RIビームによる核反応と核応答

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  - Asymptotic Normalization Constant and Spectroscopic Factor
  - Examples
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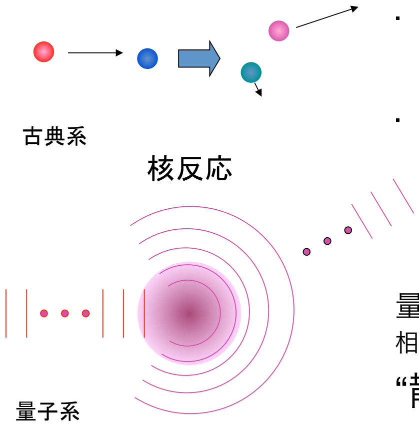
  - Exothermic reaction
  - Transferring various quantum numbers

核反応測定



核反応の測定

- 量子(ミクロ)系の情報を、古典(マクロ)系における物理量－エネルギー・運動量－から得る
- 反応確率を、運動量とエネルギーの関数としてみる

量子系と古典系を結ぶ物理量:
相対波動関数の自由空間での漸近形

"散乱(反応)の量子論"

核構造と核反応

量子多体系の"波動関数"の性質

- 密度分布、形、配位、相互作用、相関、集団性、応答…

核構造モデル

- 座標空間・配位空間における モデル波動関数

核反応論(モデル)

相対波動関数の自由空間での漸近形を反応に関与する相互作用と核構造から求め、反応で得られる物理量を得る

運動量空間における波動関数

座標空間と運動量空間

$$\Phi(\vec{p}_1,\vec{p}_2,\cdots,\vec{p}_A)=\prod_i\left(\frac{1}{2\pi\hbar}\right)^{3/2}\int d^3r_i\exp\left(\frac{i\vec{p}_i\cdot\vec{r}_i}{\hbar}\right)\Psi(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A)$$

$$\Psi(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A)=\prod_i\left(\frac{1}{2\pi\hbar}\right)^{3/2}\int d^3p_i\exp\left(-\frac{i\vec{p}_i\cdot\vec{r}_i}{\hbar}\right)\Phi(\vec{p}_1,\vec{p}_2,\cdots,\vec{p}_A)$$

運動量空間 ⇔ 座標空間：Fourier変換

- その状態が運動量 $\{p_i\}$ を持つ確率振幅
- 座標空間の波動関数を平面波展開したときの係数が運動量空間の波動関数
- 平面波を基底にとったときの波動関数の表現
- エネルギー固有状態の波動関数を、運動量固有状態で展開
- デルタ関数の Fourier 変換は平面波

座標空間と運動量空間

多体波動関数(3A次元)と密度(3次元)

座標空間における密度⇔運動量空間における形状因子 $F(q)$

$$\rho(\vec{r})=\langle\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A|\sum_i\delta^3(\vec{r}-\vec{r}_i)|\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A\rangle$$

$$=\frac{1}{(2\pi)^3}\int d^3q\exp(i\vec{q}\cdot\vec{r})\sum_i\langle\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A|\exp(-i\vec{q}\cdot\vec{r}_i)|\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A\rangle$$

$$\equiv\frac{1}{(2\pi)^3}\int d^3q\exp(i\vec{q}\cdot\vec{r})F(q)$$

$$F(q)=\sum_i\langle\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A|\exp(-i\vec{q}\cdot\vec{r}_i)|\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A\rangle$$

$$=\sum_i\langle\vec{r}_i|\exp(-i\vec{q}\cdot\vec{r}_i)|\vec{r}_i\rangle\quad\text{for single slater determinant}$$

密度：単位体積中に存在する粒子の個数の期待値

運動量

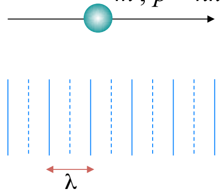
ドブロイ波長

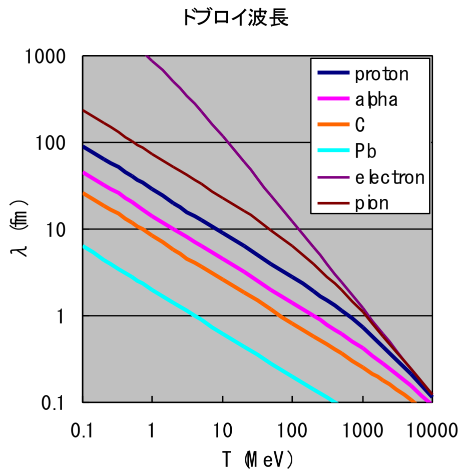
$$\hat{\vec{p}}=-i\hbar\vec{\nabla}$$

$$\vec{p}=\hbar\vec{k}=\frac{h}{\lambda}$$

for plane wave: $\exp(i\vec{k}\cdot\vec{r})$

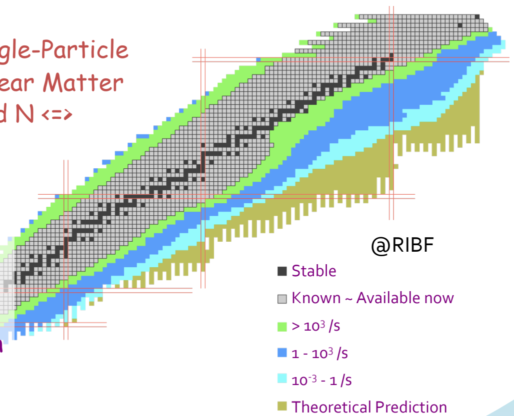
$m\,;\,\vec{p}=\hbar\vec{k}$





Exotic nuclei

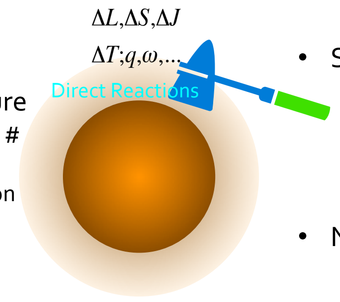
- Structure and Dynamics of Nuclei as functions of N and Z in a wide area of nuclear chart
  - Collectivities / Single-Particle
  - Properties of Nuclear Matter
  - Decoupling of P and N <=> Isospin symmetry
  - …
- New Magic Numbers
- Shape Coexistence
- Neutron Skin/Halo
- Di-neutron correlation
- Giant/Pigmy Resonances
- …



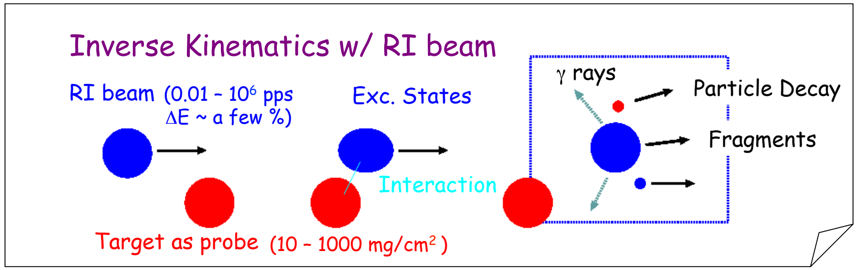
Studies in Neutron/Proton-rich nuclei

- Size/r-distribution
  - Skin/Halo
- Shell Structure
  - New magic #
  - Isospin / Deformation
- New modes
  - IVE1
  - ISE0, ISE1
- etc.

Mean field / Correlation ...



- Size/r-distribution
  - σR, elastic scat.
- Shell Structure
  - Mass / S_n, S_{2n}
  - Inelastic scatt.
    - Low lying states
  - Knockout / Transfer
- New modes
  - Coulex
  - Inelastic scatt.
  - CEX
- etc.



- Formation of Excited States of Exotic Nuclei
  - Direct reactions and their selectivities
- In-beam spectroscopy measuring decay products
  - Invariant-mass/γ-ray spectroscopy
    - Particle detectors at forward angles (kin. focus.)
    - Gamma detectors surrounding target (Doppler shift)
    - [Recoil particle detectors]

Transition Probabilities

$$M_{if} = \left\langle E_f J_f \pi_f T_f; \xi_f \left\| O(lsj\tau;\xi) \right\| E_i J_i \pi_i T_i; \xi_i \right\rangle$$

Cross Section $\propto |M_{if}|^2$; Lifetime $\propto 1/|M_{if}|^2$ — observables

$|E_i J_i \pi_i T_i; \xi_i\rangle$ and/or $|E_f J_f \pi_f T_f; \xi_f\rangle$ — to be studied

$O(lsj\tau;\xi)$: Propety of Reaction / Decay Processes — selectivity

Determine Quantum numbers and C.S. (lifetime)

- *Configuration / Collectivities*
- *Single particle / Correlation energies*

Probes for direct reactions

- Heavy Nuclei: Strong Coulomb Field
  - Coulomb Excitation, Coulomb Dissociation
  - E1, E2, (M1) / Isovector

$$O(E\lambda) \propto \sum_j \left(\frac{1}{2} - \tau_j\right) r_j^\lambda Y_\lambda(\hat{r}_j)$$

- H, D, ^{4}He [Liquid targets]
  - Inelastic Scattering
    - Isovector (H) / Isoscaler(H, D, ^{4}He)
    - Spin-Flip (H, D) / Spin-Non-Flip (H, D, ^{4}He)

$$O = O(L,S,J,T)$$

$$\sum_j r_j^\ell Y_L(\hat{r}_j) \; ; \; \sum_j \sigma_j r_j^\ell Y_L(\hat{r}_j)$$

$$\sum_j \tau_j r_j^\ell Y_L(\hat{r}_j) \; ; \; \sum_j \sigma_j \tau_j r_j^\ell Y_L(\hat{r}_j)$$

$$\ell = L, L+2$$

  - Charge Exchange
    - Fermi type (H) / Gamow-Teller type (H, D)
  - Nucleon Transfer
    - (α,t), (α,^{3}He), (d,p), (d,n) ... Reaction
    - Knockout

$$O \propto a^+(l_f, s_f, j_f)$$

$$O \propto a(l_i, s_i, j_i)$$

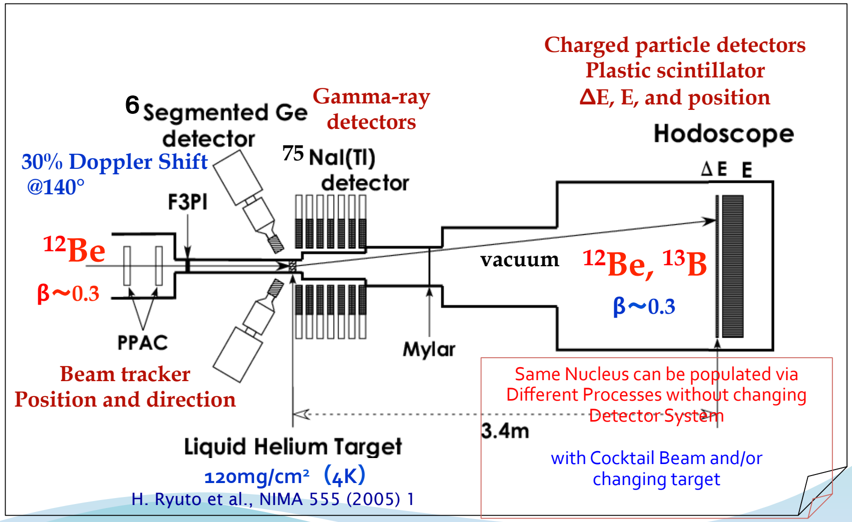
- Other (Be, C, ...)
  - Inelastic Scattering
  - Knockout / Fragmentation

Observables – reaction/decay measurements

- Yields (Cross Sections) / Lifetime / Width
  - Spectra : As a function of Excitation Energy (+ incident energy)
  - Properties of populated states (<- Selectivity)
- Angular Distribution / Momentum Transfer
  Reliable Reaction Models with small numbers of parameters
  - Assignment of L -> J^{π}
  - Eikonal Model [Knockout]
  - Virtual Photon / DWBA / Coupled Channels [Coulex, Inelastic, Transfer]
  - Optical Potential / Transition Density
    - Folding Model with Density Dependent Effective Interaction
- Angular Correlation / Alignments
  - Assignment of J^{π}

Cross sections as a function of …

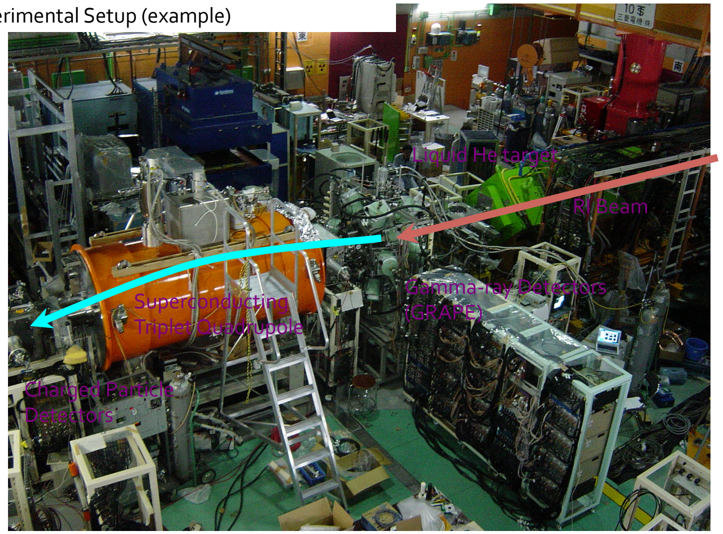
Typical Setup of Experiment (before RIBF)



Same Nucleus can be populated via Different Processes without changing Detector System

with Cocktail Beam and/or changing target

Experimental Setup (example)



2体の重心系（a+A=αチャンネル）のハミルトニアン、固有関数

$$H = h_a(\xi_a) + h_A(\xi_A) + h_\alpha(\vec{r}_\alpha, \xi_a, \xi_A) = H_\alpha + V_\alpha$$

$$H\Psi = E\Psi \,;\, h_\alpha(\vec{r}_\alpha, \xi_a, \xi_A) = T_\alpha(\vec{r}_\alpha) + V_\alpha(\vec{r}_\alpha, \xi_a, \xi_A)$$

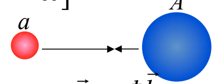
$$h_\alpha(\vec{r}_\alpha, \xi_a, \xi_A) \to T_\alpha(\vec{r}_\alpha) = -\frac{\hbar^2}{2\mu_\alpha}\nabla_\alpha^2 \quad [\vec{r}_\alpha \to \infty]$$

$$h_a(\xi_a)\phi_a(\xi_a) = \varepsilon_a \phi_a(\xi_a) \,;\, h_A(\xi_A)\phi_A(\xi_A) = \varepsilon_A \phi_A(\xi_A)$$

$$\Phi_\alpha = \phi_a(\xi_a)\phi_A(\xi_A)\varphi(\vec{k}_\alpha, \vec{r}_\alpha)$$

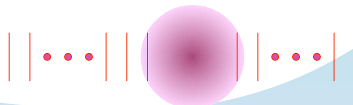
: an eigen function of H_{tot} at $[\vec{r}_\alpha \to \infty]$

$$\varphi(\vec{k}, \vec{r}) = \frac{1}{(2\pi)^{3/2}} \exp(i\vec{k}\cdot\vec{r})$$



$\mu_\alpha \,;\, \vec{p}_\alpha = \hbar\vec{k}_\alpha$

: Incoming Plane wave



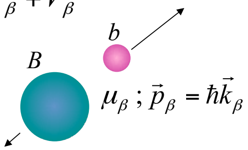
$$E_{tot} = \varepsilon_a + \varepsilon_A + \frac{\hbar^2 k_\alpha^2}{2\mu_\alpha}$$

$(a+A) \rightarrow b+B$ 反応： $b+B$ チャンネルのハミルトニアン

$$H = h_a(\xi_a) + h_A(\xi_A) + h_\alpha(\vec{r}_\alpha, \xi_a, \xi_A)$$
$$= h_b(\xi_b) + h_B(\xi_B) + h_\beta(\vec{r}_\beta, \xi_b, \xi_B) = H_\beta + V_\beta$$

$$h_\beta(\vec{r}_\beta, \xi_b, \xi_B) = T_\beta(\vec{r}_\beta) + V_\beta(\vec{r}_\beta, \xi_b, \xi_B)$$

$$h_\beta(\vec{r}_\beta, \xi_b, \xi_B) \rightarrow T_\beta(\vec{r}_\beta) = -\frac{\hbar^2}{2\mu_\beta}\nabla_\beta^2 \quad [\vec{r}_\beta \rightarrow \infty]$$

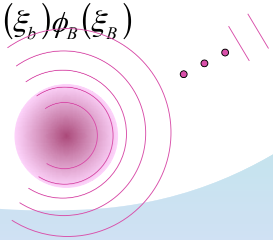


$$h_b(\xi_b)\phi_b(\xi_b) = \varepsilon_b\phi_b(\xi_b) \; ; \; h_B(\xi_B)\phi_B(\xi_B) = \varepsilon_B\phi_B(\xi_B)$$

$$\Phi_\beta = \varphi(\vec{k}_\beta, \vec{r}_\beta)\phi_b(\xi_b)\phi_B(\xi_B) \text{ and}$$

$$\Phi_\beta^{sc} = \frac{1}{(2\pi)^{3/2}}\frac{\exp(ik_\beta r_\beta)}{r_\beta} f_{\beta\alpha}(\vec{k}_\alpha, \vec{k}_\beta)\phi_b(\xi_b)\phi_B(\xi_B)$$

: eigen functions at $[\vec{r}_\beta \rightarrow \infty]$



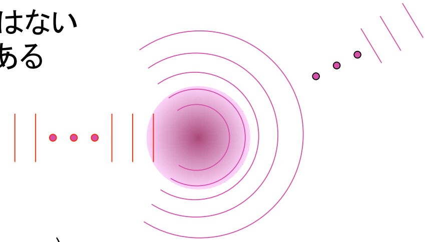
$$E_{tot} = \varepsilon_a + \varepsilon_A + \frac{\hbar^2 k_\alpha^2}{2\mu_\alpha} = \varepsilon_b + \varepsilon_B + \frac{\hbar^2 k_\beta^2}{2\mu_\beta}$$

$a+A \rightarrow (b+B)$ 反応の境界条件

この反応を含む、$a+A$で入射する核反応を記述する波動関数は、漸近形が以下の条件を満たさなければならない

- すべての開いたチャンネルには、原点から外向きに広がる波がある
- 入射チャンネルだけに入射平面波がある
- 入射平面波以外に内向きの波はない
- 閉じたチャンネルの振幅は0である

これらの条件を満たす波動関数



$$\Psi_\alpha^{(+)} \rightarrow \varphi_\alpha(\vec{k}_\alpha, \vec{r}_\alpha)\phi_a\phi_A + \frac{1}{(2\pi)^{3/2}}\sum_\beta \frac{\exp(ik_\beta r_\beta)}{r_\beta} f_{\beta\alpha}(\vec{k}_\alpha, \vec{k}_\beta)\phi_\beta + (> 2\text{body})$$

$$\phi_\beta = \phi_b\phi_B$$

$f_{\beta\alpha}(\vec{k}_\alpha, \vec{k}_\beta)$: Scattering Amplitude

Lippmann-Schwinger Equation

境界条件を含んだ積分方程式

$$H\Psi_\alpha^{(+)}(E) = E\Psi_\alpha^{(+)}(E)$$

$$H = (h_a + h_A + T_\alpha) + V_\alpha = H_\alpha + V_\alpha$$

$$\Psi_\alpha^{(+)}(E) = \Phi_\alpha(E) + \frac{1}{E - H_\alpha + i\eta}V_\alpha\Psi_\alpha^{(+)}(E)$$

$$= \frac{i\eta}{E - H + i\eta}\Phi_\alpha(E)$$

$$\Psi_\alpha^{(-)}(E) = \Phi_\alpha(E) + \frac{1}{E - H_\alpha - i\eta}V_\alpha^*\Psi_\alpha^{(-)}(E)$$

