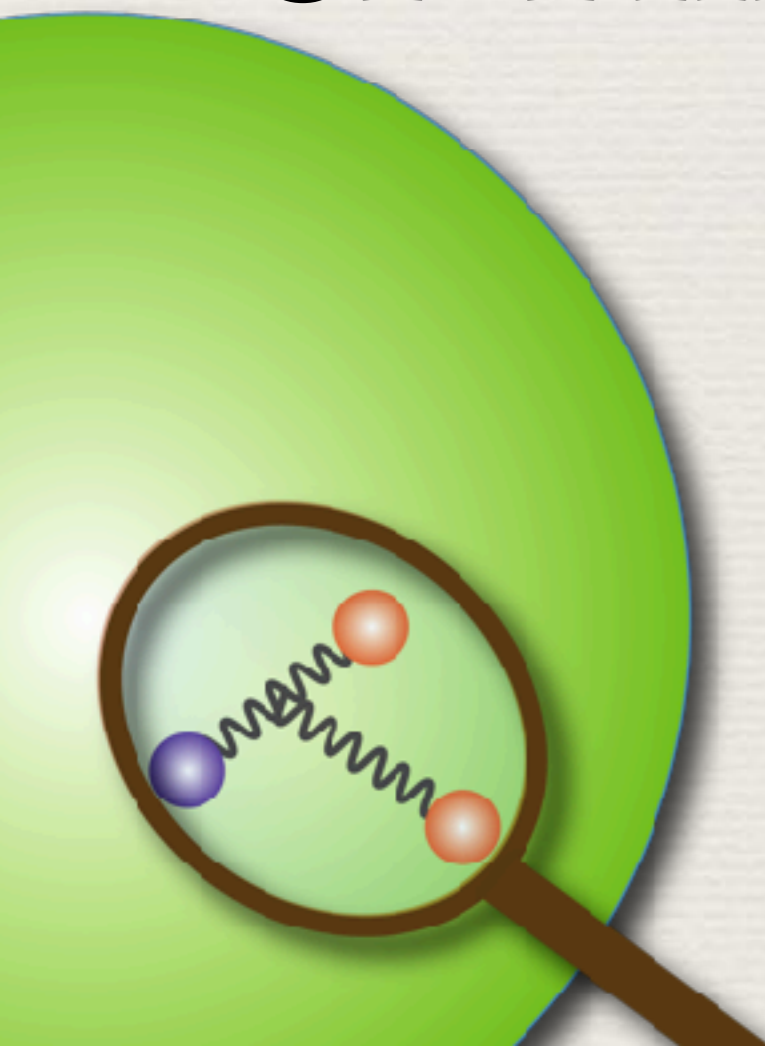


Experimental Study of Three-Nucleon Forces

Part I

Department of Physics
Tokyo Institute of Technology

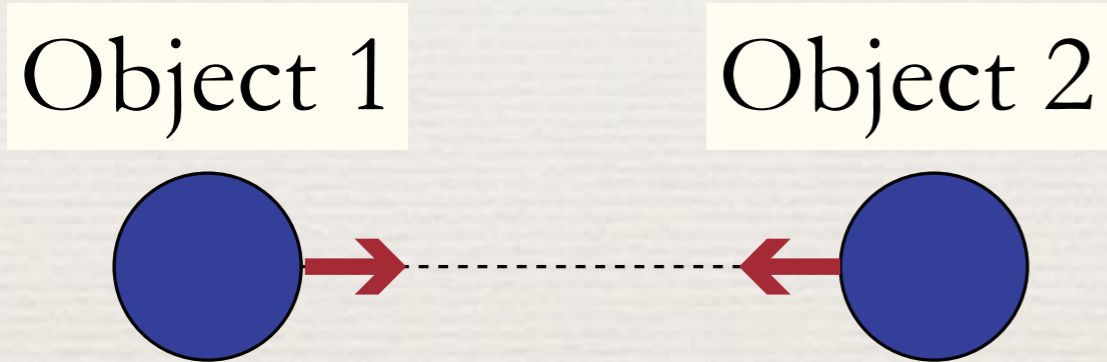
Kimiko Sekiguchi



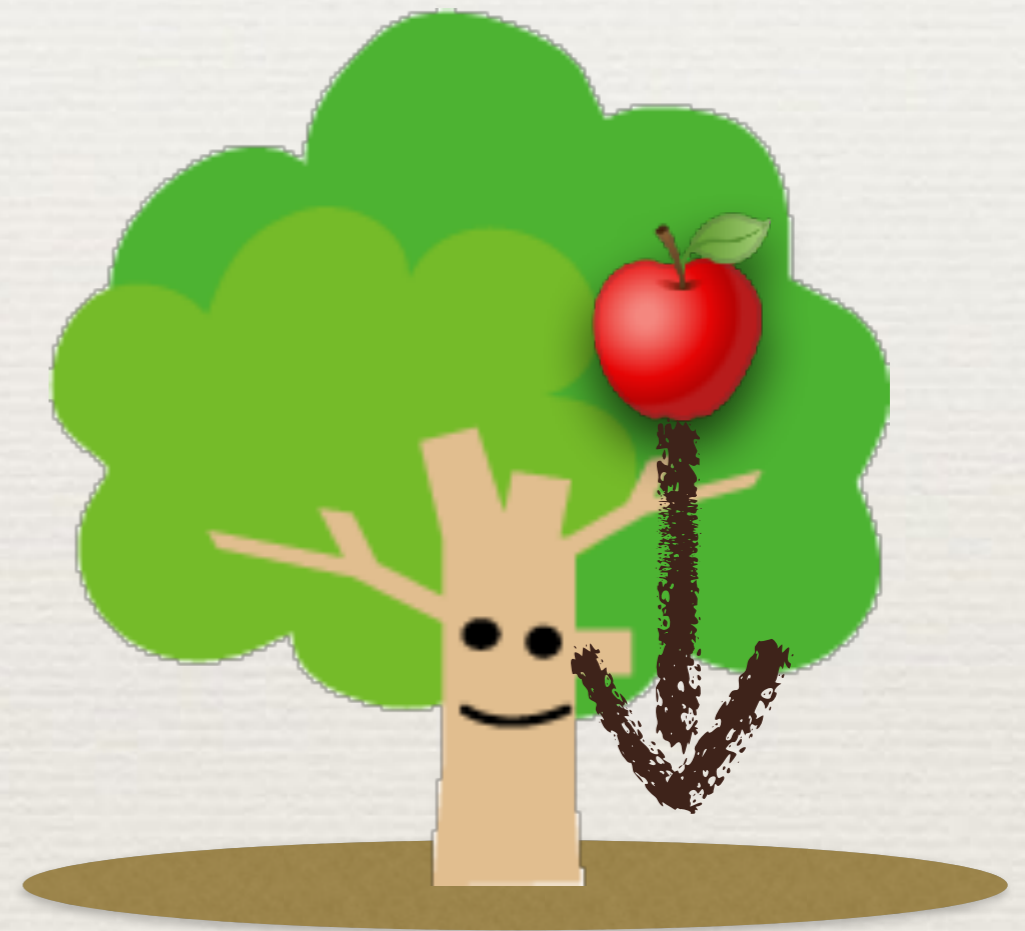
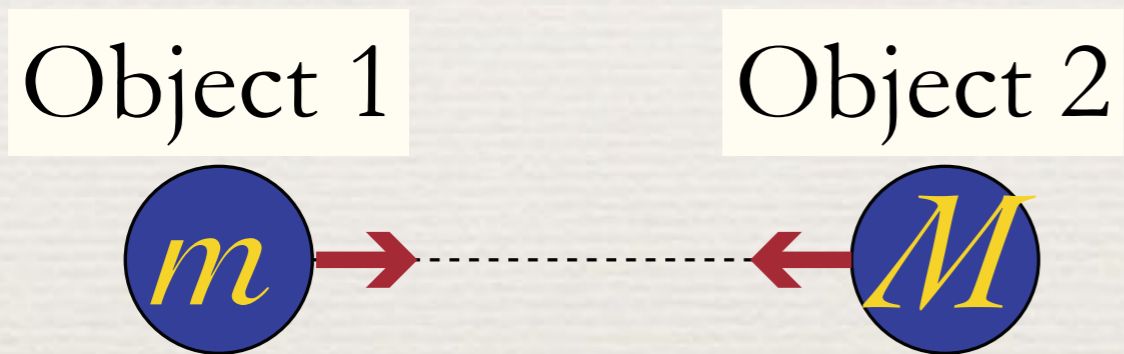
Nuclear Physics

- started in the beginning of the 20th century.
- Today's talk is on forces acting among 3-nucleons.

