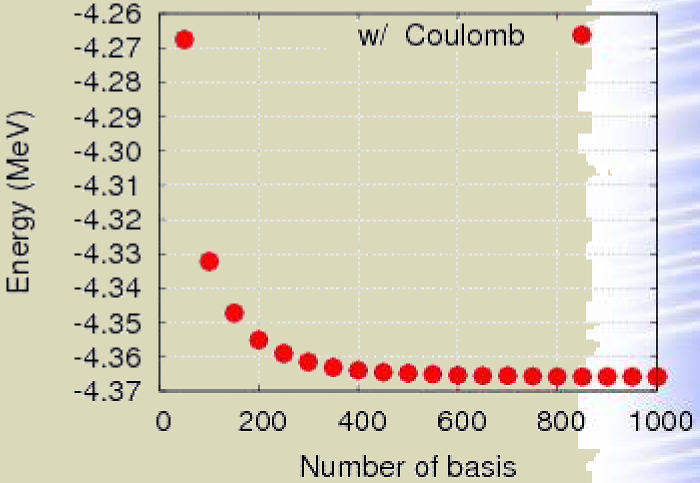
In order to obtain the accurate solution of the four-nucleon bound state with explicitly utilizing the the tensor potential, we consider nonzero orbital angular momentum states $(L, S)J^\pi = (1, 1)0^+$ and $(2, 2)0^+$ in addition to the $(0, 0)0^+$ configuration. We employ the global vector representation[11] for these nonzero orbital angular momentum states. Therefore, the angular part of the basis function is given by

$$\theta_{(LL')_k}(\mathbf{x}; (uu')_k) = v_k^{L_k} {v'_k}^{L'_k} [Y_{L_k}(\hat{\mathbf{v}}_k) \times Y_{L'_k}(\hat{\mathbf{v}}'_k)]_{L_k}, \qquad \begin{pmatrix} \mathbf{v} \\ \mathbf{v}' \end{pmatrix}_k = \sum_{i=1}^{A-1} \mathbf{x}_i \begin{pmatrix} u \\ u' \end{pmatrix}_{ki}. \tag{14}$$

The validity of the present choice of basis function is examined for several realistic NN potentials[11]. The A_{kij} and $(u, u')_{ki}$ are the nonlinear variational parameters which are determined by the stochastic variational method[12].

Results of few-body calculation

- **Inputs:**
  - m=1161.0 MeV,
  - hbar c = 197.3269602 MeV fm
  - hbar c/e^2 = 137.03599976
  - V_NN is treated as a Serber-type potential.



- **Results:**
  - B(4He)=4.37 MeV (w/ Coulomb)
    - Probabilities of (S, P, D) waves = (98.6%, 0.003%, 1.3%)
    - cf. roughly speaking (S,P,D)~(<90%, <0.1%, >10%) for a realistic NN force
  - B(4He)=5.09 MeV (w/o Coulomb)
    - Probabilities of (S, P, D) waves = (98.6%, 0.003%, 1.4%)

Plan of research

QCD

Physical point calculation with large volume ~(9fm)^3

Baryon interaction

J-PARC
hyperon-nucleon (YN) scattering

Structure and reaction of (hyper)nuclei

Equation of State (EoS) of nuclear matter

Neutron star and supernova