内向き球面波をもつ解

$$S_{\beta\alpha} = \left\langle \Psi_\beta^{(-)}(E) \middle| \Psi_\alpha^{(+)}(E) \right\rangle$$

散乱行列(S行列):
平面波 Φ_β が見出される確率振幅

T 行列、微分断面積、散乱振幅

$$T_{\beta\alpha}(\vec{k}_\alpha, \vec{k}_\beta) = \left\langle \Phi_\beta(\vec{k}_\beta, \vec{r}_\beta, \xi_\beta) \middle| V_\beta(\vec{r}_\beta, \xi_\beta) \middle| \Psi_\alpha^{(+)}(\vec{k}_\alpha, \vec{r}_\alpha, \xi_\alpha, \cdots) \right\rangle$$

: Post Form

$$= \left\langle \Psi_\beta^{(-)}(\vec{k}_\beta, \vec{r}_\beta, \xi_\beta, \cdots) \middle| V_\alpha(\vec{r}_\alpha, \xi_\alpha) \middle| \Phi_\alpha(\vec{k}_\alpha, \vec{r}_\alpha, \xi_\alpha) \right\rangle$$

: Prior Form

$$\frac{d\sigma_{\beta\alpha}}{d\Omega_\beta} = \frac{v_\beta}{v_\alpha}\left| f_{\beta\alpha}(\vec{k}_\alpha, \vec{k}_\beta) \right|^2$$

$$f_{\beta\alpha}(\vec{k}_\alpha, \vec{k}_\beta) = -\frac{(2\pi)^2\mu_\beta}{\hbar^2}T_{\beta\alpha}(\vec{k}_\alpha, \vec{k}_\beta)$$

T行列が計算できれば、微分断面積が計算できる

平面波ボルン近似 (PWBA)

$$\Psi_\alpha^{(+)}\left(\vec{k}_\alpha,\vec{r}_\alpha,\xi_\alpha,\cdots\right)=\Phi_\alpha\left(\vec{k}_\alpha,\vec{r}_\alpha,\xi_\alpha\right)+\ldots$$ 外向き球面波

$$\Psi_\beta^{(-)}\left(\vec{k}_\beta,\vec{r}_\beta,\xi_\beta,\cdots\right)=\Phi_\beta\left(\vec{k}_\beta,\vec{r}_\beta,\xi_\beta\right)+\ldots$$ 内向き球面波

$$T_{\beta\alpha}\left(\vec{k}_\alpha,\vec{k}_\beta\right)\approx\left\langle\Phi_\beta\left(\vec{k}_\beta,\vec{r}_\beta,\xi_\beta\right)\middle|V_\beta\left(\vec{r}_\beta,\xi_\beta\right)\middle|\Phi_\alpha\left(\vec{k}_\alpha,\vec{r}_\alpha,\xi_\alpha\right)\right\rangle$$

: Post Form

$$\approx\left\langle\Phi_\beta\left(\vec{k}_\beta,\vec{r}_\beta,\xi_\beta\right)\middle|V_\alpha\left(\vec{r}_\alpha,\xi_\alpha\right)\middle|\Phi_\alpha\left(\vec{k}_\alpha,\vec{r}_\alpha,\xi_\alpha\right)\right\rangle$$

: Prior Form

$$\exp(i\vec{q}\cdot\vec{r})=4\pi\sum_{lm}i^l j_l(qr)Y_{lm}(\hat{r})Y^*_{lm}(\hat{q})$$

Plane-Wave Born Approximation (PWBA)

$$\Psi_\alpha^{(+)}\left(\vec{k}_\alpha,\vec{r},\xi,\cdots\right)=\Phi_\alpha\left(\vec{k}_\alpha,\vec{r},\xi\right)+\ldots$$

$$\Psi_\beta^{(-)}\left(\vec{k}_\beta,\vec{r},\xi,\cdots\right)=\Phi_\beta\left(\vec{k}_\beta,\vec{r},\xi\right)+\ldots$$ Plane Wave + ...

$$T_{\beta\alpha}\left(\vec{k}_\alpha,\vec{k}_\beta\right)\approx\left\langle\Phi_\beta\left(\vec{k}_\beta,\vec{r},\xi\right)\middle|V_\beta\left(\vec{r},\xi\right)\middle|\Phi_\alpha\left(\vec{k}_\alpha,\vec{r},\xi\right)\right\rangle \ : \text{Post Form}$$

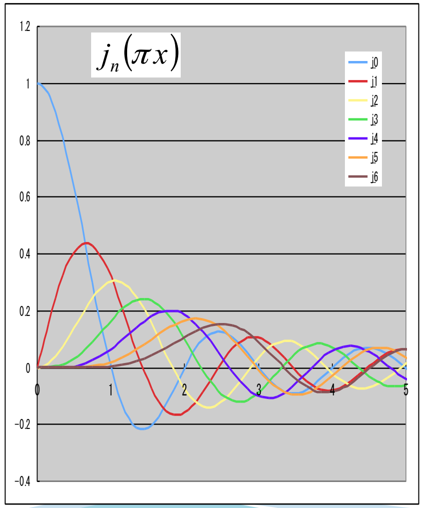
$$=\left\langle\Phi_\beta\left(\vec{k}_\beta,\vec{r},\xi\right)\middle|V_\alpha\left(\vec{r},\xi\right)\middle|\Phi_\alpha\left(\vec{k}_\alpha,\vec{r},\xi\right)\right\rangle \ : \text{Prior Form}$$

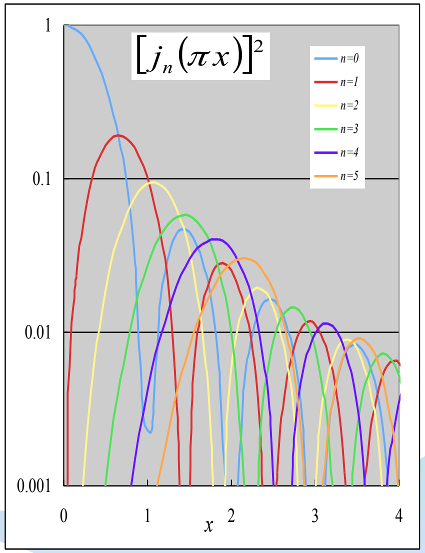
$$=\frac{1}{(2\pi)^3}\int d^3r\,V(\vec{r})\exp[i\vec{q}\cdot\vec{r}]\ ,\ \text{where}\ \vec{q}=\vec{k}_\alpha-\vec{k}_\beta$$

$$V(\vec{r})=\left\langle\phi_\beta(\xi)\middle|V_\alpha(\vec{r},\xi)\middle|\phi_\alpha(\xi)\right\rangle=\left\langle\phi_\beta(\xi)\middle|V_\beta(\vec{r},\xi)\middle|\phi_\alpha(\xi)\right\rangle$$

T matrix: Fourier Transform of Interaction (V)

球ベッセル関数



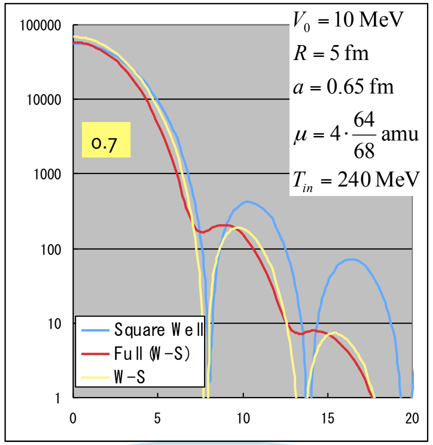


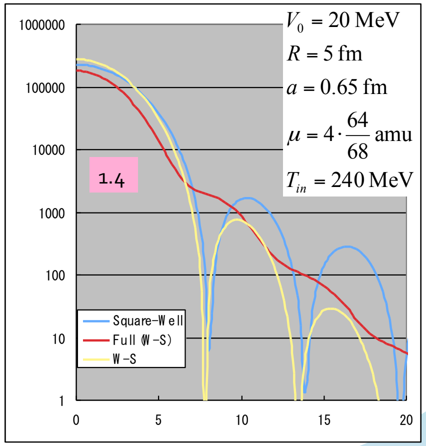
弾性散乱の平面波ボルン近似 (例)

Criterion (c.f. Schiff p.326)

$$V(r)=-\frac{V_0}{1+\exp[(r-R)/a]}$$

$$-\frac{\mu}{\hbar^2 k}\int_0^\infty\left(e^{2ikr}-1\right)V(r)dr\approx\frac{\mu V_0}{\hbar^2}\frac{R}{k}=\frac{V_0 R}{\hbar v_{rel}}$$





非弾性散乱の平面波ボルン近似 (PWBA)

$$T^{l}_{\alpha'\alpha}\left(\vec{k},\vec{k}'\right) \approx \left\langle \Phi_{\alpha'}\left(\vec{k}',\vec{r}\right)\middle| V_l\left(\vec{r}\right)\middle|\Phi_{\alpha}\left(\vec{k},\vec{r}\right)\right\rangle$$

$$= \frac{1}{(2\pi)^3}\int d^3r\, V_T(r)\, Y^*_{lm}(\hat{r}) \exp\left[i\left(\vec{k}-\vec{k}'\right)\cdot\vec{r}\right]$$

$$= \frac{1}{2\pi^2}\int r^2\,dr\, j_l(qr)\, V_T(r)\, Y_{lm}(\hat{q}) \approx \frac{V_0\beta_l R^3}{2\pi^2}\, j_l(qR)\, Y_{lm}(\hat{q}) \quad [a\to 0]$$

l により、山谷の位置が特徴づけられる

$$T^{l=2}_{\alpha'\alpha}\left(\vec{k},\vec{k}'\right) \approx \frac{V_0\beta_2 R^3}{2\pi^2} Y_{l0}(\hat{q}) \exp\left(-(qa)^2\right) \times$$

$$\left[j_2(qR) + 4\left(\frac{a}{R}\right)^2 qR\, j_1(qR) + 4\left(\frac{a}{R}\right)^4 (qR)^2 j_0(qR)\right]$$

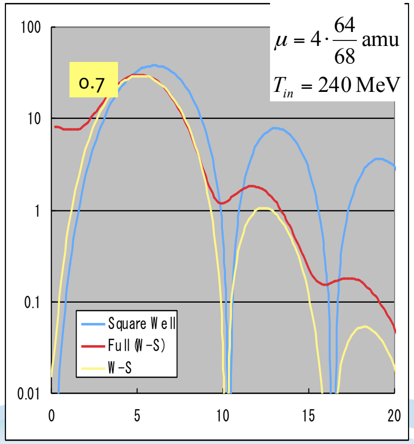
$$V_T(r) = -\beta_l \frac{r^{l-1}}{R^{l-2}}\frac{dV(r)}{dr} \quad : \text{ Tassie Form}$$

$$V(r) = \int_r^{\infty} dt \frac{V_0}{\sqrt{2\pi}\left(2a^2\right)} \exp\left[-\frac{(t-R)^2}{4a^2}\right] \approx \frac{V_0}{1+\exp\left[(r-R)/a\right]}$$

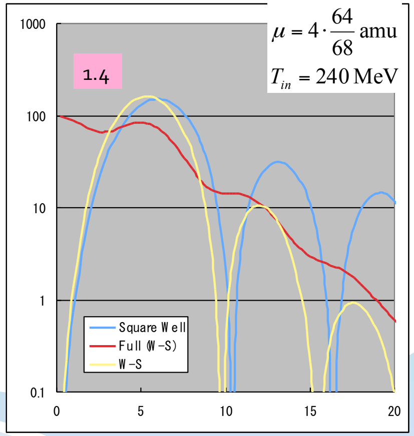
非弾性散乱(L=2)の平面波ボルン近似 (例)

$$V_{tr}(r) = -\beta\, r \frac{dV(r)}{dR}$$

$V_0 = 10$ MeV, $R = 5$ fm, $a = 0.65$ fm, $\mu = 4\cdot\frac{64}{68}$ amu, $T_{in} = 240$ MeV



$V_0 = 20$ MeV, $R = 5$ fm, $a = 0.65$ fm, $\mu = 4\cdot\frac{64}{68}$ amu, $T_{in} = 240$ MeV



Eikonal Approximation (Glauber model)

- For Kinetic Energy >> Interaction Potential

$$H = \left(h(\xi) + T(\vec{r})\right) + V(\vec{r},\xi)$$

$$h(\xi)\phi_\alpha(\xi) = \varepsilon_\alpha\phi_\alpha(\xi) \;;\; h(\xi)\phi_{\alpha'}(\xi) = \varepsilon_{\alpha'}\phi_{\alpha'}(\xi) \quad : \text{ Internal}$$

$$\frac{\hbar^2k_\alpha^2}{2\mu} \approx \frac{\hbar^2k_{\alpha'}^2}{2\mu} >> \varepsilon_{\alpha'} - \varepsilon_\alpha : \text{ adiabatic approx. } ;\; |V| << \frac{\hbar^2k_\alpha^2}{2\mu}$$

$$\Psi_\alpha^{(+)} \approx \phi_\alpha(\xi)\varphi_\alpha\left(\vec{k}_\alpha,\vec{r}\right)\exp\left[-\frac{i}{\hbar v_\alpha}\int_{-\infty}^{z} dz' V\left(\vec{b},z',\xi\right)\right]$$

$$V\Psi_\alpha^{(+)} \approx i\hbar v_\alpha e^{i\vec{k}_\alpha\cdot\vec{r}} \frac{\partial}{\partial z}\left\{\exp\left[-\frac{i}{\hbar v_\alpha}\int_{-\infty}^{z} dz' V\left(\vec{b},z',\xi\right)\right]\right\}$$

Eikonal Approximation (Glauber model)

$$T_{\alpha'\alpha} = \frac{\hbar v_\alpha}{i(2\pi)^3}\int d^2b \exp\left(i\vec{q}\cdot\vec{b}\right)\left\langle \phi_{\alpha'}(\xi)\middle| 1-\Gamma\left(\vec{b},\xi\right)\middle|\phi_\alpha(\xi)\right\rangle_\xi$$

$$\Gamma\left(\vec{b},\xi\right) = \exp\left[i\chi\left(\vec{b},\xi\right)\right] \quad : \text{ Profile function}$$

Including multiple scattering

$$\chi\left(\vec{b},\xi\right) = \frac{-1}{\hbar v_\alpha}\int_{-\infty}^{\infty} dz\, V\left(\vec{b},z,\xi\right) \quad : \text{ Phase shift function}$$

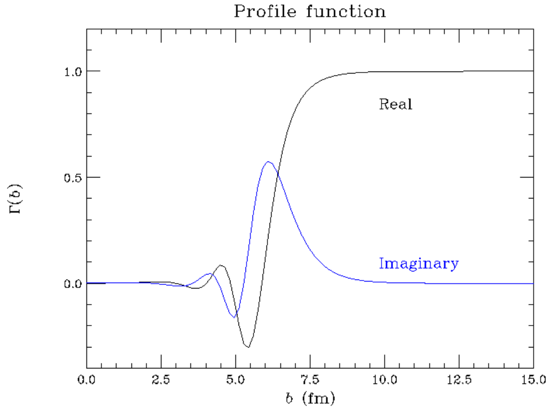
- For axial symmetric interaction

$$T_{\alpha'\alpha} = \frac{\hbar v_\alpha}{i(2\pi)^2}\int_0^\infty b\,db\, J_0(qb)\left\langle \phi_{\alpha'}(\xi)\middle| 1-\Gamma(b,\xi)\middle|\phi_\alpha(\xi)\right\rangle_\xi$$

Elastic :

$$f_{\alpha\alpha}(\theta) = ik_\alpha\int_0^\infty b\,db\, J_0(qb)\left[1-\Gamma(b)\right] \to \frac{iR}{\theta}J_1(k_\alpha R\theta) \quad : \text{ Black Disk}$$

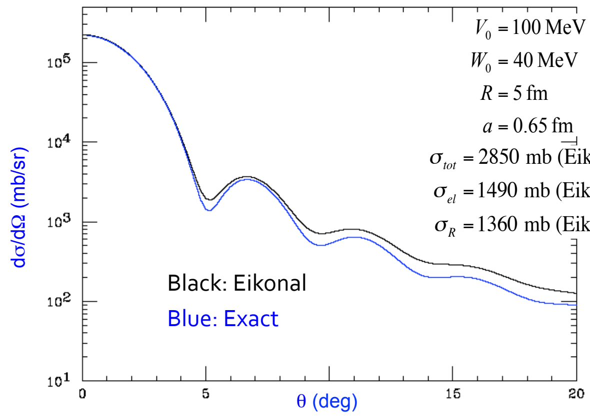
Profile Function (example)



$V_0 = 100$ MeV
$W_0 = 40$ MeV
$R = 5$ fm
$a = 0.65$ fm
$\mu = 4 \cdot \frac{64}{68}$ amu
$T_{in} = 240$ MeV
$k = 6.39$ fm^{-1}
$\beta = 0.359$

- Absorption (W_0)
- Phase shift (V_0)

Differential Cross Section (example)



$V_0 = 100$ MeV
$W_0 = 40$ MeV
$R = 5$ fm
$a = 0.65$ fm
$\mu = 4 \cdot \frac{64}{68}$ amu
$T_{in} = 240$ MeV
$k = 6.39$ fm^{-1}
$\beta = 0.359$
$\sigma_{tot} = 2850$ mb (Eikonal), 2860 mb (Exact)
$\sigma_{el} = 1490$ mb (Eikonal), 1430 mb (Exact)
$\sigma_R = 1360$ mb (Eikonal), 1430 mb (Exact)

Menu

- Introduction
  - Inverse Kinematics and In-beam Spectroscopyy
- Inelastic-type reaction
  - Characteristics of dσ/dΩ and alignment
  - Folding model for analyzing data
  - Examples
- Transfer reaction
  - Matching Condition
  - Asymptotic Normalization Constant and Spectroscopic Factor
  - Examples
- New types of reactions using RI beams as new probes
  - Exothermic reaction
  - Transferring various quantum numbers

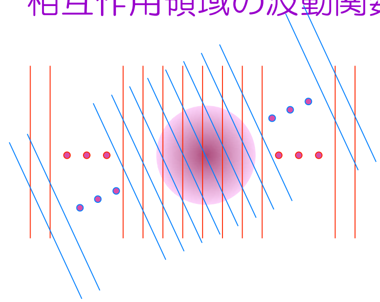
Born近似のCriterion:

$$-\frac{\mu}{\hbar^2 k}\int_0^\infty \left(e^{2ikr}-1\right)V(r)\,dr \approx \frac{\mu V_0}{\hbar^2}\frac{R}{k} = \frac{V_0 R}{\hbar v_{rel}} << 1$$

$$\hbar v_{rel} \sim 60-200 \text{ MeV fm};\ V_0 R \sim 40 \times A_{proj} \times 3 \text{ MeV fm}$$

ほとんどみたされない！

相互作用領域の波動関数は平面波とかなり違う



PWBAは、吸収の効果（別のチャンネルへのflux）をとりこめない

$$mfp = \frac{\hbar}{\sqrt{2\mu W}} \text{ fm}$$

$$\sim \frac{1.4}{A_p} \text{ fm (for } W \sim 20 A_p \text{ MeV)}$$

$\Psi_\alpha^{(+)}\left(\vec{k}_\alpha, \vec{r}_\alpha, \xi_\alpha, \cdots\right)$, $\Psi_\beta^{(-)}\left(\vec{k}_\beta, \vec{r}_\beta, \xi_\beta, \cdots\right)$ のよりよい近似は？

