Few-neutron systems

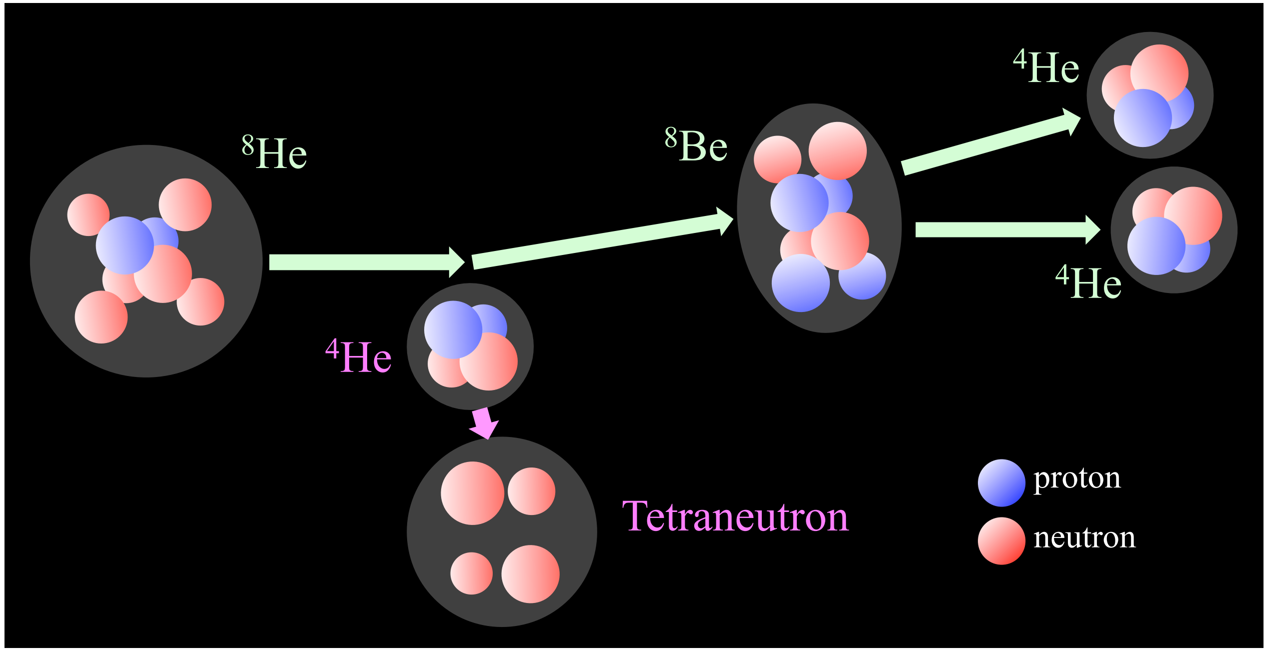
S. Shimoura
Center for Nuclear Study,
The University of Tokyo

contents:

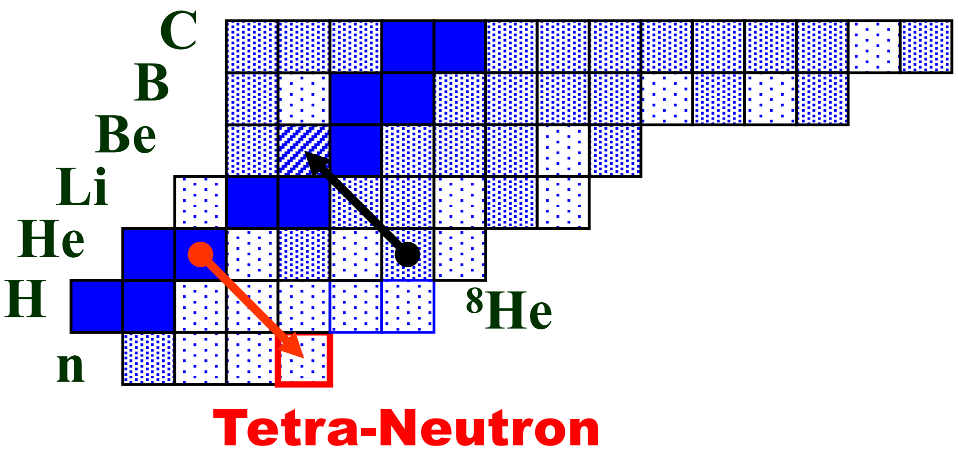
Remarks relating to Tetra-neutron exp.

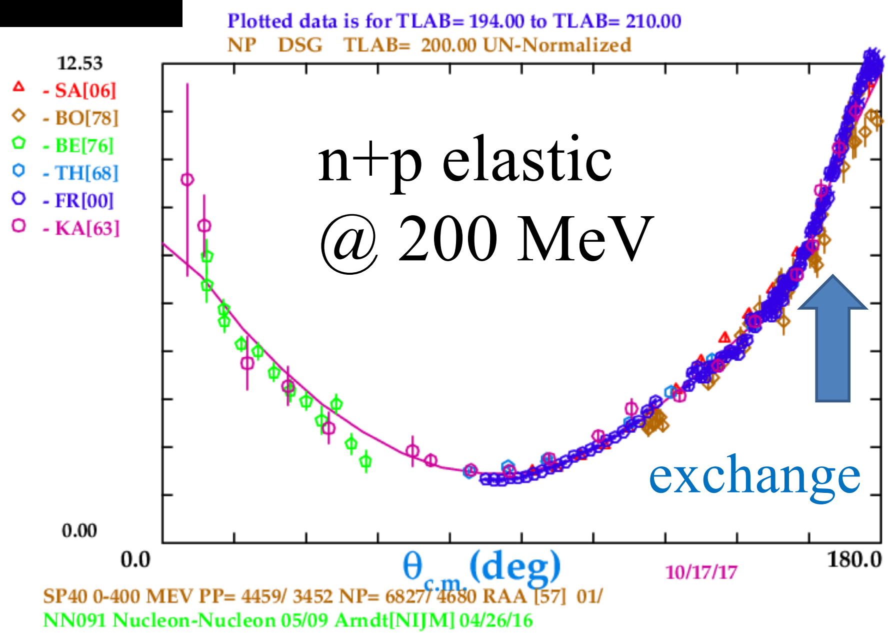
- Results and Analysis of 4n exp.
  Exothermic double-charge exchange (^{8}He,^{8}Be)
  Continuum spectrum with correlation based on idea of wave packets
- Remarks on reaction to the continuum
  - revisit to effective range theory
    - different FSI spectra for same scattering length and effective range
  - reaction based on wave packets (numerical exp.)
    - time evolution of wave packet
    - 1-dim 3-body calculation

Tetra-neutron system produced by exothermic double-charge exchange reaction

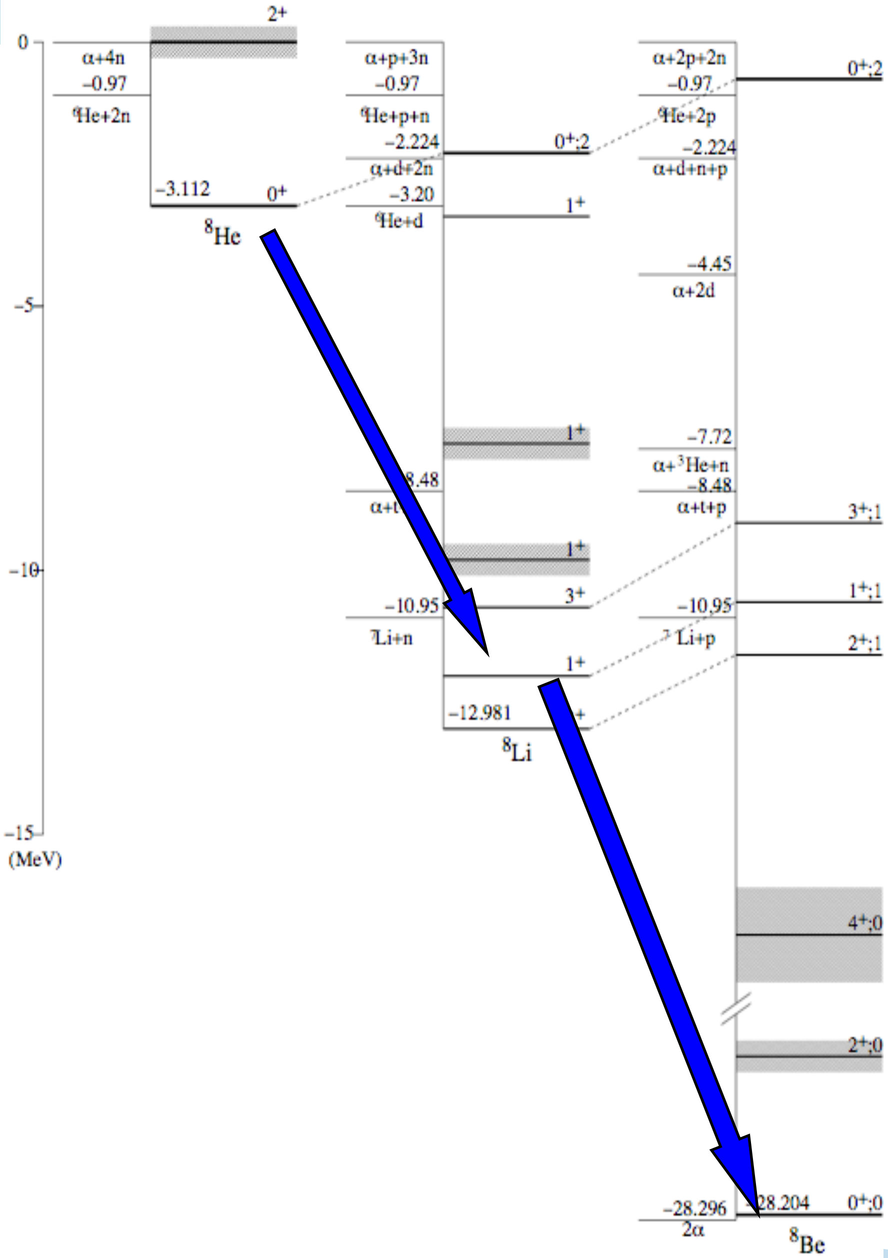


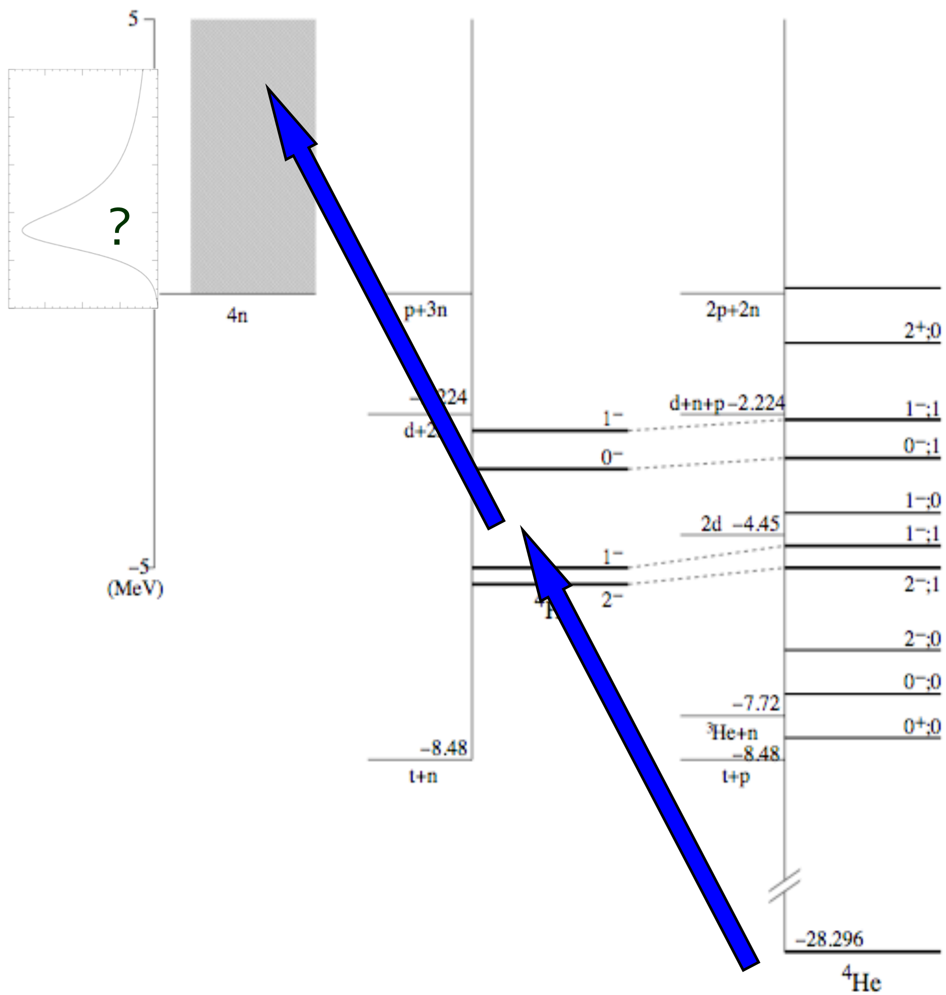
Almost recoil-less condition with ^{4}He(^{8}He,^{8}Be)4n reaction at 200 A MeV (0.63 c)





Level diagrams

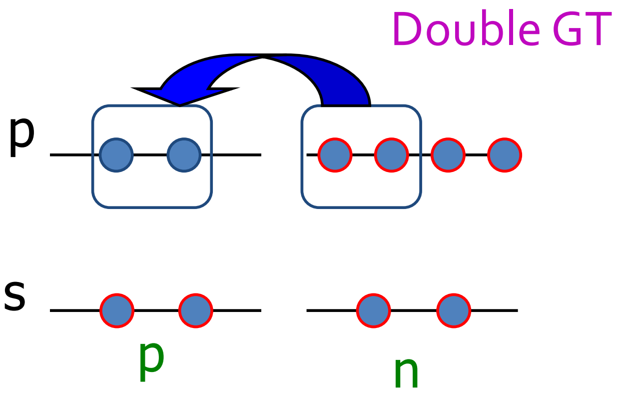




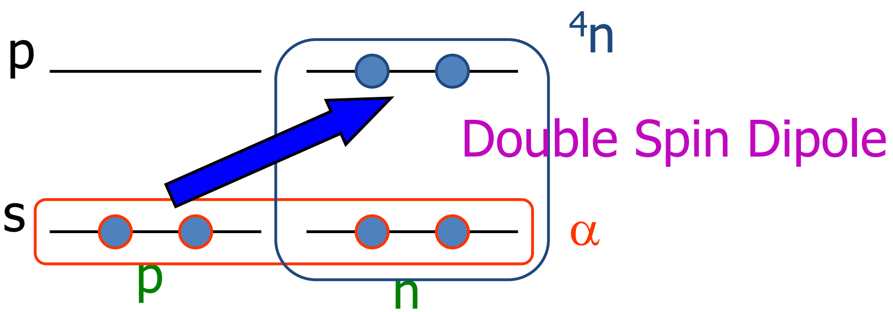
q_{min} ~ 10MeV/c

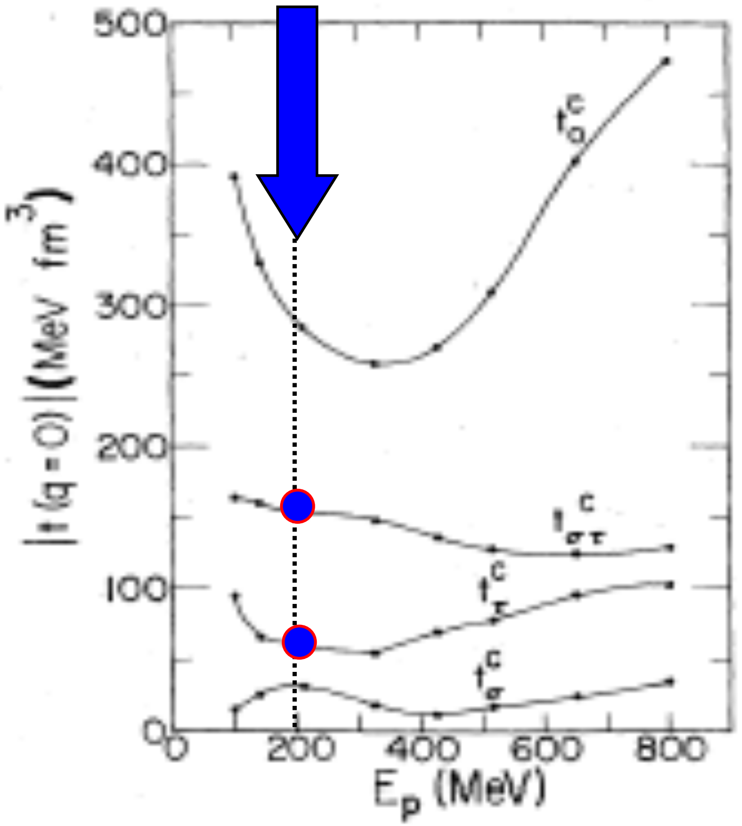
Reaction Mechanism

$^{8}He \rightarrow {}^{8}Be$



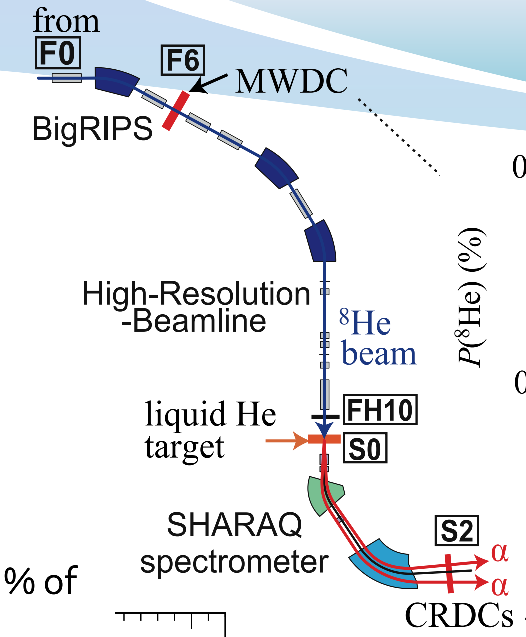
$^{4}He \rightarrow 4n$

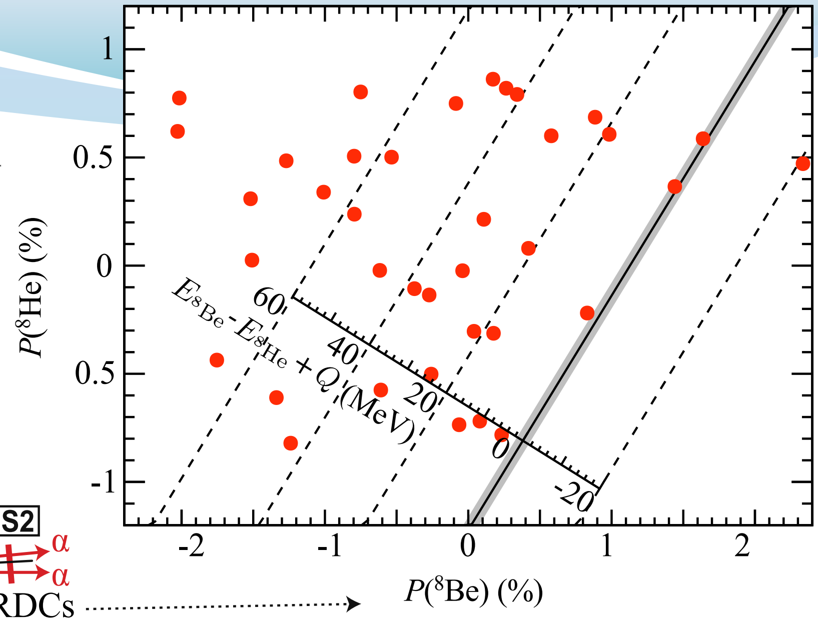




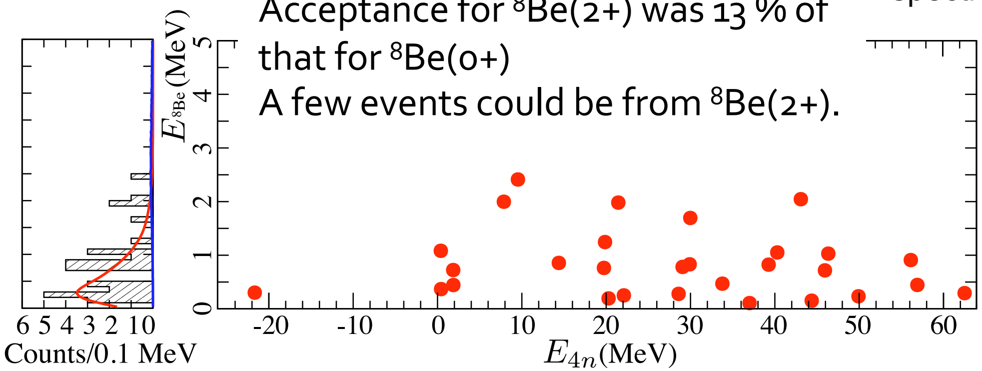
$$\left[\left(\vec{\tau}_{\mathrm{p}}\cdot\vec{\tau}_{\mathrm{t}}\right)\left(\vec{\sigma}_{\mathrm{p}}\cdot\vec{\sigma}_{\mathrm{t}}\right)r_{\mathrm{t}}Y_1\left(\hat{r}_{\mathrm{t}}\right)\right]^2$$

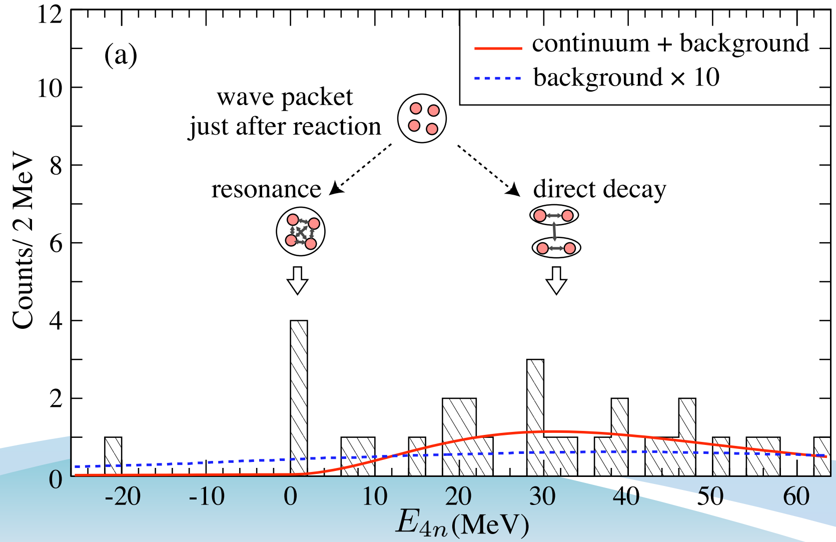
Experimental Results

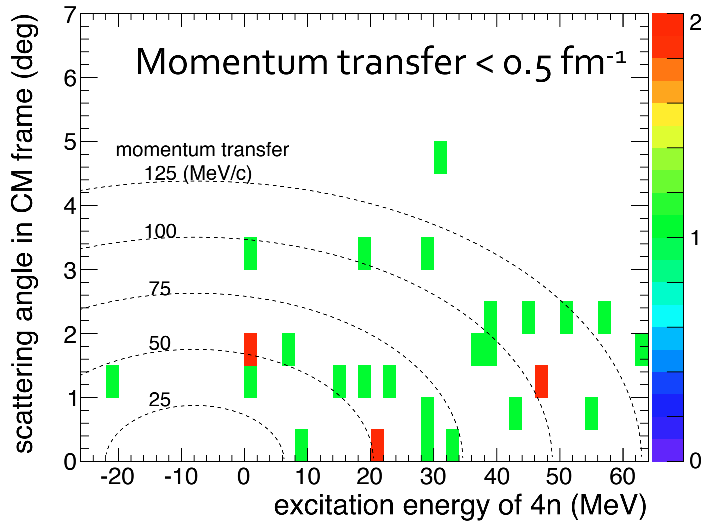




Acceptance for ^{8}Be(2+) was 13 % of that for ^{8}Be(0+)
A few events could be from ^{8}Be(2+).

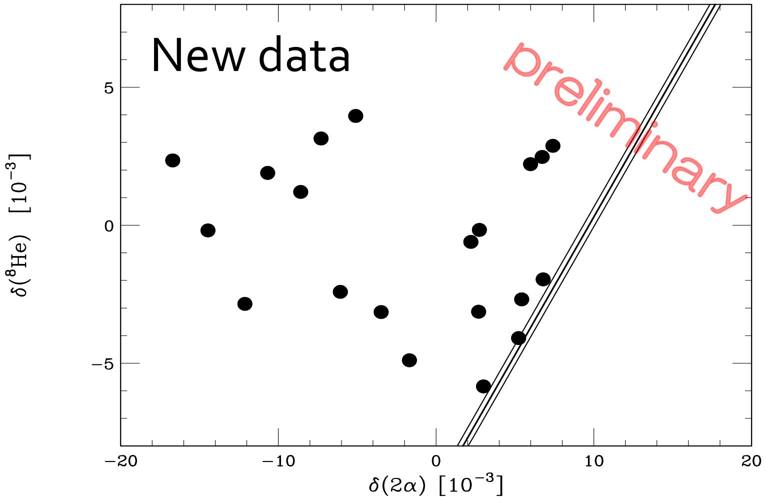






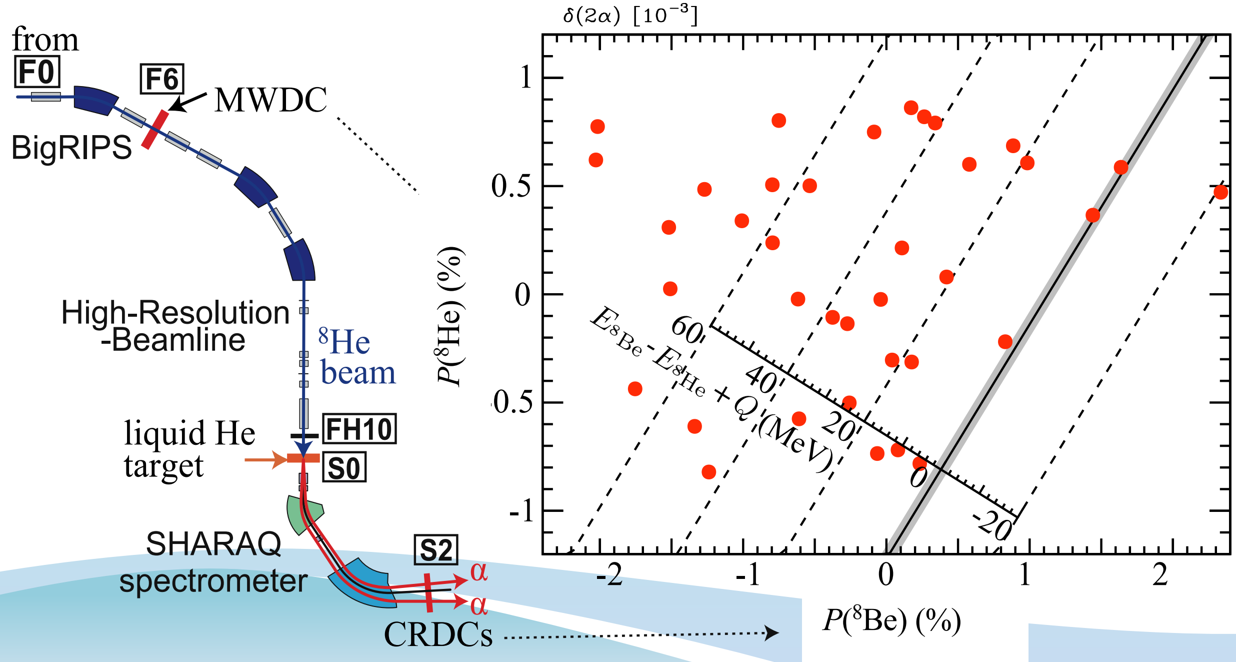
Look like having two components:
Continuum + Peak (?)
? The 4 counts just above threshold can be explained by the fluctuation of continuum or not?