Forces - basic concept-



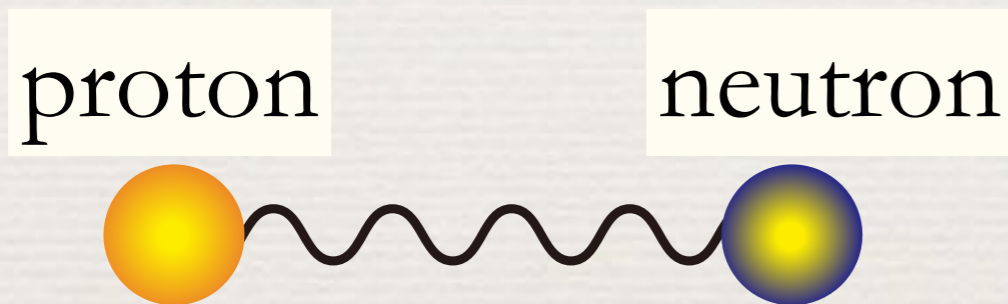
Forces - basic concept-



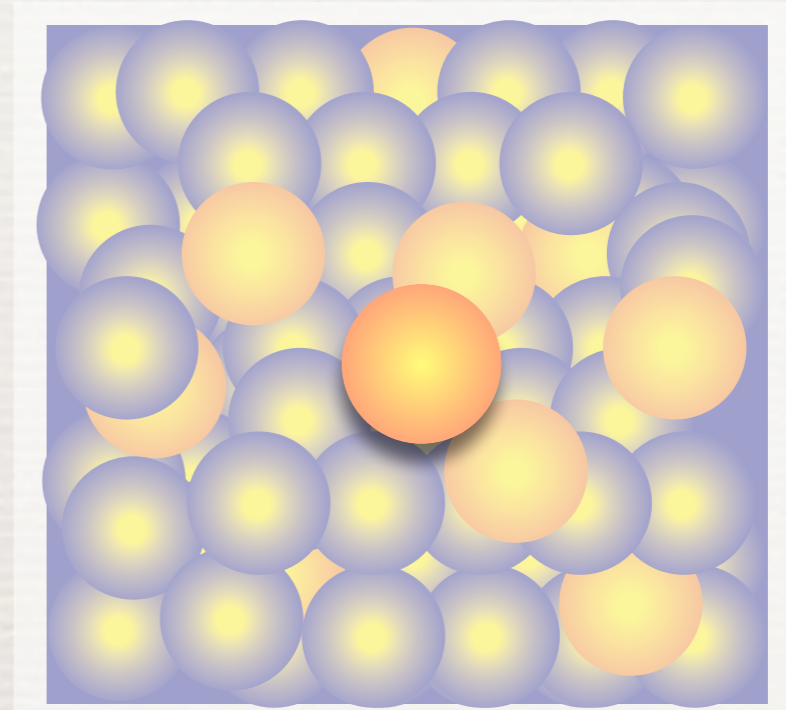
$$F = G \frac{mM}{r^2}$$

$$\sum_i F_i = \dots = mg$$

Forces - basic concept-



2-nucleon
Forces



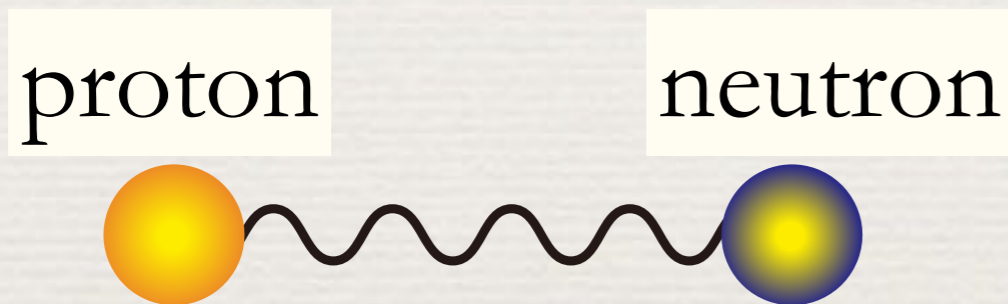
Forces in Medium

$$V_{2N}$$



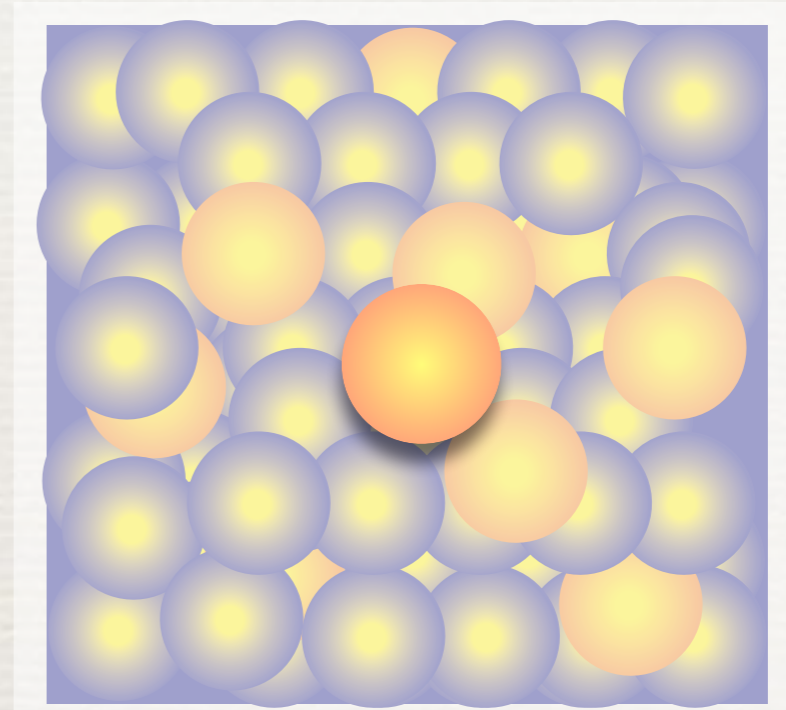
$$V = \sum_{i,j} V_{ij}$$

Forces - basic concept-

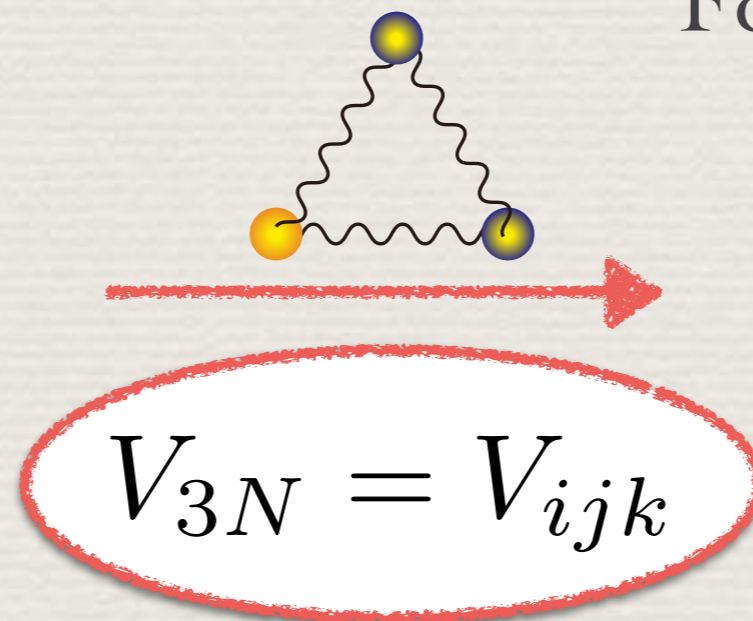


2-nucleon
Forces

$$V_{2N}$$



Forces in Medium



$$V = \sum_{i,j} V_{ij}$$

Earth-Moon-Satellite Gravitational Interactions

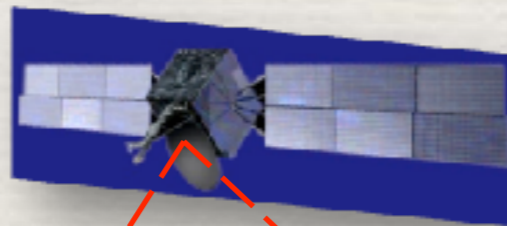
Two-Body Interaction : Gravity

$$H = \frac{1}{2} \left(\frac{P_m^2}{m} + \frac{P_M^2}{M} \right) + \frac{GMm}{r}$$

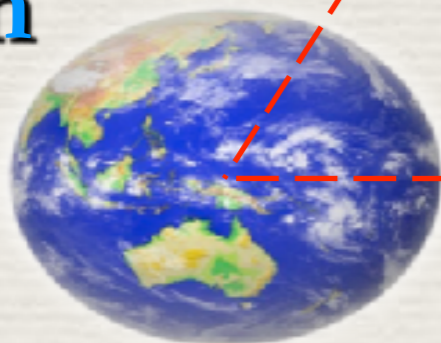
Three-Body Interaction :

$$H = \frac{1}{2} \left(\frac{P_E^2}{m_E} + \frac{P_M^2}{m_M} + \frac{P_G^2}{m_G} \right) - G \frac{m_E m_M}{r_{EM}} - G \frac{m_E m_G}{r_{EG}} - G \frac{m_M m_G}{r_{MG}} + V(r_E, r_M, r_G)$$

Satellite



Earth



Moon



by the polarizations of the ocean water of the earth by the moon's gravity

Triplet of Atoms van der Waals Type Three-Body Force

Two-Body Interaction : Electro-Magnetic

$$V_{12} = \frac{C\alpha^3}{r_{12}^6}$$

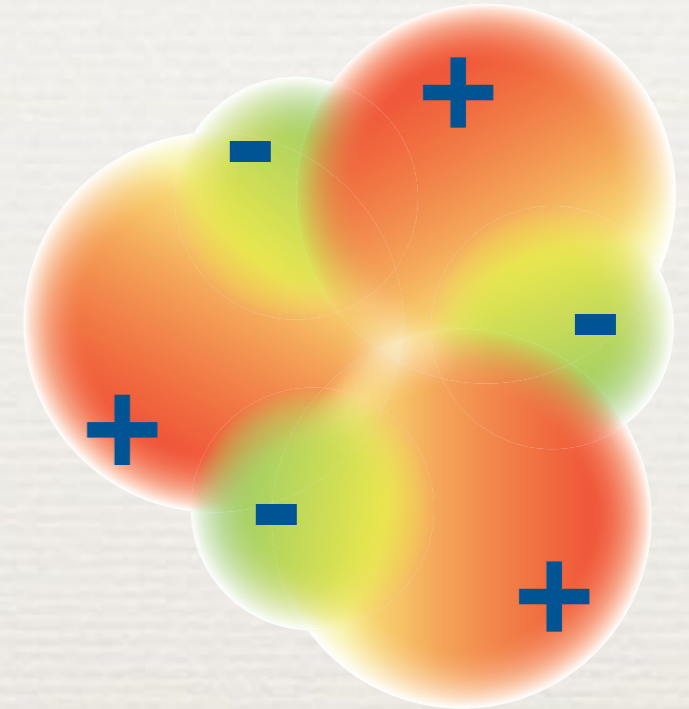
Three-Body Interaction :

$$V_{123} = C \frac{3 \cos \gamma_1 \cos \gamma_2 \cos \gamma_3 + 1}{r_{12}^3 r_{23}^3 r_{31}^3}$$

**Effects of Polarization
of the electron density distribution**

Axilrod-Teller-Muto three-body expression

B.M. Axilrod and E. Teller, J. Chem. Phys. 11, 299 (1943).
Y. Muto, J. Phys.-Math. Soc. Japan, 17, 629 (1943).

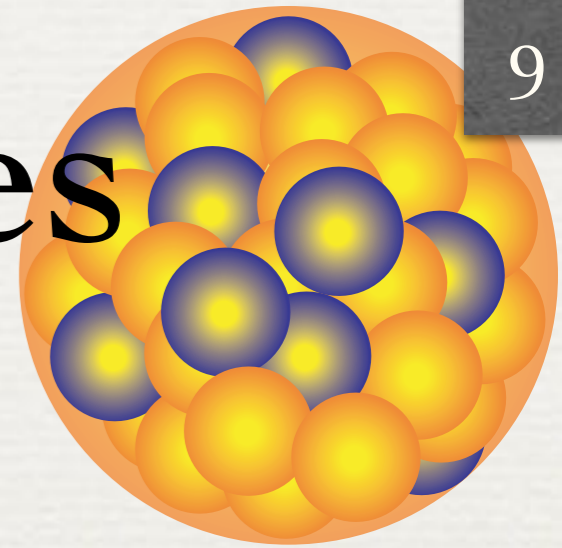


Interaction Energy [kcal/mol]

	2BF	3BF
(NH ₃) dimer	-1.43	0.00
(H ₂ O) dimer	-1.80	-0.01
Benzene - H ₂ O	-2.35	0.15
Benzene - NH ₃	-2.15	0.14

O. A. von Lilienfeld, and A. Tkatchenko, J. Chem. Phys. 132, 234109 (2010).

Three-Body Forces in Nuclei



- Nucleus : a compact system of nucleons (protons, neutrons)
- Nuclear Force : Strong Interaction ... Short and Strong
- Effects of Three Body Forces in Nuclei

— Where and How ? —

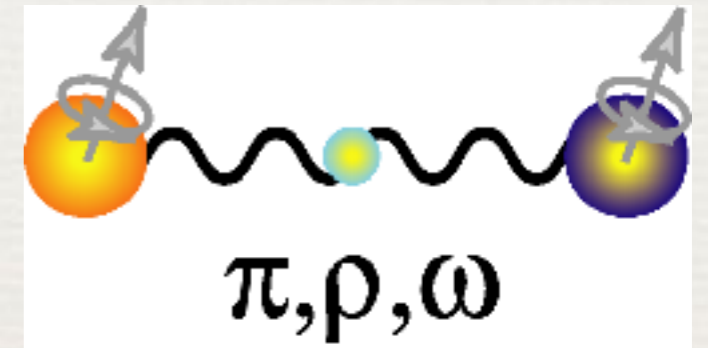
	Solar System	Atom	Nucleus
Length	10^8 m	10^{-10} m	10^{-15} m
Interaction	Gravity	Electro-Magnetic	Strong
Coupling Constant	10^{-38}	10^{-2}	1
$\frac{V(3BF)}{V(2BF)}$	0.001%	a few %	?

Before Three-Body Forces in Nuclei ...