弾性散乱が光学ポテンシャルで記述できるなら

$\Psi_\alpha^{(+)}\left(\vec{k}_\alpha,\vec{r}_\alpha,\xi_\alpha,\cdots\right)$, $\Psi_\beta^{(-)}\left(\vec{k}_\beta,\vec{r}_\beta,\xi_\beta,\cdots\right)$ の近似として

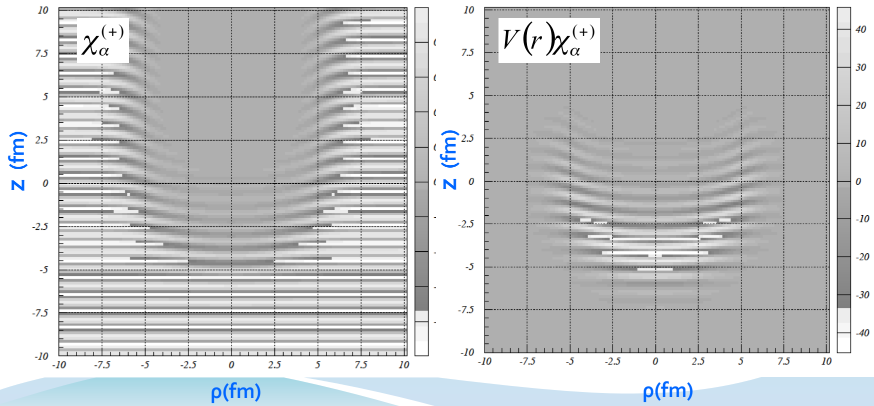
- 弾性散乱チャネルの波動関数、散乱振幅はポテンシャル問題を解けばよい。
- a+A → b+B 反応を記述するΨの主要成分が弾性散乱だとすると、Ψの近似として、ポテンシャル問題の解を用いればよさそう。
- Ψを、(平面波＋球面波)ではなく、(弾性散乱による散乱波)+ (球面波)と書き直す。(弾性散乱による散乱波)を、歪曲波と呼ぶ。
- DWBAの計算コードなどでは、歪曲波を多重極展開して求めるが、エネルギーの高い反応では、部分波の角運動量が大きくなり、見通しがよくない。kb~(l+1/2)
- 以後、Eikonal近似で得られた波動関数を用いた記述を試みる。角運動量表示による厳密なものは教科書(Satchler, 河合・吉田など)を参照のこと

歪曲波（Eikonal 近似）

$$V(r) = -\frac{V_0 + iW_0}{1+\exp[(r-R)/a]}$$

$V_0 = 100$ MeV
$W_0 = 40$ MeV
$R = 5$ fm
$a = 0.65$ fm

$\mu = 4\cdot\frac{64}{68}$ amu
$T_{in} = 240$ MeV
$k = 6.39$ MeV/c
$\beta = 0.359$



歪曲波を用いた表式　相互作用の繰り込み（光学ポテンシャル）

$$H\Psi_\alpha^{(+)} = E\Psi_\alpha^{(+)}$$

$$H = \left(h_a + h_A + T_\alpha\right) + V_\alpha = H_\alpha + V_\alpha = H_\alpha + U_\alpha + \hat{V}_\alpha$$

$$\chi_\alpha^{(+)}(\vec{r}) = \varphi\left(\vec{k}_\alpha,\vec{r}_\alpha\right) + \frac{1}{E-\left(T_\alpha + U_\alpha\right)+i\eta}U_\alpha\chi_\alpha^{(+)}(\vec{r})$$ 外向き歪曲波

$$\chi_\alpha^{(-)}(\vec{r}) = \varphi\left(\vec{k}_\alpha,\vec{r}_\alpha\right) + \frac{1}{E-\left(T_\alpha + U_\alpha\right)-i\eta}U_\alpha\chi_\alpha^{(-)}(\vec{r})$$ 内向き歪曲波

$$T_{\beta\alpha}\left(\vec{k}_\alpha,\vec{k}_\beta\right) = \left\langle \chi_\beta^{(-)}\left(\vec{k}_\beta,\vec{r}_\beta\right)\phi\left(\xi_\beta\right)\middle|\hat{V}_\beta\left(\vec{r}_\beta,\xi_\beta\right)\middle|\Psi_\alpha^{(+)}\left(\vec{k}_\alpha,\vec{r}_\alpha,\xi_\alpha,\cdots\right)\right\rangle$$

: Post Form

$$= \left\langle \Psi_\beta^{(-)}\left(\vec{k}_\beta,\vec{r}_\beta,\xi_\beta,\cdots\right)\middle|\hat{V}_\alpha\left(\vec{r}_\alpha,\xi_\alpha\right)\middle|\chi_\alpha^{(+)}\left(\vec{k}_\alpha,\vec{r}_\alpha\right)\phi\left(\xi_\alpha\right)\right\rangle$$

: Prior Form

正確な表式

歪曲波ボルン近似(DWBA)

$$\Psi_\alpha^{(+)}\left(\vec{k}_\alpha,\vec{r}_\alpha,\xi_\alpha,\cdots\right) = \chi_\alpha^{(+)}\left(\vec{k}_\alpha,\vec{r}_\alpha\right)\phi_\alpha\left(\xi_\alpha\right) + \ldots$$

$$\Psi_\beta^{(-)}\left(\vec{k}_\beta,\vec{r}_\beta,\xi_\beta,\cdots\right) = \chi_\beta^{(-)}\left(\vec{k}_\beta,\vec{r}_\beta\right)\phi_\beta\left(\xi_\beta\right) + \ldots$$

$$T_{\beta\alpha}\left(\vec{k}_\alpha,\vec{k}_\beta\right) \approx \left\langle \chi_\beta^{(-)}\left(\vec{k}_\beta,\vec{r}_\beta\right)\phi_\beta\left(\xi_\beta\right)\middle|\hat{V}_\beta\left(\vec{r}_\beta,\xi_\beta\right)\middle|\chi_\alpha^{(+)}\left(\vec{k}_\alpha,\vec{r}_\alpha\right)\phi_\alpha\left(\xi_\alpha\right)\right\rangle$$

: Post Form

$$= \left\langle \chi_\beta^{(-)}\left(\vec{k}_\beta,\vec{r}_\beta\right)\phi_\beta\left(\xi_\beta\right)\middle|\hat{V}_\alpha\left(\vec{r}_\alpha,\xi_\alpha\right)\middle|\chi_\alpha^{(+)}\left(\vec{k}_\alpha,\vec{r}_\alpha\right)\phi_\alpha\left(\xi_\alpha\right)\right\rangle$$

: Prior Form

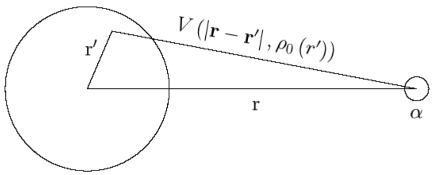
$$\equiv \left\langle \chi_\beta^{(-)}\left(\vec{k}_\beta,\vec{r}_\beta\right)\middle|F_{\beta\alpha}^{(\gamma)}\left(\vec{r}_\alpha,\vec{r}_\beta\right)\middle|\chi_\alpha^{(+)}\left(\vec{k}_\alpha,\vec{r}_\alpha\right)\right\rangle \quad (\gamma = \alpha \text{ or } \beta)$$

$$F_{\beta\alpha}^{(\gamma)}\left(\vec{r}_\alpha,\vec{r}_\beta\right) = \left\langle \phi_\beta\left(\xi_\beta\right)\middle|\hat{V}_\gamma\left(\vec{r}_\alpha,\vec{r}_\beta,\xi_{\alpha\beta}\right)\middle|\phi_\alpha\left(\xi_\alpha\right)\right\rangle_{\xi_{\alpha\beta}}$$: Form Factor

歪曲波と形状因子を計算すればよい

歪曲波をつくる光学ポテンシャル

現象論的ポテンシャル (B-G, CH89, etc.)
Folding 模型 (JLM, etc.)



$$U(r) = \int d\mathbf{r}' V(|\mathbf{r}-\mathbf{r}'|, \rho_0(r'))\, \rho_0(r'),$$

$$\begin{aligned} V(|\mathbf{r}-\mathbf{r}'|, \rho_0(r')) &= -V\left(1+\beta_V \rho_0(r')^{2/3}\right)\exp\left(-|r-r'|^2/\alpha_V\right) \\ &\quad -iW\left(1+\beta_W \rho_0(r')^{2/3}\right)\exp\left(-|r-r'|^2/\alpha_W\right), \end{aligned}$$

Alpha particle at 140-400 MeV: U ~ 130 – 60 MeV, W ~ 25 – 40 MeV
Proton at 50-200 MeV : U ~ 50 – a few MeV, W ~ 10 – 20 MeV (see JLM)

非弾性散乱の形状因子

相対座標、内部座標、相互作用演算子が共通

$$T_{\alpha'\alpha}\left(\vec{k}_\alpha, \vec{k}_{\alpha'}\right) = \left\langle \chi^{(-)}_{\alpha'}\left(\vec{k}_{\alpha'}, \vec{r}\right)\middle| F^{(\gamma)}_{\alpha'\alpha}(\vec{r}) \middle| \chi^{(+)}_{\alpha}\left(\vec{k}_\alpha, \vec{r}\right)\right\rangle$$

$$F^{(\gamma)}_{\alpha'\alpha}(\vec{r}) = \left\langle \phi_{\alpha'}(\xi)\middle|\hat{V}_\gamma(\vec{r},\xi)\middle|\phi_\alpha(\xi)\right\rangle_\xi$$

巨視的模型

$$U_\alpha(\vec{r},R) \approx U_{\alpha'}(\vec{r},R)$$

$$\hat{V}_\gamma(\vec{r},\xi) = U_\alpha(\vec{r},R) - U_\alpha(\vec{r},R_0)$$

$$R = R(\Omega) = R_0\left(1 + \sum_{\ell m} \alpha_{\ell m} Y^*_{\ell m}(\Omega)\right)$$

非弾性散乱（振動模型の形状因子）

表面振動

$$\hat{V}_\gamma(\vec{r},\xi) = R_0 \left.\frac{dU(r,R)}{dR}\right|_{R=R_0} \sum_{\ell m} \alpha_{\ell m} Y^*_{\ell m}(\Omega)$$

$$= -R_0 \frac{dU(r,R_0)}{dr} \sum_{\ell m} \alpha_{\ell m} Y^*_{\ell m}(\Omega)$$

$$\alpha_{\ell m} = \frac{1}{\sqrt{2\ell+1}} \beta_\ell \left\{ a^+_{\ell m} + (-)^m a_{\ell,-m} \right\}$$

1フォノン励起の形状因子

$$F_{\alpha'\alpha}(\vec{r}) = \left(I_A M_A \ell m \mid I_{A^*} M_{A^*}\right) \frac{\beta_\ell R_0}{\sqrt{2\ell+1}} \left.\frac{dU(r,R)}{dR}\right|_{R=R_0} Y^*_{\ell m}(\Omega)$$

$$F^{Tas}_{\alpha'\alpha}(\vec{r}) = \left(I_A M_A \ell m \mid I_{A^*} M_{A^*}\right) \frac{\beta_\ell R_0}{\sqrt{2\ell+1}} \left(\frac{r}{R_0}\right)^{\ell-1} \left.\frac{dU(r,R)}{dR}\right|_{R=R_0} Y^*_{\ell m}(\Omega)$$

T行列は、βR に比例する: 断面積の大きさ⇔$(\beta R)^2$

非弾性散乱（回転模型の形状因子）

軸対称変形

$$R = R(\Omega') = R_0\left(1 + \sum_{\lambda} \alpha_{\lambda 0} Y^*_{\lambda 0}(\Omega')\right)$$

$$U(\vec{r},R) = \sum_{\ell} \hat{V}_\ell\left(r,\{\alpha_{\lambda 0}\}\right) Y^*_{\ell 0}(\Omega')$$

W–S 形状因子のメモ
（前回配布）を参照

$$\hat{V}_\ell\left(r,\{\alpha_{\lambda 0}\}\right) = \frac{1}{4\pi} \int d\Omega' U(\vec{r},R) Y_{\ell 0}(\Omega')$$

偶遇核回転励起 (0 -> I) の形状因子

$$F_{\alpha'\alpha}(\vec{r}) = \sqrt{\frac{8\pi^2}{2I+1}} \hat{V}_I\left(r,\{\alpha_{\lambda 0}\}\right) Y^*_{I m}(\Omega)$$

奇核などK量子数をもつ場合

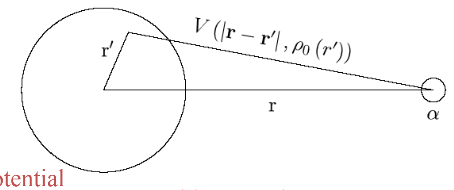
$$F_{\alpha'\alpha}(\vec{r}) = \left(K K \ell 0 \mid I' K\right) \sqrt{\frac{8\pi^2}{2\ell+1}} \hat{V}_\ell\left(r,\{\alpha_{\lambda 0}\}\right) Y^*_{\ell m}(\Omega)$$

非弾性散乱（微視的アプローチ）

遷移密度からスタート

$$M_{a\to b} = \left\langle \Psi_b(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A)\middle|F\middle|\Psi_a(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A)\right\rangle$$
$$= \sum_i \left\langle \Psi_b(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A)\middle|f(\vec{r}_i)\middle|\Psi_a(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A)\right\rangle$$
$$= \sum_i \int d^3r\, f(\vec{r}) \left\langle \Psi_b\middle|\delta^3(\vec{r}-\vec{r}_i)\middle|\Psi_a\right\rangle$$
$$= \int d^3r\, f(\vec{r}) \sum_i \int \prod_j d^3r_j\, \delta^3(\vec{r}-\vec{r}_i)\Psi_b^+\Psi_a$$
$$= \int d^3r\, f(\vec{r})\rho_{tr}(\vec{r})$$
$$\rho_{tr}(\vec{r}) = \sum_i \int \prod_j d^3r_j\, \delta^3(\vec{r}-\vec{r}_i)\Psi_b^+\Psi_a$$

非弾性散乱（微視的アプローチ）



Transition Potential
=形状因子 F

Transition Density

$$\delta U(r,E) = \int d\mathbf{r}'\,\delta\rho_L(\mathbf{r}',E)\left[V(|\mathbf{r}-\mathbf{r}'|,\rho_0(r')) + \rho_0(r')\frac{\partial V(|\mathbf{r}-\mathbf{r}'|,\rho_0(r'))}{\partial\rho_0(r')}\right],$$

Transition density に対する Collective model

Compression mode

$$\delta\rho_{L=0}(r,E) = -\alpha_0(E)\left(3 + r\frac{d}{dr}\right)\rho_0(r),$$
$$\delta\rho_{L=1}(r,E) = -\frac{\alpha_1(E)}{R}\left[3r^2\frac{d}{dr} + 10r - \frac{5}{3}\langle r^2\rangle\frac{d}{dr} + \epsilon\left(r\frac{d^2}{dr^2} + 4\frac{d}{dr}\right)\right]\rho_0(r),$$
$$\delta\rho_{L\geq 2}(r,E) = -\alpha_l(E)\, r^{l-1}\frac{d}{dr}\rho_0(r),$$

Tassie model

非弾性散乱（運動量表示）

$$T_{\alpha'\alpha}(\vec{k}_\alpha,\vec{k}_{\alpha'}) = \left\langle \chi^{(-)}_{\alpha'}(\vec{k}_{\alpha'},\vec{r})\middle|F^{(\gamma)}_{\alpha'\alpha}(\vec{r})\middle|\chi^{(+)}_{\alpha}(\vec{k}_\alpha,\vec{r})\right\rangle$$
$$= \int d^3q\, D(\vec{q})\,\tilde{\rho}_{tr}(\vec{q})\,\tilde{V}_{aN}(\vec{q})$$

相互作用に密度依存がない Folding 模型のフーリエ変換

$$F_{\alpha'\alpha}(\vec{r}) = \int d^3r'\,\rho_{tr}(\vec{r}')V_{aN}(|\vec{r}-\vec{r}'|)$$
$$\tilde{F}_{\alpha'\alpha}(\vec{q}) = \tilde{\rho}_{tr}(\vec{q})\tilde{V}_{aN}(\vec{q})$$

→ tρ 近似

歪曲波のフーリエ変換

$$D(\vec{q}) = \int d^3r\, e^{-i\vec{q}\cdot\vec{r}}\chi^{(-)}_{\alpha'}{}^{*}(\vec{k}_{\alpha'},\vec{r})\chi^{(+)}_{\alpha}(\vec{k}_\alpha,\vec{r})$$
$$\to \delta^3(\vec{k}_\alpha - \vec{k}_{\alpha'} - \vec{q}) \quad : \text{Plane wave limit}$$

非弾性散乱（歪曲波の効果）

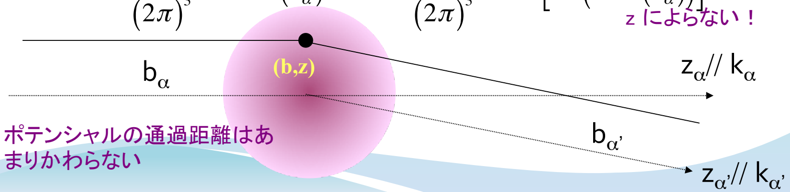
Eikonal 近似による歪曲波で D(q) を評価してみよう

$$\chi^{(+)}_{\alpha}(\vec{k}_\alpha,\vec{r}) = \phi_\alpha(\vec{k}_\alpha,\vec{r})\exp\left[-\frac{i}{\hbar v_\alpha}\int_{-\infty}^{z_\alpha} dz_\alpha' U(\vec{b}_\alpha,z_\alpha')\right]$$
$$\chi^{(-)}_{\alpha'}(\vec{k}_{\alpha'},\vec{r}) = \phi_{\alpha'}(-\vec{k}_{\alpha'},\vec{r})\exp\left[-\frac{i}{\hbar v_{\alpha'}}\int_{\infty}^{z_{\alpha'}} dz_{\alpha'}' U^*(\vec{b}_{\alpha'},z_{\alpha'}')\right]$$

if $v_\alpha \approx v_{\alpha'}$ & forward scattering

$$\chi^{(-)}_{\alpha'}{}^{*}(\vec{k}_{\alpha'},\vec{r})\chi^{(+)}_{\alpha}(\vec{k}_\alpha,\vec{r}) \approx \frac{\exp\left[i(\vec{k}_\alpha-\vec{k}_{\alpha'})\cdot\vec{r}\right]}{(2\pi)^3}\exp\left[-\frac{i}{\hbar v_\alpha}\int_{-\infty}^{\infty} dz_\alpha' U(\vec{b}_\alpha,z_\alpha')\right]$$
$$= \frac{\exp\left[i(\vec{k}_\alpha-\vec{k}_{\alpha'})\cdot\vec{r}\right]}{(2\pi)^3}\Gamma(b_\alpha) = \frac{\exp\left[i(\vec{k}_\alpha-\vec{k}_{\alpha'})\cdot\vec{r}\right]}{(2\pi)^3}\left[1-(1-\Gamma(b_\alpha))\right]$$

z によらない！



ポテンシャルの通過距離はあまりかわらない

非弾性散乱（歪曲波の効果）

Eikonal 近似による歪曲波で D(q) を評価してみよう

$$D(\vec{q}) \approx \delta^3\left(\vec{k}_\alpha - \vec{k}_{\alpha'} - \vec{q}\right) - \frac{\delta\left(\left(\vec{k}_\alpha - \vec{k}_{\alpha'} - \vec{q}\right)_{//}\right)}{(2\pi)} \int b\,db\,J_0\left(\left(\vec{k}_\alpha - \vec{k}_{\alpha'} - \vec{q}\right)_\perp b\right)\left(1-\Gamma(b)\right)$$

$$= \delta^3\left(\vec{q}_0 - \vec{q}\right) - \frac{\delta\left(\left(\vec{q}_0 - \vec{q}\right)_{//}\right)}{(2\pi)} \int b\,db\,J_0\left(\left(\vec{q}_0 - \vec{q}\right)_\perp b\right)\left(1-\Gamma(b)\right)$$