Very preliminary result for restricted 2α geometry at focal plane of SHARAQ



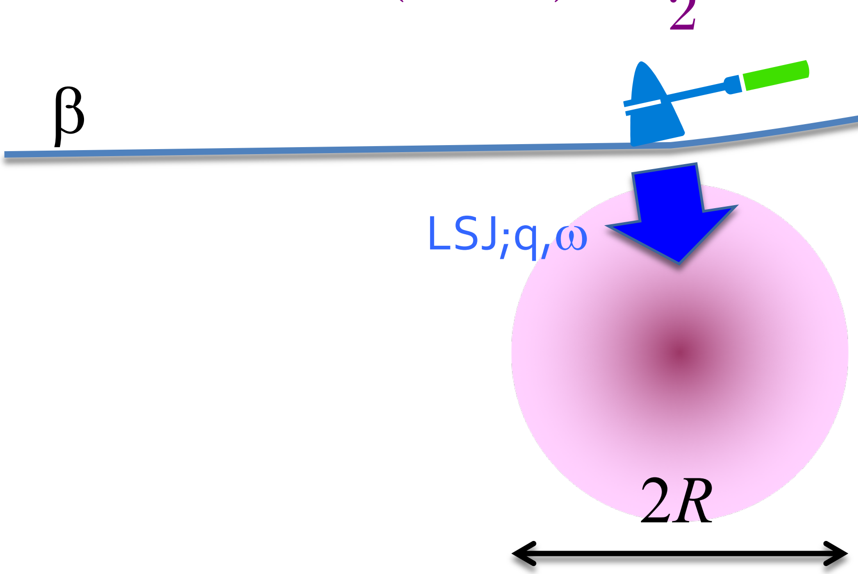
accuracy ~ 100 keV;
resolution ~ +/-2 MeV,
which is to be improved

Although low statistics, similar spectrum where a few events are just above threshold.



Reaction time & excitation energy for intermediate-energy "inelastic-type scattering"

$$\omega \ll \mu c^2 (\gamma - 1) \simeq \frac{1}{2}\mu c^2 \beta^2$$



$$\Delta E \cdot \Delta t \sim 2\pi\hbar$$

$$\omega_{\rm max} \sim \frac{2\pi\hbar \cdot \beta c}{2R} \simeq 100\beta \ \ {\rm MeV}$$

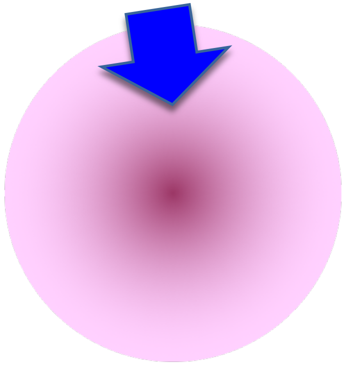
Off energy shell

E/A ~ 200 MeV : β ~ 0.6 : ω_{max} ~ 60 MeV

$$O(lsj\tau;\xi)\left|E_i J_i \pi_i T_i;\xi_i\right\rangle = \sum\!\!\!\!\!\!\int_f M_{if}(E_f)\left|E_f J_f \pi_f T_f;\xi_f\right\rangle$$ Response

$\left|M_{if}(E_f)\right|^2$: Energy Spectrum

“Transition” as time-dependent action



$$i\hbar\frac{\partial}{\partial t}\Psi(t) = (H + V_R(t))\,\Psi(t)$$

$$\Psi(t) = \sum_i a_i(t)\,\psi_i \exp(-iE_i t/\hbar)$$

$$H\psi_i = E_i\psi_i$$

$$a_0(-\infty) = 1 \quad ; \quad a_i(-\infty) = 0 \quad \text{for } i > 0$$

$$|a_i(+\infty)|^2 : \text{Energy spectrum after reaction}$$

$$\sum_i i\hbar\,\dot{a}_i(t)\,\psi_i \exp(-iE_i t/\hbar) = \sum_i a_i(t)\,V_R(t)\,\psi_i \exp(-iE_i t/\hbar)$$

$$i\hbar\,\dot{a}_k(t) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{t^2}{2\Delta T^2}\right) \times \sum_i a_i(t)\,\langle\psi_k|\mathcal{O}|\psi_i\rangle \exp\left(-\frac{i(E_i - E_k)t}{\hbar}\right)$$

$$V_R(t) = \frac{\mathcal{O}}{\sqrt{2\pi}}\exp\left(-\frac{t^2}{2\Delta T^2}\right)$$

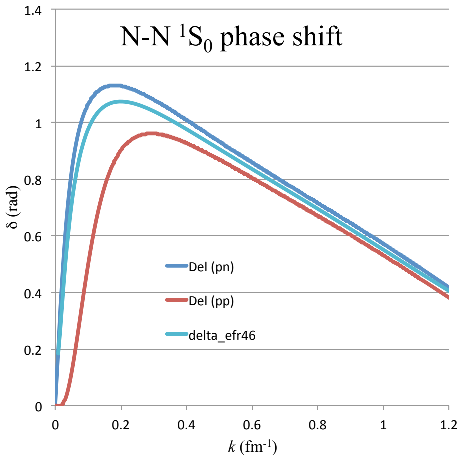
Perturbation

$$a_i(-\infty) \ll 1 \quad \text{for } i > 0$$

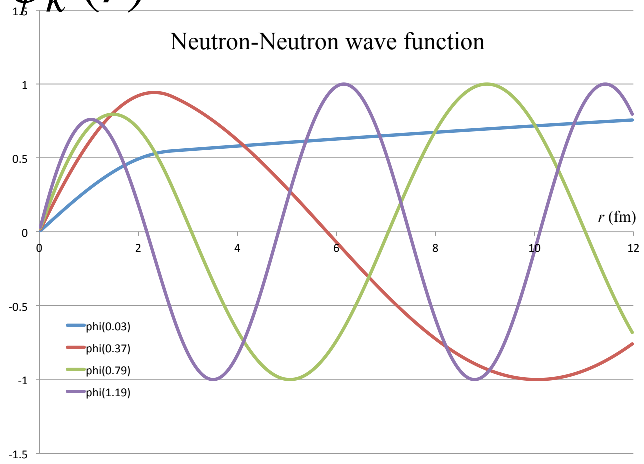
$$a_0(+\infty) - a_0(-\infty) \simeq -i\frac{\Delta T}{\hbar}\langle\psi_0|\mathcal{O}|\psi_0\rangle$$

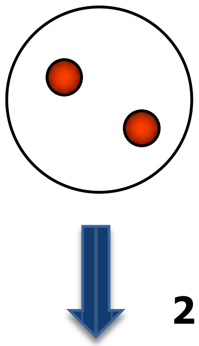
$$a_k(+\infty) \simeq -i\frac{\Delta T}{\hbar}\langle\psi_k|\mathcal{O}|\psi_0\rangle \exp\left(-\frac{(E_{i0}\Delta T)^2}{2\hbar^2}\right)$$

NN case with FSI



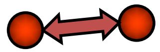
$\phi_k(r)$





2n wave packet just after a certain reaction
ϕ_0~ **Gaussian**

^{2}n



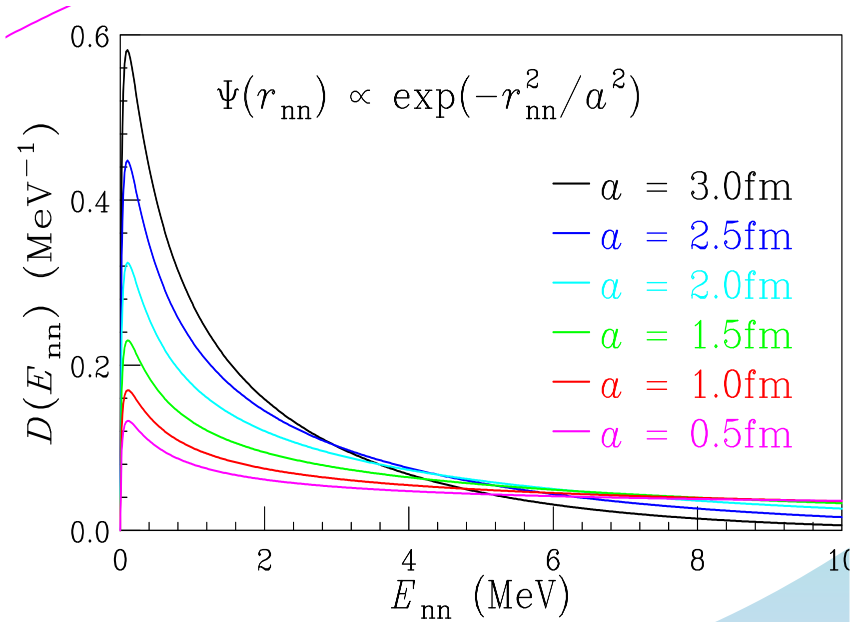
Scattering state of correlated neutron pair

Density of State

$$D\left(E_{\rm nn}\right)=\frac{\left|A\left(k\right)\right|^{2}}{k}\;;\;E_{\rm nn}=\frac{\hbar^{2}k^{2}}{m_{\rm N}}$$

$$A\left(k\right)=\int drr\Psi\left(r\right)\phi_{k}\left(r\right)$$

Expand Ψ_0 with correlated n-n scattering wave $\phi_k(r)$
$A(k)$'s are used instead of Fourier component



Effective Range Theory :

$\phi_k(r) \sim \sin\delta(k) \times f(r)$ for small r

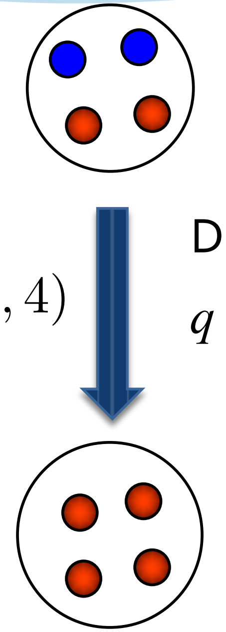
$D \sim (\sin\delta)^2/k$ (Watson-Migdal approx.)

Direct Part

$$\Phi_0 \propto \mathcal{A}\left[\left(r_\alpha^2 - r_{12}^2\right)\exp\left(-\frac{r_\alpha^2}{a^2} - \frac{r_{12}^2}{2a^2} - \frac{r_{34}^2}{2a^2}\right)\chi(1,2)\,\chi(3,4)\right]$$

$$\propto \left(\frac{4r_\alpha^2}{a^2} - \frac{r_{12}^2}{a^2} - \frac{r_{34}^2}{a^2}\right)\exp\left[-\frac{r_\alpha^2}{a^2} - \frac{r_{12}^2}{2a^2} - \frac{r_{34}^2}{2a^2}\right]\chi(1,2)\,\chi(3,4)$$

$$+\frac{4\vec{r}_{12}\cdot\vec{r}_{34}}{a^2}\exp\left[-\frac{r_\alpha^2}{a^2} - \frac{r_{12}^2}{2a^2} - \frac{r_{34}^2}{2a^2}\right]\vec{X}(1,2)\cdot\vec{X}(3,4)$$



^{4}He ~ $\Phi[(0\mathrm{s})^4]$

DCX

q << 200 MeV/c

$$\vec{r}_\alpha = \frac{\vec{r}_1 + \vec{r}_2}{2} - \frac{\vec{r}_3 + \vec{r}_4}{2}$$

$$\chi(i,j) = \frac{1}{\sqrt{2}}\left(\uparrow(i)\downarrow(j) - \downarrow(i)\uparrow(j)\right)$$

$$\vec{X}(i,j) = \begin{pmatrix} \uparrow(i)\uparrow(j) \\ \frac{1}{\sqrt{2}}\left(\uparrow(i)\downarrow(j) + \downarrow(i)\uparrow(j)\right) \\ \downarrow(i)\downarrow(j) \end{pmatrix}$$

4n wave packet just after DCX
$\Phi_0 \sim \boldsymbol{r}_1 \cdot \boldsymbol{r}_2\, \Phi[(0\mathrm{s})^4]$

Fourier Transform: $(\boldsymbol{r}_{12}, \boldsymbol{r}_{34}, \boldsymbol{r}_\alpha) \rightarrow (\boldsymbol{k}_{12}, \boldsymbol{k}_{34}, \boldsymbol{k})$

$$\int \left|\mathcal{A}\tilde{\Phi}_0\right|^2 d^3k\, d^3k_{12}\, d^3k_{34}\, \delta(E - \epsilon - \epsilon_{12} - \epsilon_{34}) \propto X^{11/2}\exp(-X)$$

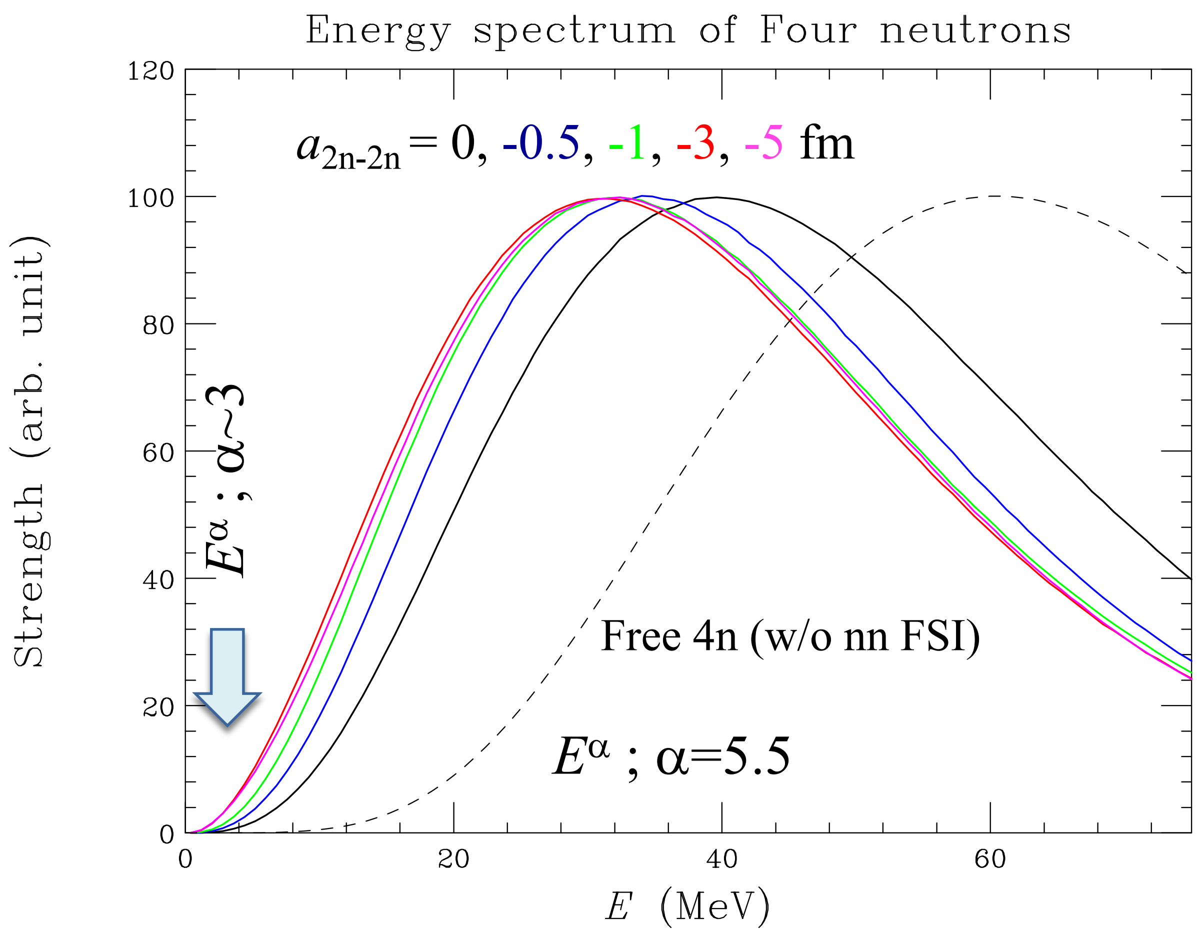
Peak at $X = 11/2$; $E \sim 60$ MeV

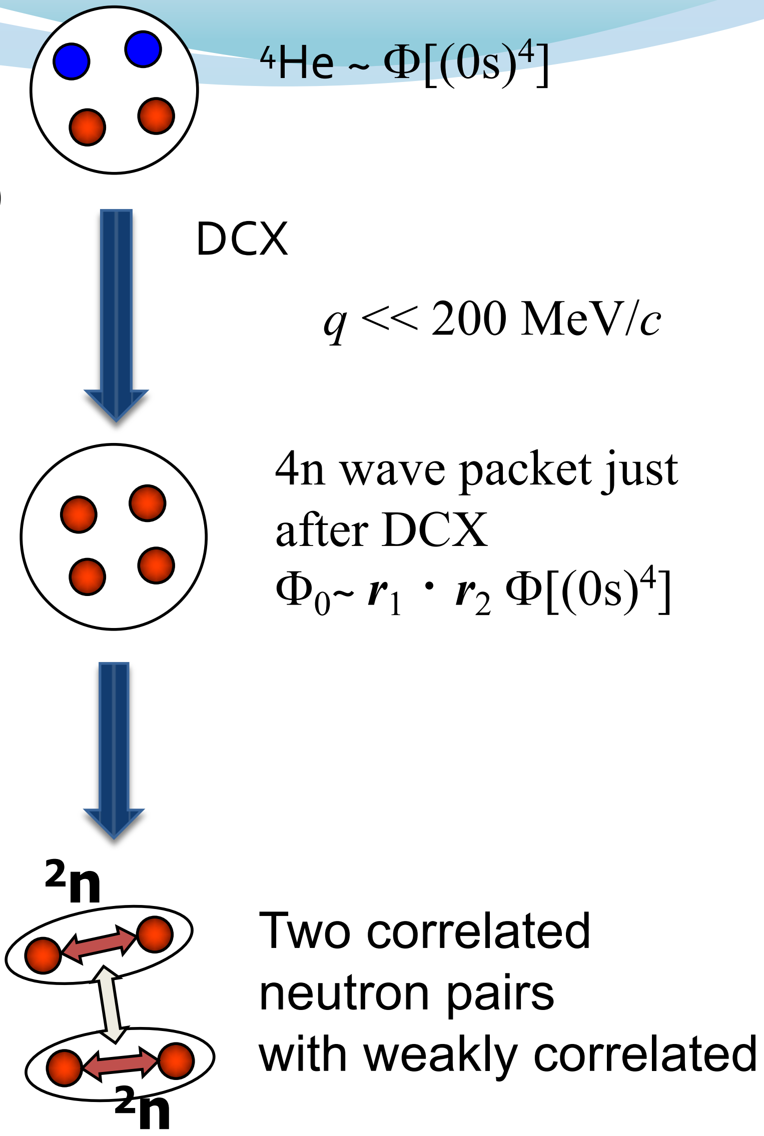
$$X = E/\epsilon_a \qquad \epsilon_a = \frac{\hbar^2}{m_N a^2} = 11\,\mathrm{MeV}$$

Direct Part

Continuum spectrum with n-n FSI

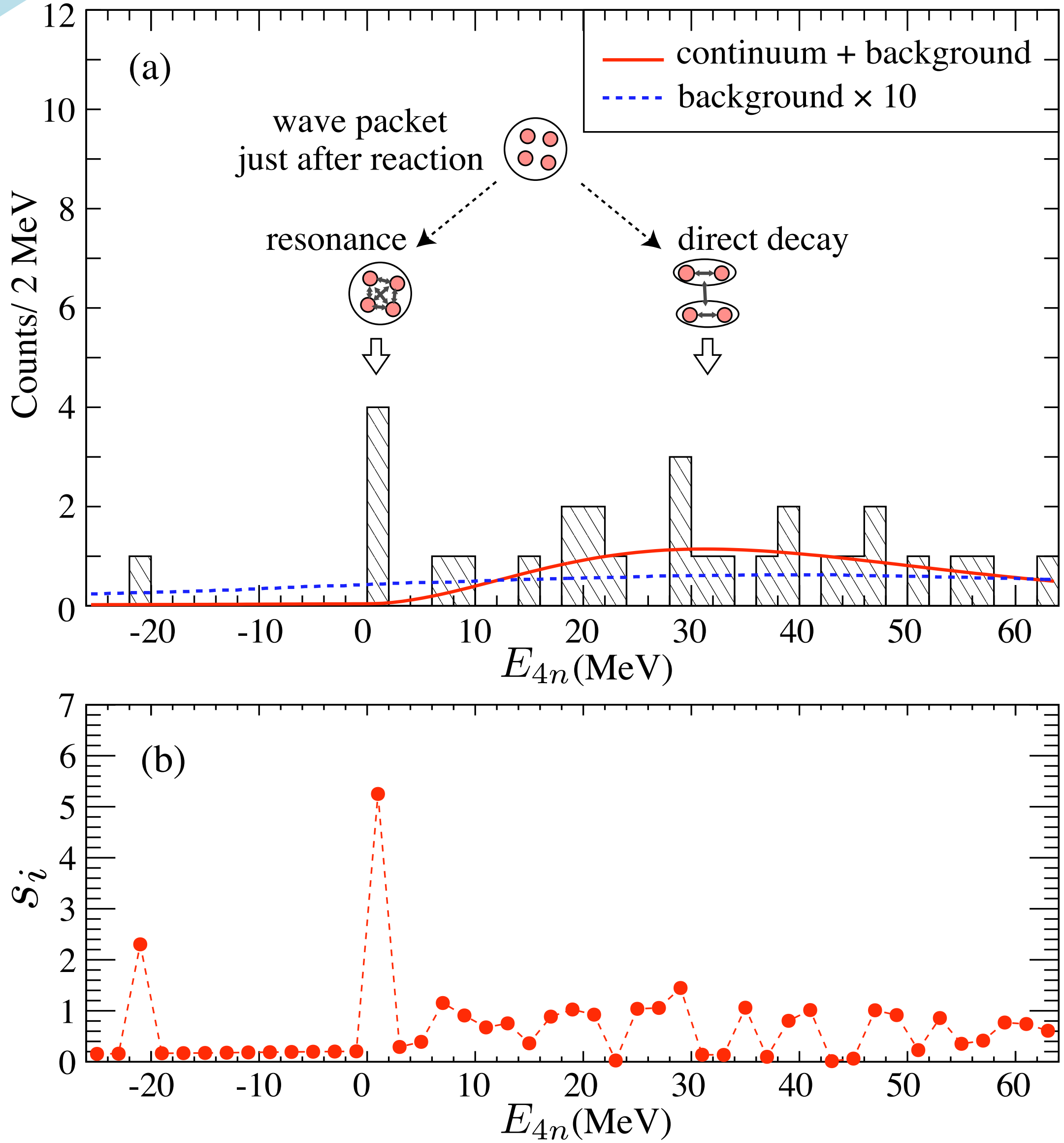
c.f.
L.V. Grigorenko, N.K. Timofeyuk, M.V. Zhukov, Eur. Phys. J. A 19, 187 (2004)





Correlation is taking into account for 2n-2n relative motion by using scattering length

Fit with direct component & BG



Energy spectrum is expressed by the continuum from the direct decay and (small) experimental background except for four events at $0<E_{4n}<2$ MeV

The Four events suggest a possible resonance at

0.83 ± 0.65(stat.) $\pm$ 1.25(sys.) MeV

with width narrower than 2.6 MeV (FWHM). [4.9σ significance]

Integ. cross section $\theta_{cm}< 5.4$deg:

$3.8^{+2.9}_{-1.8}$ nb

- likelihood ratio test

$$\chi^2_\lambda = -2\ln\left[L(\boldsymbol{y};\boldsymbol{n})/L(\boldsymbol{n};\boldsymbol{n})\right]$$

- Significance:

$$s_i = \sqrt{2[y_i - n_i + n_i \ln(n_i/y_i)]}$$

n_i : num. of events in the i-th bin

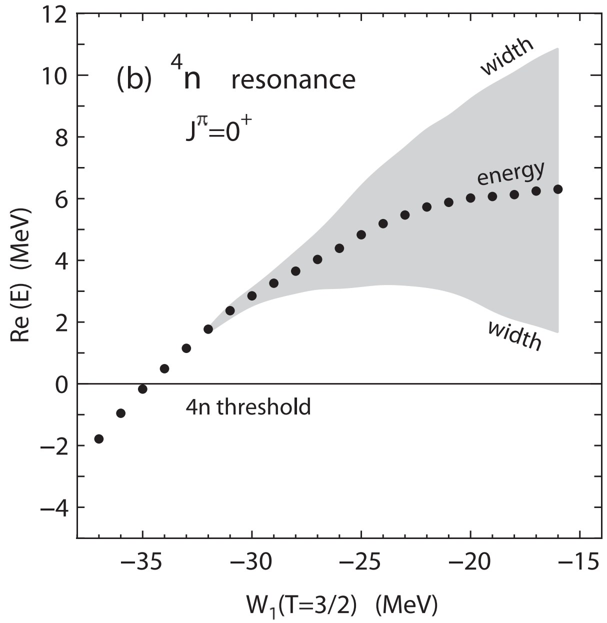
y_i : trial function in the i-th bin

- Look Elsewhere Effect

$$\mu^n e^{-\mu}/n! \simeq 10^{-6} \quad \text{for} \quad \mu = 0.07, \ n = 4$$

Recent theoretical works

E. Hiyama et al., PRC 93, 044004 (2016)



$$V_{ijk}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^{2} W_n(T) e^{-(r_{ij}^2+r_{jk}^2+r_{ki}^2)/b_n^2} \mathcal{P}_{ijk}(T),$$

Too strong attraction is necessary for 4n resonance, which makes 4H bound!

A.M. Shirokov et al., PRL 117, 182502 (2016)

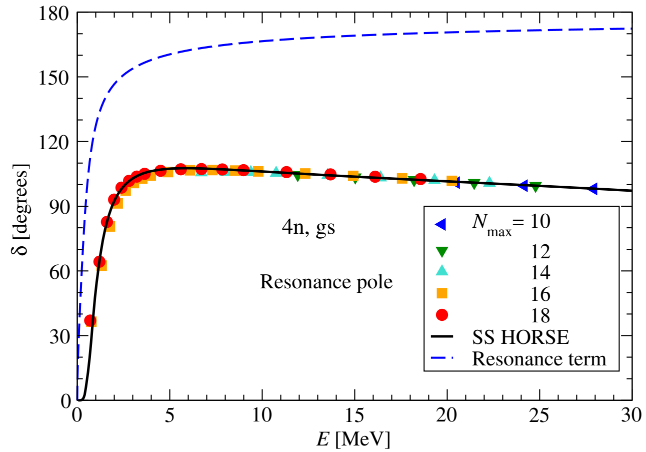


FIG. 2. The 4 → 4 scattering phase shifts: parametrization with a single resonance pole (solid line) and obtained directly from the selected NCSM results using Eq. (2) (symbols). The dashed line shows the contribution of the resonance term.

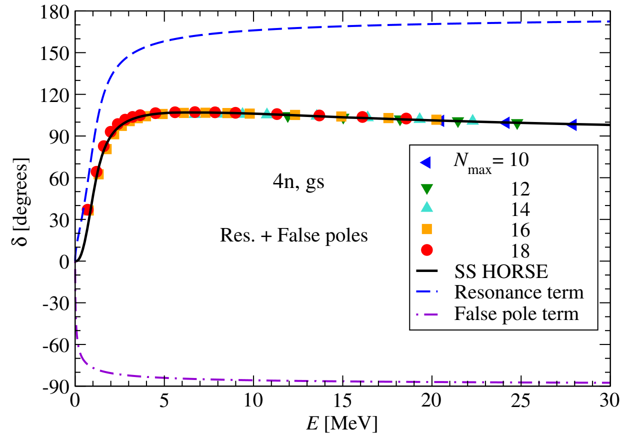


FIG. 3. The same as Fig. 2 but for the parametrization with resonance and false state poles. The dashed-dotted line shows the contribution of the false state pole term.

NCSM calculation w/ DISP16 interaction: No NNN, Non-local

4-body phase shift (HH coordinate) shows resonance around 0.8 MeV.