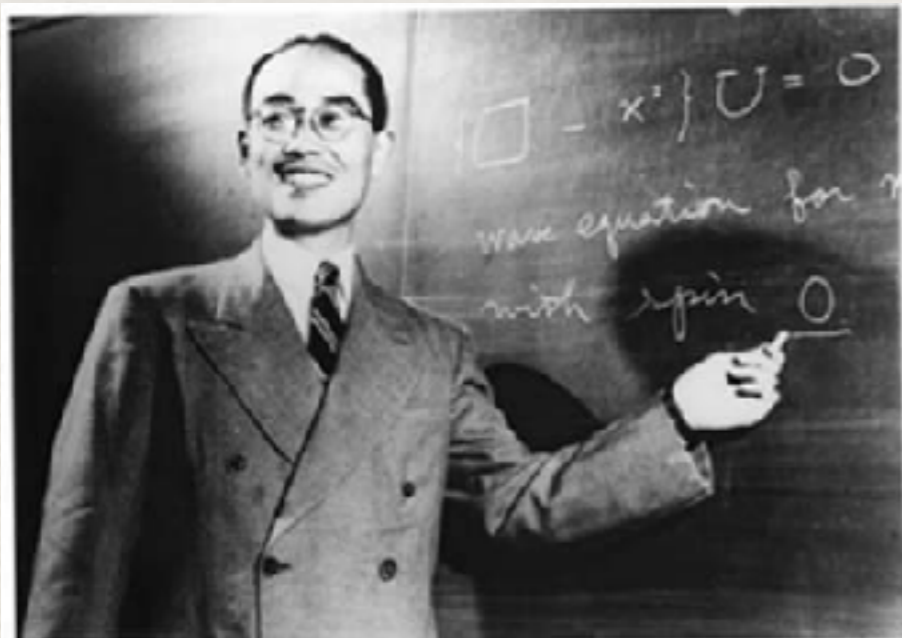
Nuclear Force \sim Yukawa's Idea \sim

Yukawa's Meson Theory

Proc. Phys. Math. Soc. Jpn. 17, 48 (1935)



Nuclear force is explained by exchanging a 'virtual particle' (meson) between **two nucleons**.



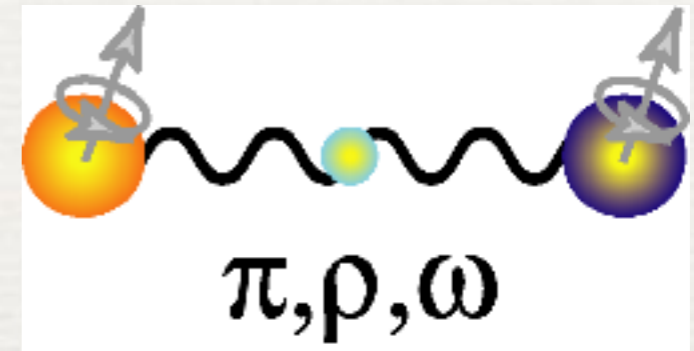
Scanned at the American
Institute of Physics

- Force \Leftrightarrow Exchange of Particles
→ Field Theory
- Origin of Strong Force
- Quantum Mechanics & Relativity

Nuclear Force

Yukawa's Meson Theory

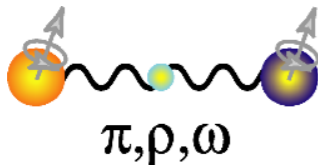
Proc. Phys. Math. Soc. Jpn. 17, 48 (1935)



Theory



One Pion Exchange Model



One Boson Exchange Model
Heavier Meson Exchange
e.g. ρ, ω

Experiment

Nucleon-Nucleon Scattering

($d\sigma/d\Omega$ and Spin Observables)

Deuteron Properties

1990's Realistic Modern Nucleon-Nucleon Forces (**2-Nucleon F**)

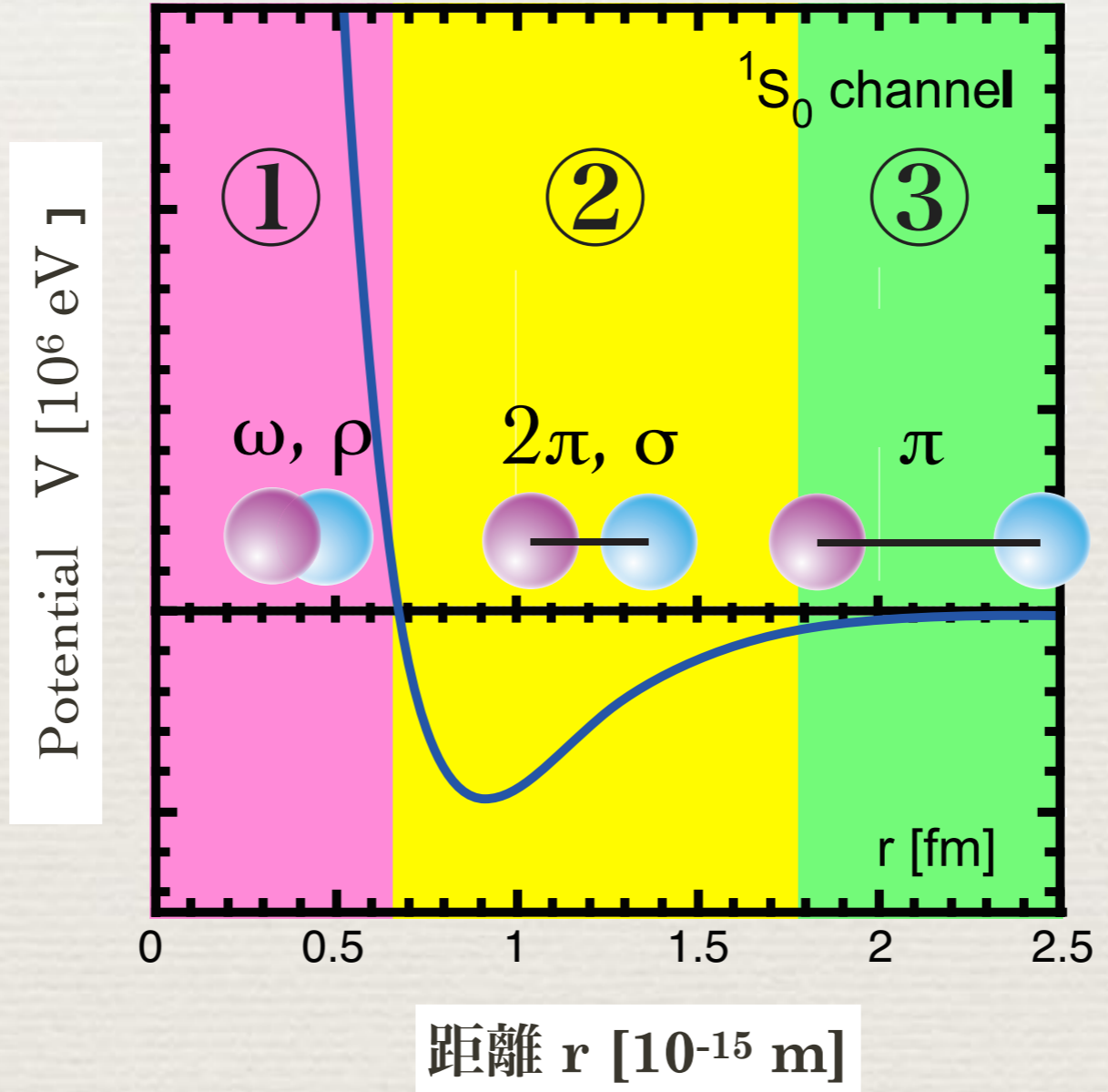
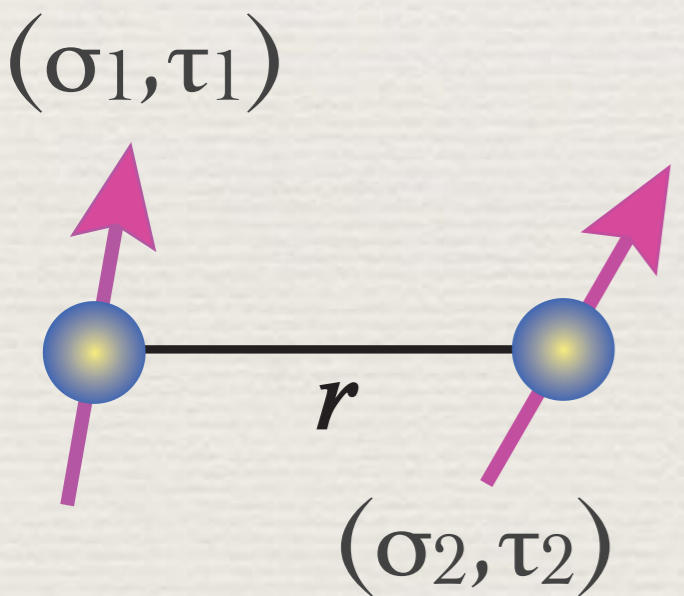
reproduce 3500 NN scattering exp. data with high precision, $\chi^2 \sim 1$.

Nuclear Force - 2-Nucleon Force -

①. Repulsive
-Short Range-

②. Attractive (strong)
-Intermediate Range-

③. Attractive (weak)
- Long Range -



σ : Nucleon Spin
 τ : Iso-spin
(p (uud) or n (ddu)?)

$$= V_0(r) + V_\sigma(r) \sigma_1 \cdot \sigma_2 + V_\tau(r) \tau_1 \cdot \tau_2 + V_{\sigma\tau}(r) (\sigma_1 \cdot \sigma_2) (\tau_1 \cdot \tau_2) + V_T S_{12} + V_{T\tau} S_{12} \tau_1 \cdot \tau_2 + V_{LS} L \cdot S + V_{LS\tau} (L \cdot S) (\tau_1 \cdot \tau_2) + \dots$$

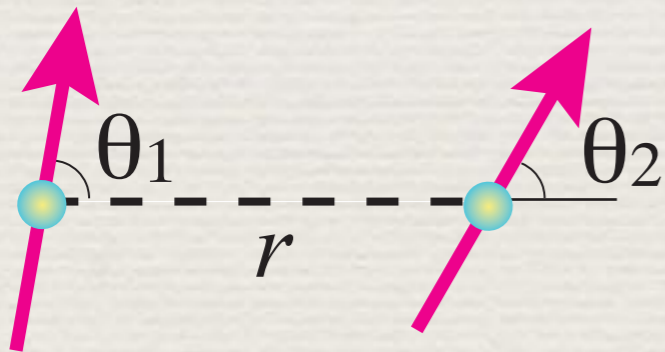
Bulk Properties of Nuclear Forces

1. short range (finite range)
2. attractive at intermediate range
3. repulsive core at short range
4. spin-dependent / non-central force
: Tensor force, Spin-Orbit force etc...