Distortion の効果 ($q=q_0$ にピーク)

～ $k_{\alpha'}$ を z 軸とした弾性散乱の散乱振幅

非弾性散乱（運動量表示）

$$T_{\alpha'\alpha}\left(\vec{k}_\alpha, \vec{k}_{\alpha'}\right) = \int d^3q\, D(\vec{q})\tilde{\rho}_{tr}(\vec{q})\tilde{V}_{aN}(\vec{q})$$

$$\tilde{V}_{aN}(\vec{q}) \approx -\frac{V\lambda^3}{\pi^{3/2}} \exp\left[-(q\lambda)^2\right] \quad (\lambda : \text{range of interaction})$$

$$\tilde{\rho}_{tr}(\vec{q}) = \tilde{G}_{\ell jm}(q) Y^*_{\ell m}(\hat{q})$$

$$\tilde{\rho}^{\ell=2}_{tr}(\vec{q}) \propto Y^*_{\ell m}(\hat{q}) \exp\left(-(qa)^2\right) \times \left[j_2(qR) + 4\left(\frac{a}{R}\right)^2 qR\,j_1(qR) + 4\left(\frac{a}{R}\right)^4 (qR)^2 j_0(qR)\right]$$

W-S density の場合

$$D(\vec{q}) \approx \delta\left(\left(\vec{q}_0 - \vec{q}\right)_{//}\right) \times \left[\delta^2\left(\left(\vec{q}_0 - \vec{q}\right)_\perp\right) - \frac{1}{(2\pi)} \int b\,db\,J_0\left(\left(\vec{q}_0 - \vec{q}\right)_\perp b\right)\left(1-\Gamma(b)\right)\right]$$

Collective Form Factor for Transition Density

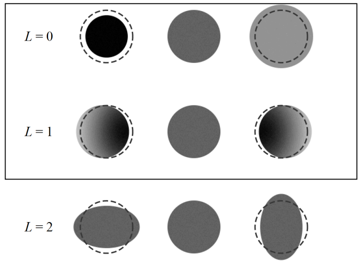
Compression mode

$$\delta\rho_{L=0}(r,E) = -\alpha_0(E)\left(3 + r\frac{d}{dr}\right)\rho_0(r),$$

$$\delta\rho_{L=1}(r,E) = -\frac{\alpha_1(E)}{R}\left[3r^2\frac{d}{dr} + 10r - \frac{5}{3}\langle r^2\rangle\frac{d}{dr} + \epsilon\left(r\frac{d^2}{dr^2} + 4\frac{d}{dr}\right)\right]\rho_0(r),$$

$$\delta\rho_{L\geq 2}(r,E) = -\alpha_l(E)\,r^{l-1}\frac{d}{dr}\rho_0(r),$$

Tassie model
(Surface vibration)

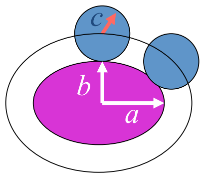


非弾性散乱から何がわかるか(変形長)

- 断面積の絶対値は、形状因子の大きさ～遷移密度の大きさで決まる。巨視的模型では、βR（変形長）の大きさ。

$$F_{\alpha'\alpha}(\vec{r}) = \left(I_A M_A \ell m \mid I_{A^*} M_{A^*}\right) \frac{\beta_\ell R_0}{\sqrt{2\ell+1}} \left.\frac{dU(r,R)}{dR}\right|_{R=R_0} Y^*_{\ell m}(\Omega)$$

- 半径cの球形プローブで、半径r、変形度βの原子核を見たときの見かけの変形度はβ' となるが、変形長は同じ

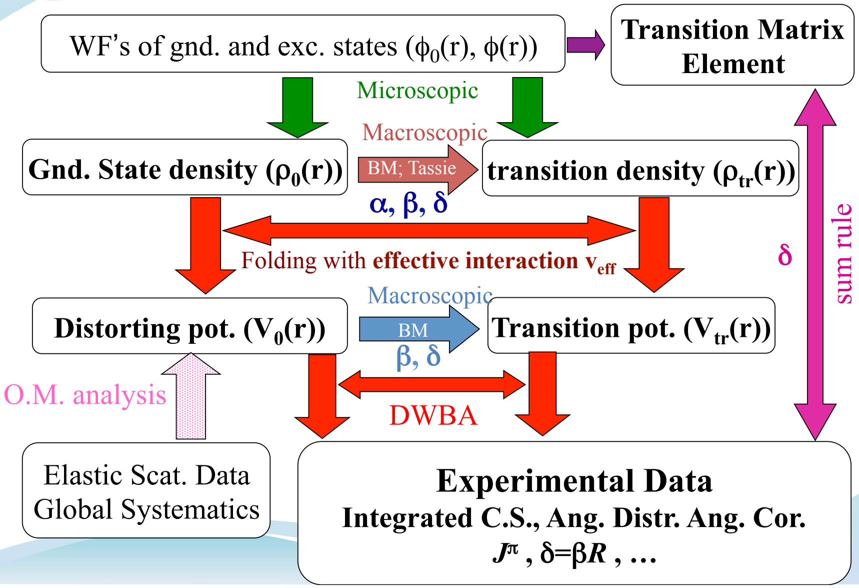


$$\beta = \text{const.}\times\frac{a-b}{a+b} = \text{const.}\times\frac{a-b}{2R}$$

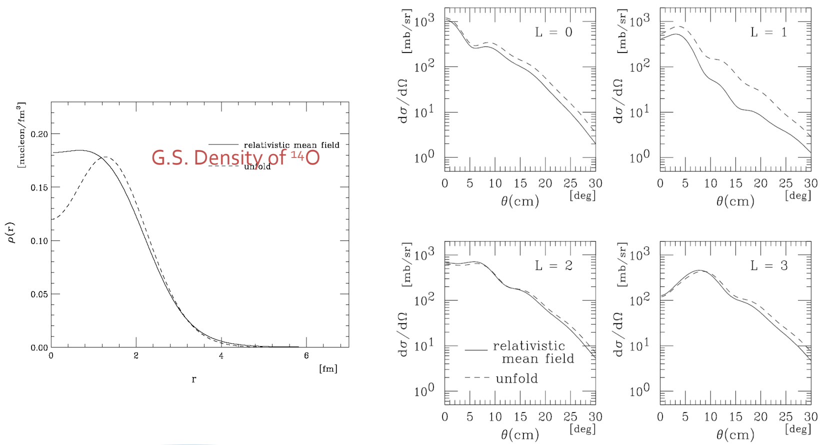
$$\beta' = \text{const.}\times\frac{a-b}{a+b+2c} = \text{const.}\times\frac{a-b}{2(R+c)}$$

$$\beta R = \beta'(R+c)$$

Analysis of Alpha inelastic excitation

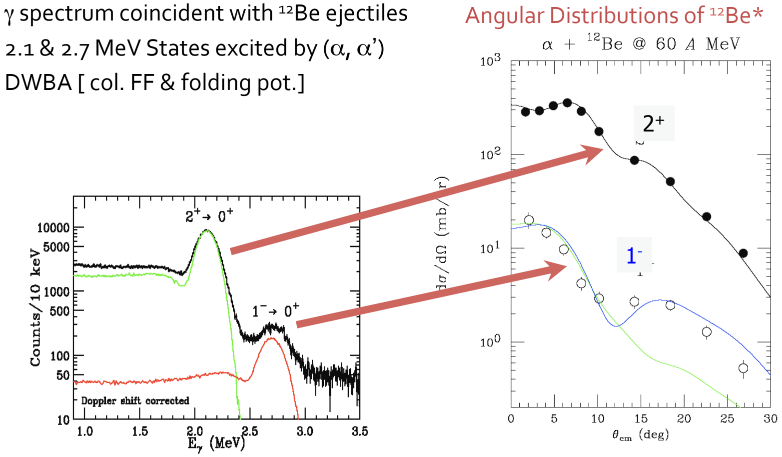


Example ^{14}O+a @ 60 A MeV

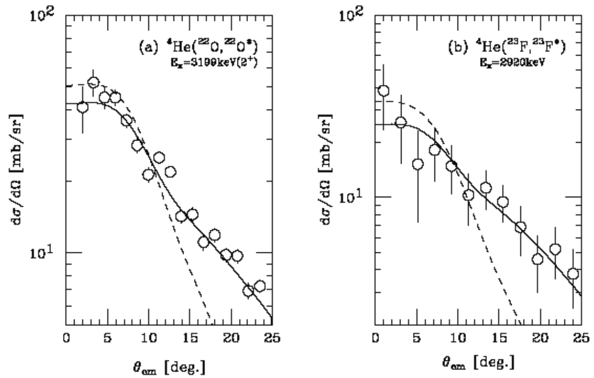


^{4}He(^{12}Be, ^{12}Be γ) at 60 A MeV

- γ spectrum coincident with ^{12}Be ejectiles
- 2.1 & 2.7 MeV States excited by (α, α')
- DWBA [col. FF & folding pot.]



^{4}He(^{22}O, ^{22}O*), ^{4}He(^{23}F, ^{23}F*)



Alignment

$$T^{PWBA}_{inelastic} = \int d^3r \exp(i\vec{q}\cdot\vec{r})\, F_L(\vec{r})$$

$$F_L(\vec{r}) = \int \psi_B{}^* \psi_b{}^* V_L(\vec{r})\, \psi_A \psi_a d\tau$$

$$= f_L(r) Y^*_{LM}(\hat{r})$$

$$T \propto \int j_L(qr)\, f_L(r)\, r^2\, dr \cdot Y_{LM}(\hat{q})$$

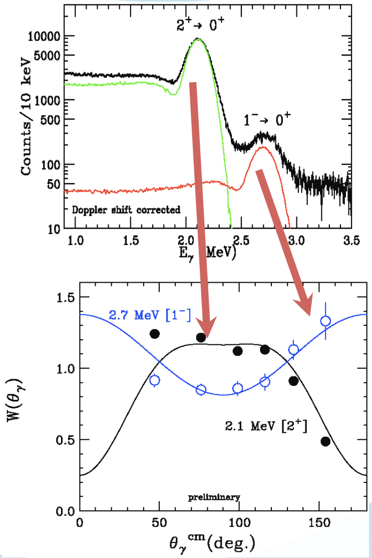
$\vec{q}$ // z-axis ⇨ M=0 : Alignment

^{4}He(^{12}Be, ^{12}Be γ) at 60 A MeV

Angular Distributions of γ (DCO)

Alignment

- 2.1 & 2.7 MeV States excited by (α, α')
- Alignments of ^{12}Be* Anisotropic Angular Distribution of γ
- Consistent with Prediction of DWBA calculation assuming 2^+ & 1^- excitation, resp.
- Confirmation of 1^- assignment for 2.7 MeV state

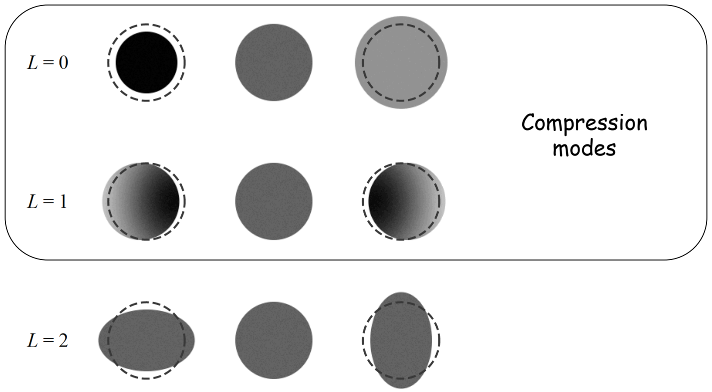


Alpha inelastic scatterings

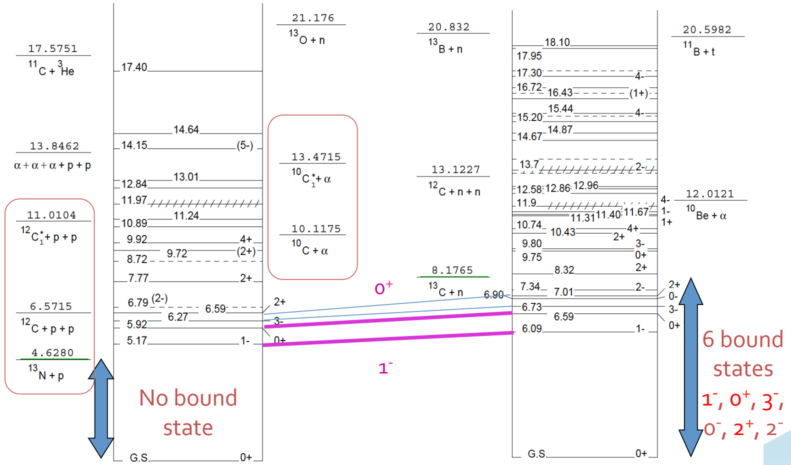
- ^{4}He(^{14}O,^{14}O*) @ 60 A MeV
  - No bound states in ^{14}O
  - ^{14}O*$\rightarrow$^{13}N+p, ^{12}C+2p, ^{12}C*+2p, ^{10}C +α, ^{10}C*+α : Invariant mass
  - Isoscalar E0 & E1 modes (compression)
    - Multipole Decomposition Analysis (**MDA**) of Isoscalar excitation
  - Effect of Continuum

Compression modes

- Isoscalar Vibration

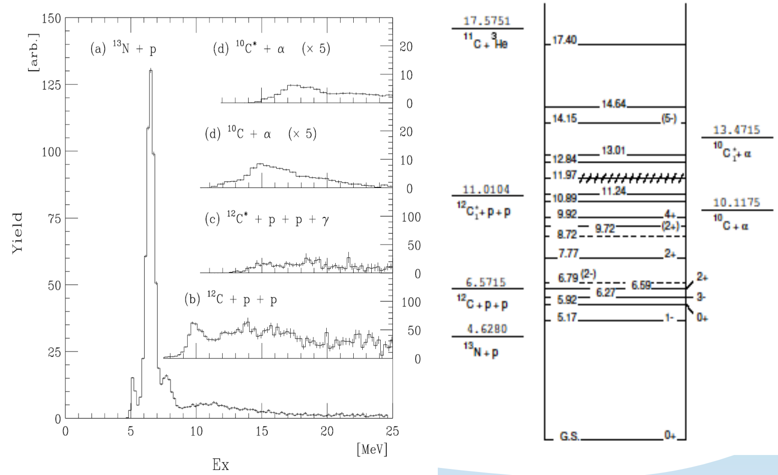


^{14}O – ^{14}C (mirror pair; sub magic)



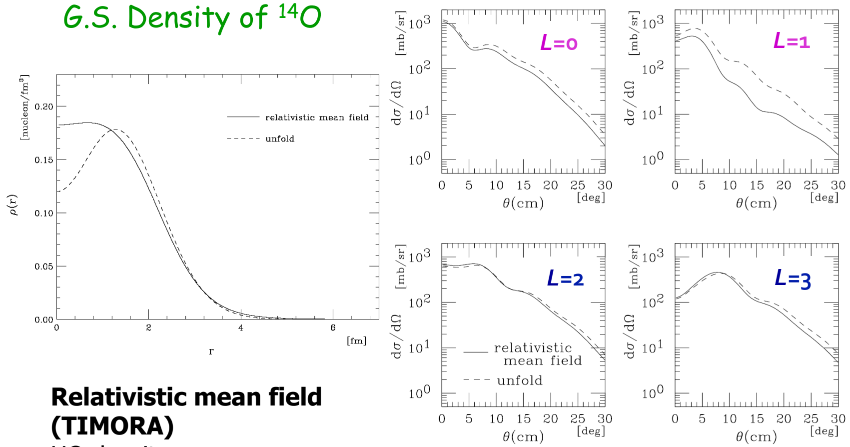
$^{4}He(^{14}O, ^{14}O^{*})$ at 60 A MeV

$^{14}O^{*} \rightarrow {}^{13}N+p$, $^{12}C+2p$, $^{12}C^{*}+2p$, $^{10}C+\alpha$, $^{10}C^{*}+\alpha$



$^{4}He(^{14}O, ^{14}O^{*})$ at 60 A MeV

G.S. Density of ^{14}O



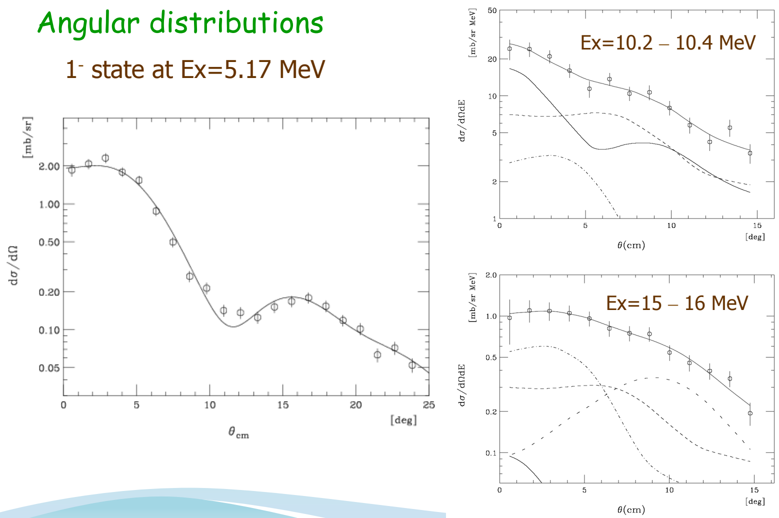
Relativistic mean field (TIMORA)
HO density
(Unfold from e-scat. on ^{14}C)

Angular distributions for two types of ^{14}O density

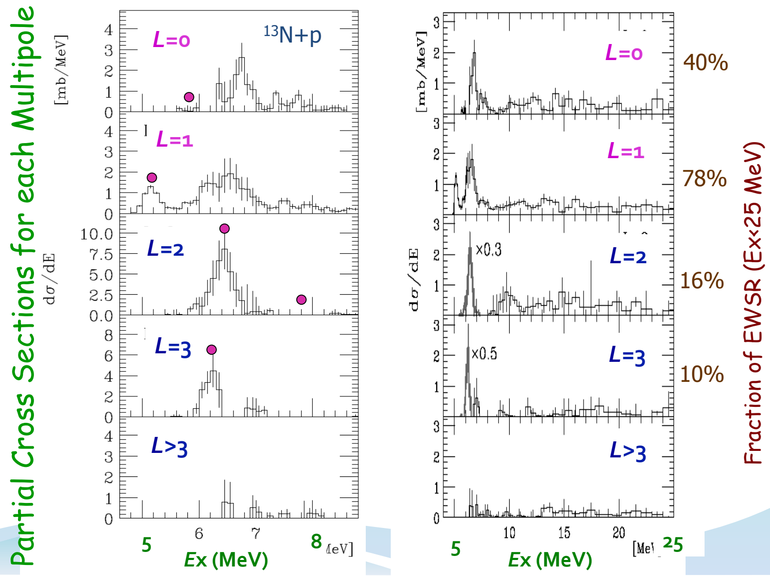
$^{4}He(^{14}O, ^{14}O^{*})$ at 60 A MeV (MDA)