Recent theoretical works

A. Deltuva, PL 782, 238 (2018)

AGS equation (momentum space)

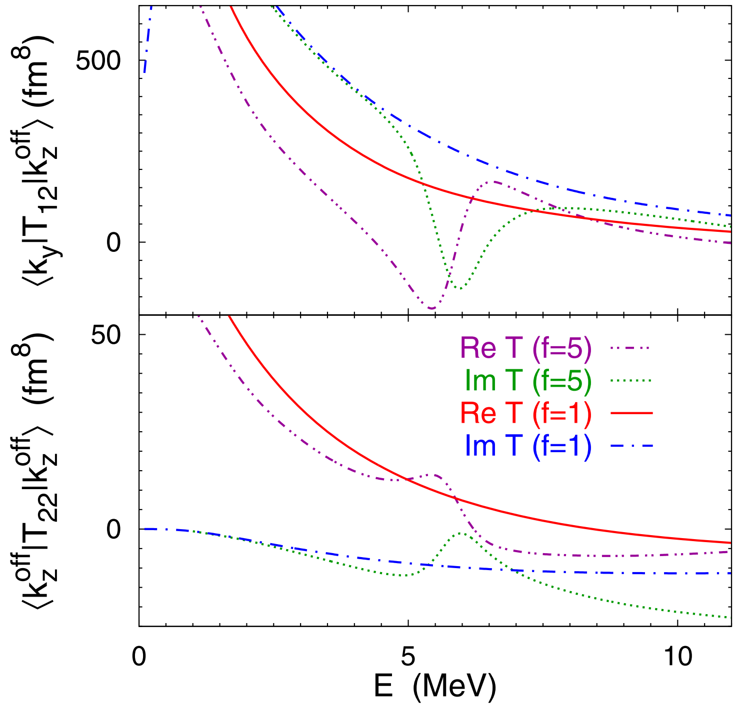
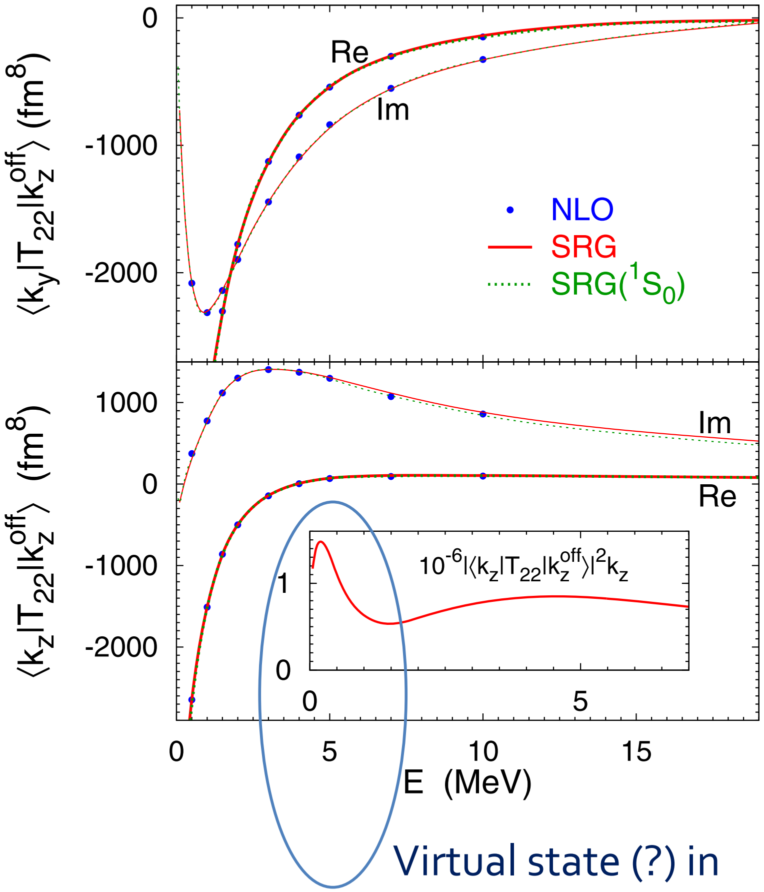


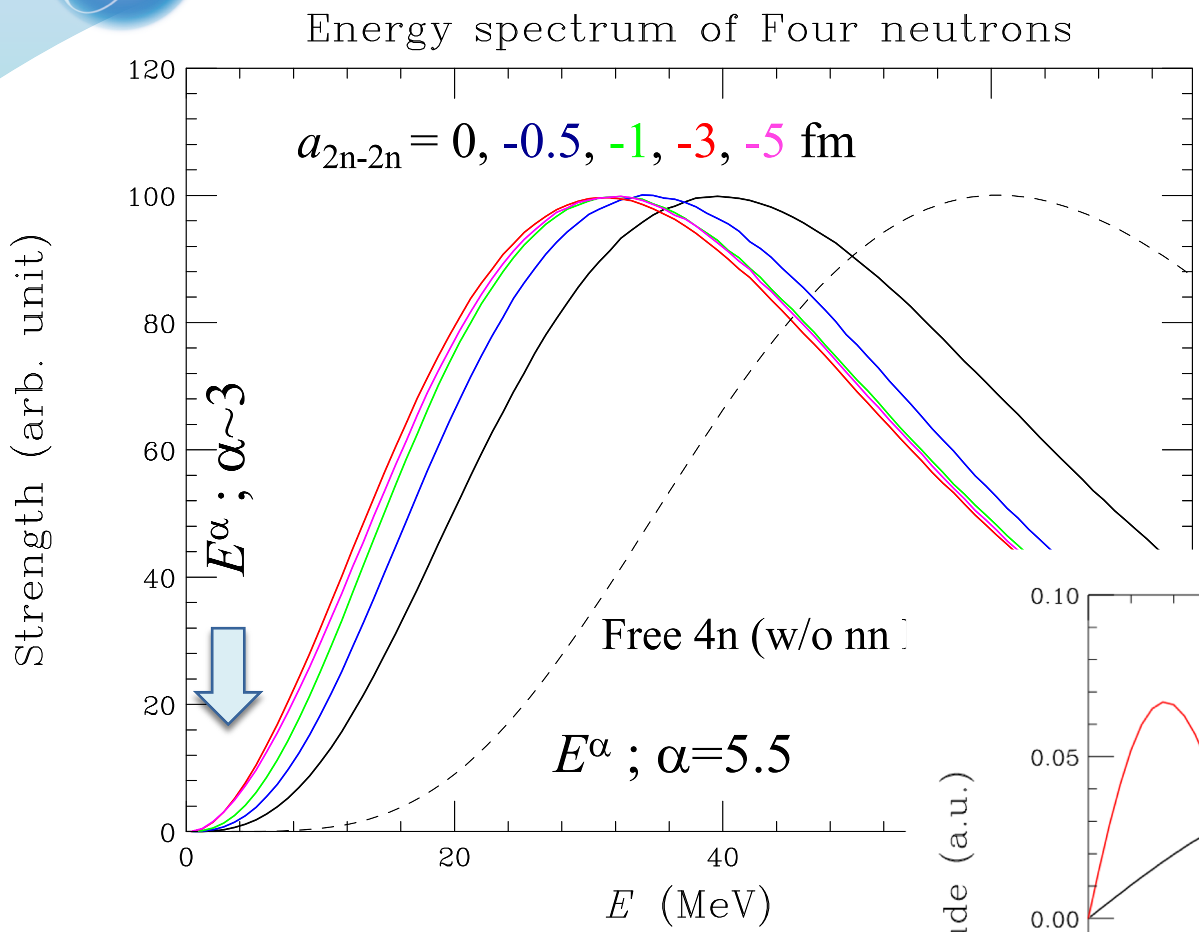
Fig. 1. (Color online) Energy dependence of real and imaginary parts of selected $\mathcal{J}^{\Pi} = 0^{+}$ four-neutron transition matrix elements calculated using the SRG potential with higher wave enhancement factors $f = 1$ and 5.

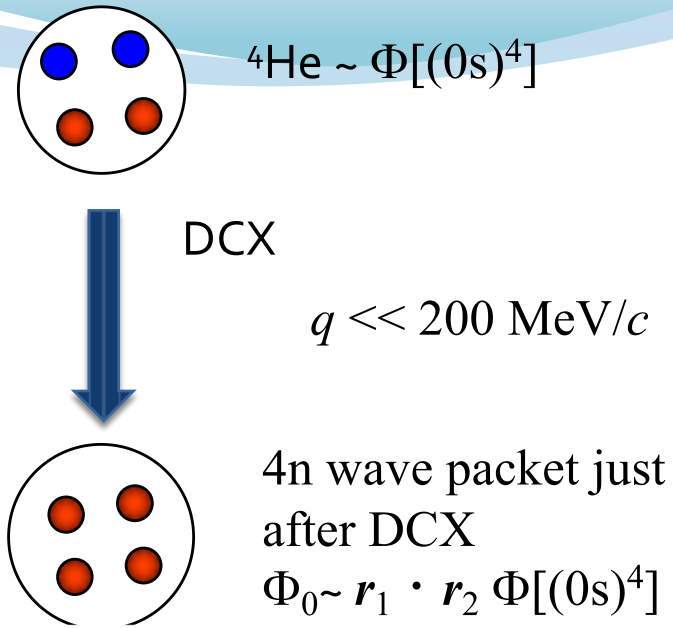
In order to produce a resonance, 5 times more attraction is necessary

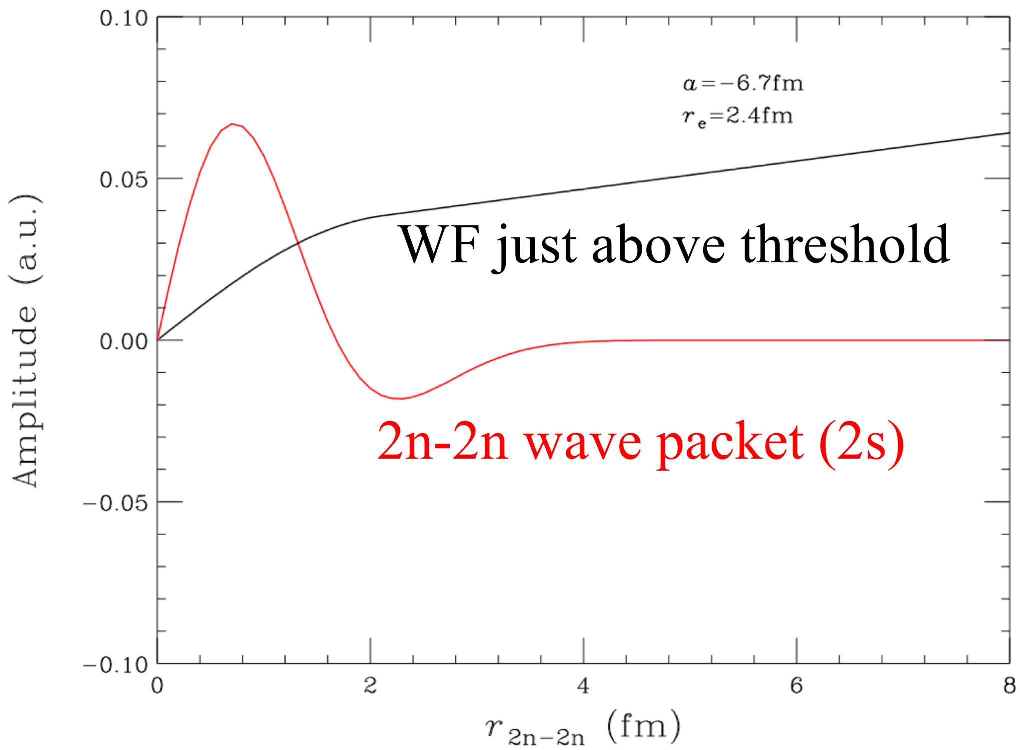


Virtual state (?) in 2n-2n scattering

Direct Part







Peak position seems to saturate for inc
attractive force between the 2n-2n pair
Pauli effect from anti-symmetrized wa
no amplitude for Pauli-forbidden scatt

Old theoretical work on α–α interaction

J. Hiura & R. Tamagaki, PTP Suppl. 52, 25 (1972)

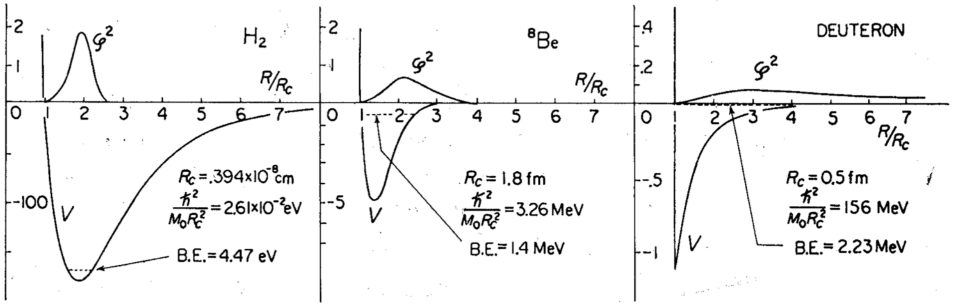


Fig. 3. Comparison of three binding forces; the binding potential for the ground state ($^1\Sigma_g$) of H_2-molecule, the α-α potential (nuclear only) for the ^{8}Be ground state and the two-nucleon potential (effective central) for the deuteron. Relative distance is given in units of the extent of short-range repulsion (R_c) and the energy unit is taken as $\hbar^2/M_0R_c^2$, where M_0 is the mass of subunits. For H_2 and the deuteron, Fig. 2-36 in Ref. 8) should be referred to. For ^{8}Be, we show the S-state potential of Endo et al. shown in Fig. 2. φ^2 are the resulting probability densities. These figures indicate the intermediate character of the two-α "molecular" states in ^{8}Be, in comparison with the other two cases.

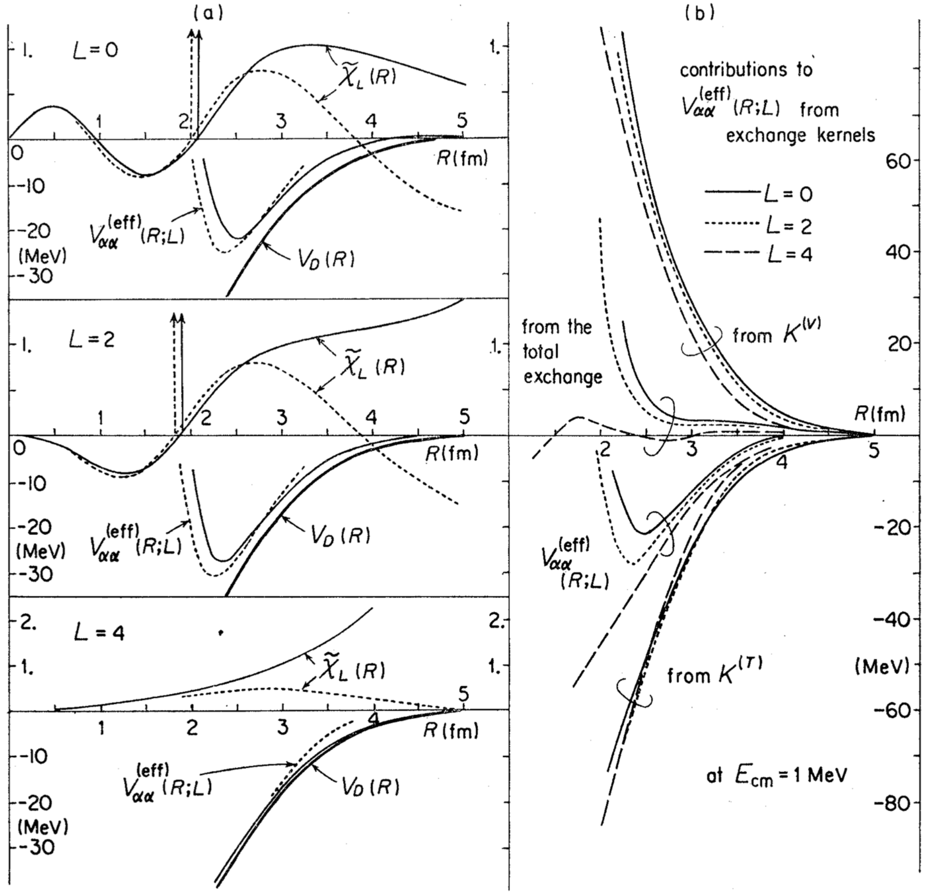
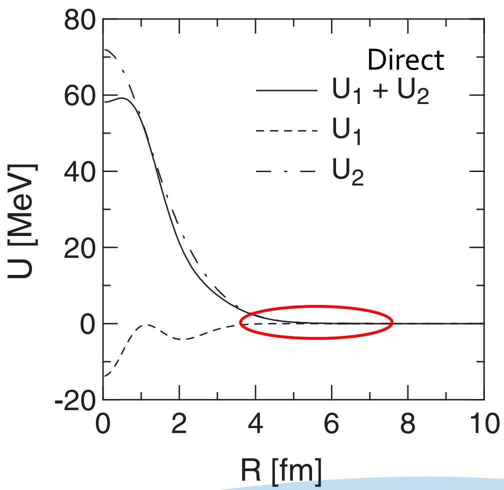
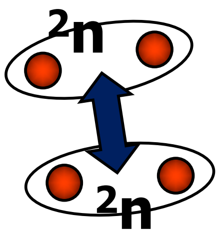


Fig. 8. Energy dependence of the relative wave functions $\tilde{\chi}_L(R)$ and angular momentum and energy dependence of the effective α-α potential $V_{\alpha\alpha}^{(eff)}$. $\tilde{\chi}_L(R)$ and $V_{\alpha\alpha}^{(eff)}(R;L)$ calculated at the two energies (—; E_{cm}=1 MeV and ···; E_{cm}=14.45 MeV) are normalized at R=1.5fm in part (a). The arrows at $R\sim2$ fm indicate the equivalent core radius for L=0 and 2. Part (b) shows the L-dependences of the contributions to $V_{\alpha\alpha}^{(eff)}(R;L)$ from the kinetic, potential and total exchange terms.

C.A. Bertulani & V. Zelevinsky, JPG 29, 2431 (2003)





Effective core due to Pauli principle

Effective repulsive core due to Pauli blocking

Direct potential is deeply attractive
W.f. has nodes in the core region orthogonal to the Pauli-forbidden state
simple Effective Range treatment may not be adequate

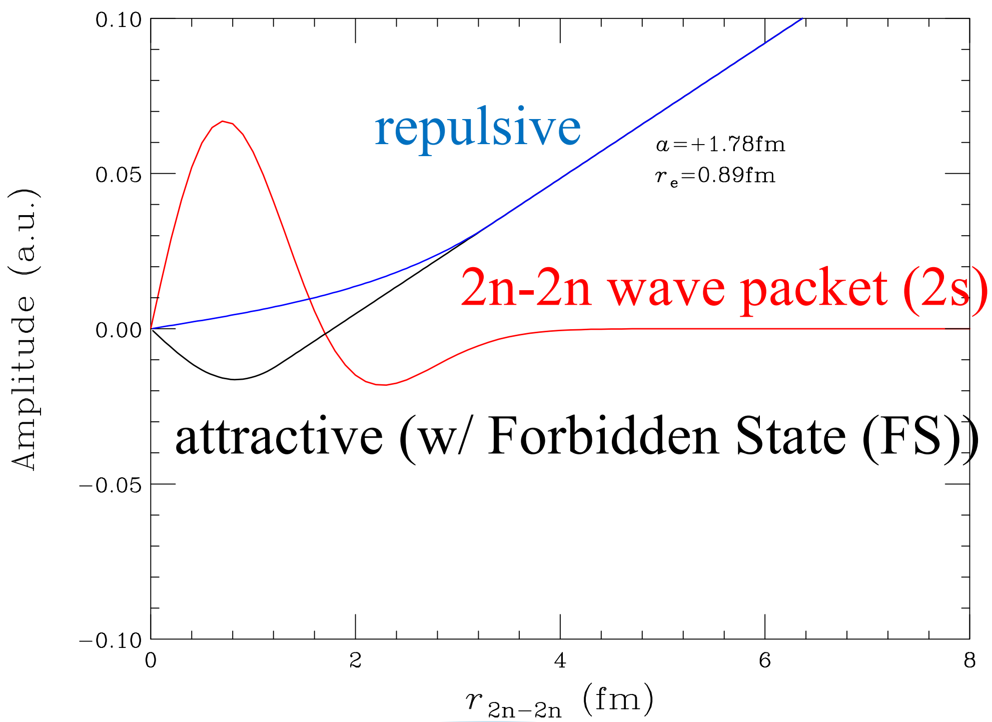
Effective Range Theory & DOS for scattering between composite particles with Pauli effect

Low-energy WFs with the same scattering length & effective range

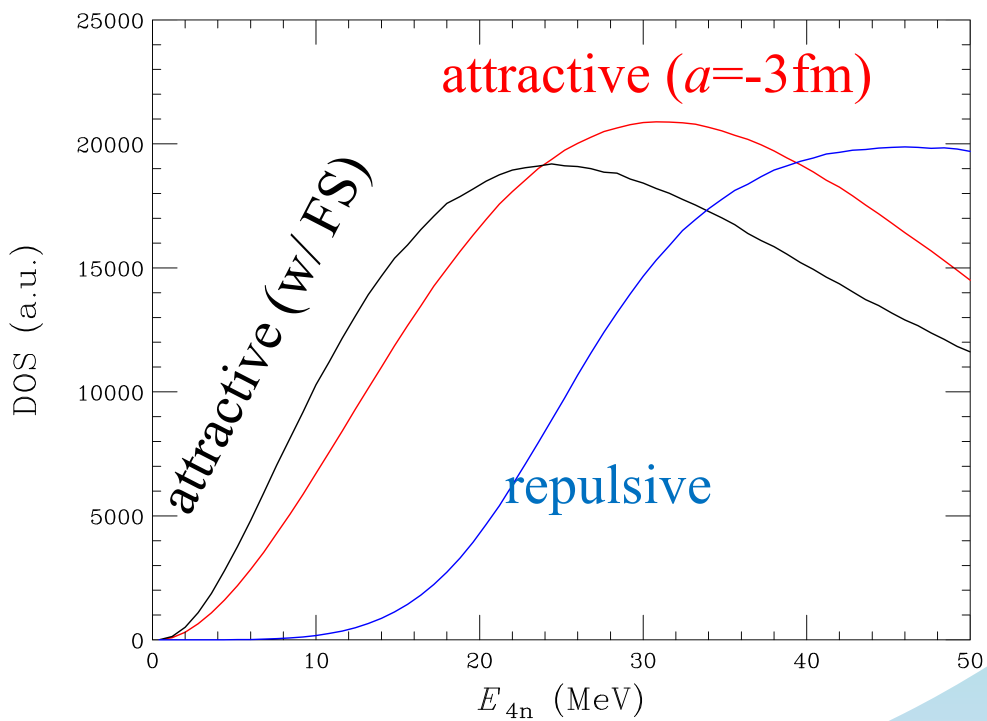
Same asymptotic behavior and energy dependence of PS at low energy

Example: a=1.78 fm, r_e = 0.89 fm

Low-energy wave function



4n Density of State



Short-range correlation, 3-body force, etc., may play roles

Scattering vs FSI in the present prescription

- Scattering observables at low energies are determined by scattering length and effective range
  - insensitive to the wave function at short range
  - structural core is equivalent to Pauli effect

but

- FSI cross sections are determined by overlap between initial wave packet and scattering wave function
  - may be sensitive to the wave function at short range if the initial wave packet is localized

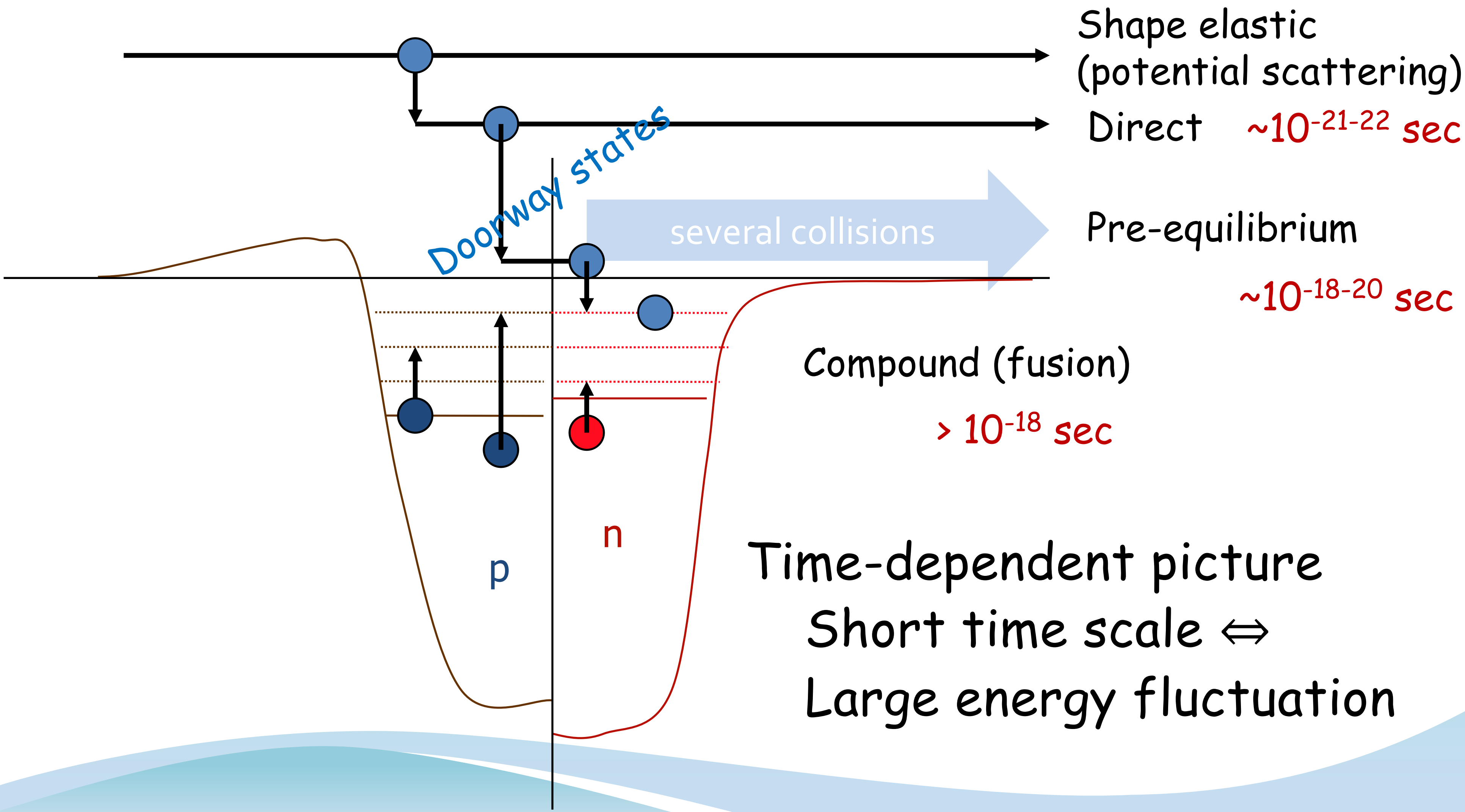
- Is present prescription justified?

- How are the many-body scattering states?

Nuclear Reaction (time-dependent)

Energy & time scale for evolution of reaction mechanism

- Shape elastic - direct - pre equilibrium - compound



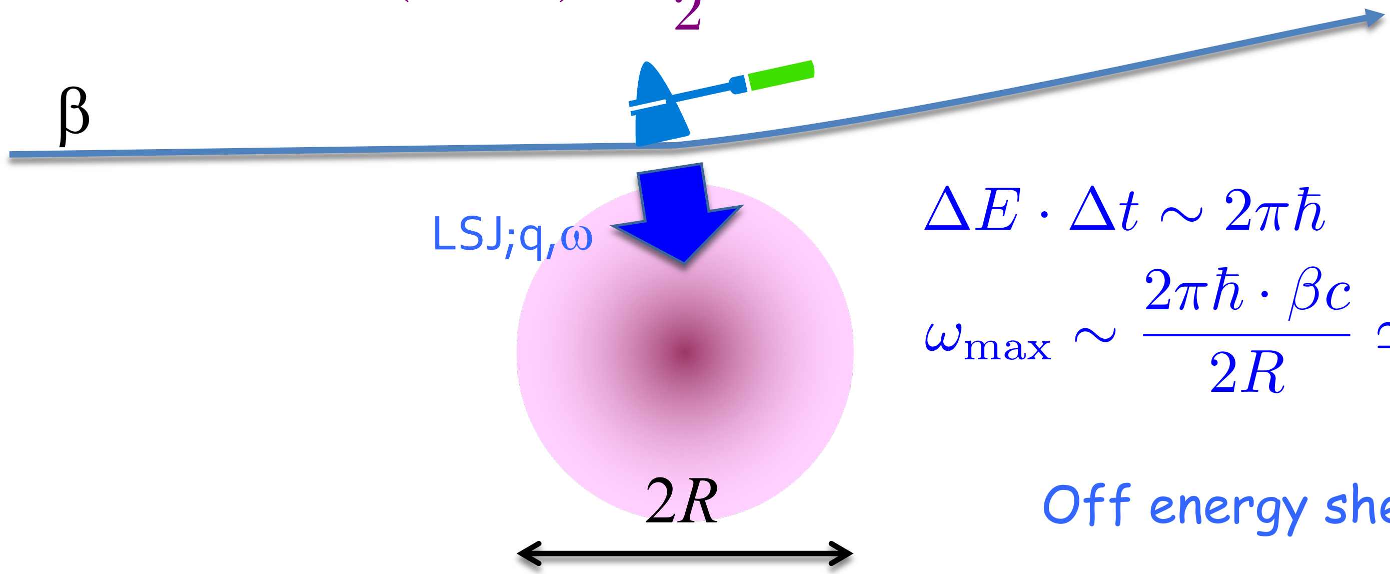
Time-dependent picture

Short time scale ⇔

Large energy fluctuation

Is this picture justified?

$$\omega \ll \mu c^2(\gamma - 1) \simeq \frac{1}{2}\mu c^2 \beta^2$$

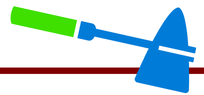


$$\Delta E \cdot \Delta t \sim 2\pi\hbar$$

$$\omega_{\max} \sim \frac{2\pi\hbar \cdot \beta c}{2R} \simeq 100\beta \ \ \mathrm{MeV}$$

Off energy shell

E/A ~ 200 MeV : β ~ 0.6 : ω_{max} ~ 60 MeV

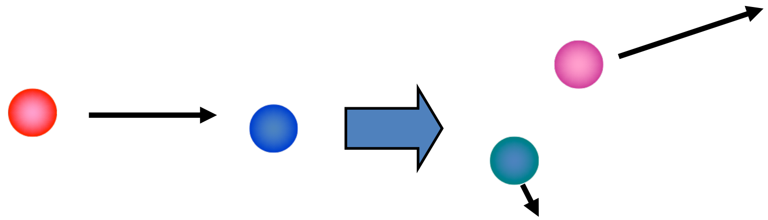


$$O(lsj\tau;\xi)\left|E_iJ_i\pi_iT_i;\xi_i\right\rangle = \sum\!\!\!\!\!\!\int_f M_{if}(E_f)\left|E_fJ_f\pi_fT_f;\xi_f\right\rangle$$ Response

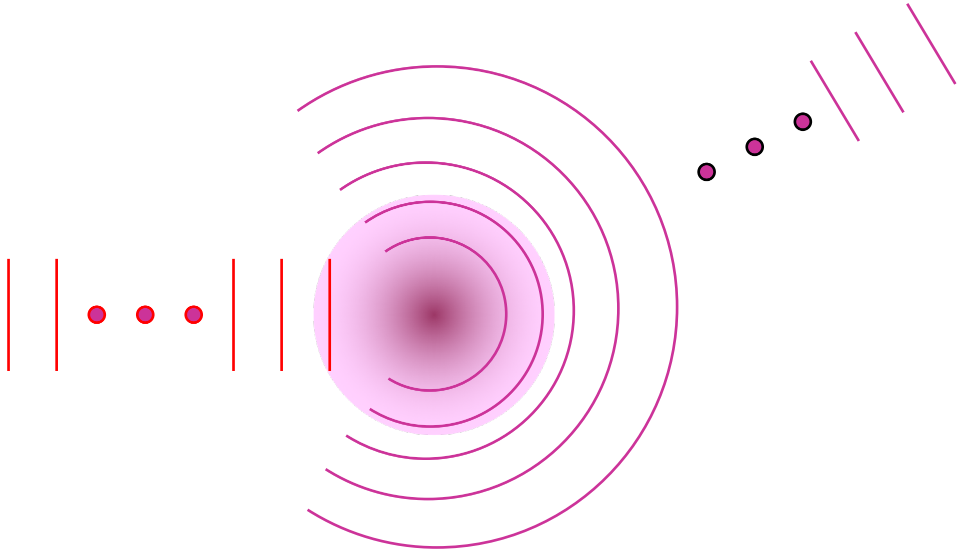
$\left|M_{if}(E_f)\right|^2$: Energy Spectrum

(standard) Picture of nuclear reaction

Nuclear Reaction



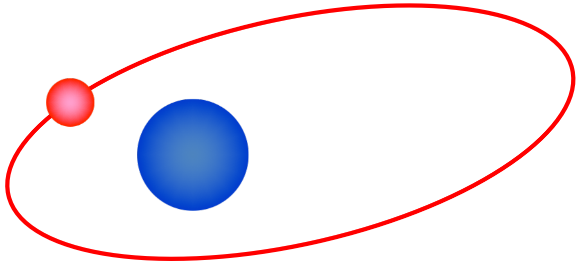
Classical system (time dependent; local)
→ (time independent) classical wave mechanics



Quantum system (time independent; infinite region)

cf.