4. spin-dependent non-central force

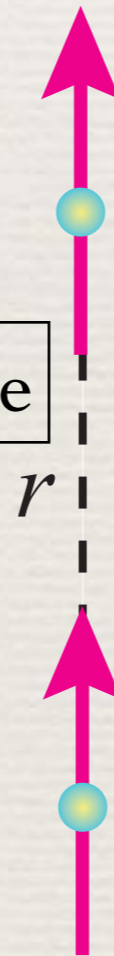
Tensor Force

$$\begin{aligned}
 V_{\text{tensor}} &\equiv (-S_{12}) \\
 &= -3 (\vec{\sigma}_1 \cdot \hat{r}) (\vec{\sigma}_2 \cdot \hat{r}) + \vec{\sigma}_1 \cdot \vec{\sigma}_2
 \end{aligned}$$

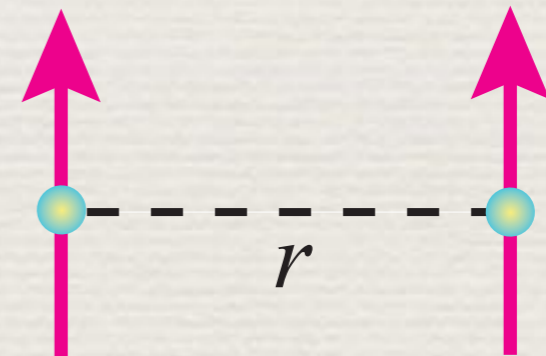


general form
expressed by r, θ_1, θ_2

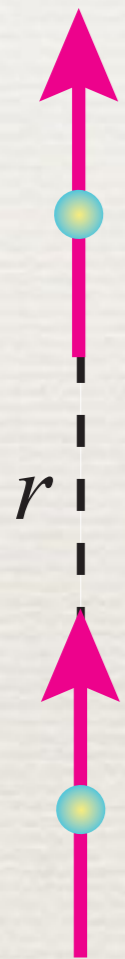
attractive



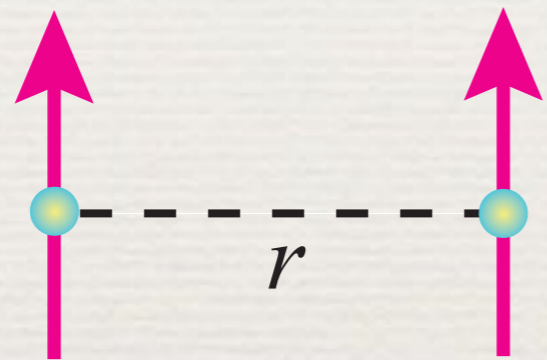
repulsive



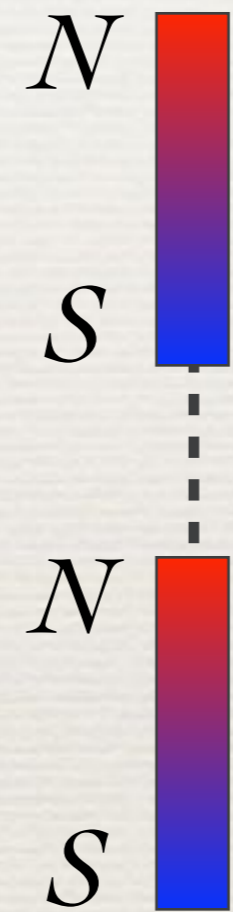
Tensor Force



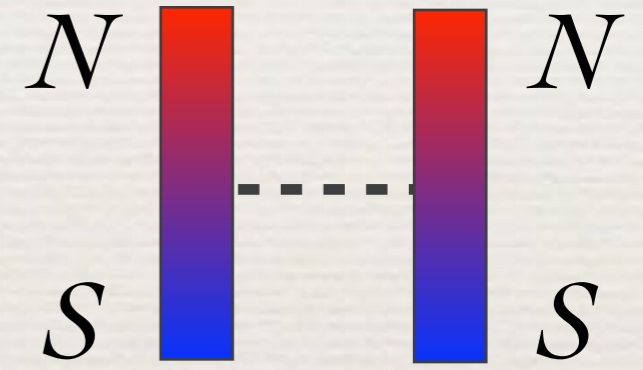
attractive



repulsive



attractive



repulsive

Tensor Force

Evidence of Tensor Force in Nuclei

1. Deuteron ($A=2, Z=1$)

- Non-zero quadrupole moment $Q \neq 0$
- Magnetic moment $\mu_p + \mu_n \neq \mu_d$

2. NN Scattering

mixing parameter ε

requires

D-state ($l=2$) contribution of two-nucleon system

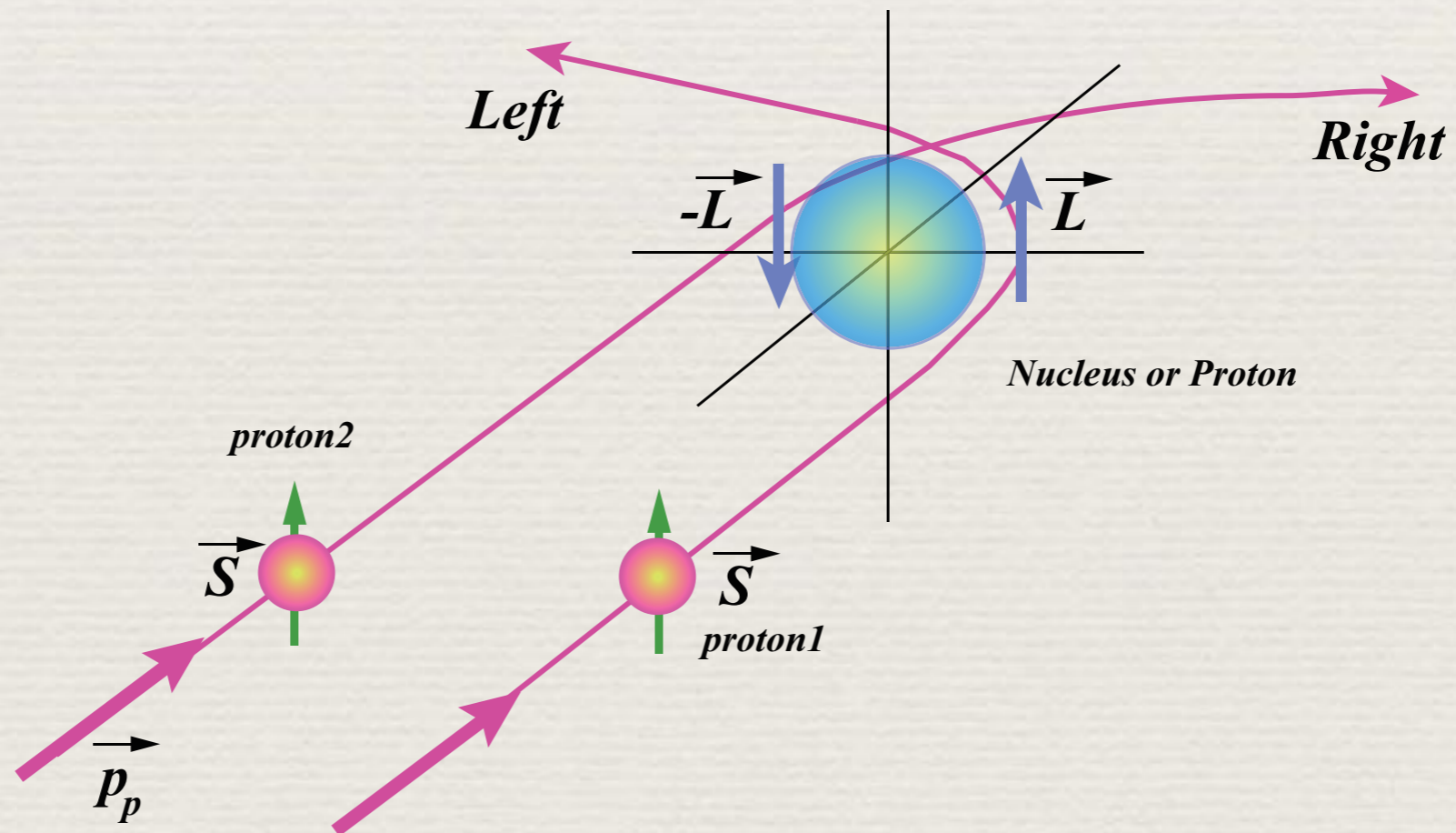


Tensor Force
(non-central force)

4. spin-dependent non-central force

Spin-Orbit Force

$$V_{LS} = \vec{L} \cdot \vec{S}$$



Spin-Orbit Force

P wave ($l=1$)

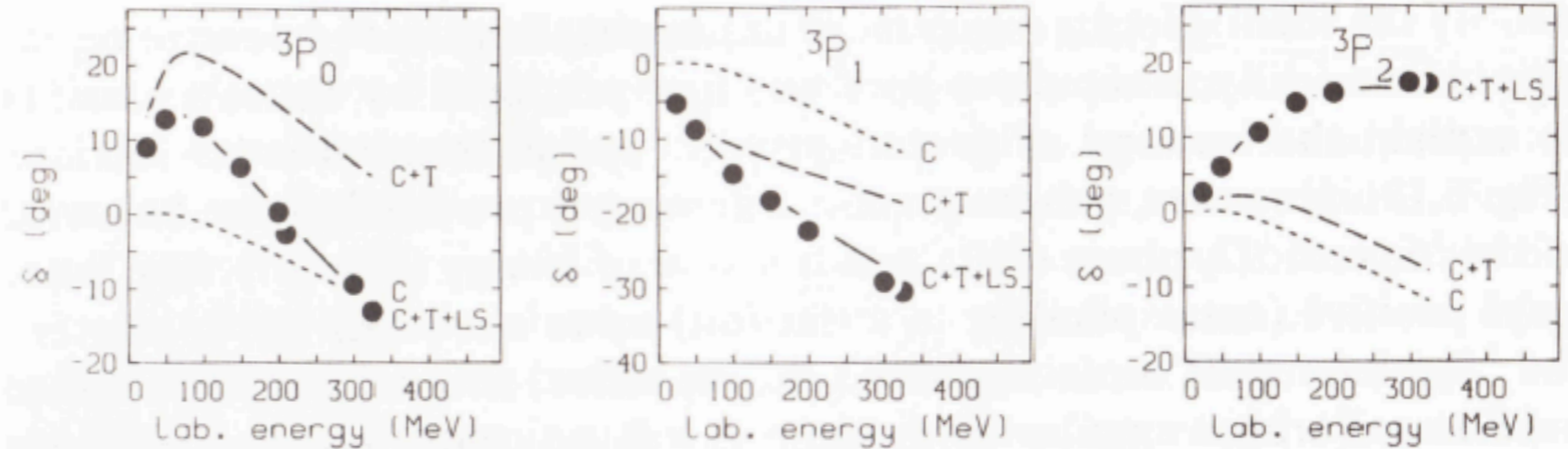
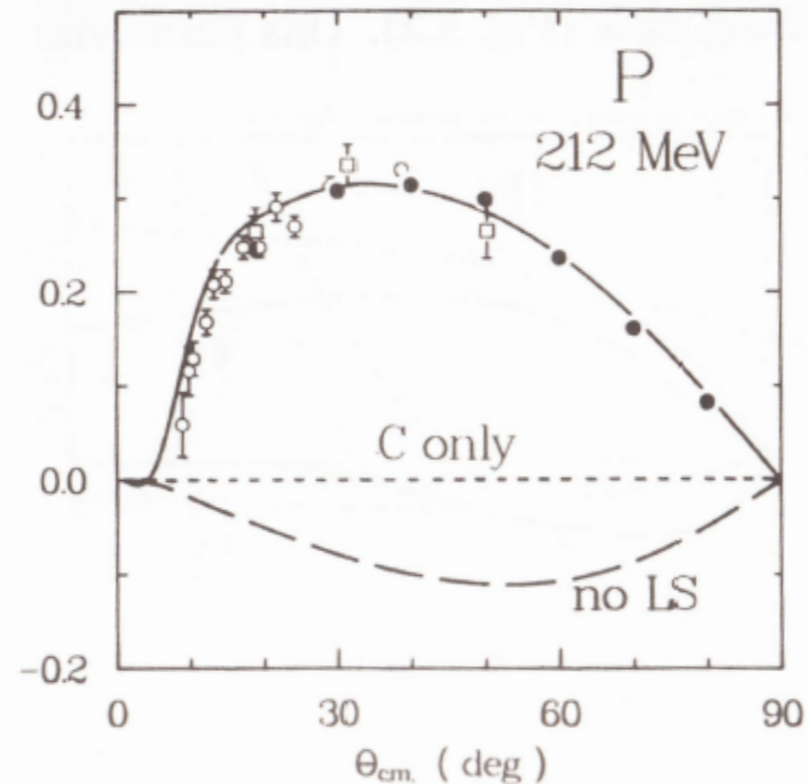


Fig. 3.3. NN phase shifts in triplet P waves. Shown are predictions using a central force only (C), central plus tensor (C + T), and central plus tensor plus spin-orbit force (C + T + LS). The dots represent energy-independent phase shift analyses (Arn+ 83, Dub+ 82).