Angular distributions

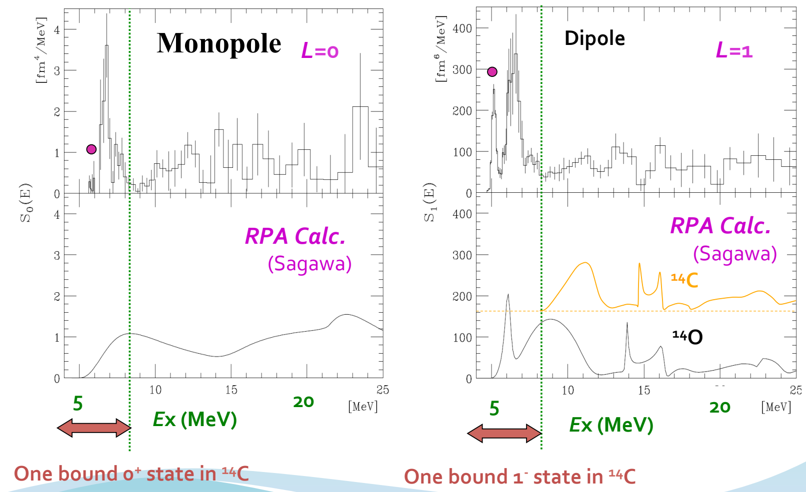
1^{-} state at Ex=5.17 MeV



^{4}He(^{14}O, ^{14}O*) - MDA

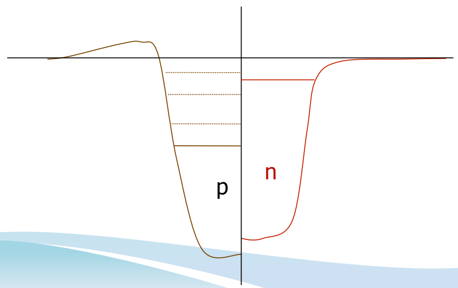


^{4}He(^{14}O, ^{14}O*) : Strength distribution



Nucleon Transfer from ^{4}He @ 30-50 A MeV

- Proton Single particle states in neutron-rich nuclei
  - **^{4}He(^{12}Be,^{13}Bγ)**
    [S. Ota et al., Phys. Lett. B 666 (2008) 311]
  - **^{4}He(^{22}O,^{23}Fγ),** [^{4}He(^{23}F,^{23}Fγ), ^{4}He(^{24}F,^{23}Fγ), He(^{25}Ne,^{23}Fγ)] utilizing cocktail beams
    [S. Michimasa et al., Phys. Lett. B 638 (2006) 146]



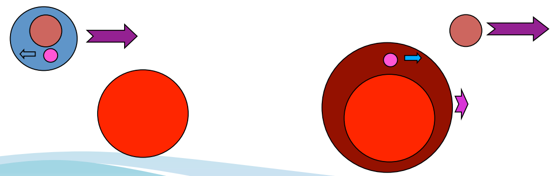
Nucleon (Proton) Transfer from Alpha

- (d,n), (^{3}He,d), (α,t), …

Incident energy higher than 30 MeV/u
✓ Thicker target (100~200 mg/cm^2)
✓ Less distortion effect
✓ Less multi-step process
✓ Same optical potential as inelastic excitation
✓ Identification by comparing with other direct reactions
But
× Momentum mismatch

Momentum Matching Condition (Classical)

- As if "Getting off from the moving train (projectile) to platform (target)"
  - Internal motion in the train (Fermi motion in the projectile) along the opposite direction of the train (target)
  - Motion on the platform (Fermi motion of the nucleon on the target) along the direction of the train

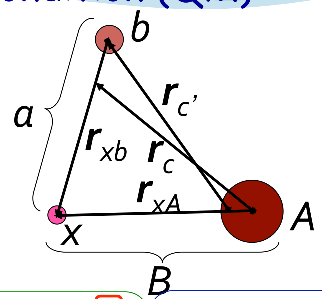


Momentum Matching Condition (QM)

$$\underset{[c]}{a + A} \rightarrow \underset{[c']}{b + B}$$

$$a = (b + x)$$

$$B = (A + x)$$



Plane Wave Born Approx.

$$T_{PW}^{\text{Post}} = \langle \phi_{c'} | V_{xb} + V_{bA} | \phi_c \rangle$$

$$T_{PW}^{\text{Prior}} = \langle \phi_{c'} | V_{xA} + V_{bA} | \phi_c \rangle$$

$$\langle \phi_{c'} | V_{xb} | \phi_c \rangle = -\frac{1}{(2\pi)^3} \frac{\hbar^2}{2\mu_{xb}} \left(k_{xb}^2 + \kappa_a^2\right) \int d^3 r_{xb} \psi_a(\vec{r}_{xb}) e^{i\vec{k}_{xb}\cdot\vec{r}_{xb}} \int d^3 r_{xA} \psi_B^*(\vec{r}_{xA}) e^{-i\vec{k}_{xA}\cdot\vec{r}_{xA}}$$

$$\langle \phi_{c'} | V_{xA} | \phi_c \rangle = -\frac{1}{(2\pi)^3} \frac{\hbar^2}{2\mu_{xA}} \left(k_{xA}^2 + \kappa_B^2\right) \int d^3 r_{xb} \psi_a(\vec{r}_{xb}) e^{i\vec{k}_{xb}\cdot\vec{r}_{xb}} \int d^3 r_{xA} \psi_B^*(\vec{r}_{xA}) e^{-i\vec{k}_{xA}\cdot\vec{r}_{xA}}$$

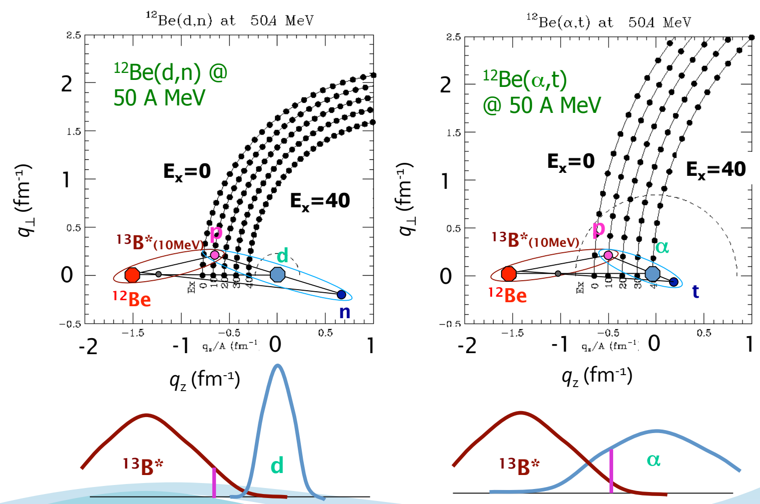
Fourier Components of wave functions of nucleon (*x*) in *a* and *B*

$$\vec{k}_{xb} = \frac{b}{a}\vec{k}_c - \vec{k}_{c'}; \quad \vec{k}_{xA} = \vec{k}_c - \frac{A}{B}\vec{k}_{c'}$$

Same value but different expressions

$$\frac{\hbar^2 \kappa_a^2}{2\mu_{xb}} = \varepsilon_a; \quad \frac{\hbar^2 \kappa_B^2}{2\mu_{xA}} = \varepsilon_B$$ Binding energies of *a* and *B*

Proton Transfer in Momentum Space

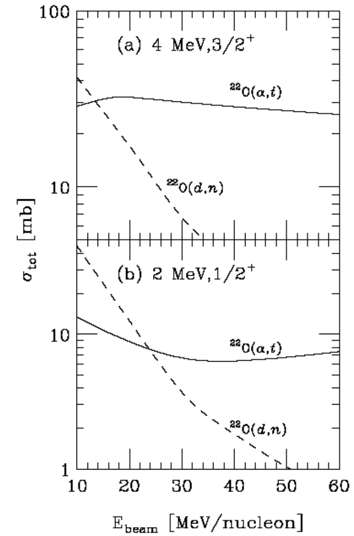


DWBA Calculation

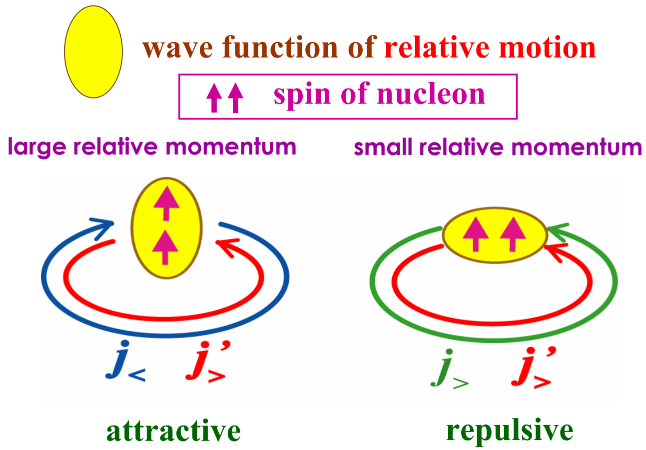
Example:

$^{22}O(\alpha,t)^{23}F$

$^{22}O(d,n)^{23}F$



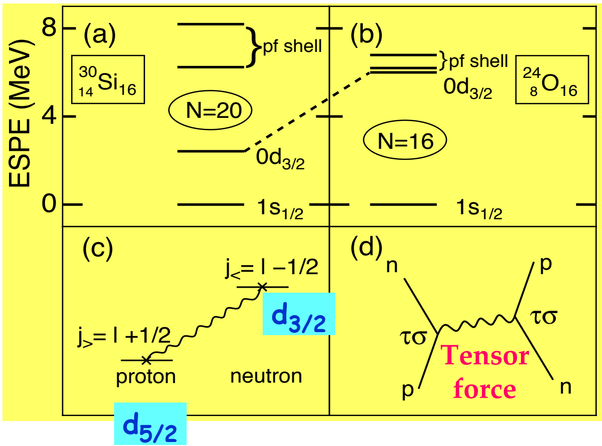
Intuitive picture of monopole effect of tensor force



$j_> = l + \frac{1}{2},\quad j_< = l - \frac{1}{2}$

TO *et al.*, Phys. Rev. Lett. 95, 232502 (2005)

N=16 gap : Ozawa, et al., PRL 84 (2000) 5493;
Brown, Rev. Mex. Fis. 39 21 (1983)

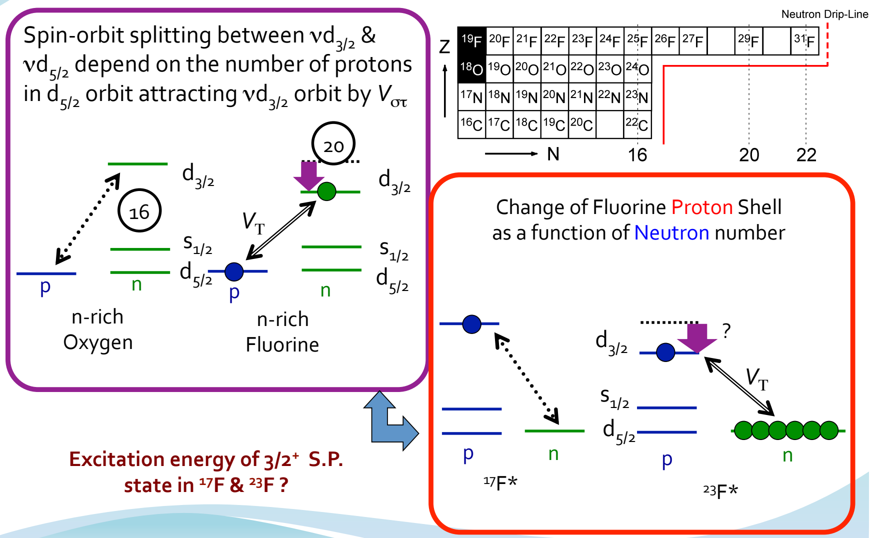


only exchange term

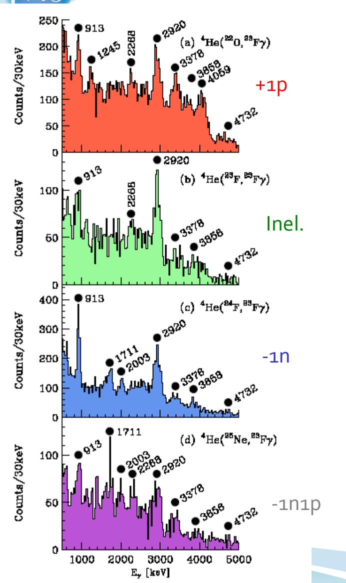
Example : Dripline of F isotopes is 6 units away from O isotopes

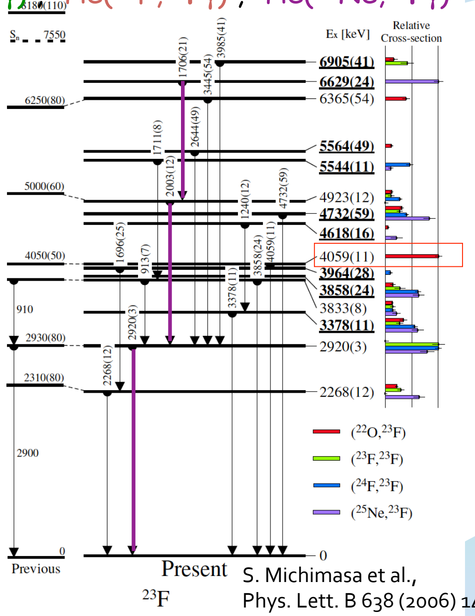
Sakurai *et al.*, PLB 448 (1999) 180, …

Exotic Magic Number at N=16 (Z~8)



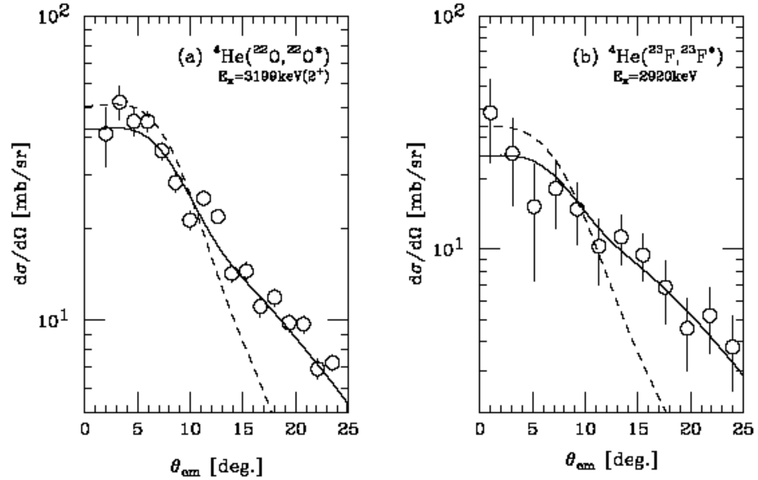
$^4He(^{22}O,^{23}F\gamma)$, $^4He(^{23}F,^{23}F\gamma)$, $^4He(^{24}F,^{23}F\gamma)$, $^4He(^{25}Ne,^{23}F\gamma)$





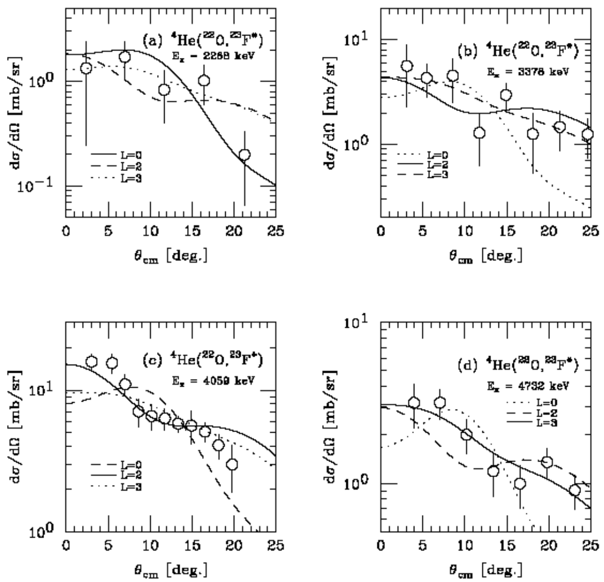
S. Michimasa et al.,
Phys. Lett. B 638 (2006) 146

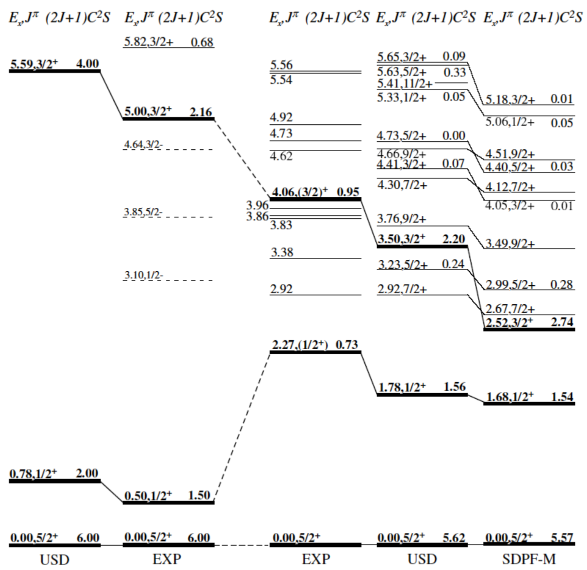
$^4He(^{22}O,\ ^{22}O^*)$, $^4He(^{23}F,\ ^{23}F^*)$



S. Michimasa et al., Phys. Lett. B 638 (2006) 146

$^{22}O(a,t)^{23}F^*$





^{17}F $\nu d_{5/2}$ empty

^{23}F $\nu d_{5/2}$ full

S. Michimasa et al., Phys. Lett. B 638 (2006) 146

SHARAQ

RI beam as probe

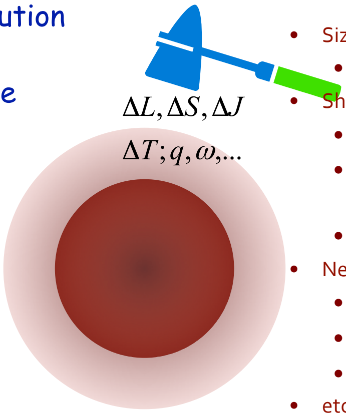
Charge Exchange

spin-isospin response

Studies of Nuclei via Direct reactions

- Size/ρ-distribution
  - Skin/Halo
- Shell Structure
  - New magic #
  - Isospin / Deformation
- New modes
  - IVE1
  - ISE0, ISE1
- etc.

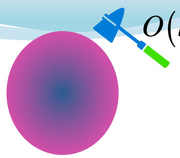
Direct Reactions



- Size/ρ-distribution
  - σ_R, elastic scat.
- Shell Structure
  - Mass / S_n, S_{2n}
  - Inelastic scatt.
    - Low lying states
  - Knockout / Transfer
- New modes
  - Coulex
  - Inelastic scatt.
  - CEX
- etc.