Bound State



Classical system
time dependent motion or
time independent (confined) orbit

↕

wave function in Quantum system

corresponding picture
can be imagine

Infinites in time & space

For bound system

Energy eigen state (constant energy)

- Time independent (only the phase evolution (exp(-iEt/hbar))
- Localized wave function

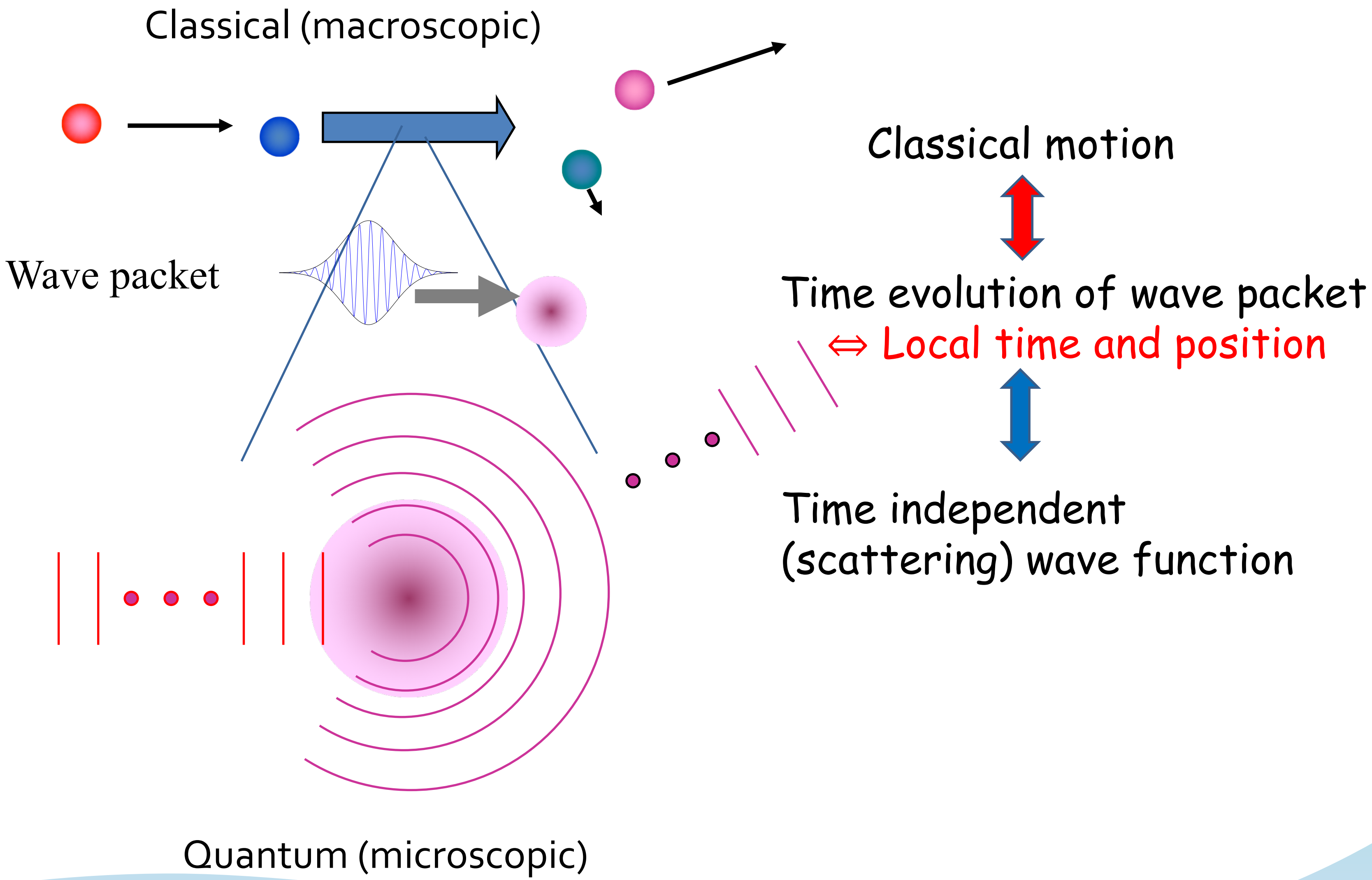
For scattering system

Energy eigen state (constant energy)

- Time independent
  - Superposition of various channels, sub-systems of which have different time evolution (exp(-i(Ea+Eb+…+Erel)t/hbar) even at asymptotic region)
- Non-local wave function
  - Asymptotic wave functions
  - Boundary conditions may be non-trivial in many-body scattering system

→ Scattering of Localized wave packet

Picture of nuclear reactions (time dependent wave packet)

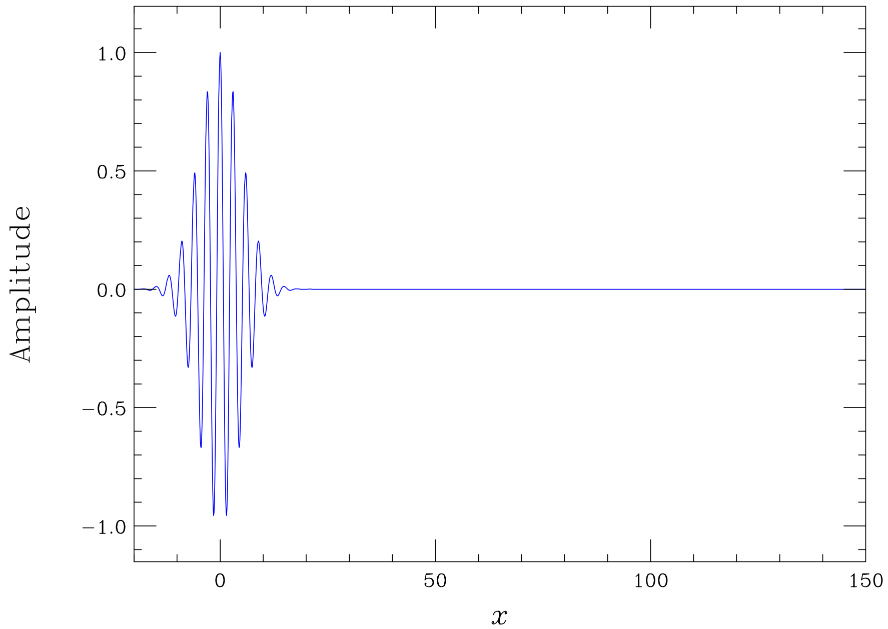


Minimal wave packet

$$\psi(x,t=0) = \left[\frac{1}{\sqrt{2\pi}\sigma_0}\right]^{1/2} \exp\left[-\frac{x^2}{4\sigma_0^2}\right] \exp(ik_0 x)$$

$$P_\psi(x,k) = \frac{1}{\pi}\int_{-\infty}^{\infty} dy\, \psi^*(x+y)\,\psi(x-y)\,\mathrm{e}^{2iky}$$

Wigner Transform



$$P_\psi(x,k;t=0) = \frac{1}{\pi}\exp\left[-\frac{x^2}{2\sigma_0^2}\right]\exp\left[-2\sigma_0^2(k-k_0)^2\right]$$

$$\delta x \cdot \delta k = (\sigma)\cdot\left(\frac{1}{2\sigma}\right) = \frac{1}{2}$$ uncertainty relation

Schroedinger eq. for wave packet

$$i\hbar \frac{\partial}{\partial t} \Psi\left(t, \{\vec{r}_i\}\right) = H\left(\{\vec{r}_i\}\right) \Psi\left(t, \{\vec{r}_i\}\right)$$

$$\Psi\left(t + \Delta t, \{\vec{r}_i\}\right) = \exp\left(-i \frac{H\left(\{\vec{r}_i\}\right)}{\hbar} \Delta t\right) \Psi\left(t, \{\vec{r}_i\}\right)$$

$$a\left(\omega\right) \psi_\omega\left(\{\vec{r}_i\}\right) = \int dt\ \Psi\left(t, \{\vec{r}_i\}\right) \exp\left(i\omega t\right)$$

$$\Psi\left(t, \{\vec{r}_i\}\right) = \int d\omega\ a\left(\omega\right) \psi_\omega\left(\{\vec{r}_i\}\right) \exp\left(-i\omega t\right)$$

- Hamiltonian : Operator of time evolution
- Time evolution of wave packet with proper boundary (initial) conditions.
- Harmony with classical picture
- Size of wave packet ~ Fluctuation of momentum ~ reaction time ~ coherent sum of various incident energies

Time evolution of minimal wave packet *(mass m)*

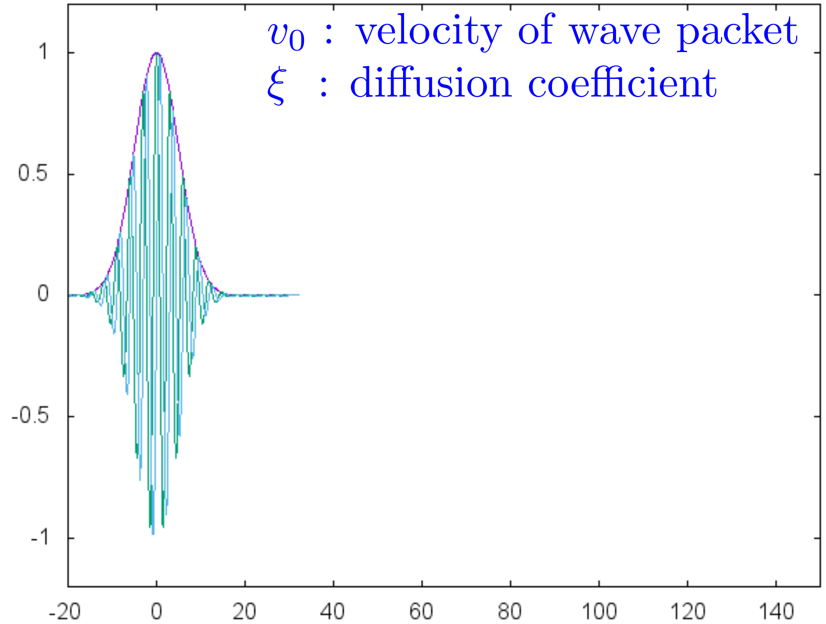
Analytical solution (tedious but very good exercise of QM)

$$\psi(x,t) = \left[\frac{1}{\sqrt{2\pi}\sigma_0 (1+i\xi t)}\right]^{1/2} \exp\left[-\frac{x^2}{4\sigma_0^2(1+i\xi t)} + \frac{i(k_0 x - \omega_0 t)}{1+i\xi t}\right]$$

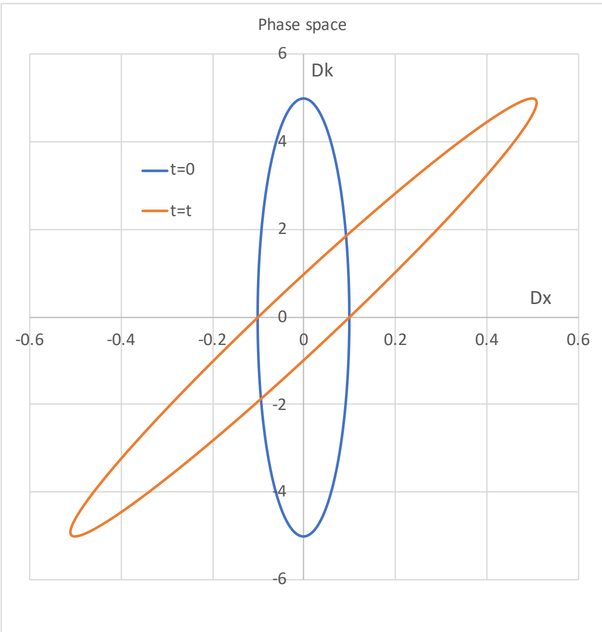
$$= \left[\frac{1}{\sqrt{2\pi}\sigma_0 (1+i\xi t)}\right]^{1/2} \exp\left[-\frac{(x-v_0 t)^2}{4\sigma_0^2\left(1+(\xi t)^2\right)} + i\left(K(x,t)\,x - \Omega(t)\,t\right)\right]$$

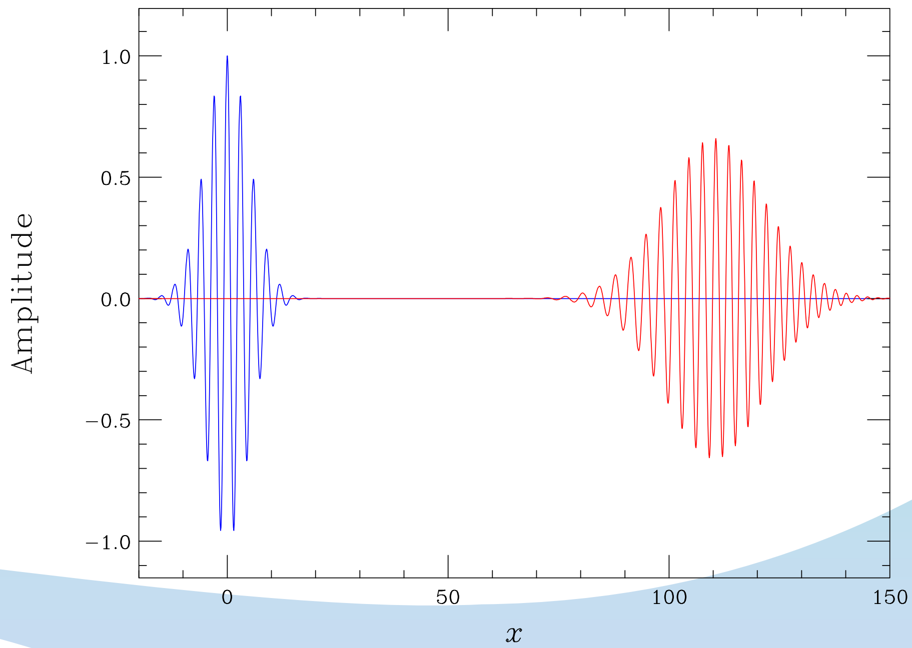
$$\xi = \frac{\hbar}{2m\sigma_0^2}\ ;\ \omega_0 = \frac{\hbar k_0^2}{2m}\ ;\ v_0 = \frac{\hbar k_0}{m}$$

$$K(x,t) = k_0 \cdot \frac{1+(\xi t)^2\,(x/(2v_0 t))}{1+(\xi t)^2}\ ;\ \Omega(t) = \frac{\omega_0}{1+(\xi t)^2}$$



Wigner Transform





$$P_\psi(x,k;t) = \frac{1}{\pi} \exp\left[-\frac{1}{2\sigma_0^2}(\Delta x - \Delta v\ t)^2\right] \exp\left[-2\sigma_0^2 \Delta k^2\right]$$

$$\Delta k = k - k_0\ ;\ \Delta v = \frac{\hbar \Delta k}{m}\ ;\ \Delta x = x - v_0 t$$

Simple two-body case (3-dim. s-wave ~ 2n case)

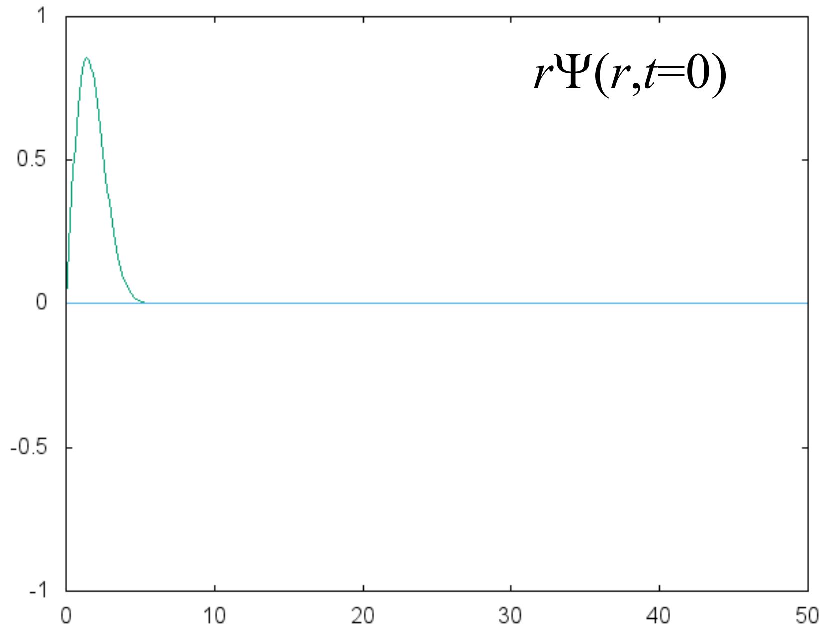
Time propagation of wave packet $\Psi(t)$:

$$i\hbar\frac{\partial}{\partial t}\Psi(t) = H\Psi(t)$$

$$\Psi(t+\Delta t) = \exp\left(-i\frac{H}{\hbar}\Delta t\right)\Psi(t)$$

$$\simeq \frac{1-\frac{i}{2}\frac{H}{\hbar}\Delta t}{1+\frac{i}{2}\frac{H}{\hbar}\Delta t}\Psi(t)$$

➡ Difference equation



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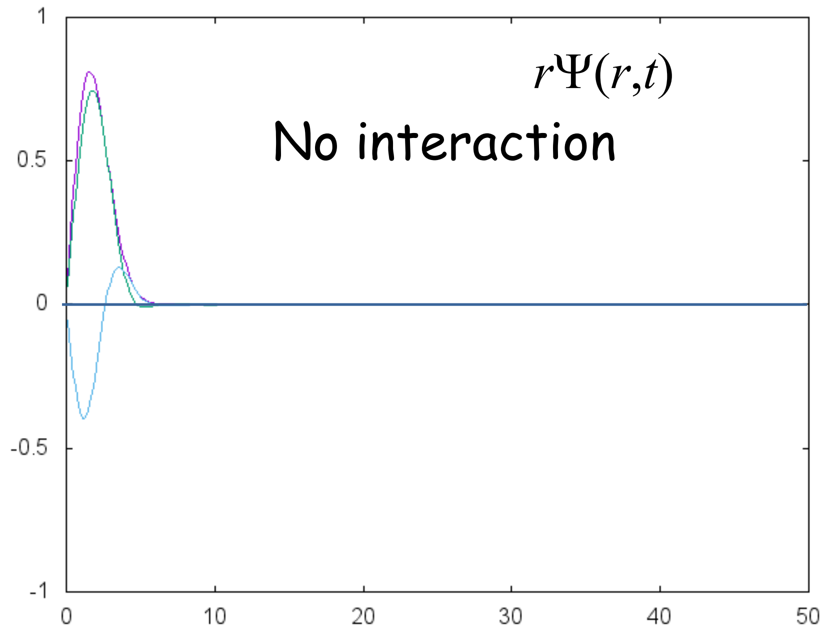
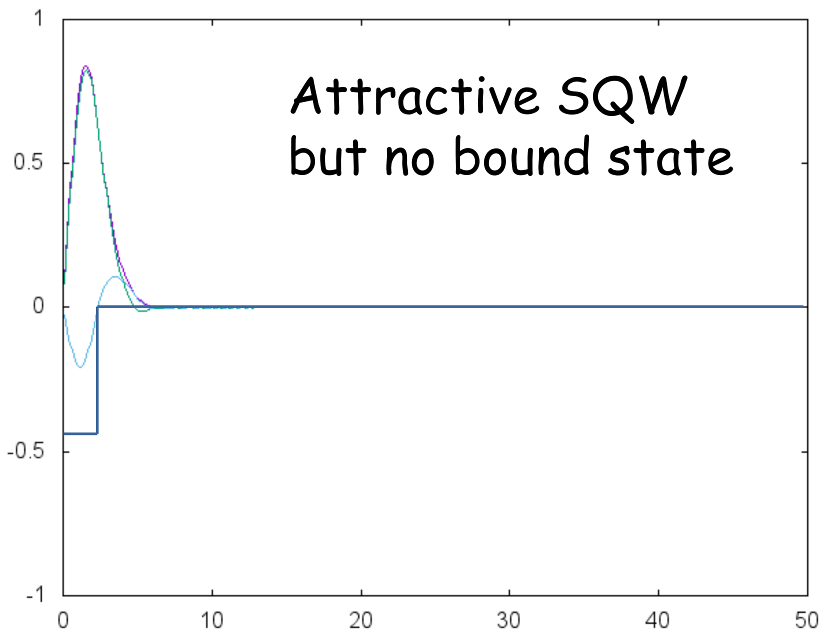
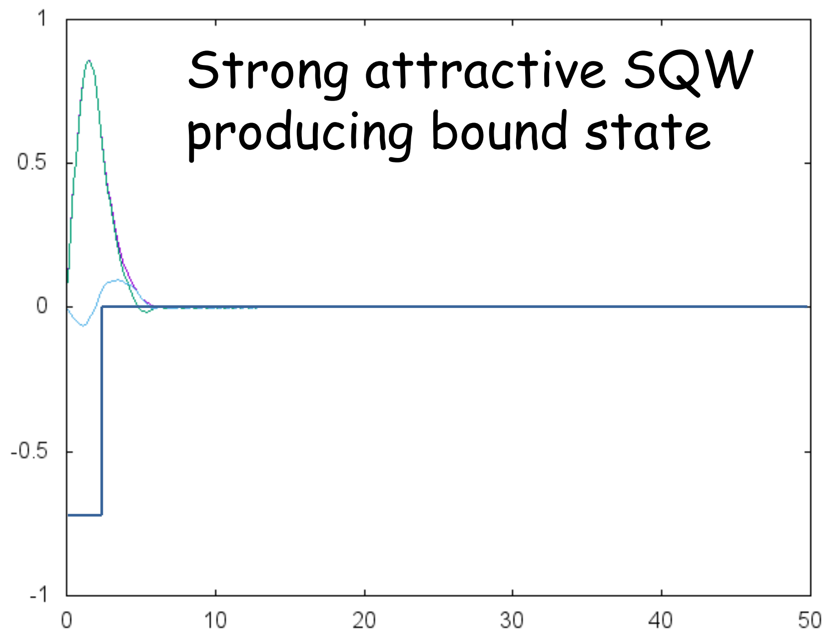
$$\simeq \frac{1-\frac{i}{2}\frac{H}{\hbar}\Delta t}{1+\frac{i}{2}\frac{H}{\hbar}\Delta t}\Psi(t)$$

➡ 差分方程式







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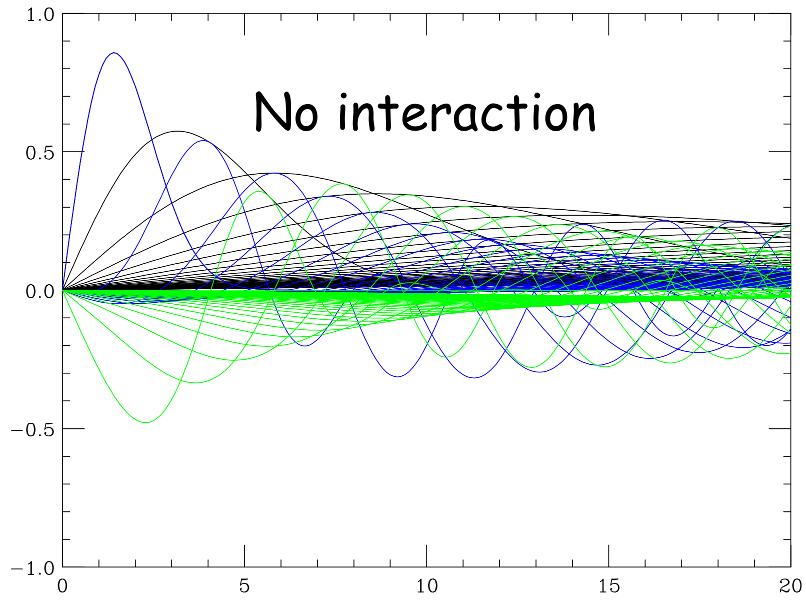
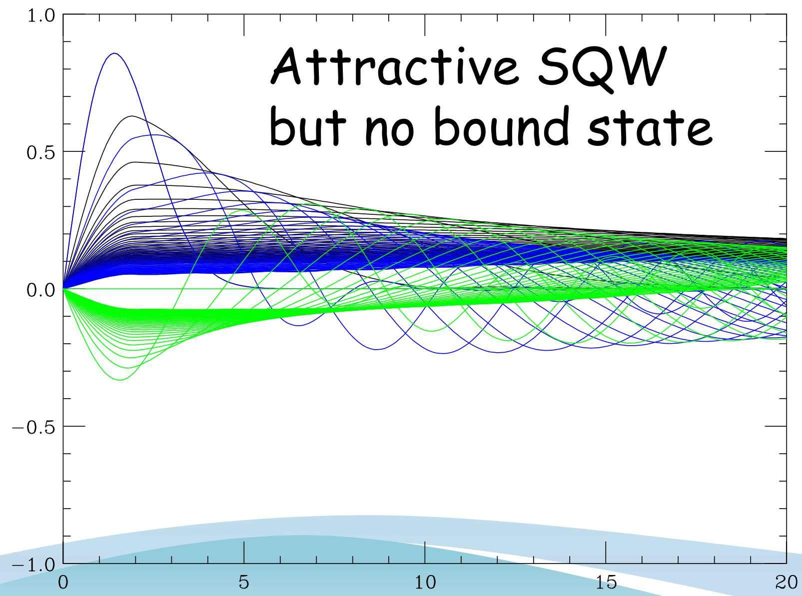
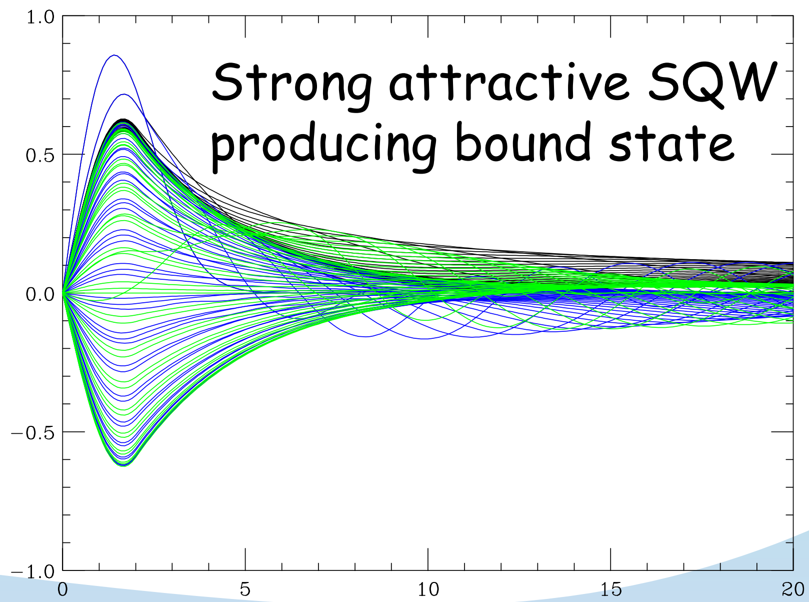
$$\Psi(t+\Delta t) = \exp\left(-i\frac{H}{\hbar}\Delta t\right)\Psi(t) \simeq \frac{1-\frac{i}{2}\frac{H}{\hbar}\Delta t}{1+\frac{i}{2}\frac{H}{\hbar}\Delta t}\Psi(t)$$