Vector Polarization



5. charge independence

Interaction of pp , nn , & pn

$pp = nn$: charge symmetry

$pp/nn = pn$: charge independence

Scattering Length a via Partial wave analysis of NN scattering at low energies

$$a_{pp} = -17.3 \pm 0.4 \text{ fm}$$

$$a_{nn} = -18.8 \pm 0.5 \text{ fm}$$

$$a_{pn} = -23.74 \pm 0.02 \text{ fm}$$



Charge independence breaking

NN potential
(e.g. Bonn Potential)

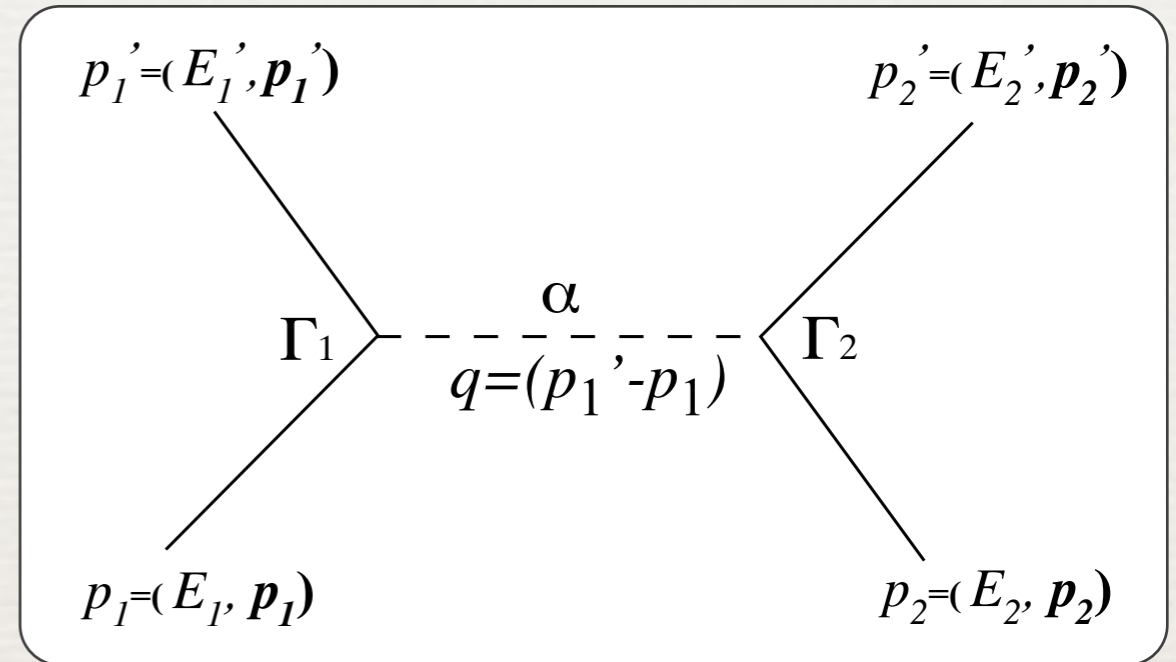
One-Pion Exchange Potential

One-Boson Exchange Potential

$$\frac{\bar{u}_1(p'_1)\Gamma_1 u_1(p_1) P_\alpha \bar{u}_2(p'_2)\Gamma_2 u_2(p_2)}{(p'_1 - p_1)^2 - m_\alpha^2}$$

$u(p)$: Dirac spinor

$\frac{P_\alpha}{(p'_1 - p_1)^2 - m_\alpha^2}$: Meson Propagator



π NN Lagrangian

$$\mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma_5 \gamma_\mu \tau \psi \cdot \partial^\mu \phi_\pi \longrightarrow \Gamma$$

One-Pion Exchange Potential

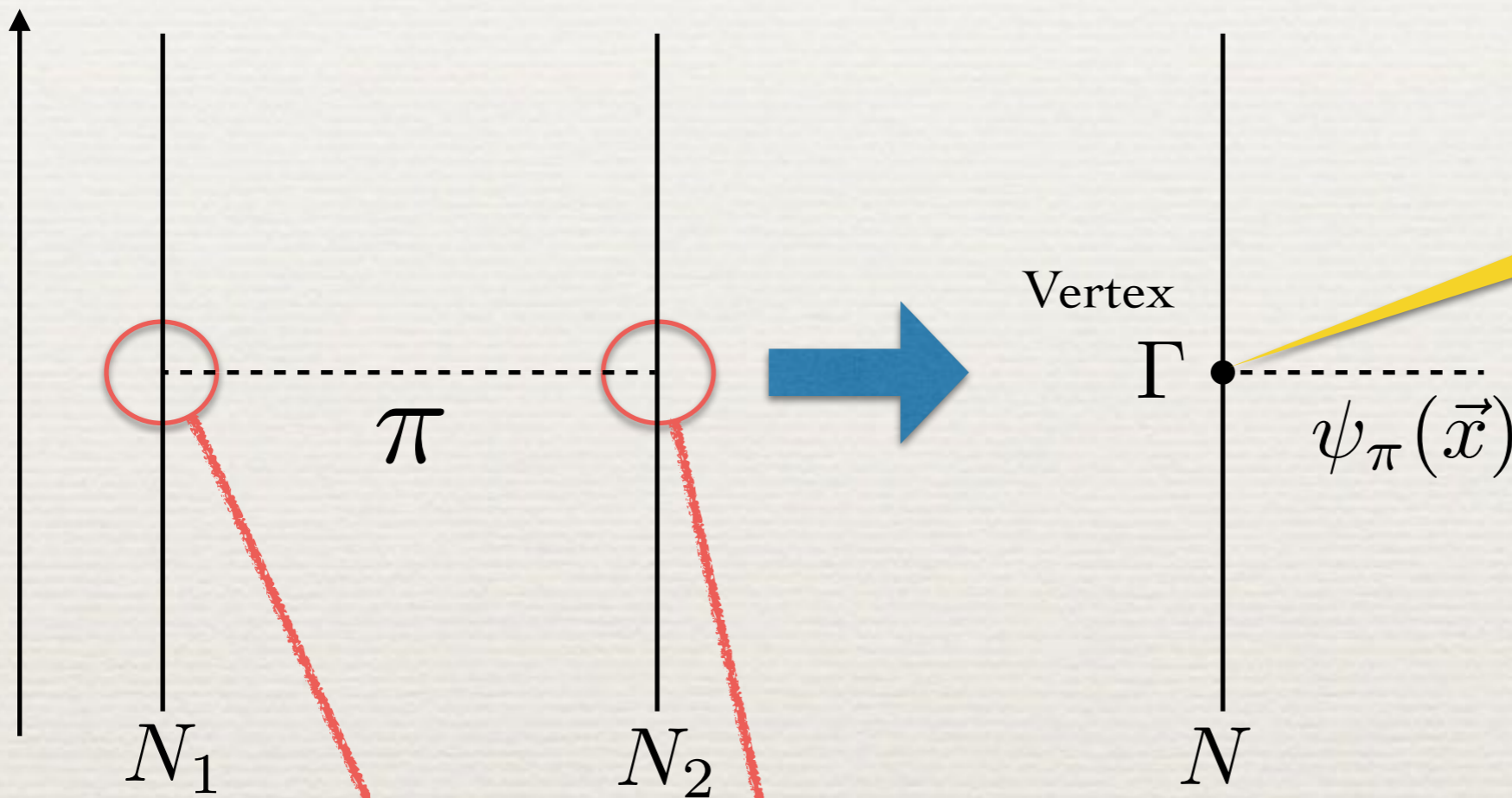
$$V_\pi = \frac{f_\pi^2}{m_\pi^2} \frac{\bar{u}(p'_1) \gamma_5 \gamma_\mu (p_1 - p'_1)^\mu u(p_1) \bar{u}(p'_2) \gamma_5 \gamma_\nu (p_2 - p'_2)^\nu u(p_2)}{(p_1 - p'_1)^2 - m_\pi^2} \tau_1 \cdot \tau_2$$

Taking the limit of non-relativity...

$$V_\pi = \frac{f_\pi^2}{3m_\pi^2} \frac{\mathbf{k}^2}{\mathbf{k}^2 + m_\pi^2} \left[-\sigma_1 \cdot \sigma_2 - S_{12}(\hat{\mathbf{k}}) \right] \tau_1 \cdot \tau_2$$

$$(p_1 - p'_1)^2 = -\mathbf{k}^2$$

time



$$\frac{(\vec{\sigma}_1 \cdot \vec{p})(\vec{\sigma}_2 \cdot \vec{p})}{m_\pi^2 + \vec{p}^2}$$

Propagator

$\psi_\pi(\vec{x})$: pseudo scalar
 \downarrow
 Γ : pseudo scalar
 $\vec{\sigma} \cdot \vec{p}$

Scalar

Finally

$$V_{\pi} = -\frac{f_{\pi NN}^2}{m_{\pi}^2} \frac{(\vec{\sigma}_1 \cdot \vec{p})(\vec{\sigma}_2 \cdot \vec{p})}{m_{\pi}^2 + \vec{p}^2} \vec{\tau}_1 \cdot \vec{\tau}_2$$

$$S_{12}(\hat{p}) = 3(\vec{\sigma}_1 \cdot \hat{p})(\vec{\sigma}_2 \cdot \hat{p}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \quad \text{Tensor Operator}$$

Then

$$V_{\pi} = \frac{f_{\pi NN}^2}{2m_{\pi}^2} \frac{\vec{p}^2}{m_{\pi}^2 + \vec{p}^2} \left(-\vec{\sigma}_1 \cdot \vec{\sigma}_2 - S_{12}(\hat{q}) \right) \vec{\tau}_1 \cdot \vec{\tau}_2$$

attractive

$\pi(138)$

$$V_{\pi} = \frac{f_{\pi}^2}{3m_{\pi}^2} \frac{\mathbf{k}^2}{\mathbf{k}^2 + m_{\pi}^2} \left[-\sigma_1 \cdot \sigma_2 - S_{12}(\hat{\mathbf{k}}) \right] \tau_1 \cdot \tau_2$$

long range
tensor (attractive)

 $\sigma(600)$

$$V_{\sigma} = g_{\sigma}^2 \frac{1}{\mathbf{k}^2 + m_{\sigma}^2} \left(-1 + \frac{\mathbf{q}^2}{2M_N^2} - \frac{\mathbf{k}^2}{8M_N^2} - \frac{\mathbf{L} \cdot \mathbf{S}}{2M_N^2} \right)$$

Intermediate range
attractive (central), LS

 $\omega(782)$

$$V_{\omega} = g_{\omega}^2 \frac{1}{\mathbf{k}^2 + m_{\omega}^2} \left(1 - 3 \frac{\mathbf{L} \cdot \mathbf{S}}{2M_N^2} \right)$$

Short range
repulsive(central), LS

 $\rho(770)$

$$V_{\rho} = \frac{f_{\rho}^2}{12M^2} \frac{\mathbf{k}^2}{\mathbf{k}^2 + m_{\rho}^2} \left(-2\sigma_1\sigma_2 + S_{12}(\hat{\mathbf{k}}) \right) \tau_1\tau_2$$

Short range tensor
(repulsive)

Realistic NN potential

- NN potentials which reproduce deuteron properties and 3000-4000 nucleon-nucleon scattering data and with high precision ($\chi^2/\text{datum} \sim 1$).
- CD Bonn : R. Machleidt, Phys. Rev. C **63**, 024001 (2001)
- Argonne V_{18} : R.B. Wiringa et al., Phys. Rev. C **51**, 38 (1995)
- Nijmegen I, II : V.G.J. Stoks et al., Phys. Rev. C **49**, 2950 (1994)