"Hit and analyze the sound"

Transition Probabilities



$$M_{if} = \langle E_f J_f \pi_f T_f; \xi_f \| O(lsj\tau;\xi) \| E_i J_i \pi_i T_i; \xi_i \rangle$$

$$\text{Cross Section} \propto |M_{if}|^2 \;\; ; \;\; \text{Lifetime} \propto 1/|M_{if}|^2$$

$O(lsj\tau;\xi)$: Propety of Reaction / Aciton / Decay Processes

sum of one-body operator

e.g.

$$O(lsj\tau;\vec{r}) = \sum_i f(r_i) T(\tau_i) [S(\sigma_i) \otimes Y_l(\hat{r}_i)]_j$$

$|E_i J_i \pi_i T_i; \xi_i\rangle$ and/or $|E_f J_f \pi_f T_f; \xi_f\rangle$ energy eigen functions

$$O(lsj\tau;\xi) |E_i J_i \pi_i T_i; \xi_i\rangle = \sum_f M_{if}(E_f) |E_f J_f \pi_f T_f; \xi_f\rangle$$

$|M_{if}(E_f)|^2$: Energy Spectrum

Response

coherent sum of wave packets made by one-body action

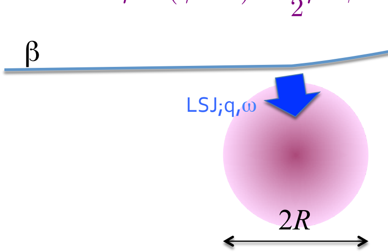
"Collective wave packet" (not always energy eigen state),

e.g. coherent sum of 1p-1h for inelastic-type excitation

Decoupling of "Scattering" and "Transition" for intermediate-energy "inelastic scattering"

Criteria for decoupling

$$\omega \ll \mu c^2 (\gamma - 1) \simeq \frac{1}{2}\mu c^2 \beta^2$$



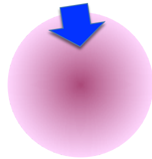
$$\Delta E \cdot \Delta t \sim 2\pi\hbar$$

$$\omega_{\max} \sim \frac{2\pi\hbar \cdot \beta c}{2R} \simeq 100\beta \ \ \mathrm{MeV}$$

Off energy shell

E/A > 100 MeV satisfies the decoupling conditions
E/A ~ 10 MeV may be marginal

"Transition" as time-dependent action



$$i\hbar\frac{\partial}{\partial t}\Psi(t) = (H + V_R(t))\,\Psi(t)$$

$$\Psi(t) = \sum_i a_i(t)\,\psi_i \exp(-iE_i t/\hbar)$$

$$H\psi_i = E_i\psi_i$$

$$a_0(-\infty) = 1 \ \ ; \ \ a_i(-\infty) = 0 \ \ \text{for } i > 0$$

$$|a_i(+\infty)|^2 : \text{Energy spectrum after reaction}$$

$$\sum_i i\hbar\,\dot{a}_i(t)\,\psi_i \exp(-iE_i t/\hbar) = \sum_i a_i(t)\,V_R(t)\,\psi_i \exp(-iE_i t/\hbar)$$

$$i\hbar\,\dot{a}_k(t) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{t^2}{2\Delta T^2}\right) \times \sum_i a_i(t)\,\langle\psi_k|\mathcal{O}|\psi_i\rangle \exp\left(-\frac{i(E_i - E_k)t}{\hbar}\right)$$

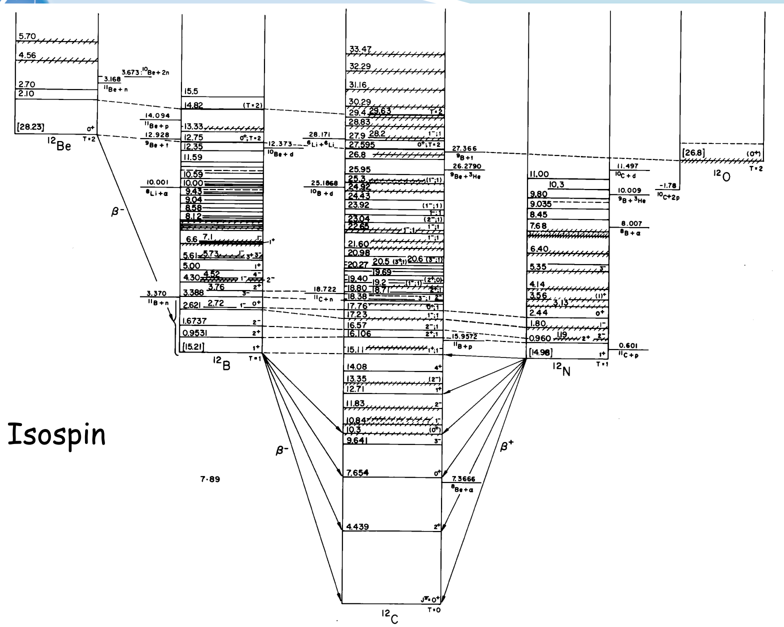
$$V_R(t) = \frac{\mathcal{O}}{\sqrt{2\pi}}\exp\left(-\frac{t^2}{2\Delta T^2}\right)$$

Perturbation

$$a_i(-\infty) \ll 1 \quad \text{for } i > 0$$

$$a_0(+\infty) - a_0(-\infty) \simeq -i\frac{\Delta T}{\hbar}\langle\psi_0|\mathcal{O}|\psi_0\rangle$$

$$a_k(+\infty) \simeq -i\frac{\Delta T}{\hbar}\langle\psi_k|\mathcal{O}|\psi_0\rangle \exp\left(-\frac{(E_{i0}\Delta T)^2}{2\hbar^2}\right)$$

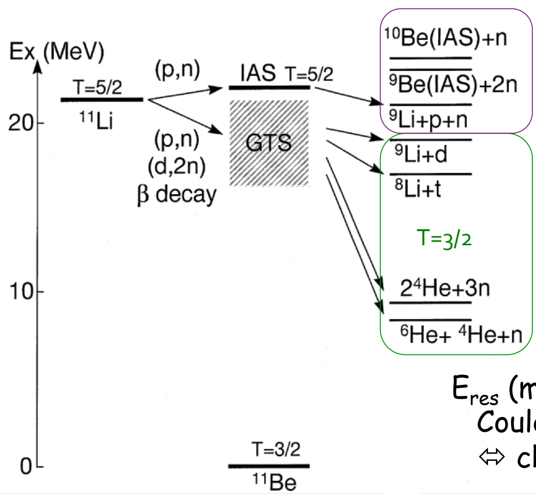


Isospin

Before exp. in SHARAQ+RIBF, CX reaction w/ inv. mass spec. was applied for

IAS of Borromean nuclei

^{11}Li: T. Teranishi et al., PLB 407 ('97) 110
^{14}Be: S. Takeuchi et al., PLB 515 ('01) 255



T=5/2

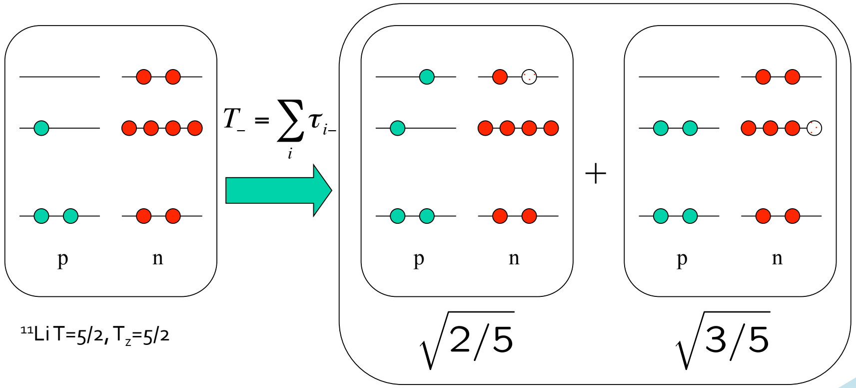
IAS appears as Resonance due to Coulomb interaction

E_{res} (mass):
Coulomb displacement E
⇔ charge distribution

IAS of Borromean nuclei

^{11}Li: T. Teranishi et al., PLB 407 ('97) 110
^{14}Be: S. Takeuchi et al., PLB 515 ('01) 255

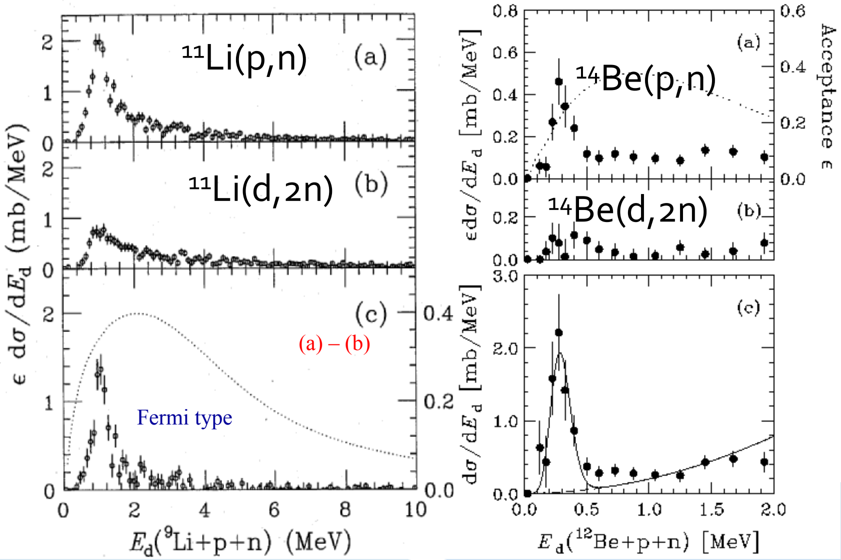
Isospin symmetry in nuclear force
isospin multiplet in nuclei



^{11}Li T=5/2, T$_z$=5/2

^{11}Be* T=5/2,T$_z$=3/2

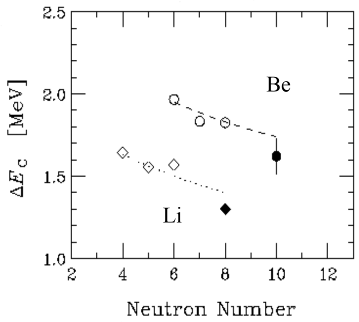
Decay Energy Spectra



Coulomb displacement energy

E_{res} = 1 MeV

E_x = 20.15 MeV, ΔE_C = 1.3 MeV



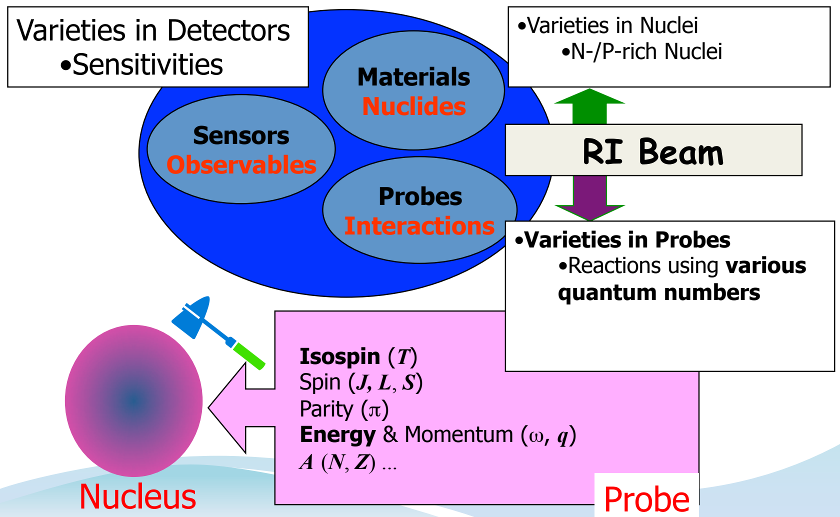
$$\Delta E_c(^{11}\mathrm{Li}) = \frac{3}{5}\Delta E_c(^{9}\mathrm{Li}) + \frac{2}{5}\Delta E_c(2N)$$

ΔE_C(2N) = 0.9 MeV

ΔE_C (2N) = Ze^2 <1/R(halo)> => <1/R(halo)>= (5 fm)$^{-1}$

Motivation of SHARAQ project

- Exp. Studies of Nuclear Many-body System



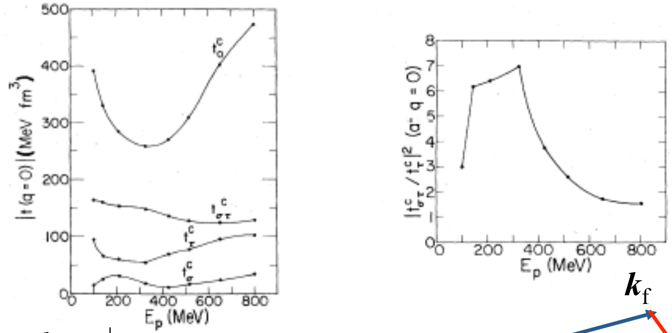
Points of View

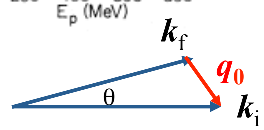
- Response of Nuclear System using Intermediate-Energy direct reactions
  - Studies using New Quantum Probe—RI beam— Large Isospin and Mass Excess、Various I^{π} (“Excited states”)
    - Controlling Transferred Momenta, Q-values, Spin, Isospin
    - ΔS, ΔT, **q-ω**
    - Accessing kinematical area/conditions inaccessible by stable nuclear beam
  - Ordinary kinematics (Missing mass spectroscopy)
    -> High Resolution Spectrometer + High Quality RI Beam
    \+ (Detectors of decaying particles)
  - Asymmetric nuclear System studied using stable probes
    - Inverse kinematics + Invariant Mass / γ-decay / Recoil and **High-resolution missing-mass spectroscopy**

We may free from conventional (kinematical) conditions

Beam Energy at RIBF (200-300 A MeV)

- Energy dependence of
  - Distortion : Central force
  - Effective Interaction : Spin-Isospin responses





$$\frac{d\sigma}{d\Omega}(\theta)\bigg|_{PWIA} \approx C\left|t_{ST}(q_0)\cdot \tilde{\rho}_{fi}(\vec{q}_0)\right|^2$$

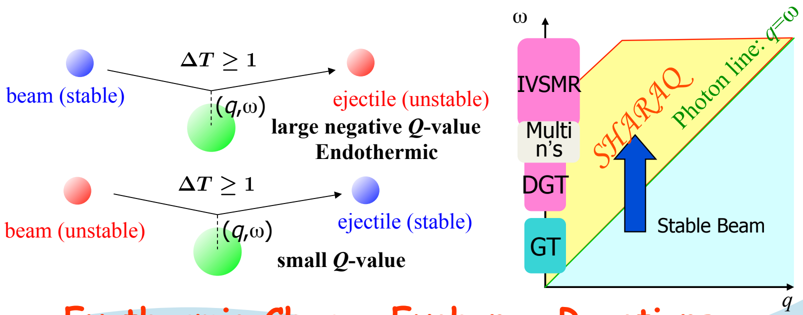
$$\tilde{\rho}_{fi}(\vec{q}) = \int d^3r\,\rho_{fi}(\vec{r})e^{i\vec{q}\cdot\vec{r}} = \int dr\,r^2 R^{tr}_{\ell}(r) j_{\ell}(qr) Y_{lm}(\hat{q})$$

SHARAQ

Spectroscopy with High-resolution Analyzer of RadioActive Quantum beams

RI Beam (E = 150—400 MeV/A) as a new PROBE to nuclear systems

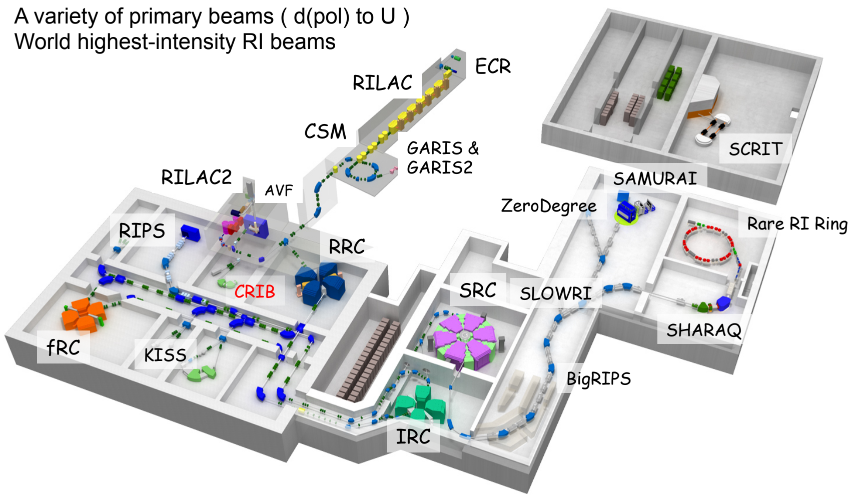
- Large Isospin — iso-tensor excitations
- Large internal energy — (q,ω) inaccessible by stable beams



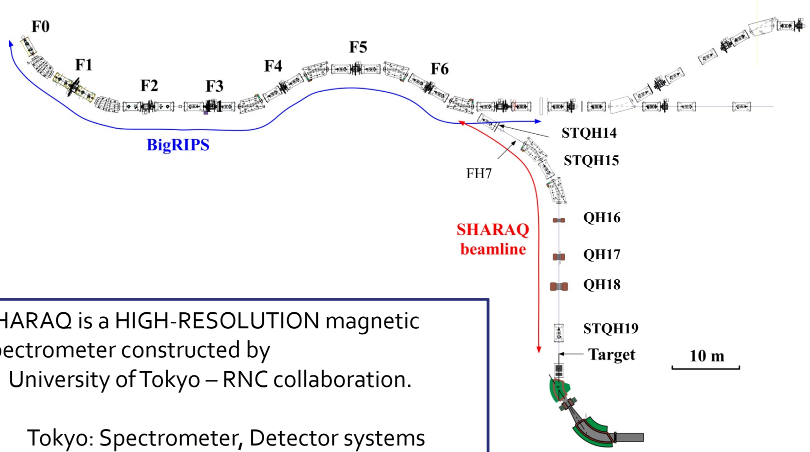
Exothermic Charge Exchange Reactions

RI Beam Factory at RIKEN

3 injectors + cascade of 4 cyclotrons
⇒ several to 345 MeV/nucleon
A variety of primary beams (d(pol) to U)
World highest-intensity RI beams



SHARAQ @ RI beam factory

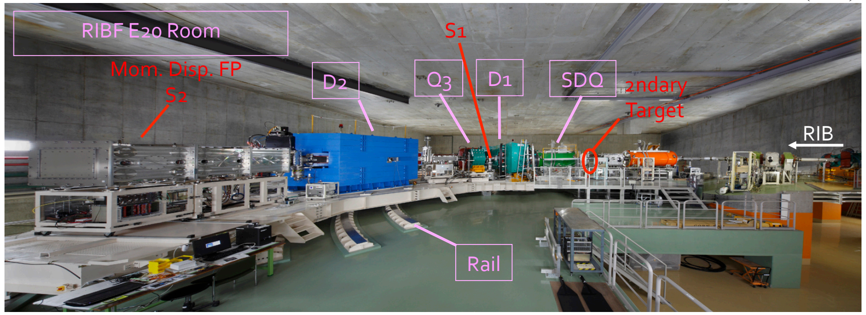


SHARAQ is a HIGH-RESOLUTION magnetic spectrometer constructed by
University of Tokyo – RNC collaboration.