➡ 差分方程式







Scattering by potential

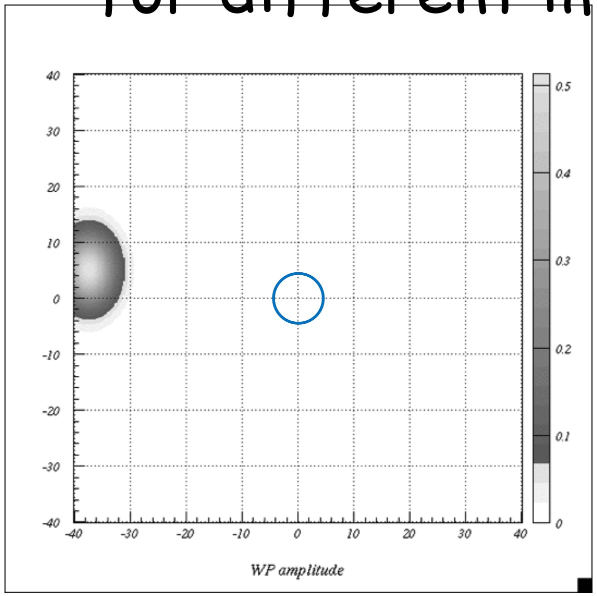
$$
\begin{aligned}
i\hbar\frac{\partial}{\partial t}\Psi\left(t,\{\vec{r}_i\}\right) &= H\left(\{\vec{r}_i\}\right)\Psi\left(t,\{\vec{r}_i\}\right) \\
\Psi\left(t+\Delta t,\{\vec{r}_i\}\right) &= \exp\left(-i\frac{H\left(\{\vec{r}_i\}\right)}{\hbar}\Delta t\right)\Psi\left(t,\{\vec{r}_i\}\right) \\
a\left(\omega\right)\psi_\omega\left(\{\vec{r}_i\}\right) &= \int dt\ \Psi\left(t,\{\vec{r}_i\}\right)\exp\left(i\omega t\right)
\end{aligned}
$$

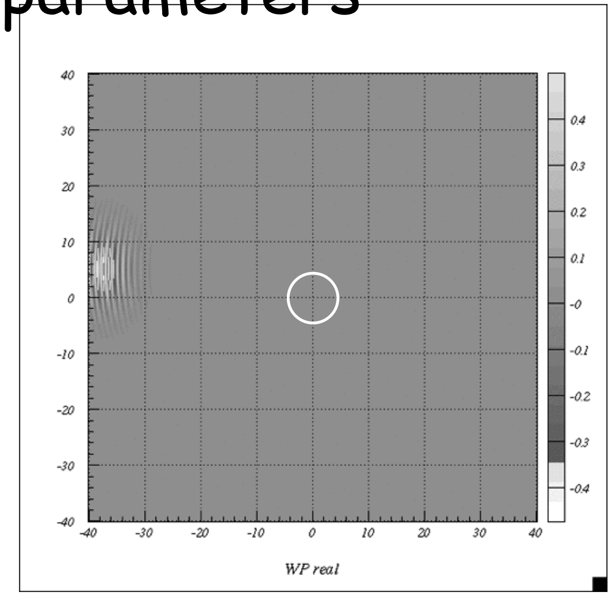
$$H = T + V$$

$$V = \frac{U + iW}{1+\exp\left[\left(r-R\right)/a\right]}$$

Scattering of wave packets (2D) for different impact parameters

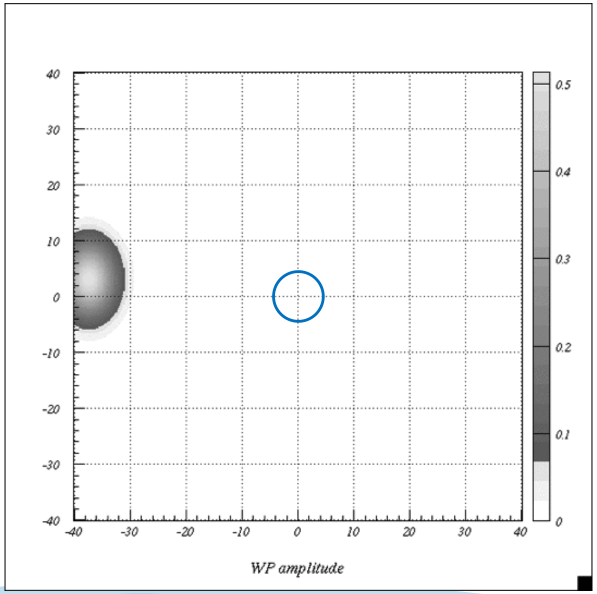
b=5 fm

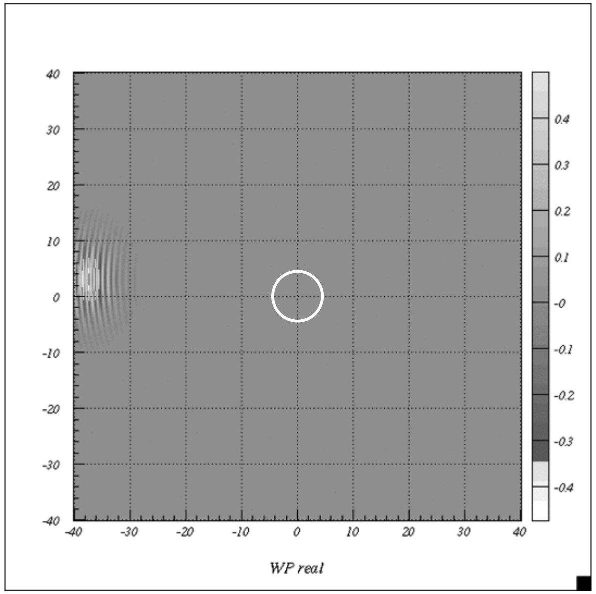




Potential
R=5 fm
a=0.65 fm

b=3 fm



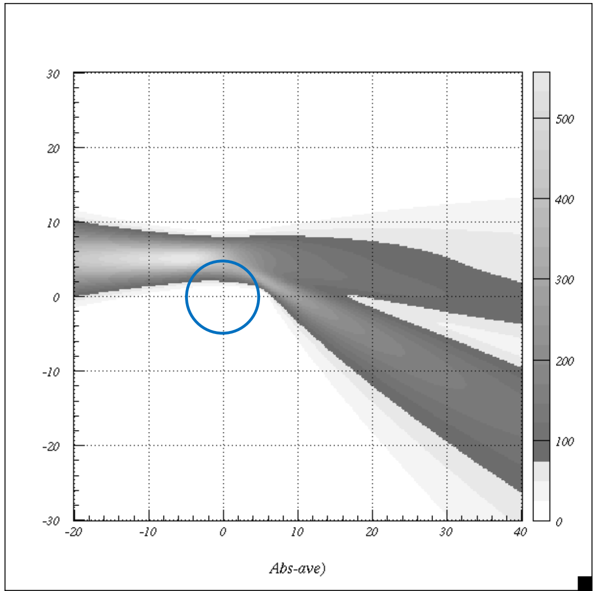


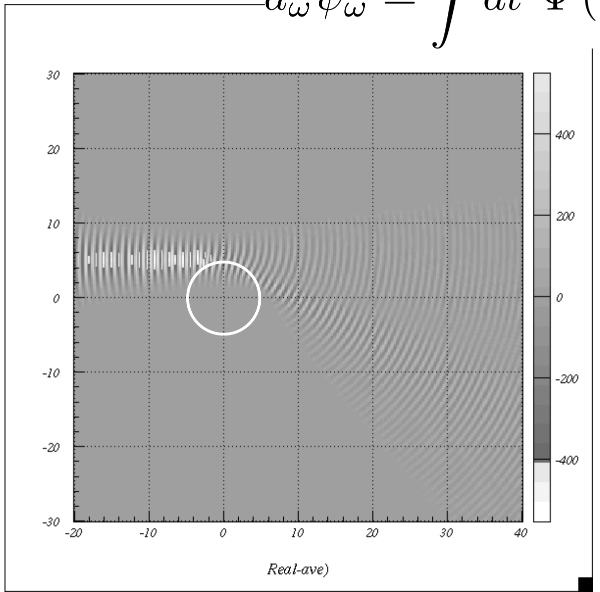
Scattering of wave packet (time integrated)

$$\Psi(t) = \int d\omega \; a(\omega)\, \psi_\omega \exp(-i\omega t)$$

$$a_\omega \psi_\omega = \int dt \;\; \Psi(t) \exp(i\omega t)$$

b=5 fm

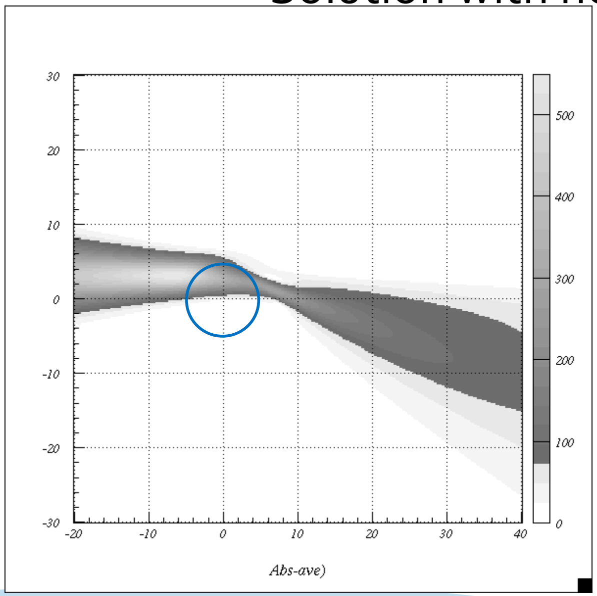


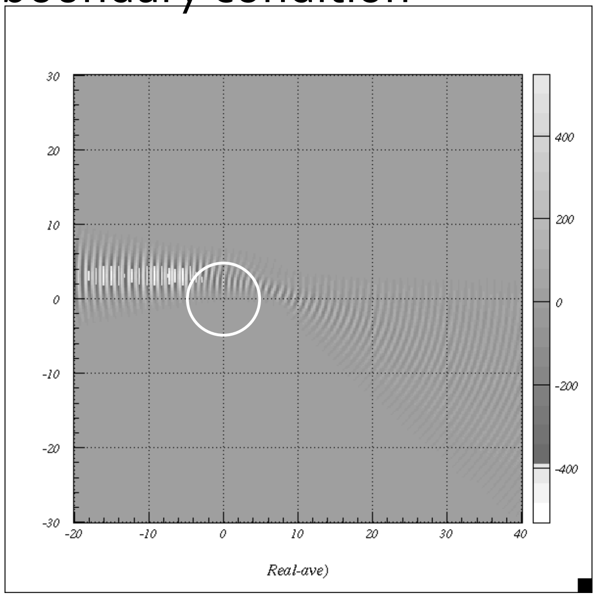


Potential
R=5 fm
a=0.65 fm

Solution with non-trivial boundary condition

b=3 fm



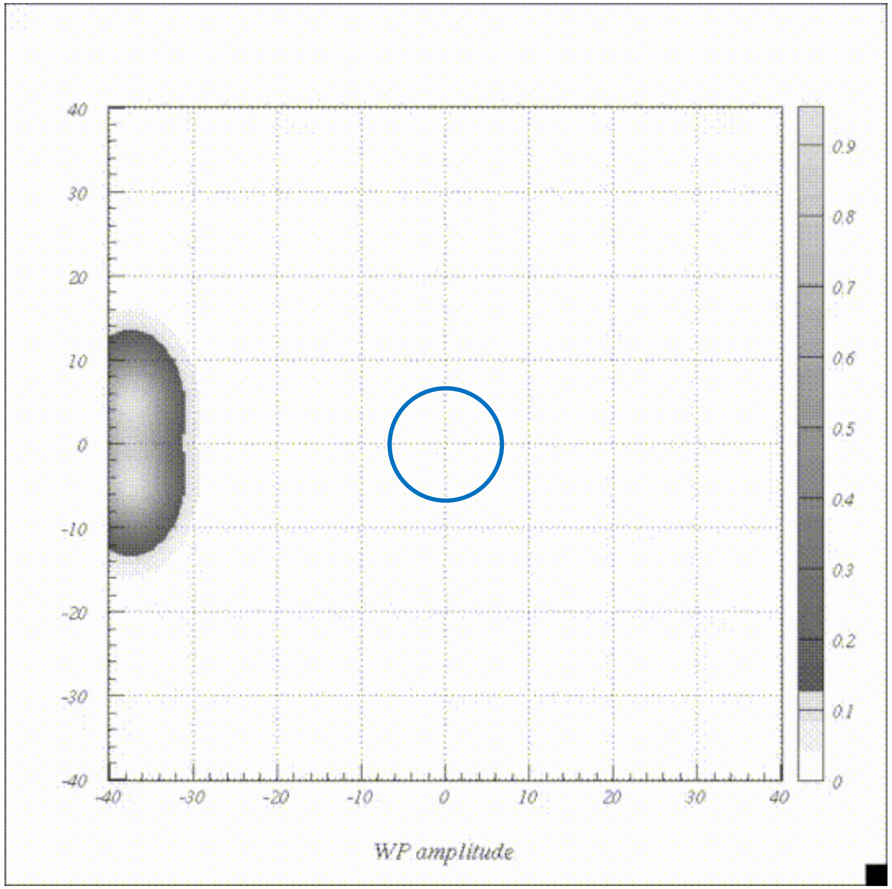


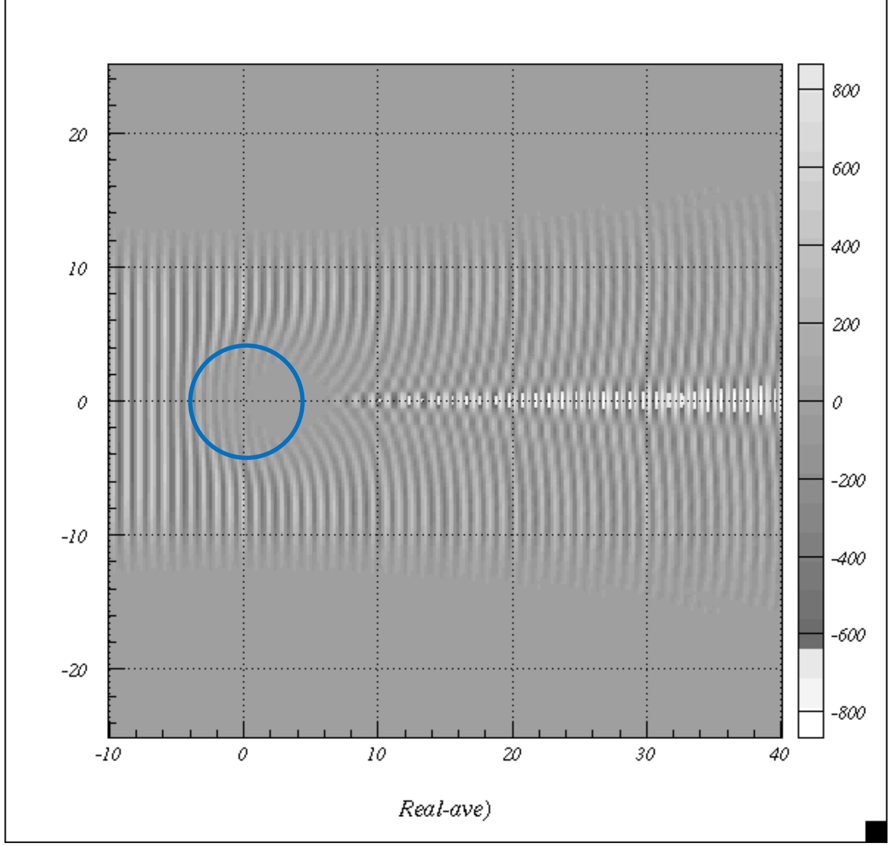
Scattering of wave packet (3D)

$$\Psi(t) = \int d\omega \; a(\omega)\, \psi_\omega \exp(-i\omega t)$$

$$a_\omega \psi_\omega = \int dt \; \Psi(t) \exp(i\omega t)$$

Potential
R=5 fm, a=0.65 fm





(Sch. Eq. after integrate with azimuthal angle)

Integral range is not necessary to be infinity, but pathing through time of wave packet.
⇔ "adiabatic" time ⇔ $i\eta$ in Green function

Lippmann-Schwinger Equation

Integral equation satisfying boundary conditions

$$H\Psi_\alpha^{(+)}(E) = E\Psi_\alpha^{(+)}(E)$$

$$H = (h_a + h_A + T_\alpha) + V_\alpha = H_\alpha + V_\alpha$$

$$\Psi_\alpha^{(+)}(E) = \Phi_\alpha(E) + \boxed{\frac{1}{E - H_\alpha + i\eta} V_\alpha \Psi_\alpha^{(+)}(E)}$$

Outgoing spherical waves

$$= \frac{i\eta}{E - H + i\eta}\Phi_\alpha(E)$$

$$\boxed{\Psi_\alpha^{(-)}(E) = \Phi_\alpha(E) + \frac{1}{E - H_\alpha - i\eta} V_\alpha^* \Psi_\alpha^{(-)}(E)}$$

Solution having incoming spherical waves

$$S_{\beta\alpha} = \left\langle \Psi_\beta^{(-)}(E) \middle| \Psi_\alpha^{(+)}(E) \right\rangle$$

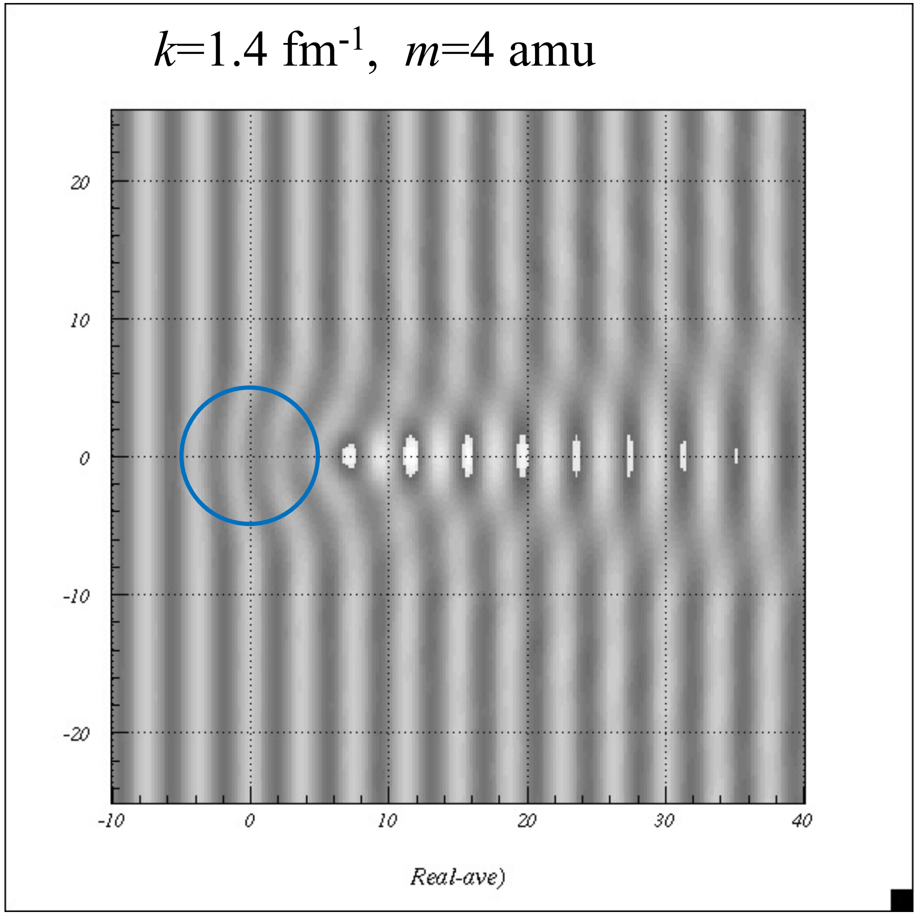
S-matrix ：
Probability amplitude overlapping plane wave, Φ_β

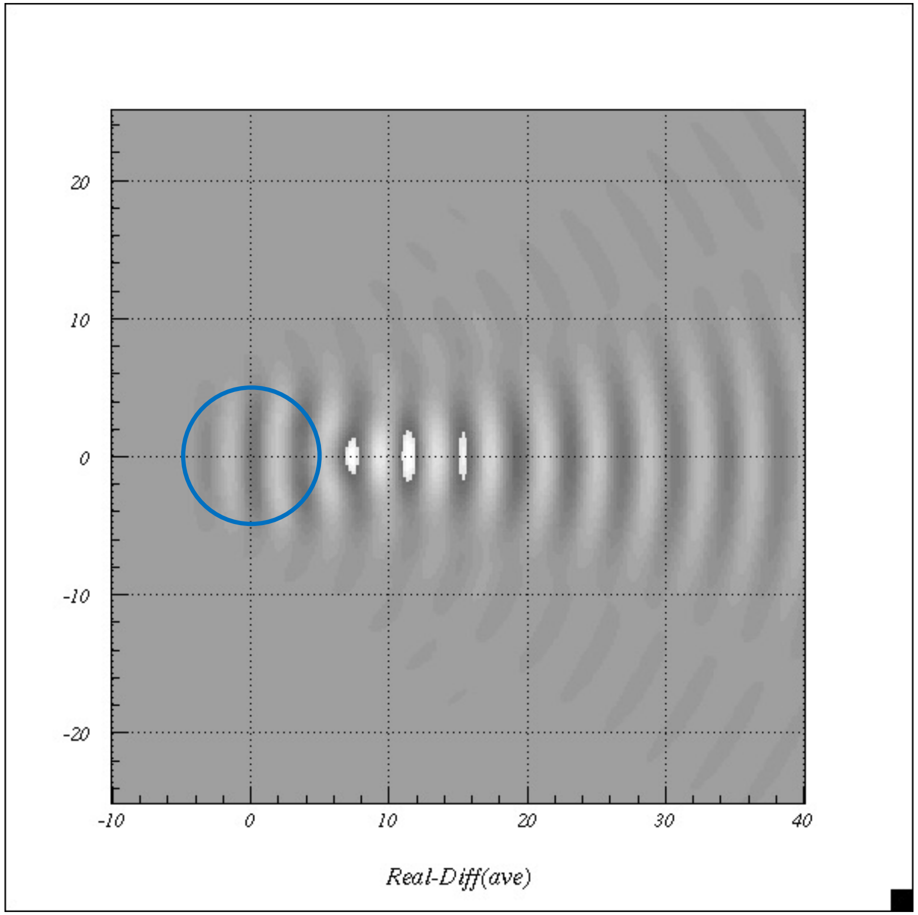
Scattering of wave packet (3D)

Constant energy
time-independent wave function
w/o multipole expansion

$$\Psi(t) = \int d\omega \ a(\omega)\,\psi_\omega \exp(-i\omega t)$$

$$a_\omega \psi_\omega = \int dt \ \Psi(t) \exp(i\omega t)$$



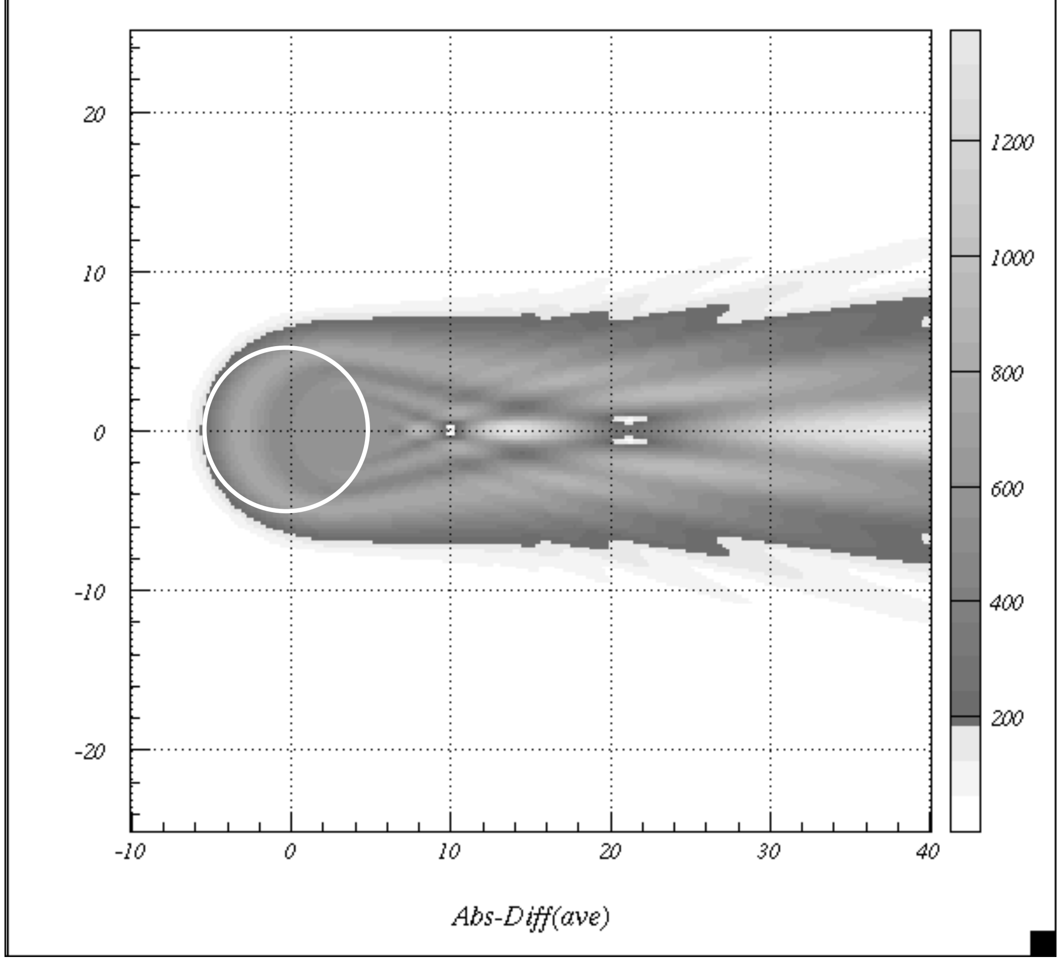


Scattering wave (spherical wave)

Integral range is not necessary to be infinity, but pathing through time of wave packet.
⇔ "adiabatic" time ⇔ $i\eta$ in Green function

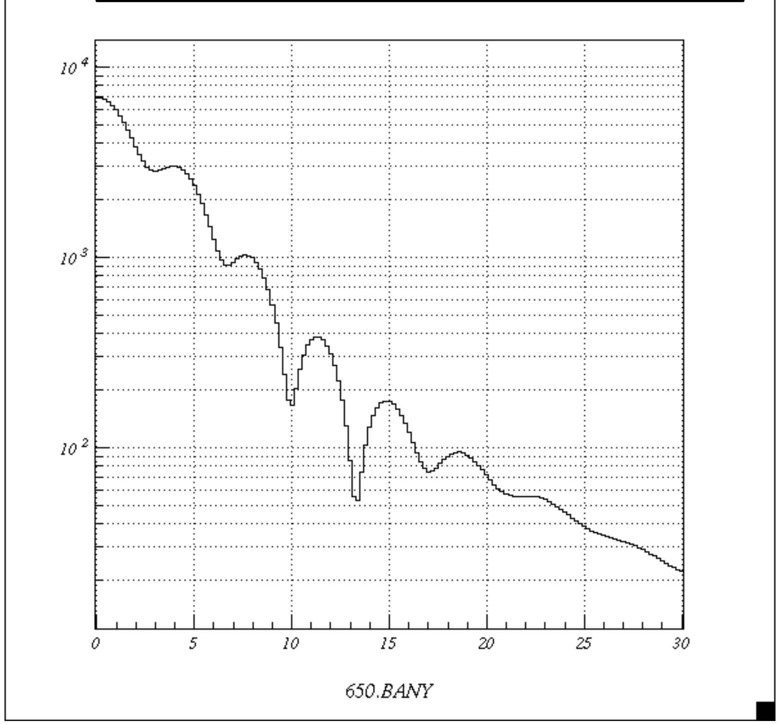
Scattering of wave packet (3D)

Amplitude of Scattering wave
(Total wave – Non-interacted wave)



Potential
R=5 fm, a=0.65 fm

Diffraction pattern
Information on Potential



Three-body case (1 dim.)