	CD-Bonn	AV_{18}	Nijm I	Nijm II	Exp.
Character	Nonlocal	Local	Nonlocal	Local	
<i>NN</i> Scattering Data					
N_{data}	3058	4301	4301	4301	
χ^2/datum	1.03	1.09	1.03	1.03	
Deuteron Properties					
Quadr. moment [fm^2]	0.270	0.270	0.2719	0.2707	0.2859(3)
Asymptotic D/S state	0.0255	0.0250	0.0253	0.0252	0.0256(4)
D-state probab. [%]	4.83	5.76	5.664	5.635	

Number of parameters to be fitted to NN data : about 40

2N Systems

Two-Nucleon Systems

- ♦ Deuteron
- ♦ Nucleon-Nucleon Scattering

<i>S</i> wave ($l=0$)				
	State	Iso-Spin	System	Scattering Length a [fm]
deuteron	3S_1	$T = 0$	np	5.424
Unbound States	1S_0	$T = 1$	np	-23.7
			pp	-17.3
			nn	-18.8

$2S+1 L_J$ J : Total Spin $J=L+S$
 S : Spin of Two fermion systems
 L : Angular Momentum

Deuteron

- Two nucleon bound system
 - $(J^\pi, T) = (1^+, 0)$
 - Binding Energy : 2.22456612(48) MeV
 - Electric quadrupole moment : $Q \neq 0$
 - Magnetic moment : $\mu_d = 0.85741(8)\mu_N$
 $\mu_p + \mu_n \neq \mu_d$
- } ${}^3S_1 + {}^3D_1$

	CD-Bonn	Argonne v_{18}	Nijmegen I	Ihaho N ³ LO(500)	Exp.
$B_d(\text{MeV})$	2.224575	2.224575	2.224575	2.224575	2.22456612(48)
Matter radius $r_d(\text{fm})$	1.966	1.967		1.975	1.975(3)
Q (fm ²)	0.270	0.270	0.272	0.275	0.2859(3)
$P_D(\%)$	4.85	5.76	5.664	4.51	—

Nucleon-Nucleon Scattering

- pp, nn, pn Systems (poor data of nn scattering)
- ♦ Observables
 - ♦ Cross Section : Magnitudes
 - ♦ Spin Observables : Spin dependence
- ♦ Scattering amplitudes

Observables

• Cross Section $\frac{d\sigma}{d\Omega}$

• Overall Strength

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

$$f(\theta) = -\frac{(2\pi)^{3/2}}{4\pi} \int \exp(-\mathbf{q} \cdot \mathbf{r}') V(r') d\mathbf{r}'$$

Fourier Transform of Nuclear Potential

➤ **Measurement** : Absolute Quantity is required (very hard !).

$$\frac{d\sigma}{d\Omega} = \frac{\text{yields}}{(\text{target thickness}) \times (\text{beam charge}) \times (\text{solid angle}) \times (\text{efficiency})}$$

• Spin Observables

– Analyzing Powers

• Vector Analyzing Power : A_y (iT_{11})

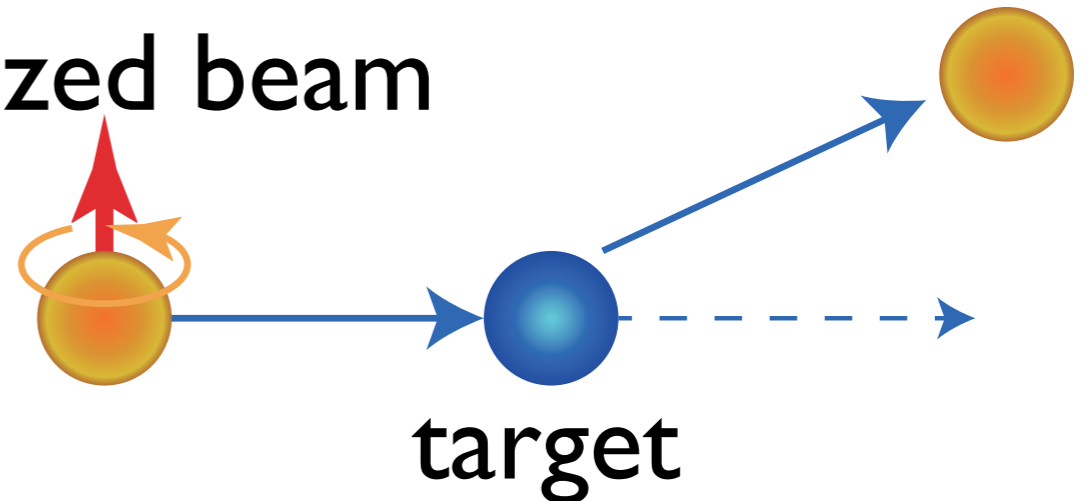
– $(\mathbf{L} \cdot \mathbf{S})$ interaction

• Tensor Analyzing Power : A_{xx}, A_{yy}, A_{xz} (T_{20}, T_{21}, T_{22})

– Tensor interaction (D-state)

– Higher order $(\mathbf{L} \cdot \mathbf{S})$ interaction

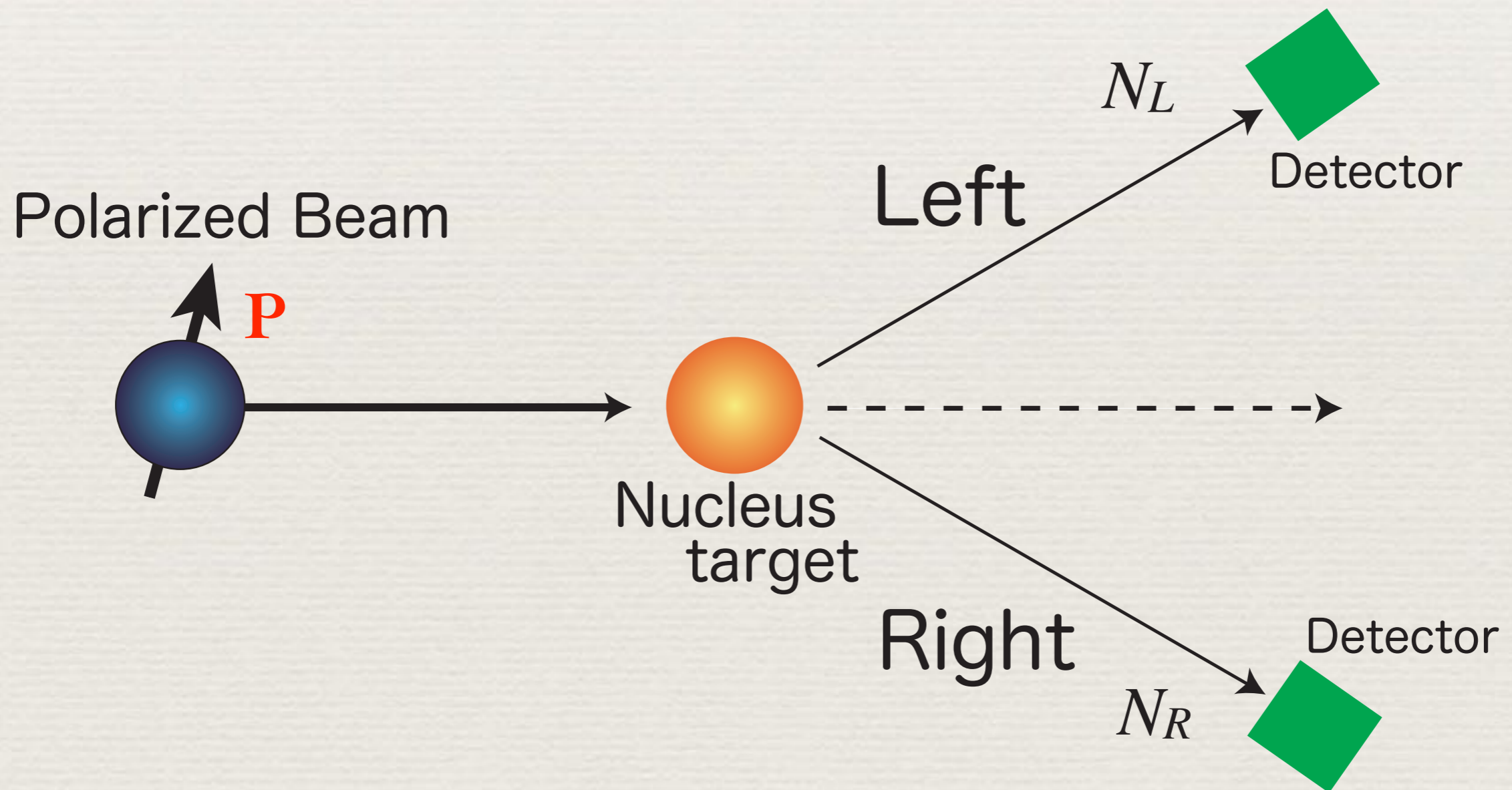
polarized beam



target

Measurement of Polarization

$$\text{Asymmetry} = P \cdot A = \frac{N_L - N_R}{N_L + N_R} \quad \begin{array}{l} P : \text{Beam Polarization} \\ A : \text{Analyzing Power} \end{array}$$



NN (pp or pn) Scattering : Partial Wave Analysis

$$f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin(\delta_l) P_l(\cos\theta)$$