Tokyo: Spectrometer, Detector systems
RNC: Beam-line, Infrastructure

SHARAQ spectrometer

T. Uesaka et al.,
NIMB B **266** (2008) 4218.
PTEP 2012, 03C007 (2012)

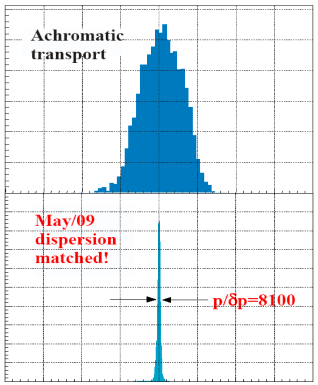


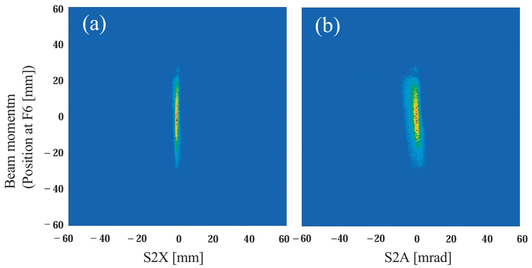
Maximum rigidity	**6.8 Tm**
Momentum resolution	**d*p*/*p* = 1/14700**
Angular resolution	**~ 1 mrad**
Momentum acceptance	**± 1%**
Angular acceptance	**~ 5 msr**

ICHOR

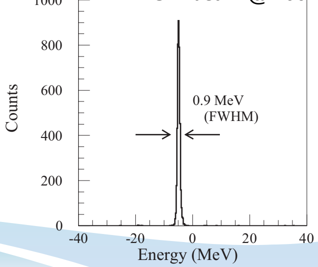
Dispersion Matching Mode

^{14}N primary beam





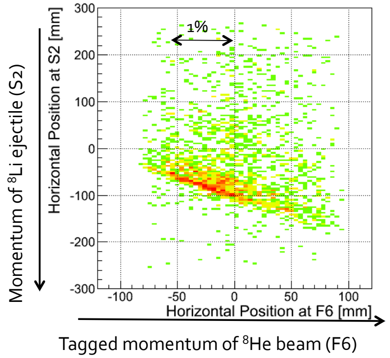
^{10}C RI beam @ 200 A MeV



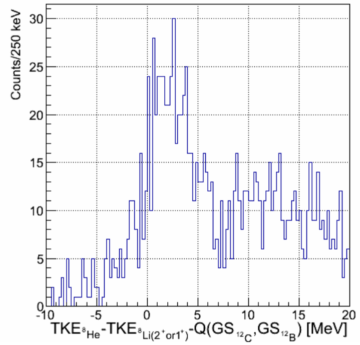
p/δp~4000

High Resolution Achromatic Mode

Beam momentum is tagged at F6 ((x|d)~7000)
Ejectile momentum is measured by SHARAQ



Preliminary momentum spectrum of (^{8}He,^{8}Li)



Preliminary missing mass spectrum (^{8}He,^{8}Li) @190 A MeV

E(^{8}He) = 760 MeV

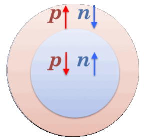
Isovector Spin Monopole Response

Spin-isospin ($\Delta S = \Delta T = 1$) modes with $\Delta L = 0$

Gamow-Teller $0\hbar\omega$ $\sum_k t_\pm(k)\sigma_\mu(k)$

Isovector Spin Monopole $2\hbar\omega$ $\sum_k t_\pm(k)\sigma_\mu(k)r(k)^2$

└ **Compression mode**



Energy centroid $\bar{E}$, width Γ of IVSMR

→ isovector spin-incompressibility

effective interaction in **spin-isospin channel**

residual interaction, V_{pp}^{IVSM}, V_{ph}^{IVSM} (?)

Sum rule (model-independent)

$$S_- - S_+ = 3(N\langle r^4\rangle_n - Z\langle r^4\rangle_p)$$

→ neutron skin thickness $\delta_{np} = \sqrt[4]{\langle r^4\rangle_n} - \sqrt[4]{\langle r^4\rangle_p}$

constraint on neutron matter equation of state

^{90}Zr, ^{208}Pb (t, ^{3}He) at 300 MeV/u

K. Miki et al., PRL 108, 262503

Primary : ^{4}He 320MeV/u 300pnA

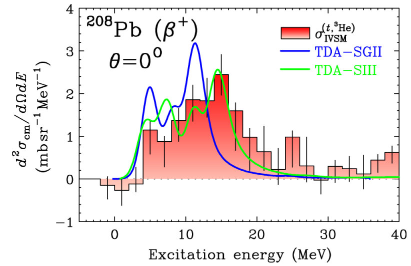
Secondary : triton 300MeV/u 10^7pps

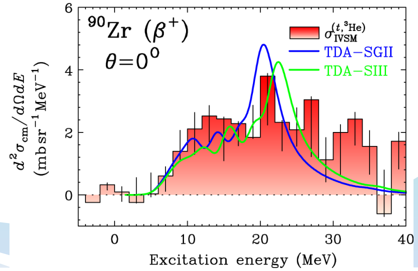
Purity > 99%

First identification of β^+-type isovector spin monopole resonances

High-intensity RI-beam + high-resolution mag. analysis

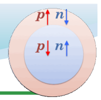
→ New probes to nuclei





Exothermic (^{12}N, ^{12}C) Reaction

β⁻ direction

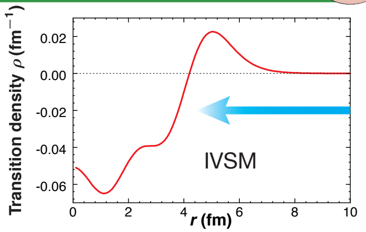


Advantages

(1) Spin-isospin selectivity

$^{12}N_{gs}(1^+; T=1) \rightarrow {}^{12}C_{gs}(0^+; T=0)$

→ $S_t = 1$, $T_t = 1$

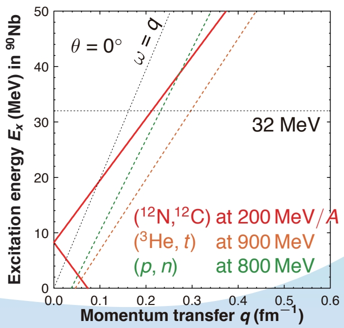


(2) Surface sensitivity

∵ strong absorption of HI reaction

probes only surface of nucleus

IVSM: transition density has a node at surface



(3) Small q for high E_x

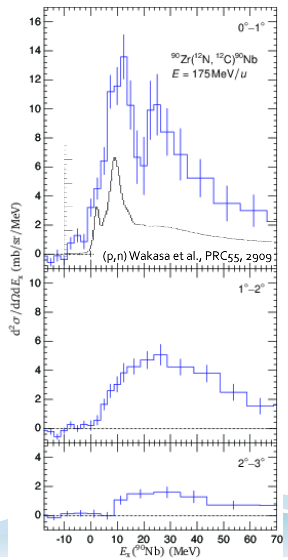
∵ large mass difference of proj. and ejec.

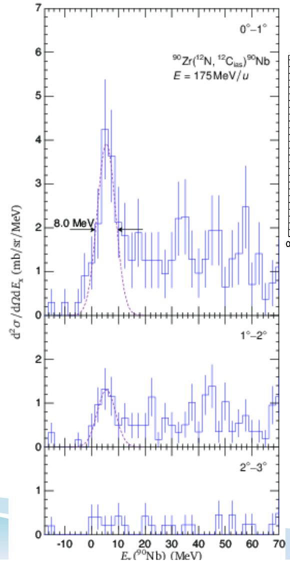
favors $\Delta L = 0$ excitations

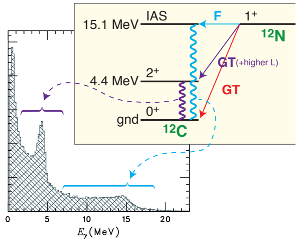
Noji et al.

(^{12}N(1$^+$;T=1,^{12}C(0$^+$;T=0): GT)

(^{12}N(1$^+$;T=1,^{12}C(1$^+$;T=1): Fermi+GT)



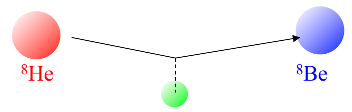




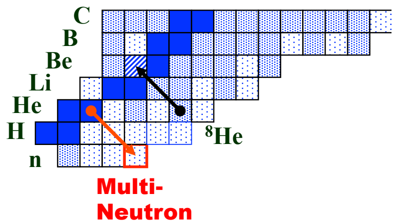
S. Noji et al.

Tetraneutron

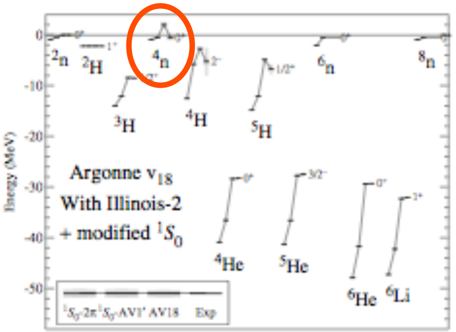
Tetra-neutron system produced by exothermic double-charge exchange reaction



Recoil-less 4n system via DCX using internal energy of ^{8}He



Susumu Shimoura
for CNS-RNC-TITech-Kyoto-RCNP-Miyazaki-IPN collaboration



S.C. Pieper et al., PRL 90, 252501 (2003)

4n in breakup of ^{14}Be : Marques et al. PRC 65 (2002) 044006

Tetra-neutron

- Multi-neutron System
  - Neutron cluster (?) in fragmentation of ^{14}Be
    PRC65, 044006 (2002)
  - NN, NNN, NNNN interactions
    - T=3/2 NNN force
      - -> 3-body force in neutron matter
    - Ab initio type calculations
  - Multi-body resonances
  - Correlations in multi-fermion scattering states

Historical Review

~ search for a bound state of 4n~

1960s fission of Uranium

- No evidence for particle stable state of tetra-neutron

J. P. Shiffer Phys. Lett. 5, 4, 292 (1963)

1980s ^{4}He(π^-, π^+) reaction

- Only upper limit of cross section was decided.

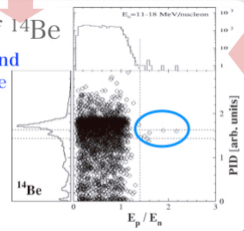
J. E. Unger, et al., Phys. Lett. B 144, 333 (1984)

Bound state: No clear evidence.

2000s Breakup of ^{14}Be

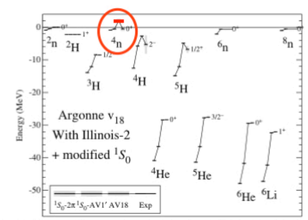
- Candidates of bound tetra-neutron were observed.

F. M. Marques, et al, Phys. Rev. C 65, 044006 (2002)



2000s Theoretical work

- ab-initio calculation
  NN, NNN interaction



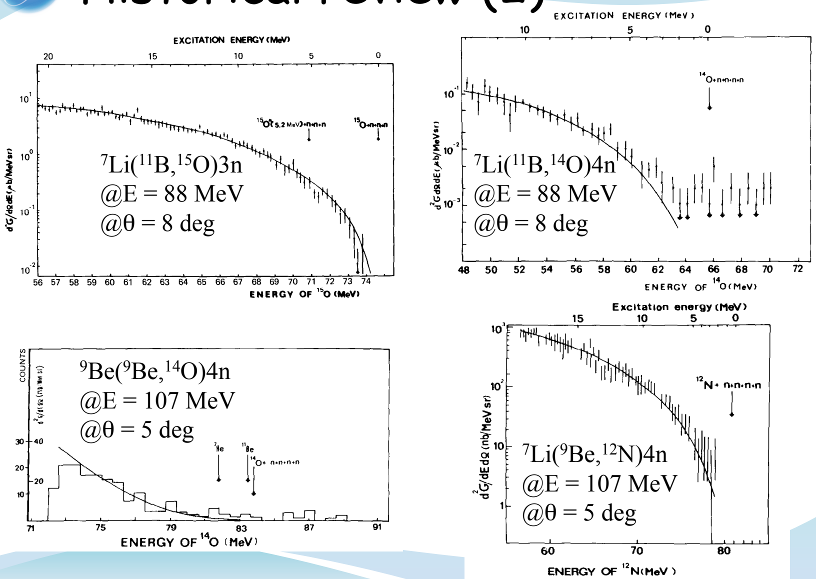
S. C. Piper, Phys. Rev. Lett. 90, 252501 (2003)

- Bound ^{4}n cannot exist
- Possible resonance stete ~2 MeV

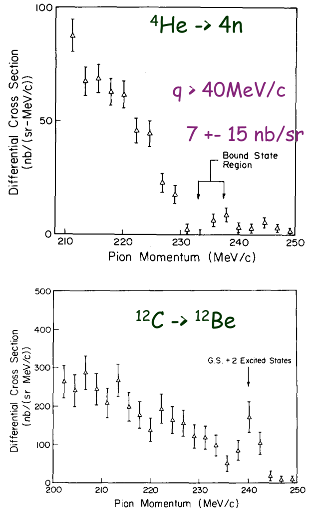
Resonance state : Possibility of the state is still an open and fascinating question.

Historical review (2)

Nucl. Phys. A477 (1988) 131



(π^-,π^+) reaction @ 165 MeV; $\theta_{\pi+}$ = 0 degree



We have measured the momentum spectrum of π^+ produced at 0° by 165 MeV π^- on ^{4}He. A $\Delta P/P$ = 1% beam of 10^6 π^- per second was provided by the P^3 line of the Los Alamos Meson Physics Facility, and a cell of 910 mg/cm^2 liquid ^{4}He with windows of 18 mg/cm^2 Kapton served as the target [15]. An

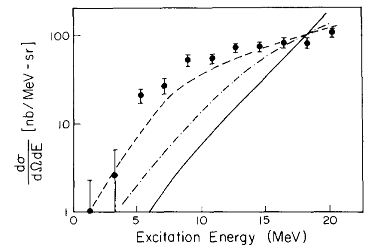
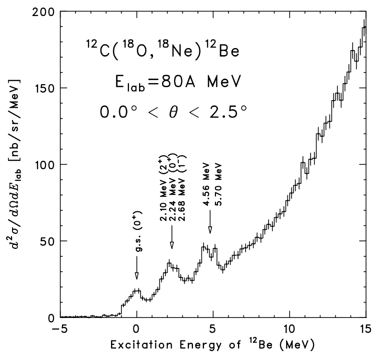


Fig. 3. The experimental results are plotted against the excitation of the final four-neutron state. The solid curve corresponds to the pure four-neutron phase space, while the dot–dashed and dashed curves are the four-neutron phase space curves with singlet state interactions in, respectively, one and both of the final state neutron pairs.

J.E. Ungar et al., PLB 144 (1987) 333

Double charge exchange (DCX) reaction of HI

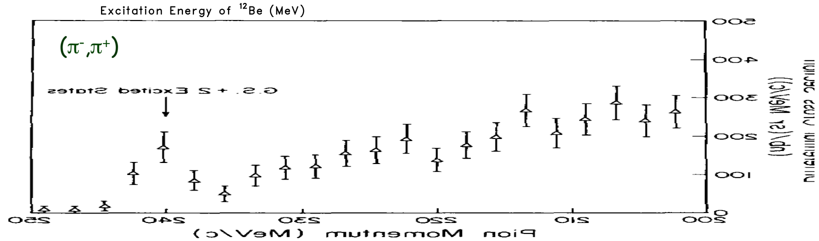


^{12}C -> ^{12}Be

Stable^{18}O beam (80A MeV) (Takaki et al.)

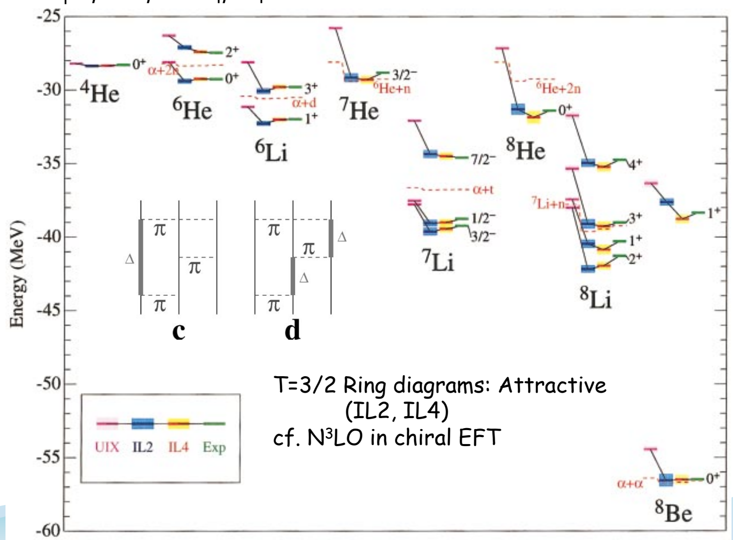
~70nb/sr (Gnd)
~200nb/sr (~2MeV)

HI DCX reaction can be used for spectroscopy for exotic nuclei
(q is not so small >80 MeV/c)

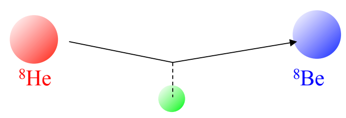


3-body force

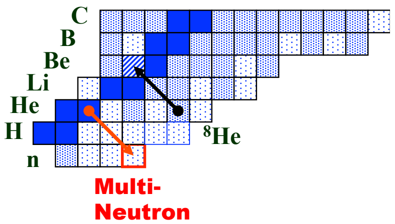
S. C. Pieper, et al., PRC 64, 014001



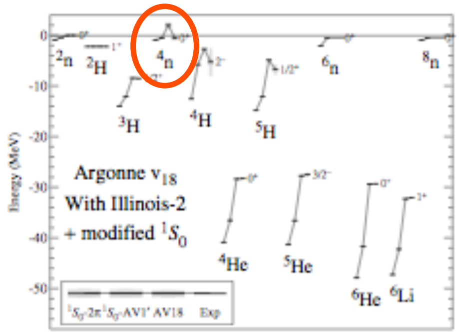
Tetra-neutron system produced by exothermic double-charge exchange reaction



Recoil-less 4n system via DCX using internal energy of ^{8}He



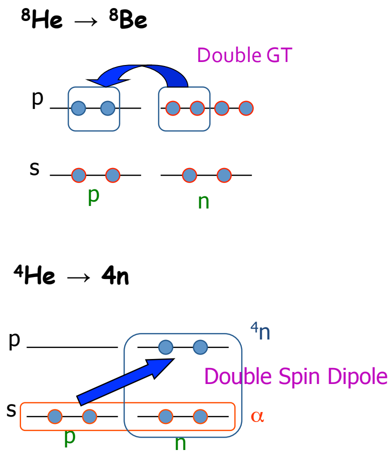
Almost recoil-less condition with ^{4}He(^{8}He,^{8}Be)4n reaction at 200 A MeV

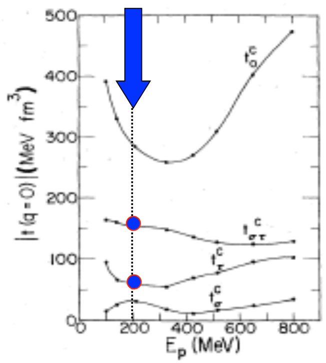


S.C. Pieper et al., PRL 90, 252501 (2003)