One-dimensional three-body system (Nucleon 1, 2 and Core 3 with infinite mass)

Hamiltonian:

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} + V_1(x_1) + V_2(x_2) + V_{12}(x_1 - x_2)$$

$$= -\frac{\hbar^2}{4m}\frac{\partial^2}{\partial X^2} - \frac{\hbar^2}{m}\frac{\partial^2}{\partial x_{12}^2} + V_1\left(X + \frac{x_{12}}{2}\right) + V_2\left(X - \frac{x_{12}}{2}\right) + V_{12}(x_{12})$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_\alpha^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_\beta^2} + V_1\left(\frac{x_\alpha + x_\beta}{\sqrt{2}}\right) + V_2\left(\frac{x_\alpha - x_\beta}{\sqrt{2}}\right) + V_{12}\left(\sqrt{2}\, x_\beta\right)$$

$$x_{12} = x_1 - x_2\ ;\ X = \frac{x_1 + x_2}{2}$$

$$x_\alpha = \frac{x_1 + x_2}{\sqrt{2}}\ ;\ x_\beta = \frac{x_1 - x_2}{\sqrt{2}}$$

Time propagation of wave packet $\Psi(t)$:

$$i\hbar\frac{\partial}{\partial t}\Psi(t) = H\Psi(t)$$

$$\Psi(t + \Delta t) = \exp\left(-i\frac{H}{\hbar}\Delta t\right)\Psi(t)$$

Cf. N. Watanabe and M. Tsukada, Phys. Rev. E 62 (2000) 2914

Initial condition of wave packet $\Psi(t)$:

$$\text{case 1:}\ \Psi(0) = \exp(-\kappa_2 |x_2|)\exp\left[-\frac{(x_1 - x_0)^2}{4(\Delta x_1)^2} + ik_0 x_1\right]$$

(p,p), (p,n), (p,pn), (p,d)

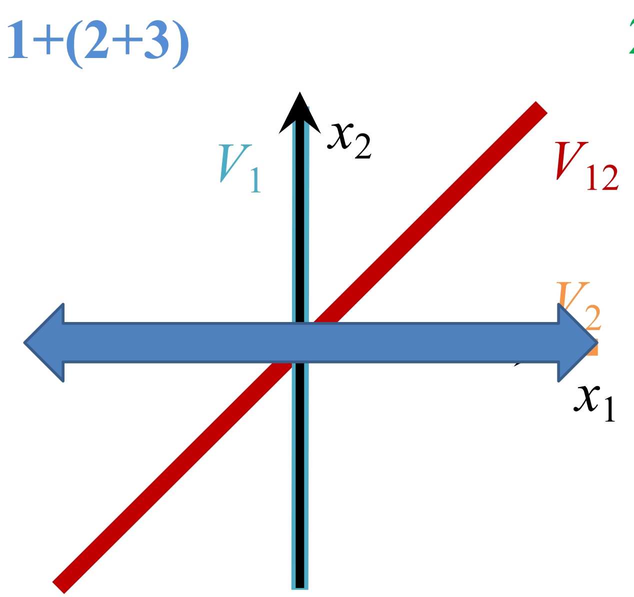
$$\text{case 2:}\ \Psi(0) = \exp(-\eta |x_{12}|)\exp\left[-\frac{(X - X_0)^2}{4(\Delta X)^2} + iK_0 X\right]$$

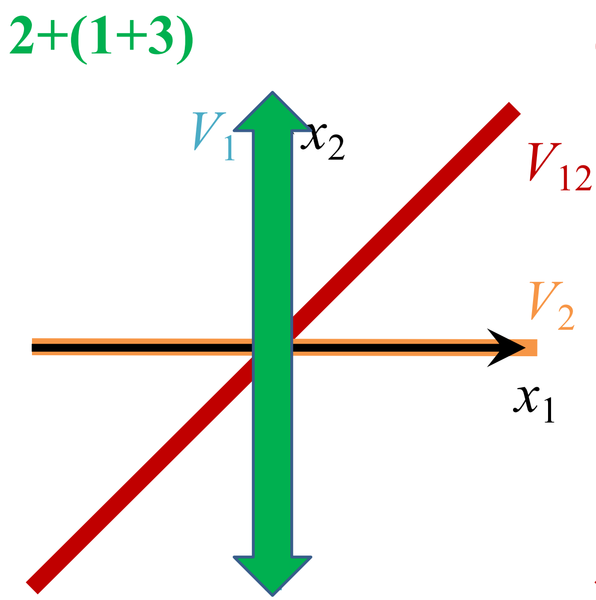
(d,d), (d,pn), (d,p), (d,n)

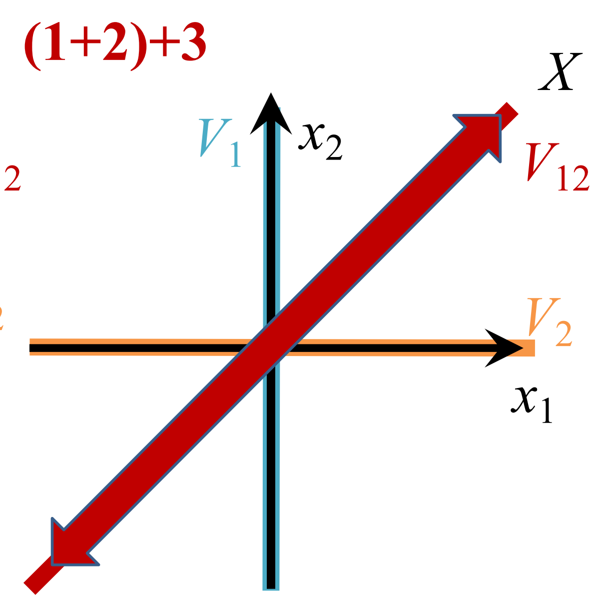
Channels: 1+(2+3); 2+(1+3); (1+2)+3; 1+2+3

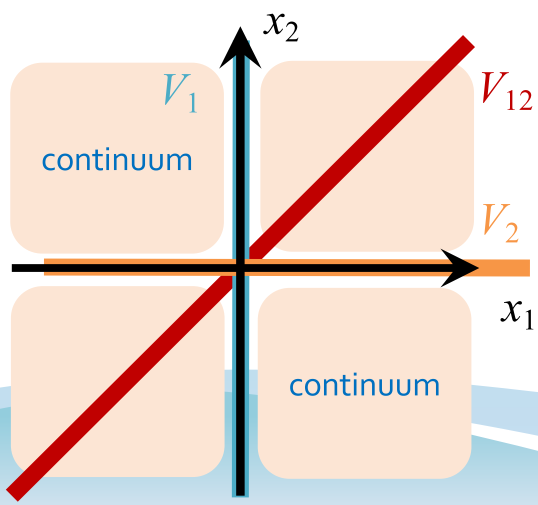
Channels of 1-dimensional 3-body system

4 Channels:









1+2+3
Continuum

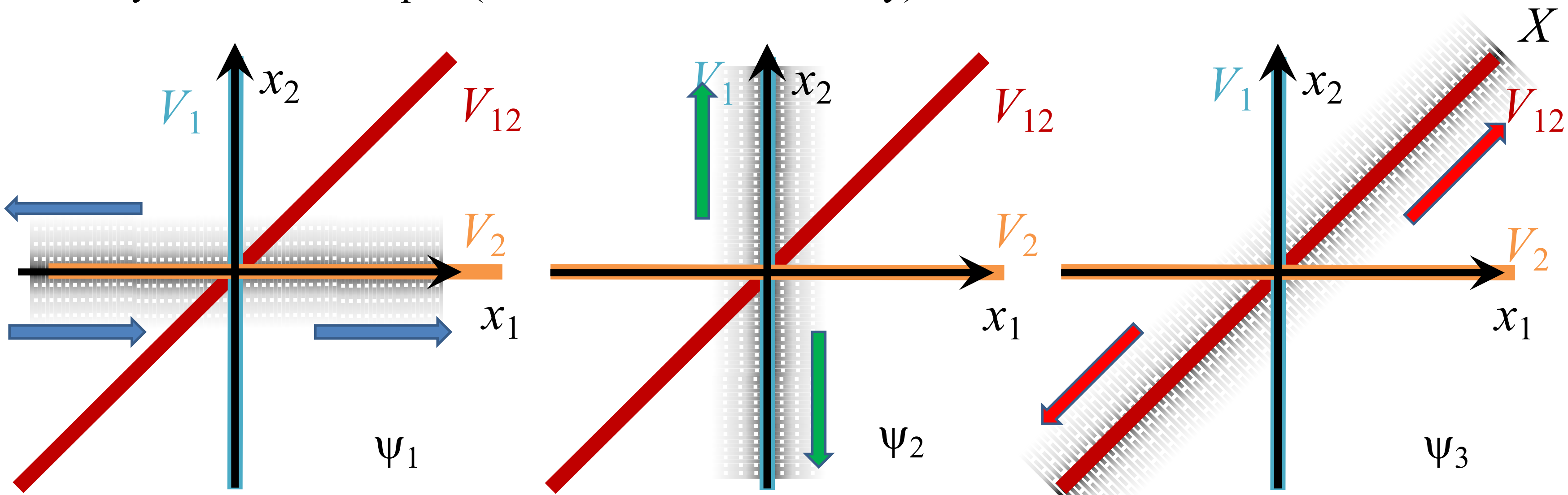
3-body scattering problem in time-independent scheme

Boundary condition

“Free” scattering wave at “asymptotic region”

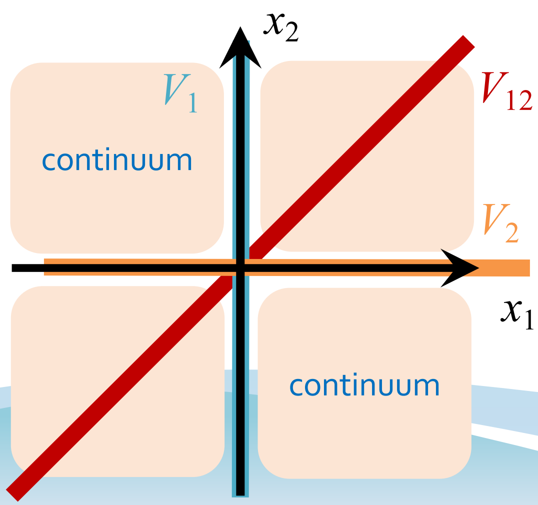
-> Faddeev equation $\Psi = \psi_1(x_1, x_2) + \psi_2(x_1, x_2) + \psi_3(x_{12}, X)$.

If 3-body channel isn’t open (all channels are two-body)



Effect of V_{12} is negligible at asymptotic region

Effects of V_1 and V_2 are negligible at asymptotic region

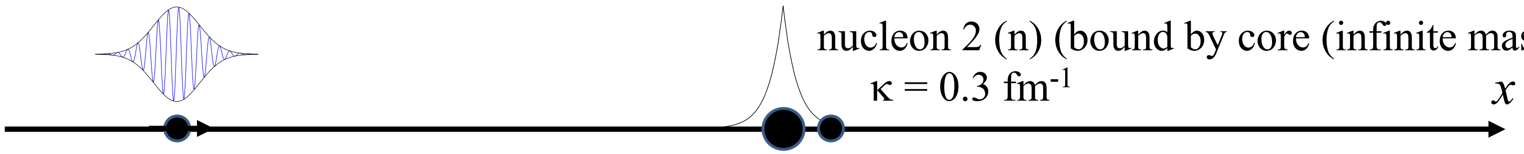


If 3-body channel is open, how to treat “continuum” and its boundary condition?

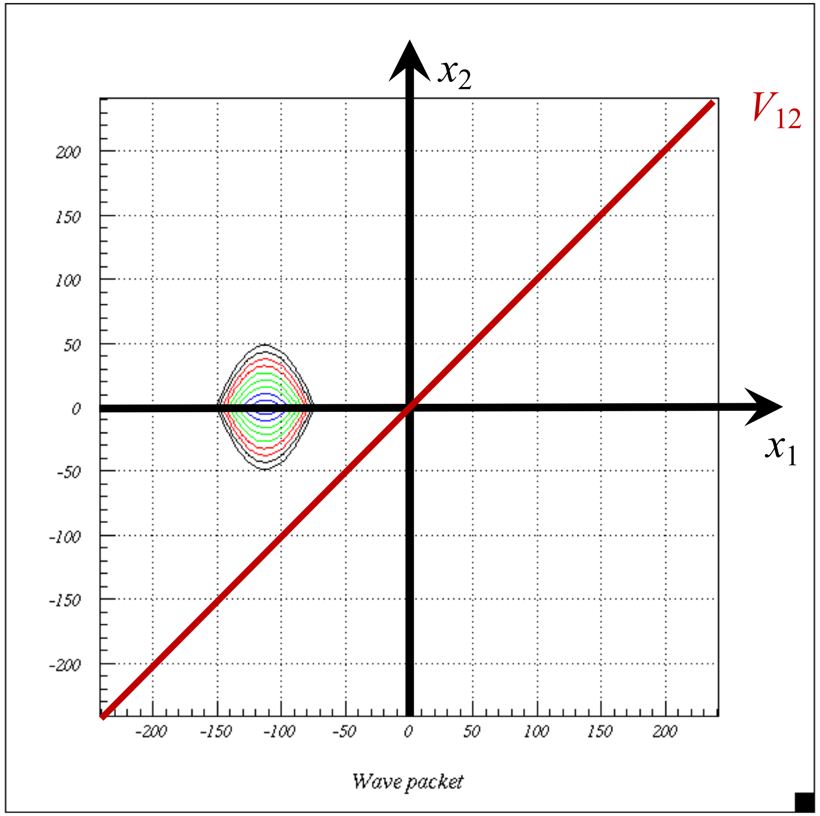
?? Extension of ψ_1 ? ψ_2 ? or ψ_3 ?

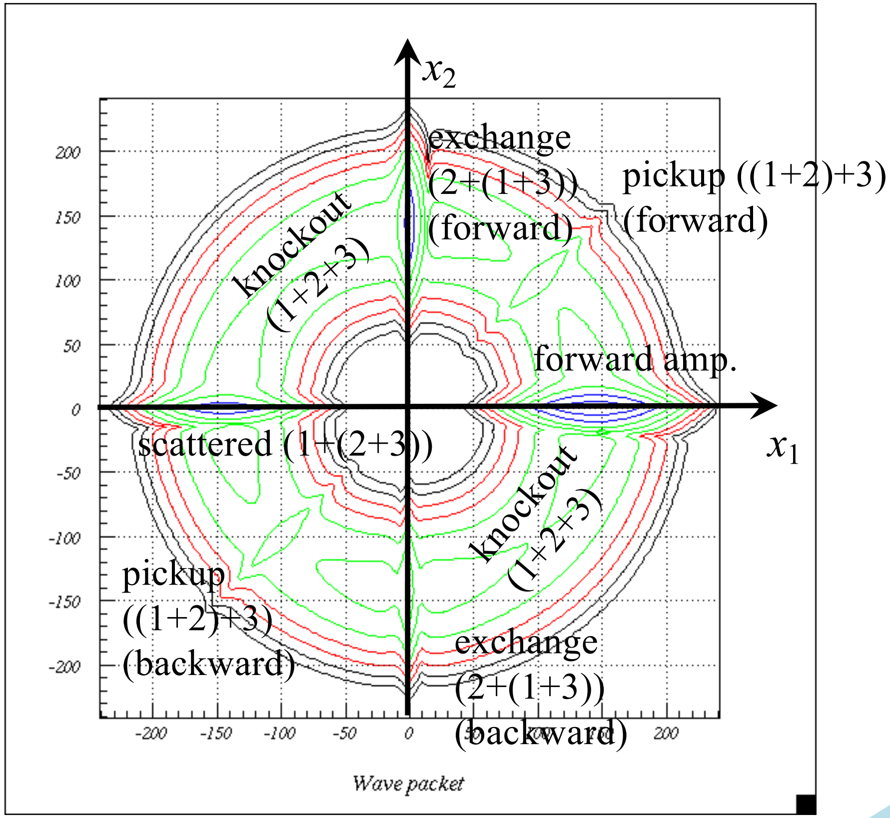
Interaction: V_{1c}, V_{2c}, V_{12} (~δ-function at x_1=0, x_2=0, x_1-x_2=0; attractive)

Case1:



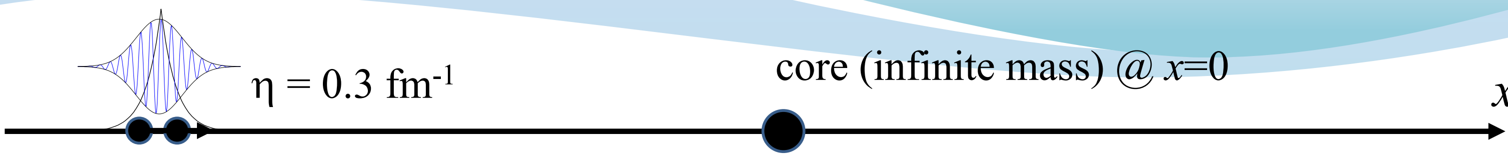
forward amp.: transmitted + forward scattering

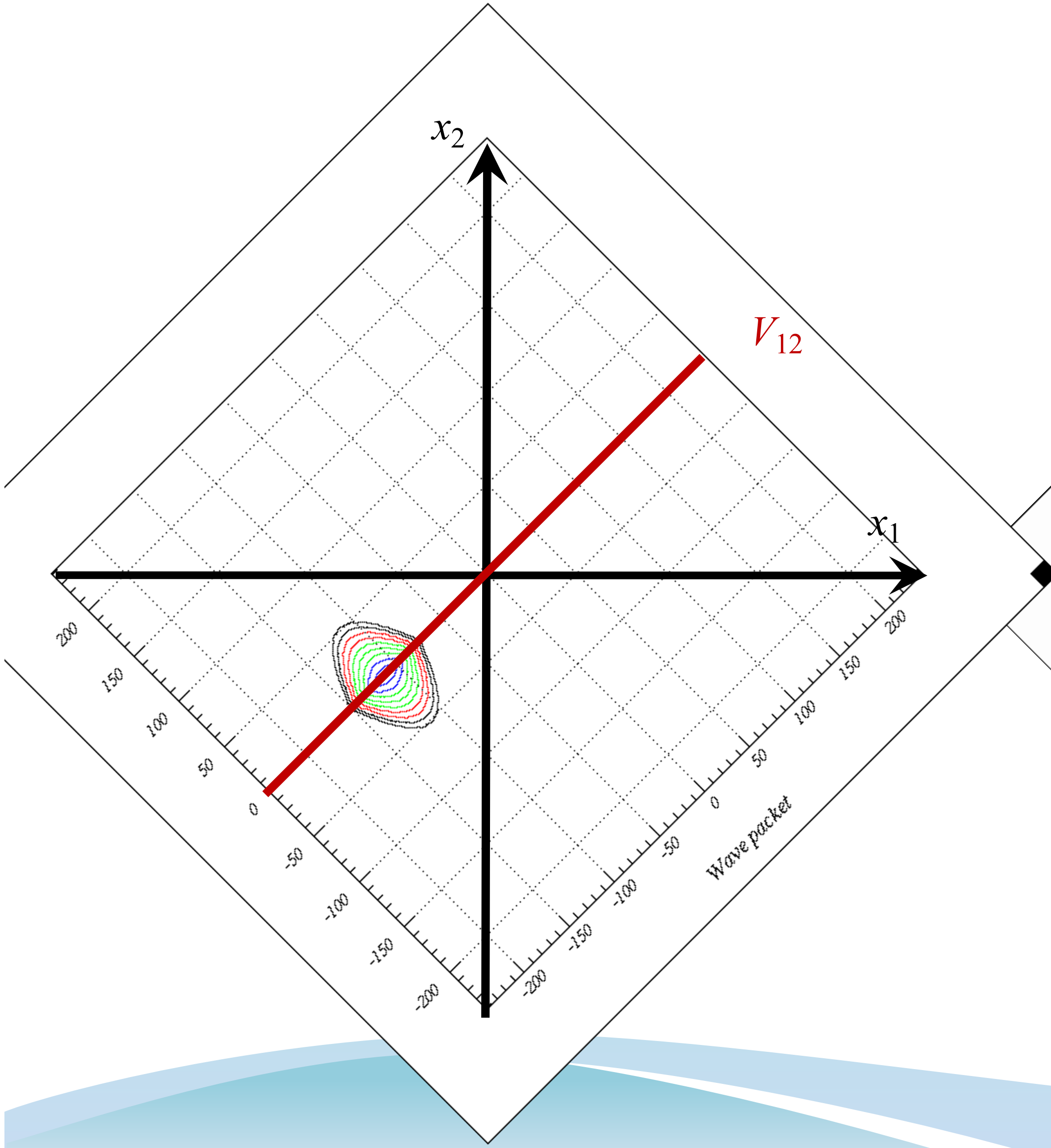


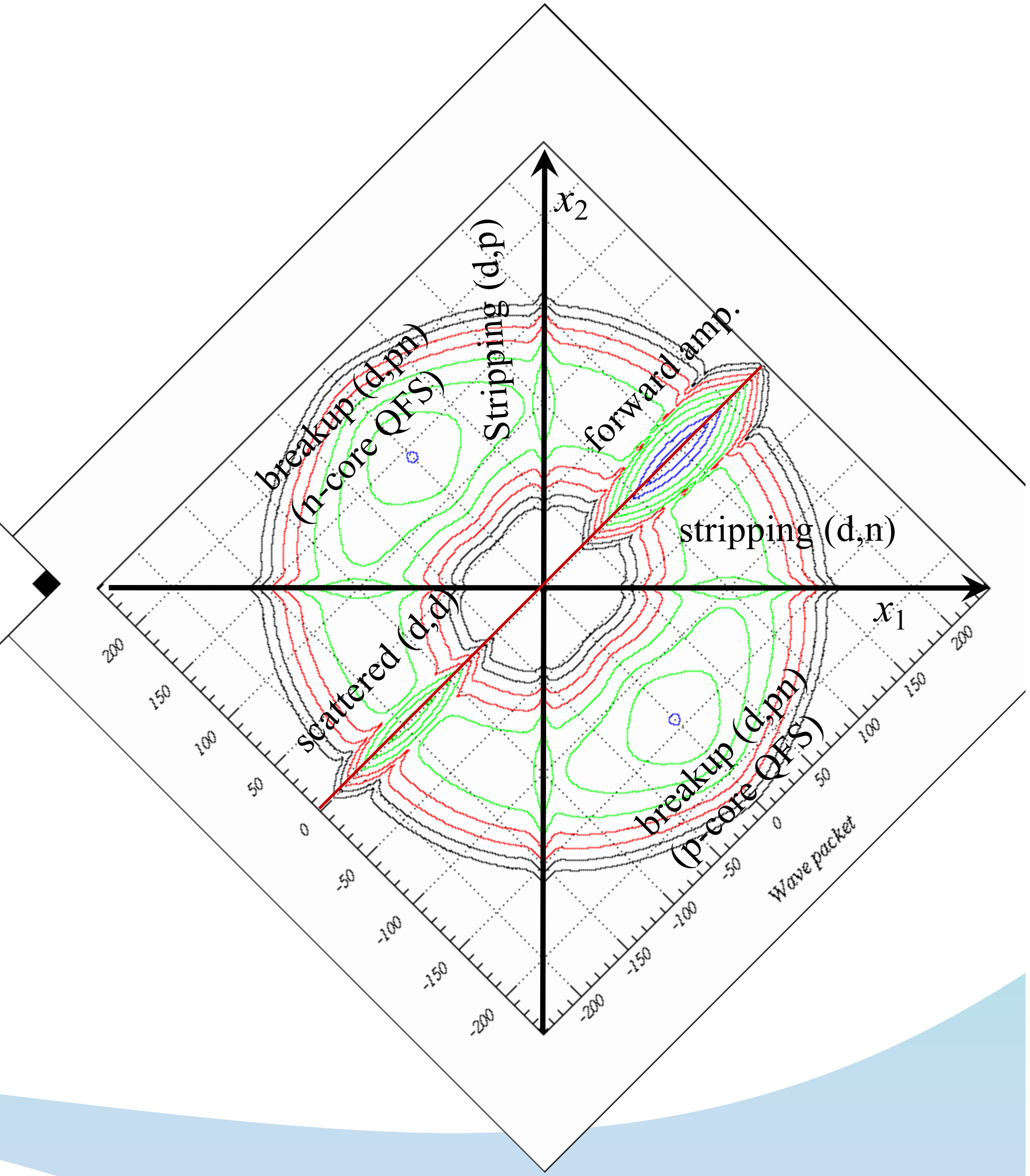


Interaction: V_1, V_2, V_{12} (δ-function; attractive)

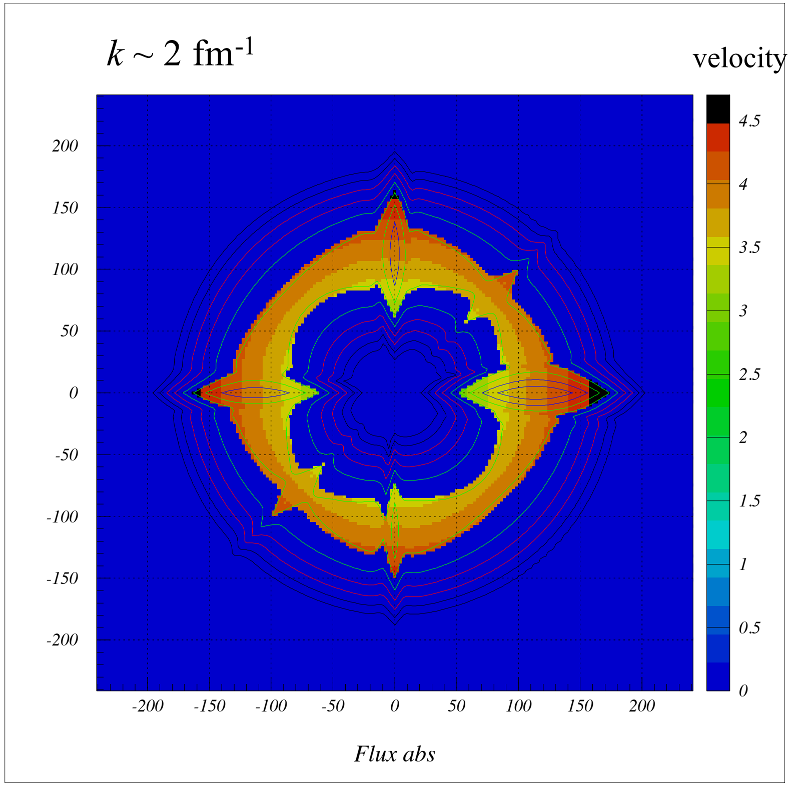
Case 2:

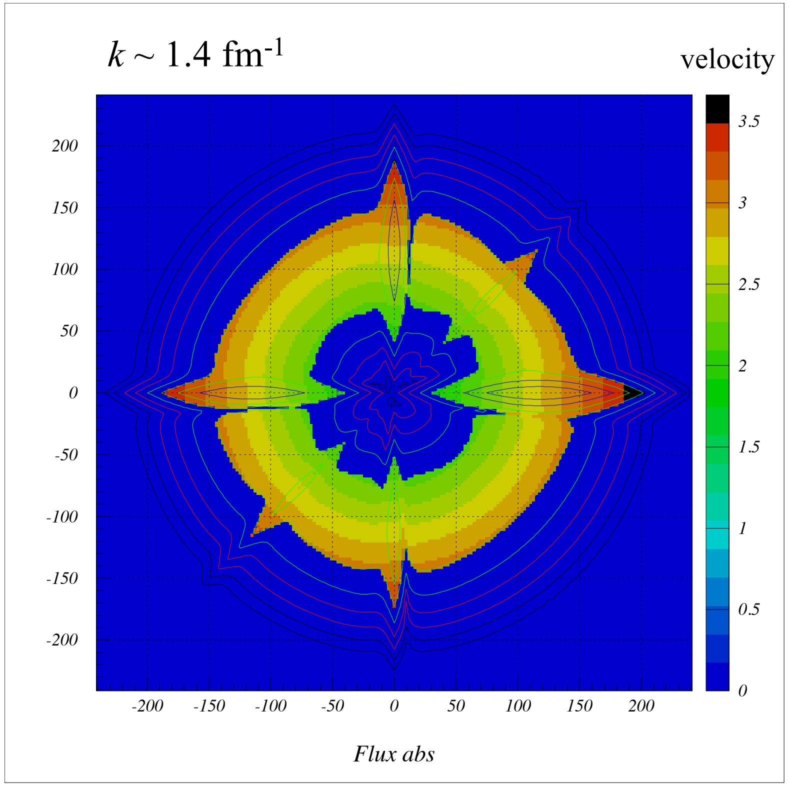






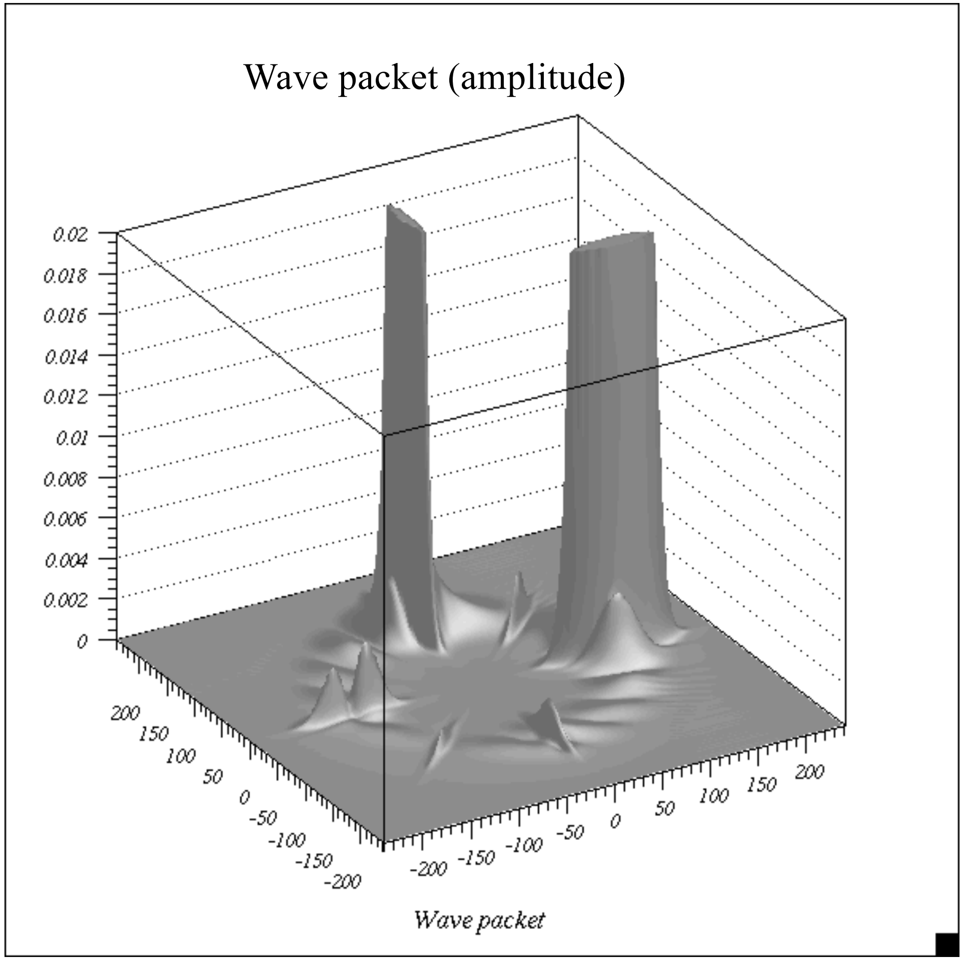
Incident energy dependence

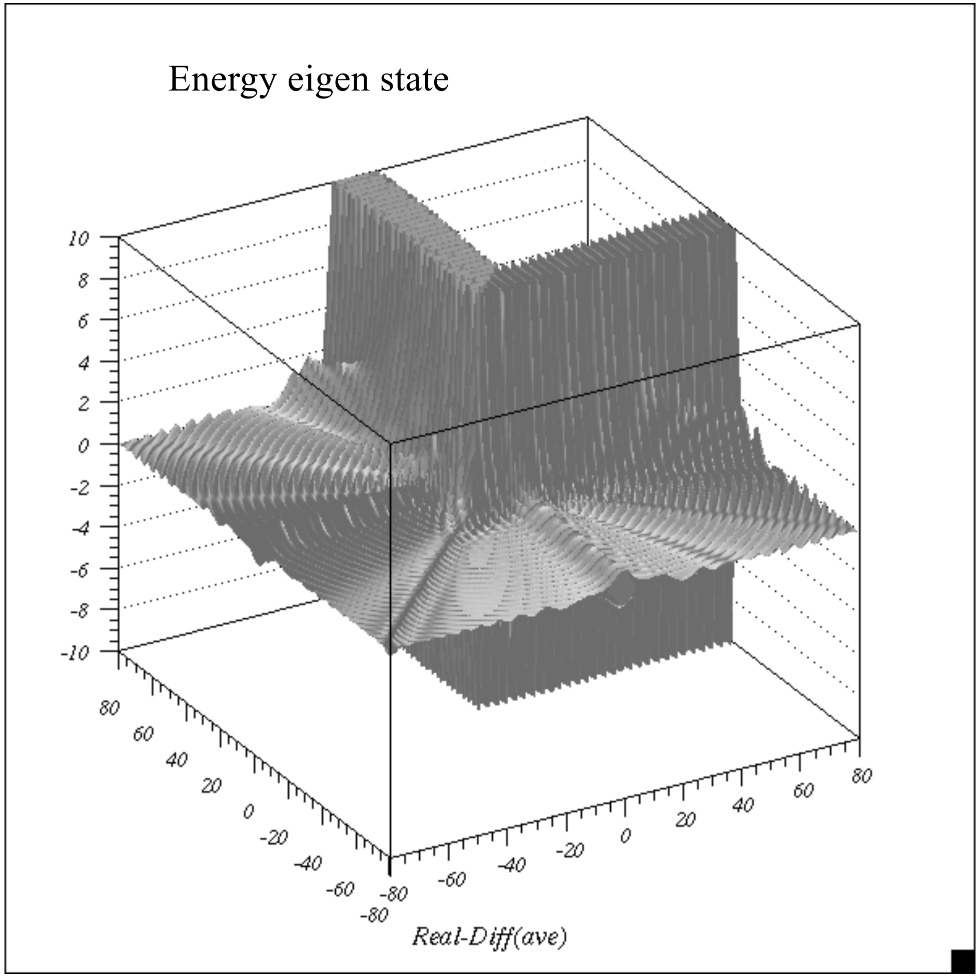




$$f_i = \frac{\hbar}{2mi}\left(\psi^* \frac{\partial \psi}{\partial x_i} - \frac{\partial \psi^*}{\partial x_i}\psi\right)$$

$$\text{velocity} \equiv \frac{\sqrt{f_1^2 + f_2^2}}{\psi^* \psi}$$





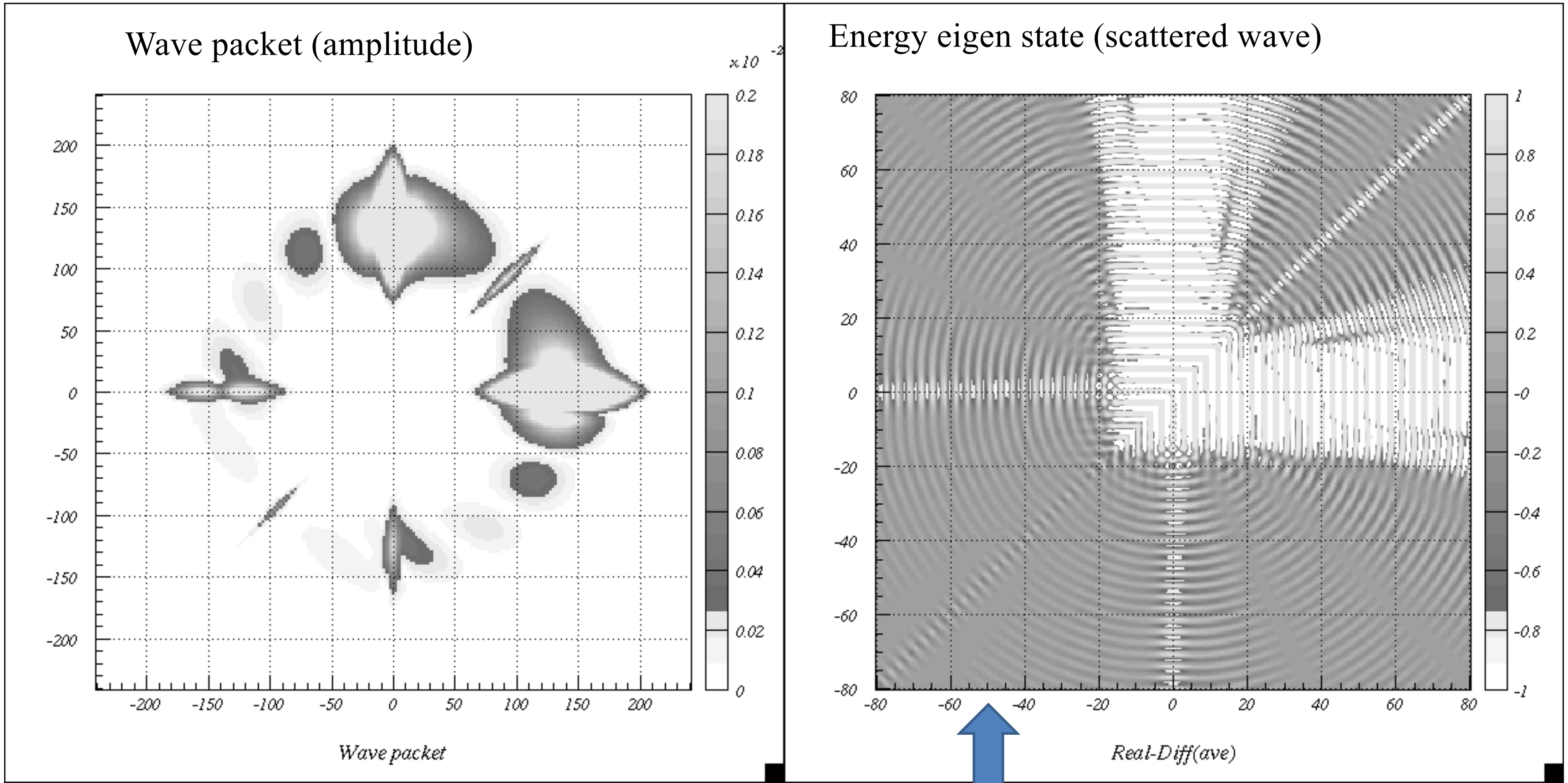
Scattering wave after subtracting
non-disturbed initial wave

Energy eigen state

Time-dependent wave packet: $$\Psi(t) = \int d\omega \ a(\omega)\, \psi_\omega \exp(-i\omega t)$$

Energy eigen state: $$a_\omega \psi_\omega = \int dt \ \Psi(t) \exp(i\omega t)$$

Energy eigen state is calculated by time dependent wave packet:



Solution of energy-independent Sch. eq. (!)

ANNALS OF PHYSICS: 74, 324–351 (1972)

The Asymptotic Form of Three-Particle Wavefunctions and the Cross Sections*

ROGER G. NEWTON†

Laboratoire de Physique Mathématique,‡
Université des Sciences, 34-Montpellier, France and
Department of Physics, Indiana University, Bloomington, Indiana

$$R^2 = r_1^2 + r_2^2$$

For freely moving particles we have $\boldsymbol{\rho}_i = 2\mathbf{k}_i t$, $\mathbf{r}_i = 2\mathbf{q}_i t$, and hence $R = 2Kt$. Thus we may set

$$K\boldsymbol{\rho}_i/R \equiv \mathbf{k}_i', \qquad K\mathbf{r}_i/R \equiv \mathbf{q}_i' \tag{2.3}$$

with the understanding that these are the asymptotic momenta appropriate to observations at $\boldsymbol{\rho}_i$ and $\mathbf{r}_i$. Thus

$$G_i^+(E; \mathbf{R}, \mathbf{R}') = \frac{e^{i\pi/4}E^{3/4}e^{iKR}}{2(2\pi)^{5/2}\, R^{5/2}}\, \psi_i^{(-)*}(\mathbf{K}'; \mathbf{R}') + o(R^{-5/2}), \tag{2.4}$$

for $r_1/r_2 = \text{const.}\ \sim 1/R^{5/2} \quad (\text{1 dim.: } \sim 1/R^{1/2})$

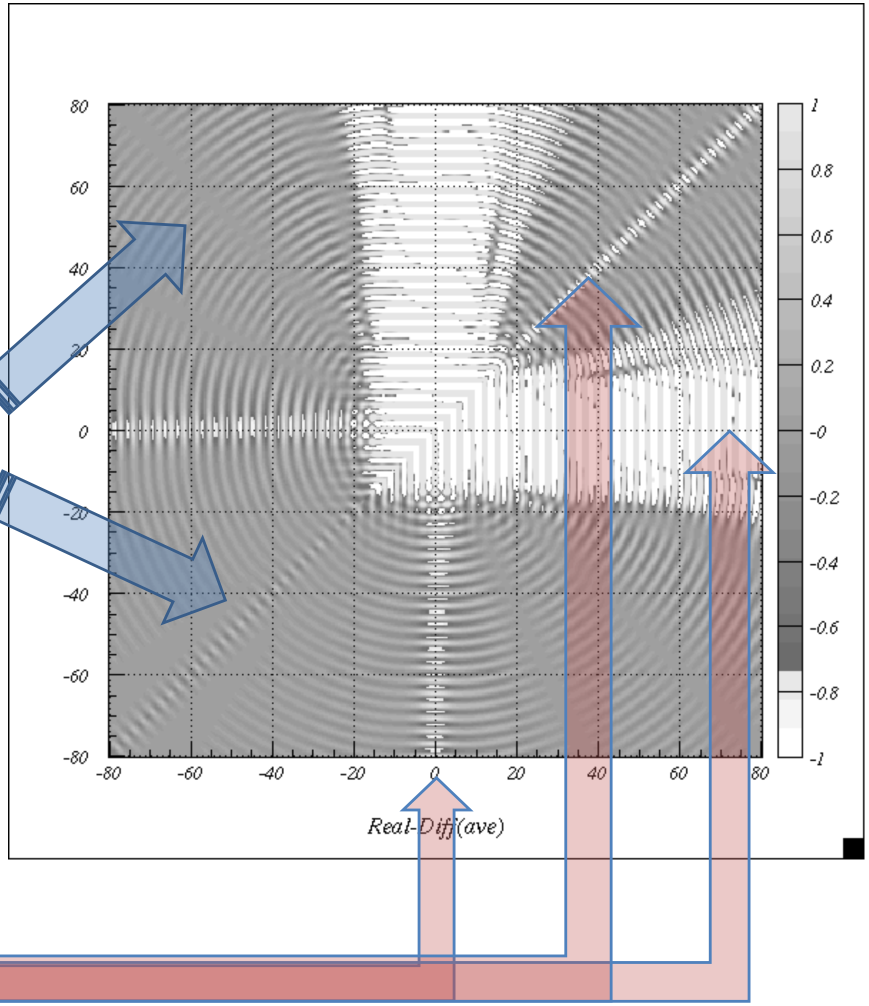
In order to derive the asymptotic form of G_i^+ for large ρ_i, with $\mathbf{r}_i$ fixed, we use another spectral decomposition:

$$G_i^+(E; \mathbf{R}, \mathbf{R}') = (2\pi)^{-3} \int (d\mathbf{k})\, \phi_i^{(\pm)}(\mathbf{k}, \mathbf{r}_i)\, \phi_i^{(\pm)*}(\mathbf{k}, \mathbf{r}_i')\, g_0^+(E - k^2; \boldsymbol{\rho}_i, \boldsymbol{\rho}_i') + (2\mu_i)^{-3/2} \sum_n \phi_{in}(\mathbf{r}_i)\, \phi_{in}^*(\mathbf{r}_i')\, g_0^+(E - E_{in}; \boldsymbol{\rho}_i, \boldsymbol{\rho}_i'),$$

$$G_i^+(E; \mathbf{R}, \mathbf{R}') = -\frac{1}{4\pi(2\mu_i)^{3/2}\, \rho_i} \sum_n e^{ik_{in}'\rho_i}\phi_{in}(\mathbf{r}_i)\, \psi_i^*(\mathbf{k}_{in}', E_n; \mathbf{R}') + O(\rho_i^{-2}), \tag{2.7}$$

where $\mathbf{k}_{in}' = \hat{\boldsymbol{\rho}}_i k_{in}$ and

$$\psi_i(\mathbf{k}_i, E_n; \mathbf{R}) = e^{i\mathbf{k}_i\cdot\boldsymbol{\rho}_i}\phi_{in}(\mathbf{r}_i). \tag{2.8}$$



for $r_1 \sim 0$ or $r_2 \sim 0$ or $r_{12} \sim 0\ :\ \sim 1/R^2 \quad (\text{1 dim.: } \sim 1)$

1-dimensional three-body case (3 equal mass)

Hamiltonian:

$$H+H_{\rm cm}=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_a^2}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_b^2}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_c^2}+V_{bc}\left(x_c-x_a\right)+V_x\left(x_b-x_c\right)+V_{ab}\left(x_a-x_b\right)$$

$$=-\frac{\hbar^2}{6m}\frac{\partial^2}{\partial X^2}-\frac{\hbar^2}{(4/3)m}\frac{\partial^2}{\partial x_{a-bc}^2}-\frac{\hbar^2}{m}\frac{\partial^2}{\partial x_{bc}^2}+V_{bc}+V_{ca}+V_{ab}$$

$$=H_{\rm cm}-\frac{\hbar^2}{m}\frac{\partial^2}{\partial x_1^2}-\frac{\hbar^2}{m}\frac{\partial^2}{\partial x_2^2}+V_{bc}+V_{ca}+V_{ab}$$

$$X=\frac{x_a+x_b+x_c}{3}\ ;\ x_{a-bc}=x_a-\frac{x_b+x_c}{2}\ ;\ x_{bc}=x_b-x_c$$

$$x_1=\frac{2}{\sqrt{3}}x_{a-bc}=\sqrt{3}\left(x_a-X\right)\ ;\ x_2=x_{bc}$$

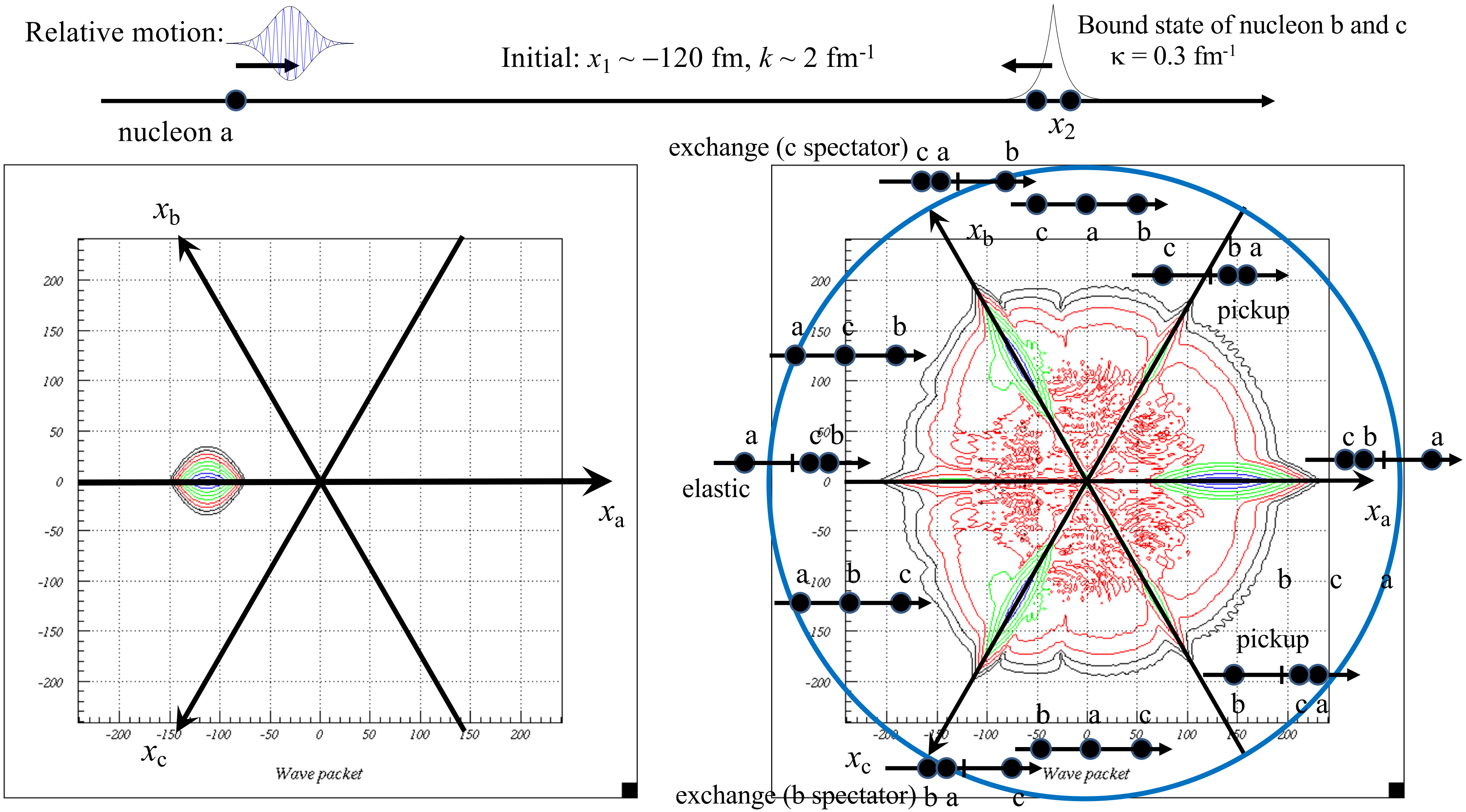
$$x_1^2+x_2^2=\frac{3}{2}\left[\left(x_b-x_c\right)^2+\left(x_c-x_a\right)^2+\left(x_a-x_b\right)^2\right]$$

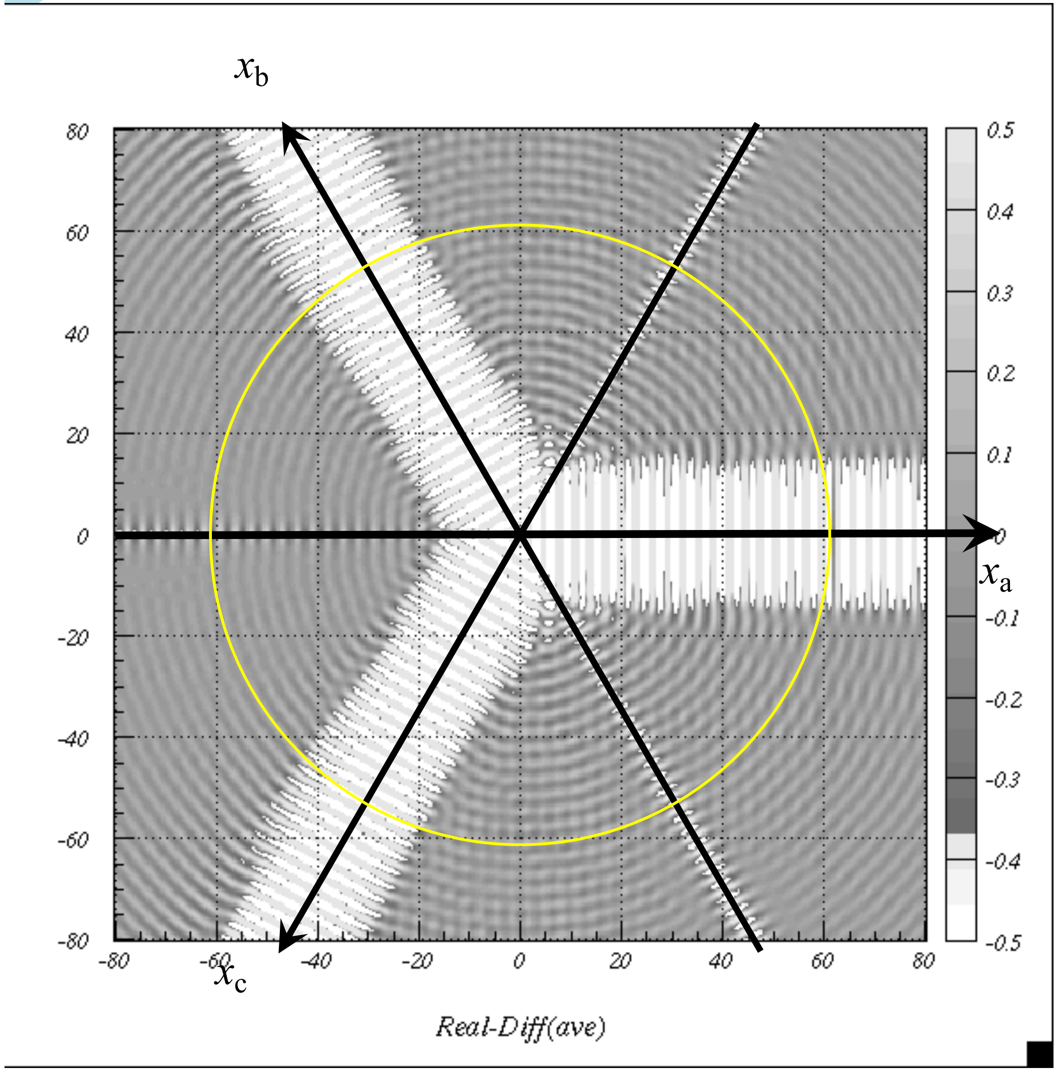
Time propagation of wave packet $\Psi(t)$:

$$i\hbar\frac{\partial}{\partial t}\Psi\left(t\right)=H\Psi\left(t\right)$$

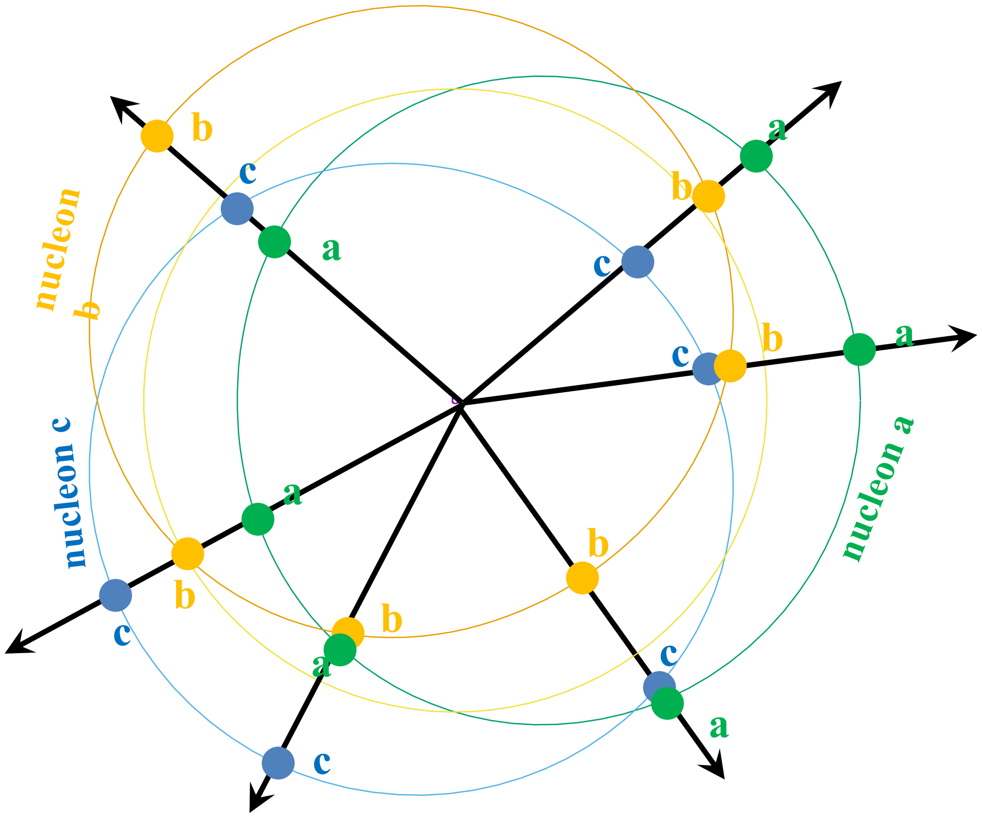
$$\Psi\left(t+\Delta t\right)=\exp\left(-i\frac{H}{\hbar}\Delta t\right)\Psi\left(t\right)$$

Interaction: V_{bc}, V_{ca}, V_{ab} (“δ-function” at origins; attractive)





x_b
x_a
x_c
80
60
40
20
0
-20
-40
-60
-80
0.5
0.4
0.3
0.2
0.1
-0.1
-0.2
-0.3
-0.4
-0.5
Real-Diff(ave)