R. MACHLEIDT

PHYSICAL REVIEW C 63 024001

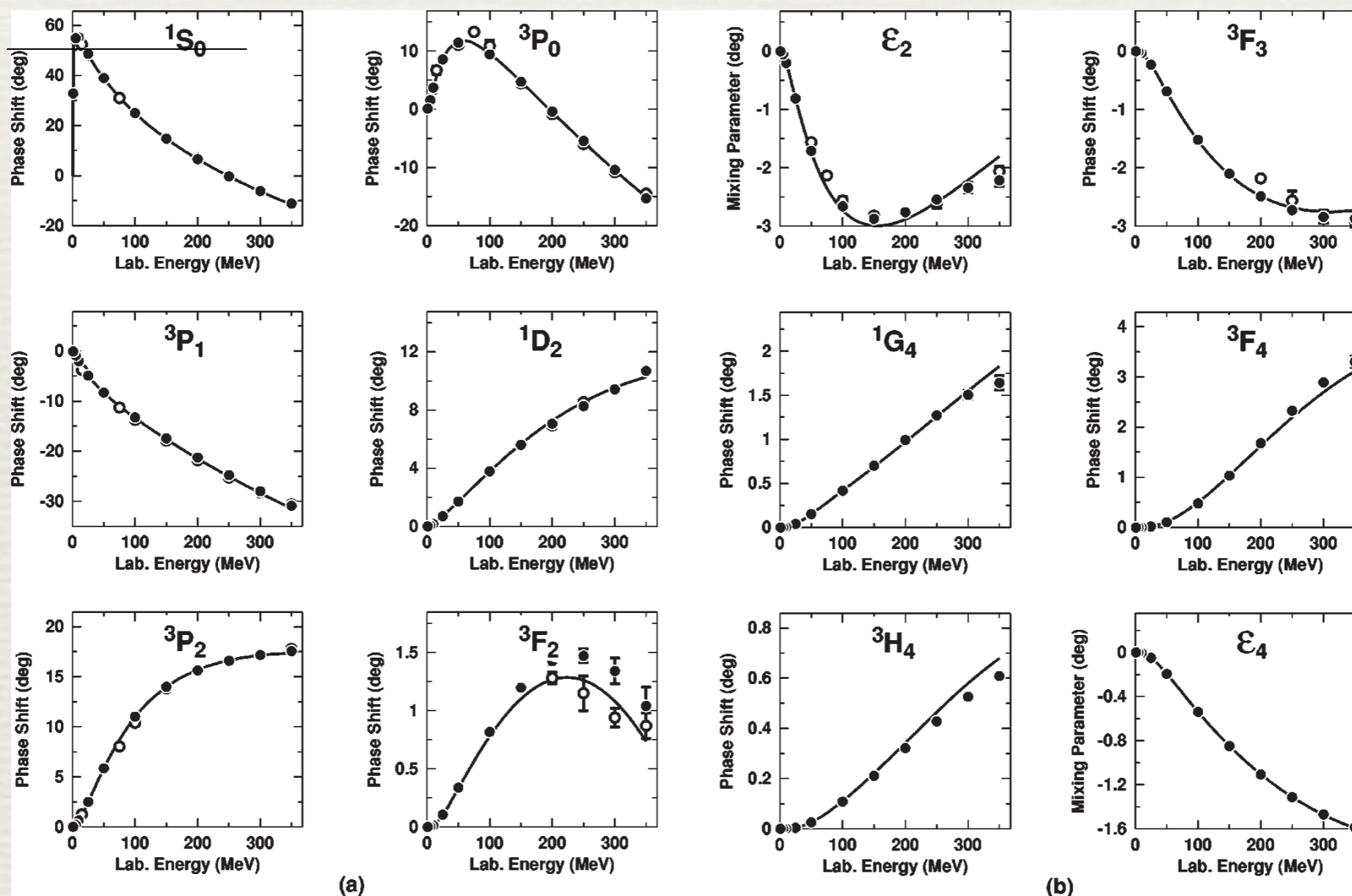


FIG. 6. pp phase parameters in partial waves with $J \leq 4$. The solid line represents the predictions by the CD-Bonn potential. The solid dots and open circles are the results from the Nijmegen multienergy pp phase shift analysis [46] and the VPI single-energy pp analysis SM99 [49], respectively.

NN Scattering

- Experiments

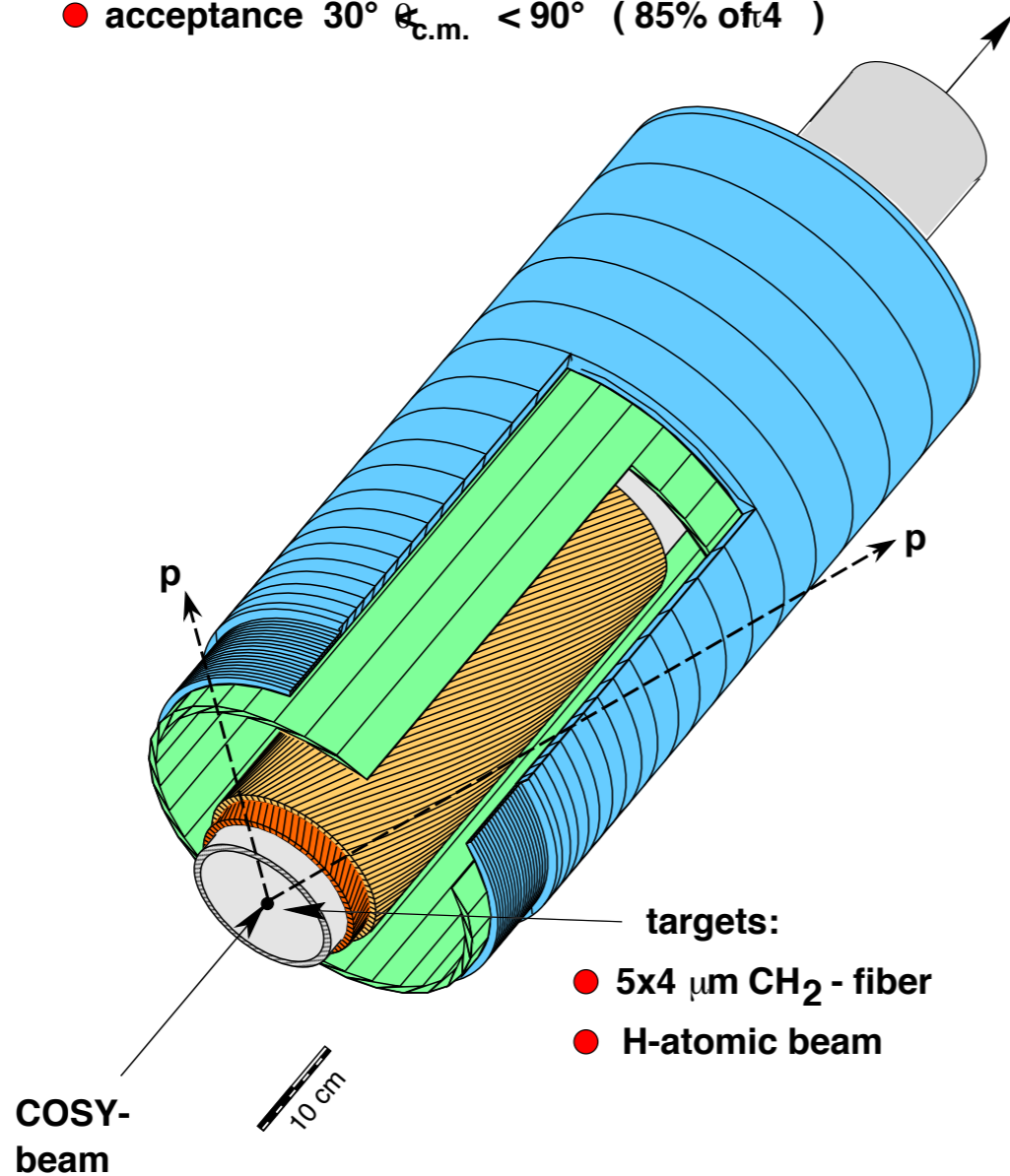
Heiko Rohdjes, HISKP Uni Bonn

6

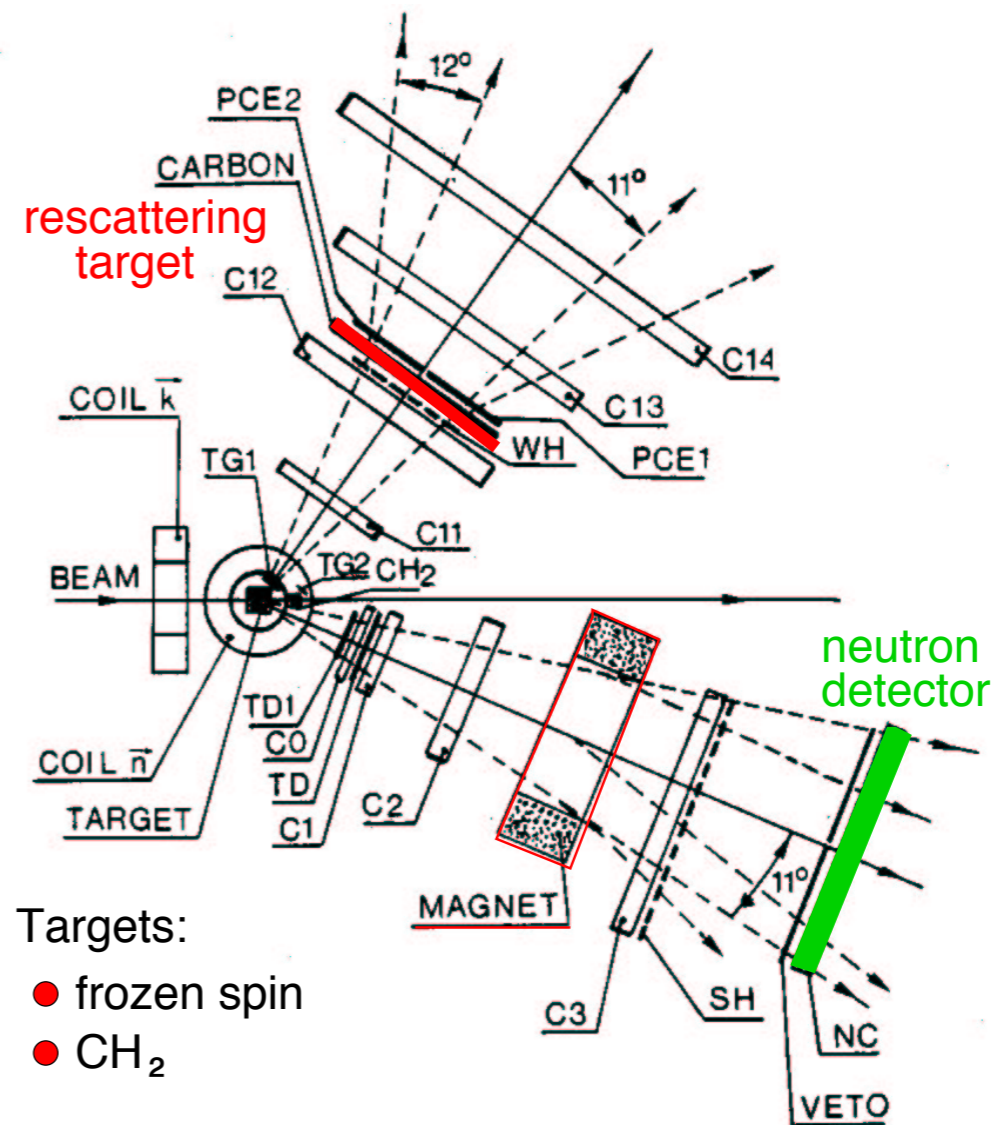
Hadron Physics at COSY, Bad Honnef 2003

EDDA@COSY

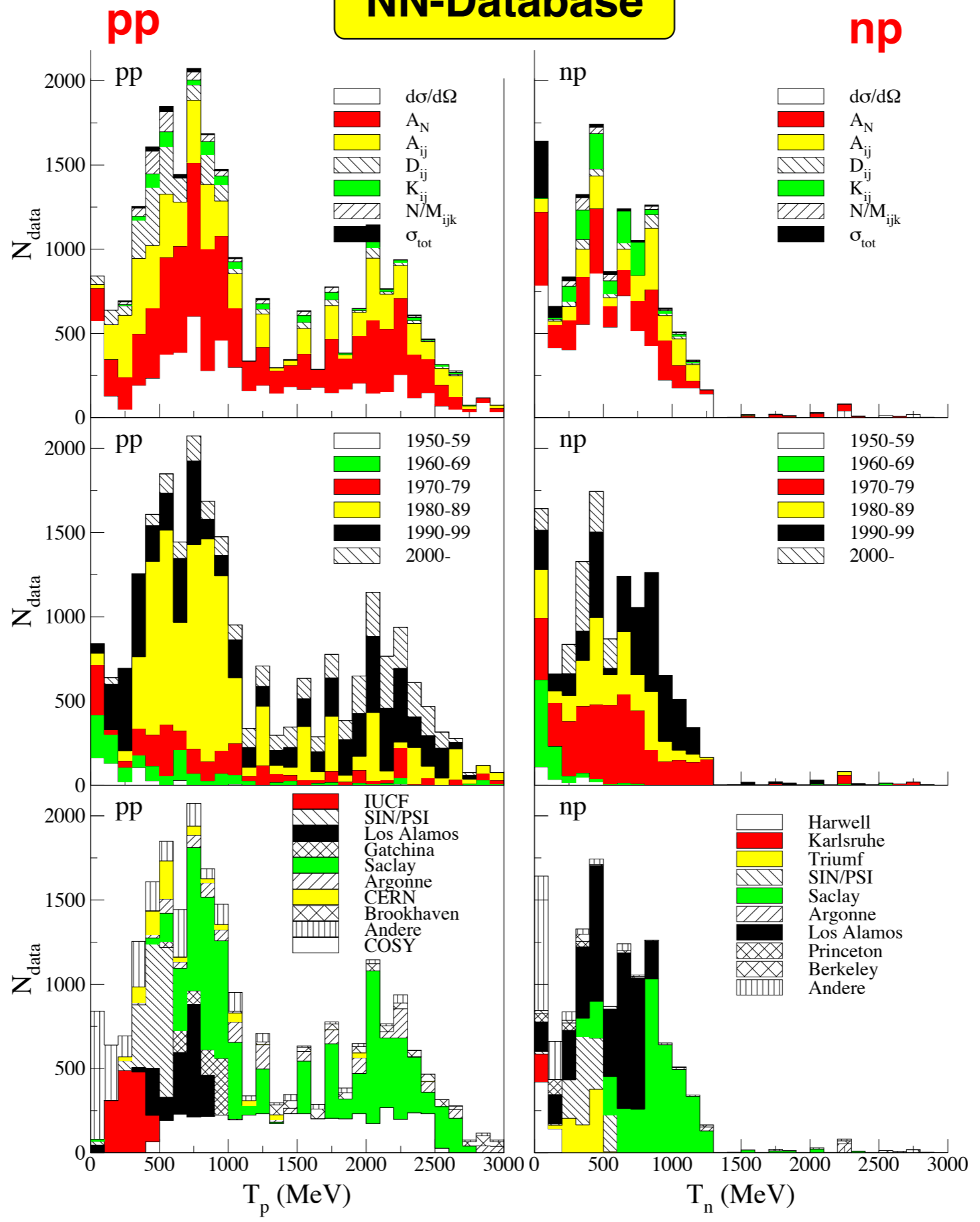
- acceptance $30^\circ < \theta_{c.m.} < 90^\circ$ (85% of 4π)