4n in breakup of ^{14}Be : Marques et al. PRC 65 (2002) 044006

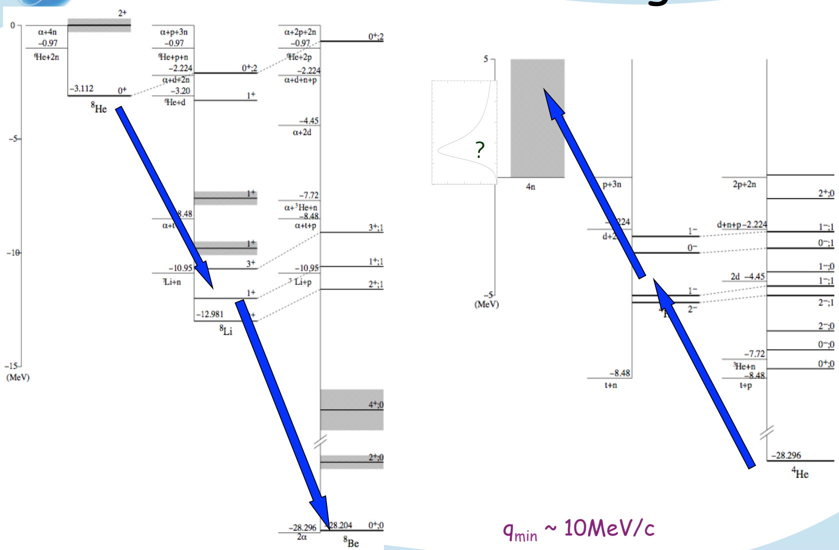
Reaction Mechanism



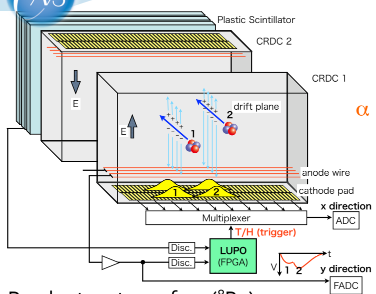


$$\left[\left(\vec{\tau}_p\cdot\vec{\tau}_t\right)\left(\vec{\sigma}_p\cdot\vec{\sigma}_t\right)r_tY_1\left(\hat{r}_t\right)\right]^2$$

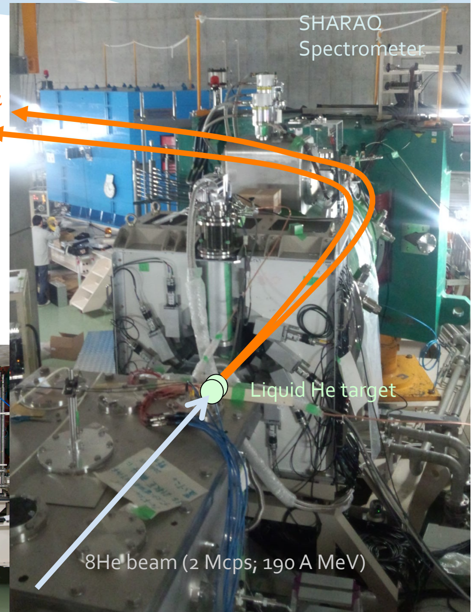
Level diagrams



q_{min} ~ 10MeV/c



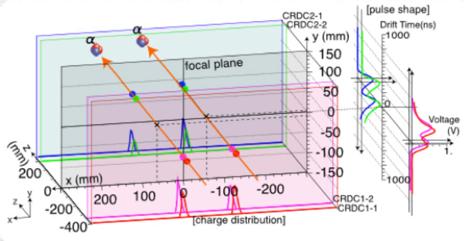
Readout system of 2α (^{8}Be)



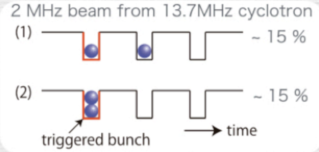
Analysis

- Selection of 4n Events
  - Extracting 2α events @SHARAQ
  - Multi-particle in high-intensity beam

Background process:
Breakup of two ^{8}He in the same beam bunch to two alpha particle
Identified by multi-hit in F6-MWDC

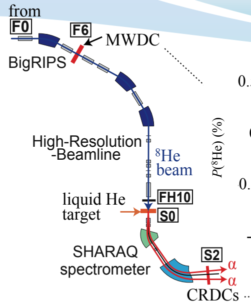


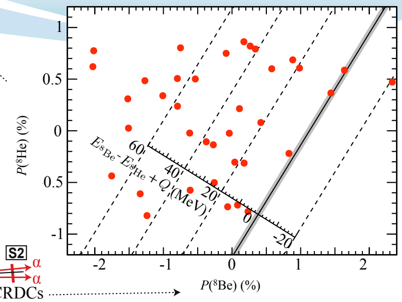
- Background Estimation
  - Shape in spectrum: random 2α
  - Number of events:
    - failure of the multi-particle rejection at MWDC
    - multi-particle in one cell of MWDC

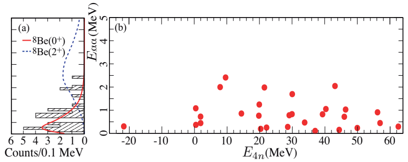


Backgrounds after analysis:
Finite efficiency of multi-hit events at F6-MWDC

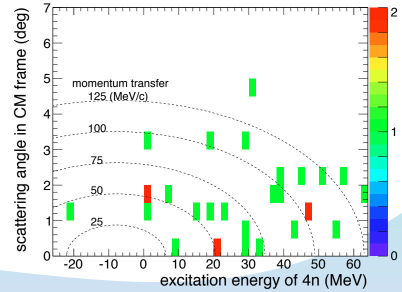
Experimental Results



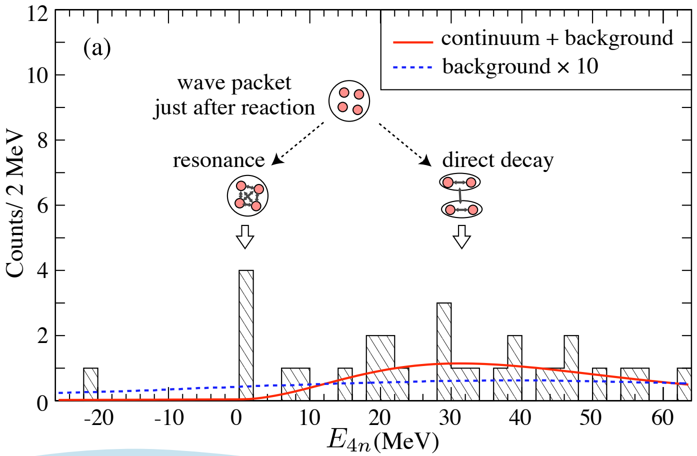




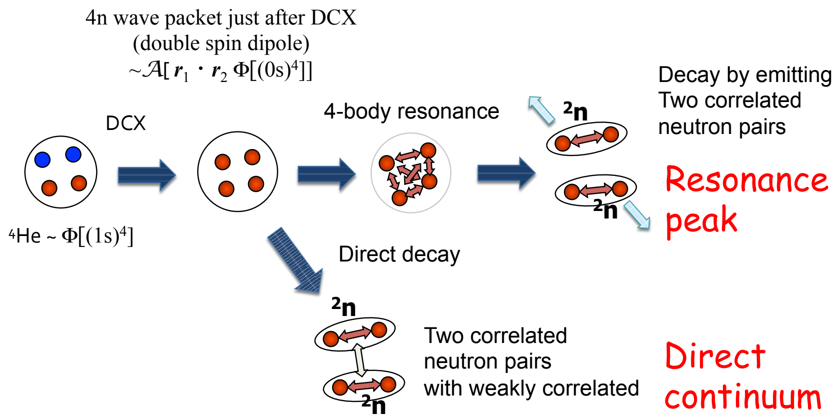
Acceptance for ^{8}Be(2+) was 13 % of that for ^{8}Be(0+)
A few events could be from ^{8}Be(2+).



Experimental Results



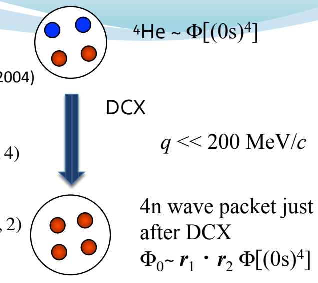
Picture of ^{4}He DCX reaction @ 200 A MeV



Direct Part

Continuum spectrum with n-n FSI

c.f.
L.V. Grigorenko, N.K. Timofeyuk, M.V. Zhukov, Eur. Phys. J. A 19, 187 (2004)



$\mathcal{A}\Phi_0(\boldsymbol{r}_{12}, \boldsymbol{r}_{34}, \boldsymbol{r}_\alpha) \sim$

$$\left[\left(\frac{r_{12}^2}{a^2}-\frac{3}{2}\right)-\left(\frac{r^2}{a^2}-\frac{3}{4}\right)\right]\exp\left[-\frac{r^2}{a^2}-\frac{r_{12}^2}{2a^2}-\frac{r_{34}^2}{2a^2}\right]\chi(1,2)\chi(3,4)$$

$$\left[\left(\frac{r_\alpha^2}{(a/\sqrt{2})^2}-\frac{3}{2}\right)-\frac{2\vec{r}_{12}\cdot\vec{r}_{34}}{a^2}\right]\exp\left[-\frac{r_\alpha^2}{a^2}-\frac{r_{12}^2}{2a^2}-\frac{r_{34}^2}{2a^2}\right]\chi(1,3)\chi(4,2)$$

$$\left[\left(\frac{r_\alpha^2}{(a/\sqrt{2})^2}-\frac{3}{2}\right)+\frac{2\vec{r}_{12}\cdot\vec{r}_{34}}{a^2}\right]\exp\left[-\frac{r_\alpha^2}{a^2}-\frac{r_{12}^2}{2a^2}-\frac{r_{34}^2}{2a^2}\right]\chi(1,4)\chi(2,3)$$

$$\vec{r}_\alpha = \frac{\vec{r}_1+\vec{r}_2}{2}-\frac{\vec{r}_3+\vec{r}_4}{2} \qquad \chi(i,j) = \frac{1}{\sqrt{2}}(\uparrow(i)\downarrow(j)-\downarrow(i)\uparrow(j))$$

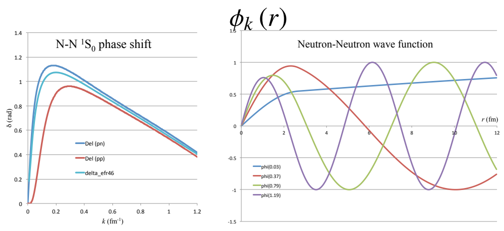
Fourier Transform: $(\boldsymbol{r}_{12}, \boldsymbol{r}_{34}, \boldsymbol{r}_\alpha) \to (\boldsymbol{k}_{12}, \boldsymbol{k}_{34}, \boldsymbol{k})$

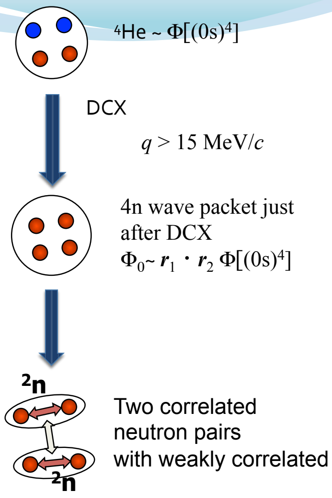
$$\int \left|\mathcal{A}\tilde{\Phi}_0\right|^2 d^3k\, d^3k_{12}\, d^3k_{34}\, \delta(E-\epsilon-\epsilon_{12}-\epsilon_{34}) \propto X^{11/2}\exp(-X)$$

Peak at $X = 11/2$; $E \sim 60$ MeV $\qquad X = E/\epsilon_a \qquad \epsilon_a = \frac{\hbar^2}{m_N a^2} = 11\text{MeV}$

NN FSI

Continuum spectrum with n-n FSI





Density of State

$$D_{ns}(\epsilon_{nn}) = \frac{\left|\hat{A}_{ns}(k)\right|^2}{k} \quad (\text{for } n=1,2)\ ;\ \epsilon_{nn} = \frac{\hbar^2 k^2}{m_N}$$

$$\hat{A}_{1s}(k) = \int_0^\infty dr\, r\,\psi_{1s}(r)\phi_k(r) = 2\left(\frac{1}{\sqrt{\pi a^3}}\right)^{1/2} k\, A_{1s}(k)$$

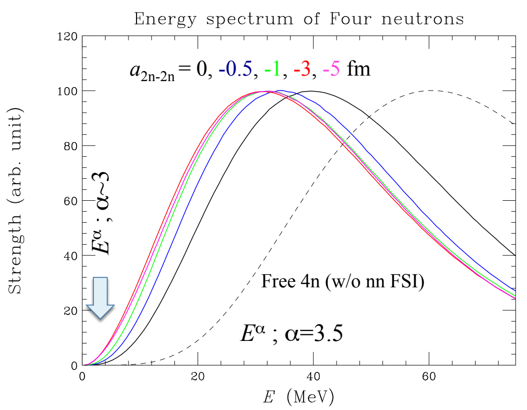
$$\hat{A}_{2s}(k) = \int_0^\infty dr\, r\,\psi_{2s}(r)\phi_k(r) = 2\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{\pi a^3}}\right)^{1/2} k\, A_{2s}(k)$$

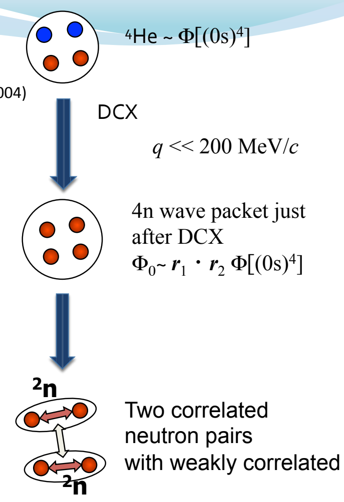
Expand $\mathcal{A}\Phi_0$ with correlated n-n scattering wave $\phi_k(r)$
$A(k)$'s are used instead of Fourier component

Direct Part

Continuum spectrum with n-n FSI

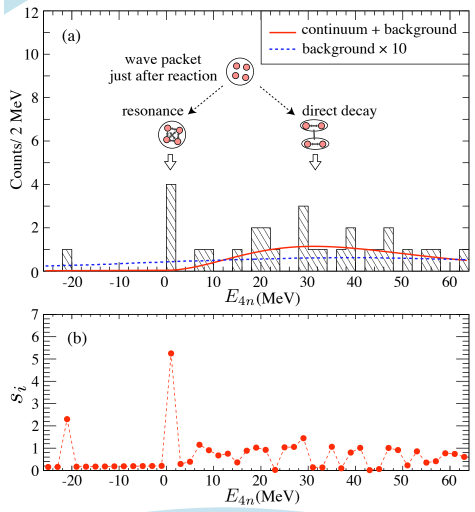
c.f.
L.V. Grigorenko, N.K. Timofeyuk, M.V. Zhukov, Eur. Phys. J. A 19, 187 (2004)





Correlation is taking into account for 2n-2n relative motion by using scattering length

Fit with direct component & BG



Energy spectrum is expressed by the continuum from the direct decay and (small) experimental background except for four events at $0<E_{4n}<2$ MeV

The Four events suggest a possible resonance at
0.83 ± 0.65(stat.) ± 1.25(sys.) MeV
with width narrower than 2.6 MeV
(FWHM). [4.9σ significance]
Integ. cross section θ_{cm}< 5.4deg:
3.8 $^{+2.9}_{-1.8}$ nb

- likelihood ratio test
  $$\chi^2_\lambda = -2\ln\left[L(\boldsymbol{y};\boldsymbol{n})/L(\boldsymbol{n};\boldsymbol{n})\right]$$
- Significance:
  $$s_i = \sqrt{2[y_i - n_i + n_i \ln(n_i/y_i)]}$$
  n_i : num. of events in the i-th bin
  y_i : trial function in the i-th bin
- Look Elsewhere Effect

$$\mu^n e^{-\mu}/n! \simeq 10^{-6} \quad \text{for} \quad \mu = 0.07,\ n = 4$$

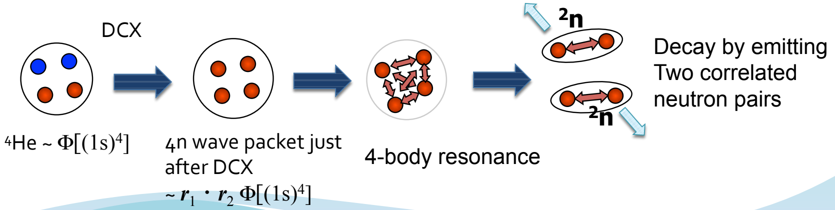
Re: Width of possible 4n resonance

$$W\left(E,\epsilon_{12},\epsilon_{34}\right) \propto \frac{2\gamma^2_{2n-2n} P\left(E-\epsilon_{12}-\epsilon_{34}\right)}{\left(E-E_0\right)^2+\left[\frac{1}{2}\Gamma\left(E\right)\right]^2} D_{nn}\left(\epsilon_{12}\right) D_{nn}\left(\epsilon_{34}\right)$$

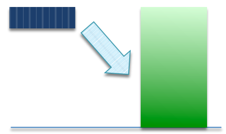
$$\Gamma\left(E\right) = 2\gamma^2_{2n-2n} \int\int d\epsilon_{12} d\epsilon_{34} P\left(E-\epsilon_{12}-\epsilon_{34}\right) D_{nn}\left(\epsilon_{12}\right) D_{nn}\left(\epsilon_{34}\right)$$

$$= 2\gamma^2_{2n-2n} P_{eff}\left(E\right)$$

$$W\left(E\right) \propto \frac{\Gamma\left(E\right)}{\left(E-E_0\right)^2+\left[\frac{1}{2}\Gamma\left(E\right)\right]^2} \qquad \gamma^2_{2n-2n} \simeq \frac{3\hbar^2}{2m_N a^2_{ch}} \simeq 8.2\ \mathrm{MeV}$$

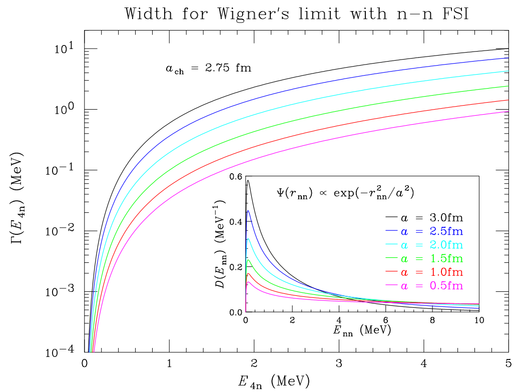


Width for Wigner's limit



Γ = 20~ 700 keV
@ E_0 ~ 1MeV

There might be sharp resonance due to small phase space for four-body decay, even for s-wave



Further experimental approach

- ^{29}F (knockout 1p) -> ^{28}O -> ^{24}O + 4n
- ^{8}He (knockout a by proton) -> 4n
- ^{4}He(^{8}He,^{8}Be)4n again with more statistics

All of three can produce recoil-less condition

Three approaches produce different initial wave packets of 4n

- resonance/continuum will be different

Summary of tetraneutron

- ^{4}He(^{8}He,^{8}Be)4n has been measured at 190 A MeV at RIBF-SHARAQ
- Missing mass spectrum with very few background
- Although statistics is low (27 evs), spectrum looks two components (continuum + peak)
- Continuum is consistent with direct breakup process from (0s)^2(0p)^2 wave packet
- Four events just above 4n threshold is statistically beyond prediction of continuum + background (4.9 σ significance)

  → candidate of 4n resonance

  at 0.83 ± 0.65(stat.) ± 1.25(sys.) MeV; Γ < 2.6 MeV
- Constraint to T=3/2 three-body force