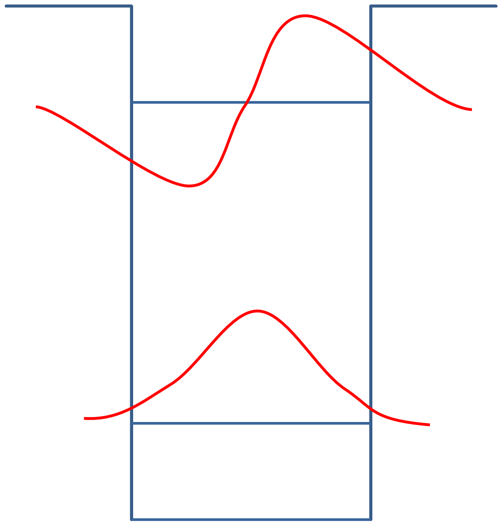


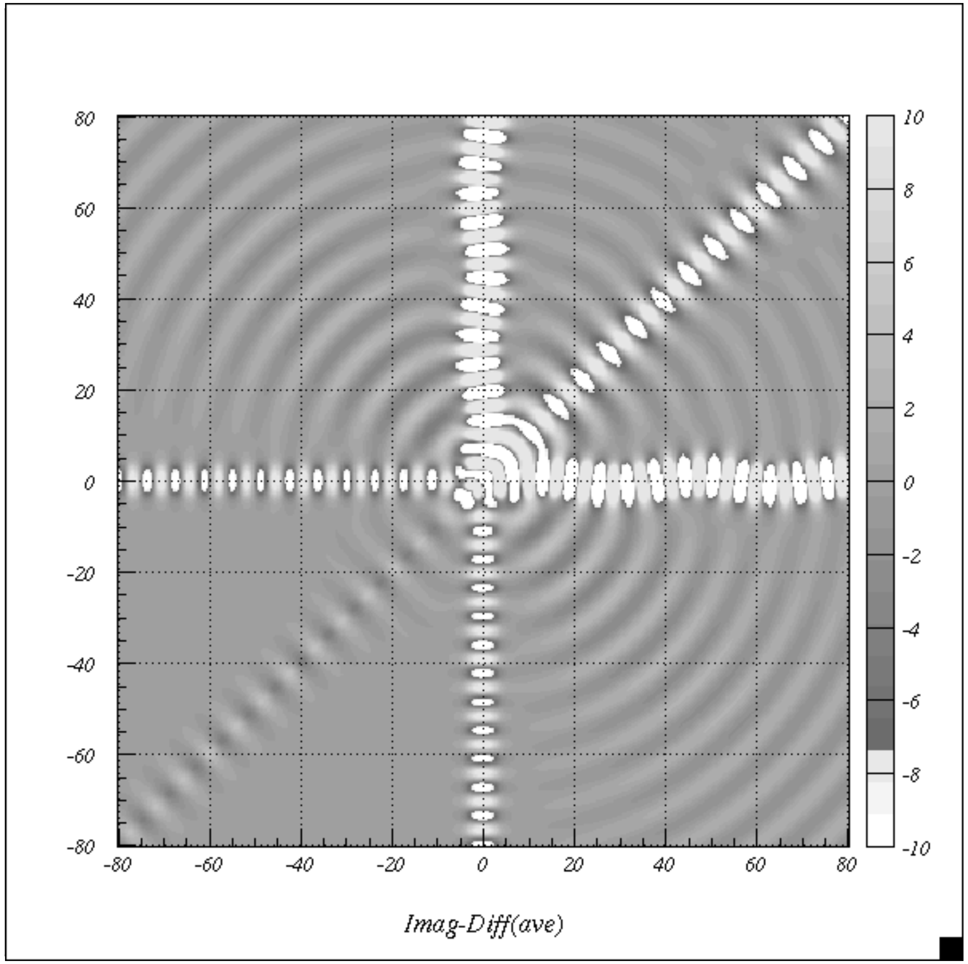
nucleon b
nucleon c
nucleon a
a
b
c

Effects of structure in (2+3) and (1+3) system

Finite range for V_1 and V_2 : Bound (or Just-above-threshold) excited state(s)

V_1 and V_2





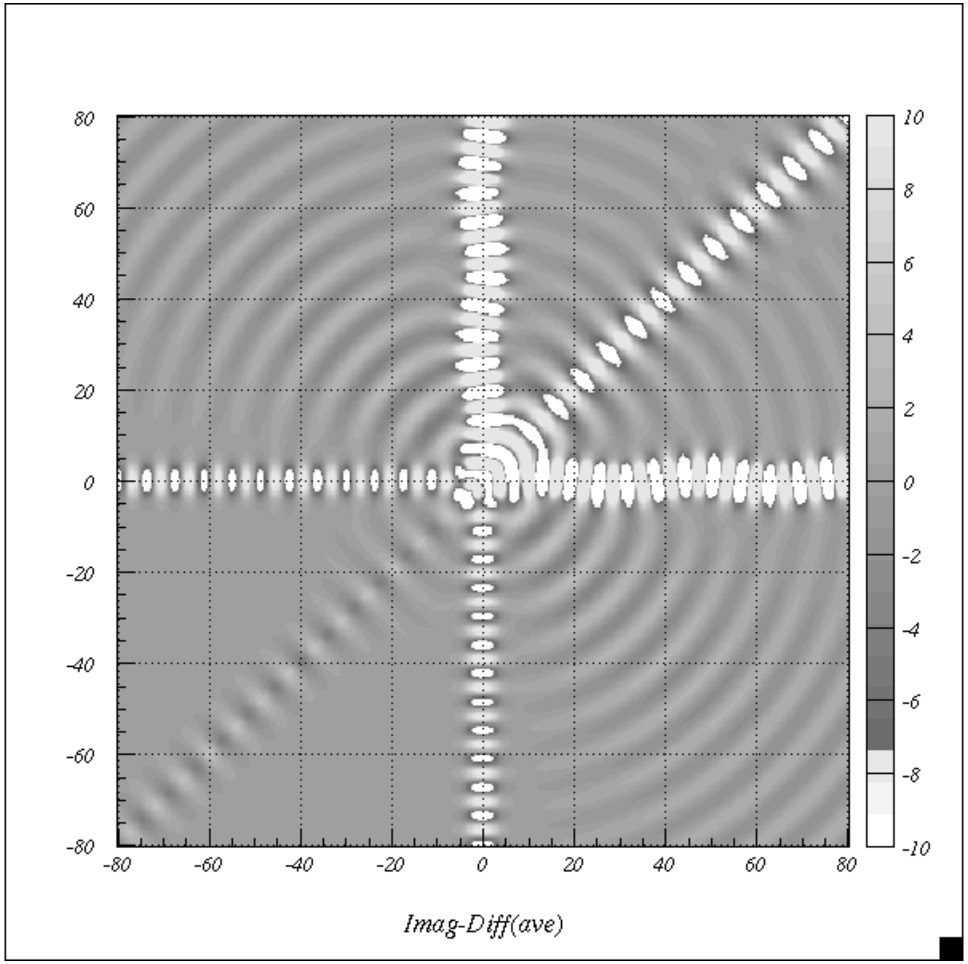
"Coupled-channel"

Parity of G.S. is +
Parity of Exc. state is –

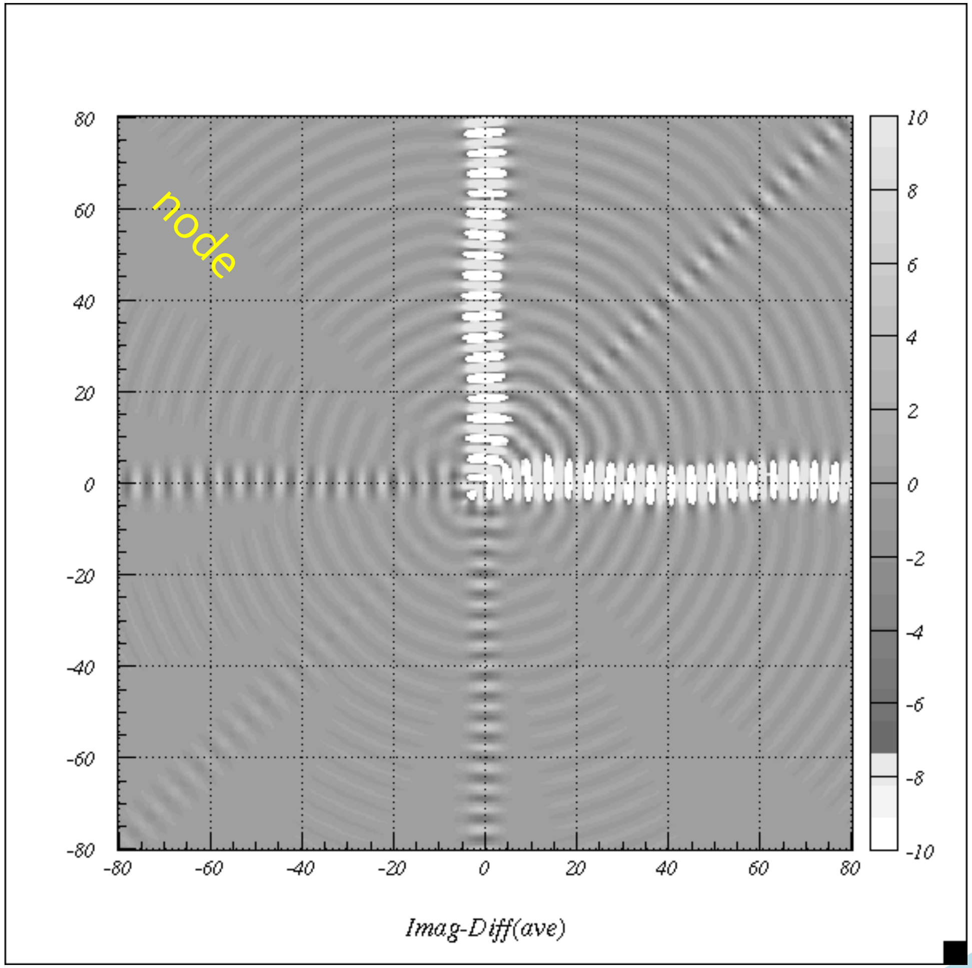
Wave numbers are slightly different and parities are opposite
Interference between elastic and inelastic scatterings

Incident energy dependence of continuum

Lower energy (K_{in}=1 fm^{-1})

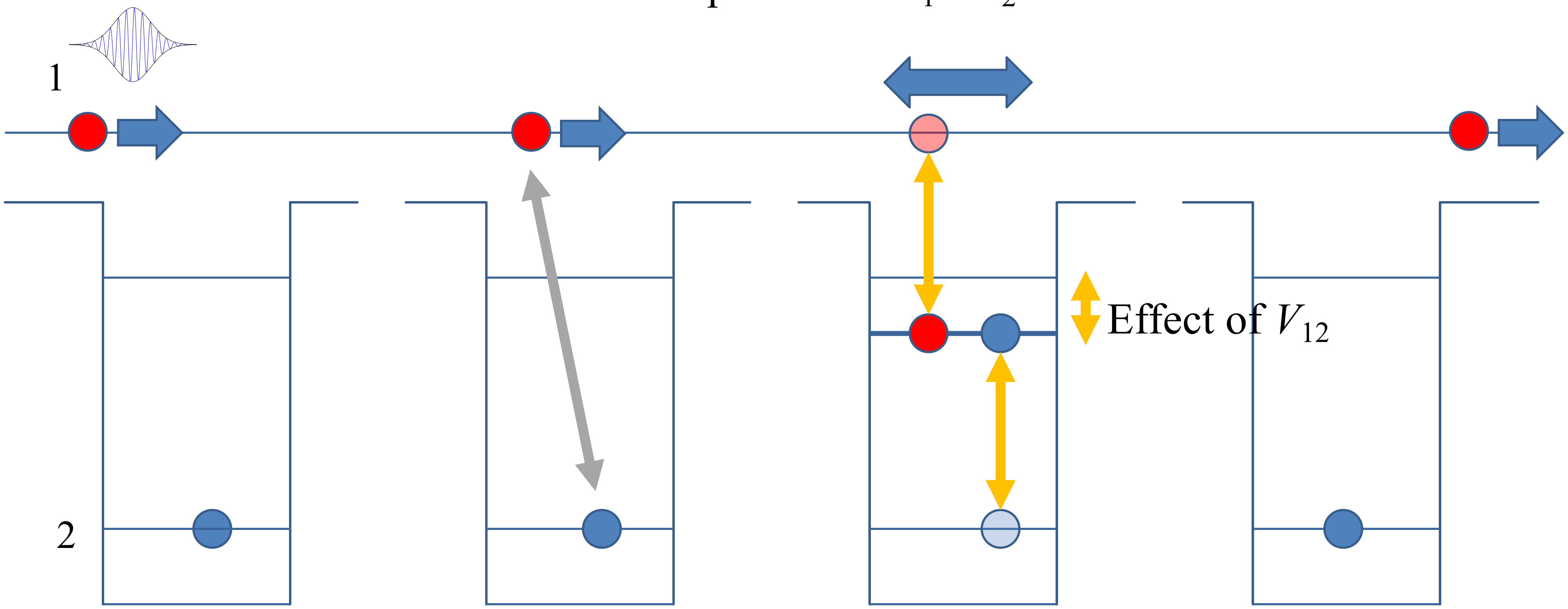


Higher energy (K_{in}=1.4 fm^{-1})

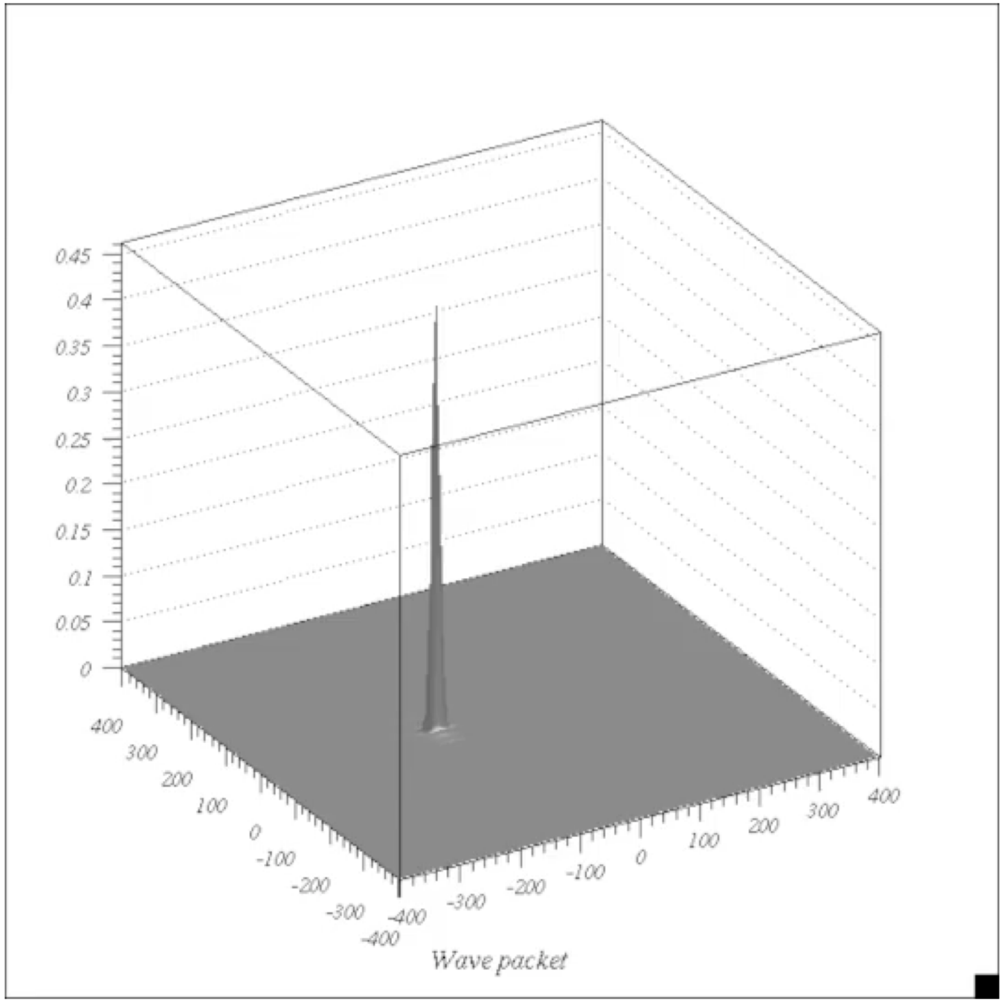


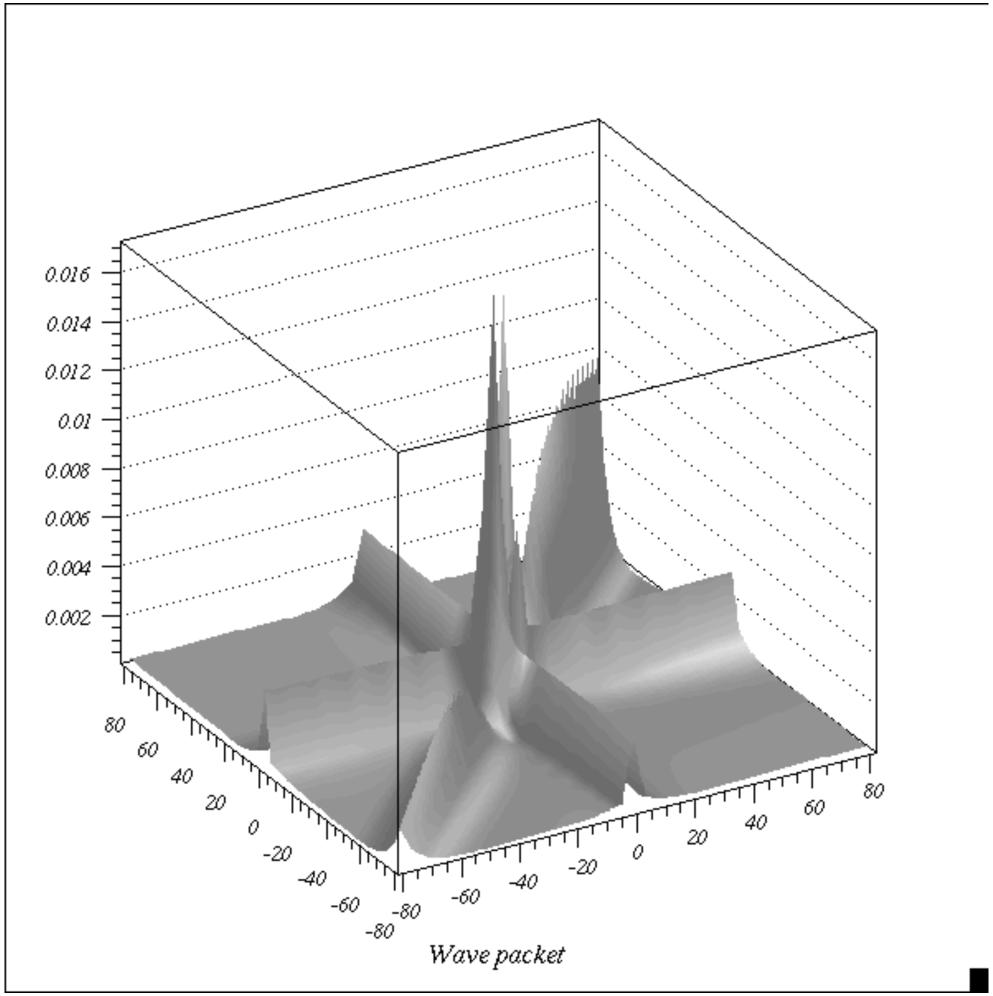
3-body Resonance (Feshbach resonance)

3-body bound state for frequency component corresponding to resonance
Amplitudes at $x_1 \sim x_2 \sim 0$ remain



3-body Resonance (closed channel)

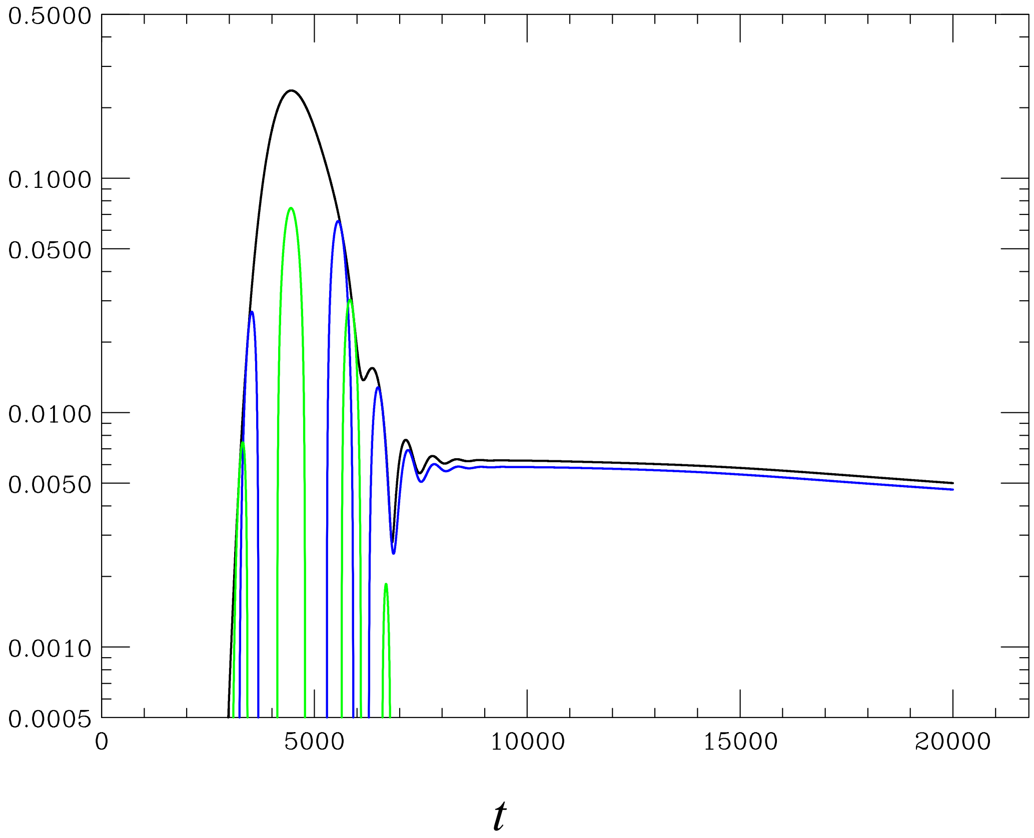


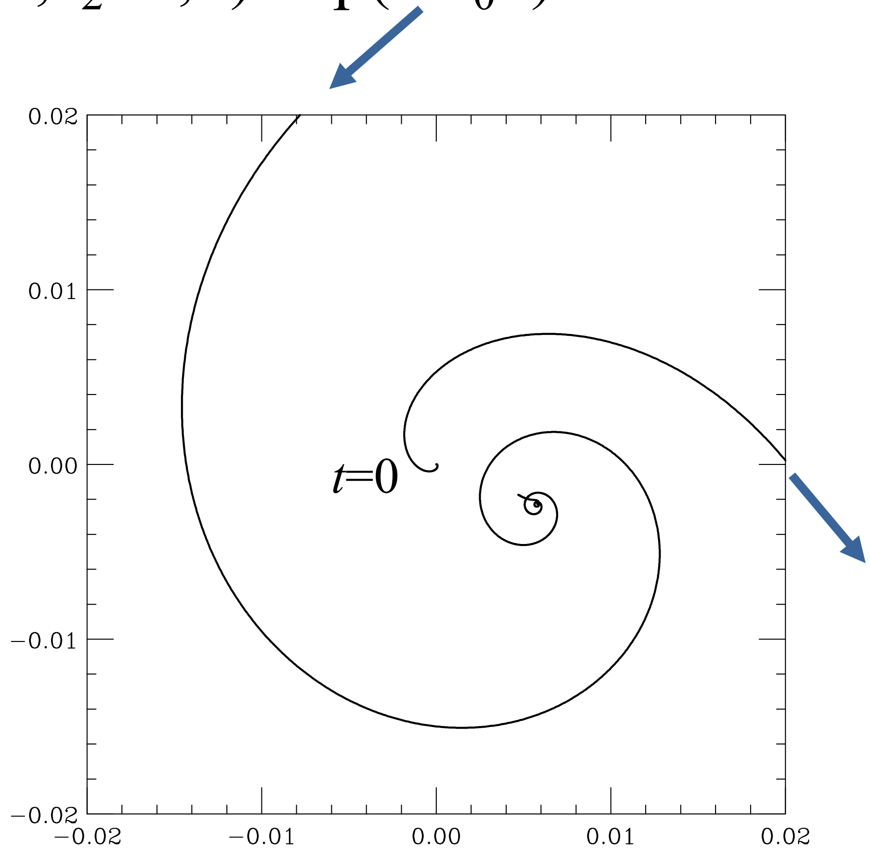


Integral range for resonance amplitude is much longer than the path-through time of w.p.
→ compound process

3-body Resonance (closed channel)

Amplitude near origin : $\psi(x_1=0, x_2=0; t)\exp(i\,\omega_0\,t)$

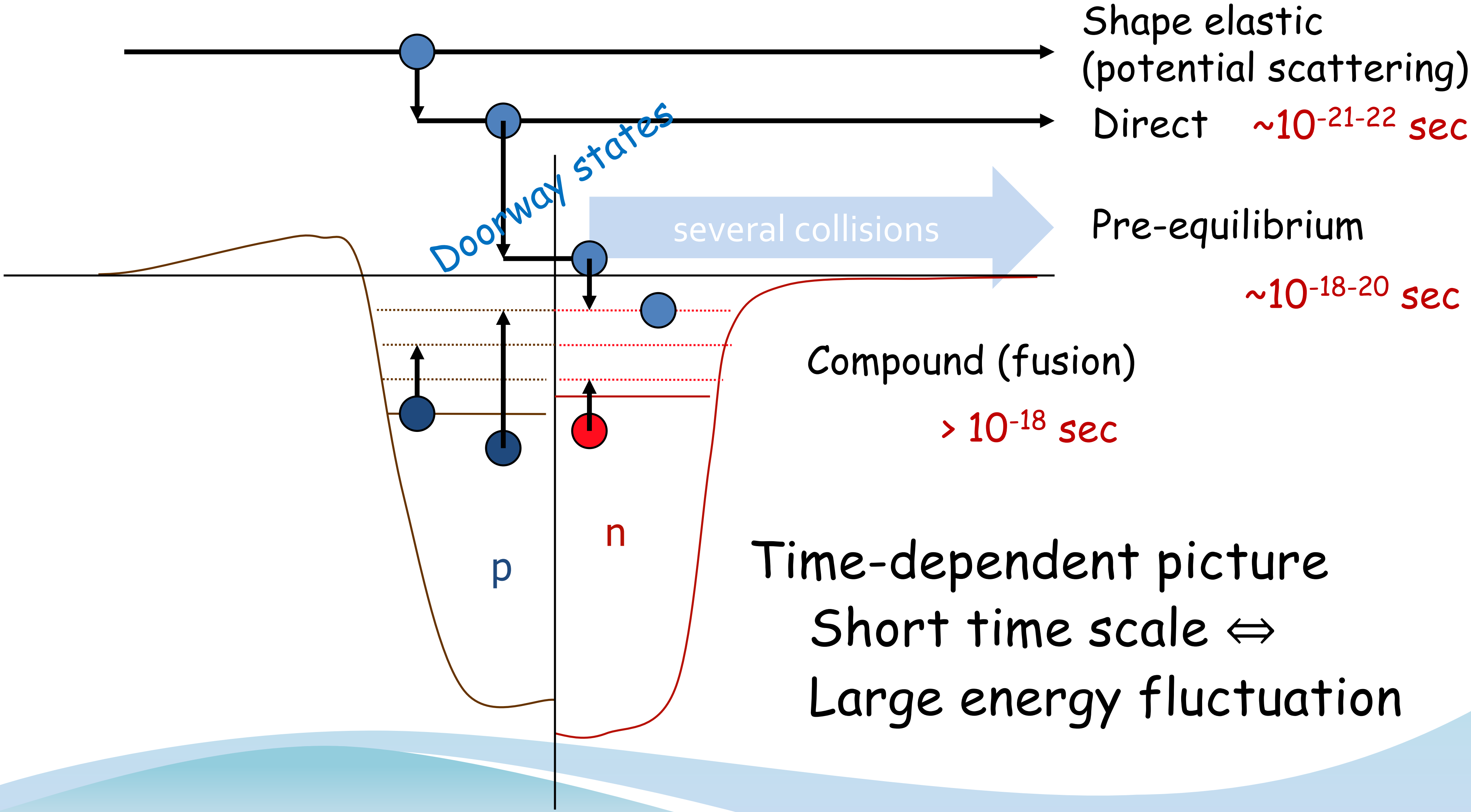




Nuclear Reaction (time-dependent)

Energy & time scale for evolution of reaction mechanism

- Shape elastic - direct - pre equilibrium - compound



Time-dependent picture

Short time scale ⇔

Large energy fluctuation

Time evolution of wave packet

Demonstration of 1-dim. 3-body wave packets

- Time evolution of wave packets gives proper boundary conditions at asymptotic regions including 3-body continuum channel.
  - seems to be not straight forward
- Coupled channel effects in binary systems are properly treated. (not quantitatively examined yet…)
- Resonant states are to be treated. (not examined yet…)
- How to extend to 3-dim. cases?

Still many things to do …