NN@Saturne II



NN-Database



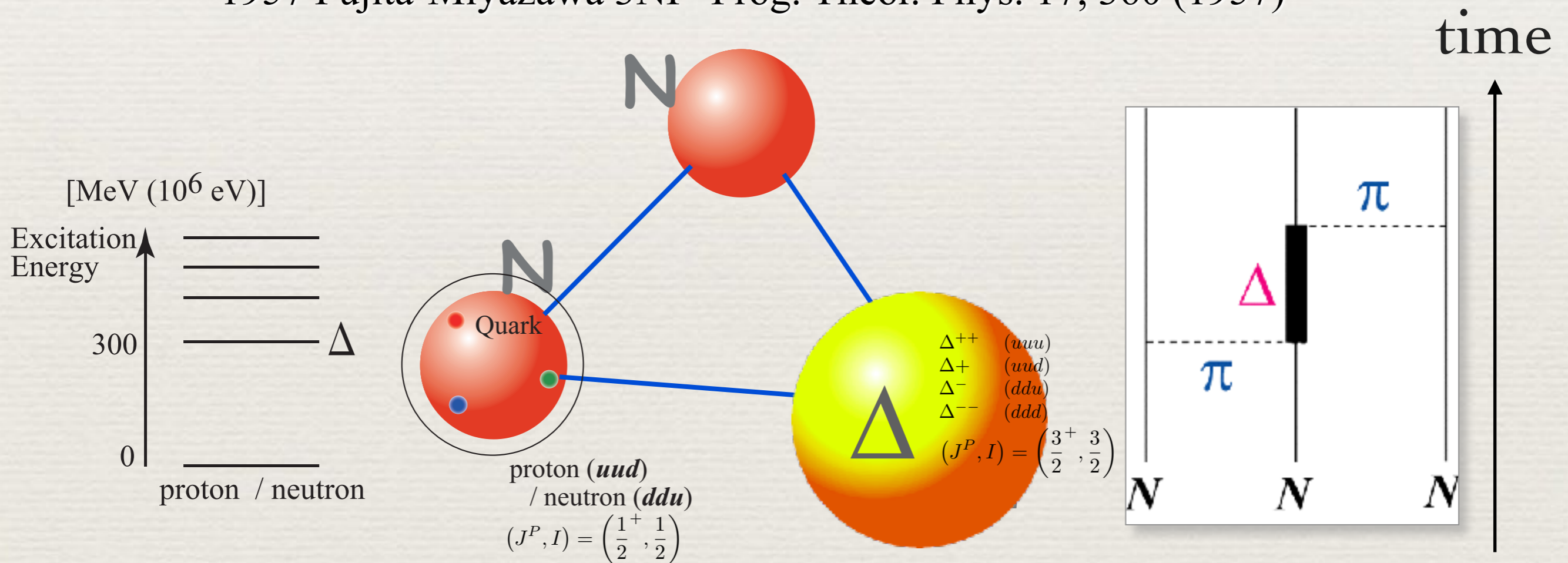
Three-Nucleon Forces

Three-Body Forces in Nuclei

- 3-Nucleon Force -

• 2π-exchange 3NF :

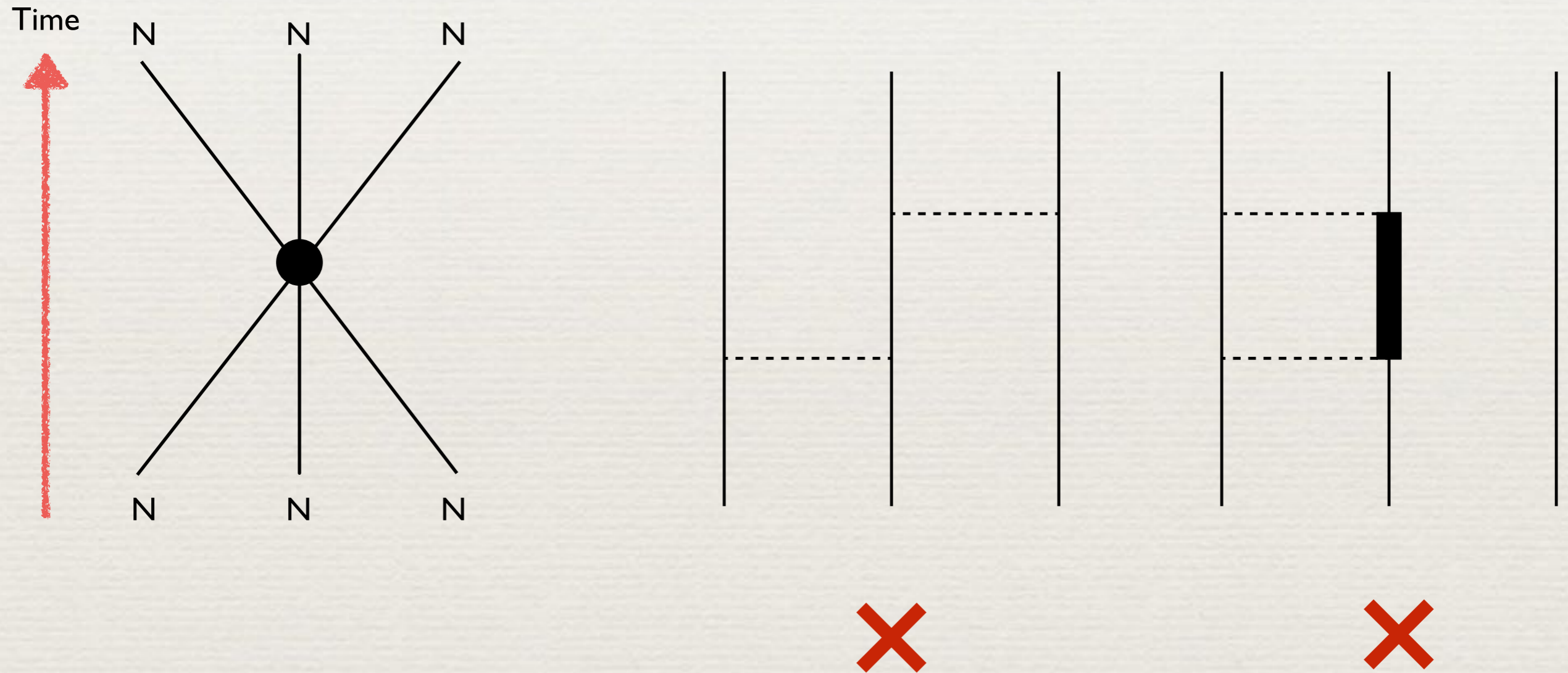
- Main Ingredients : Δ -isobar excitations in the intermediate
- 1957 Fujita-Miyazawa 3NF Prog. Theor. Phys. 17, 360 (1957)



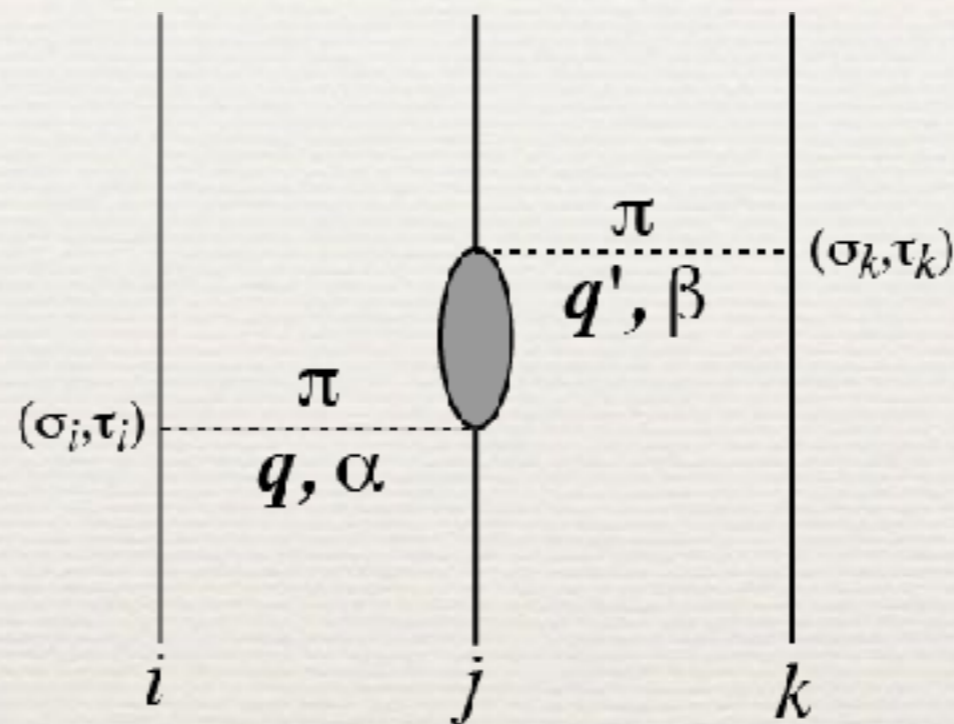
3NF naturally arises due to the inner structure of Nucleon.

Three-Nucleon Forces

- nuclear forces acting in systems more than $A > 2$ nucleons -



2-pion exchange 3NF



$$V_{3NF} = \sum_{\text{cyclic}} V_{3NF}^{(j)}, \quad \text{J.L.Friar et al., Phys. Rev. C 59, 53(1999)}$$

$$V_{3NF}^{(j)} = \frac{g^2}{4m_N^2} \frac{\vec{\sigma}_i \cdot \mathbf{q}}{\mathbf{q}^2 + m_\pi^2} \frac{\vec{\sigma}_k \cdot \mathbf{q}'}{\mathbf{q}'^2 + m_\pi^2} F_{\pi NN}^2(\mathbf{q}^2) F_{\pi NN}^2(\mathbf{q}'^2) \left[\mathcal{O}^{\alpha\beta} \tau_i^\alpha \tau_k^\beta \right],$$

$$\mathcal{O}^{\alpha\beta} = \xi^{\alpha\beta} \left[\mathbf{a} + \mathbf{b} \mathbf{q} \cdot \mathbf{q}' + \mathbf{c} (\mathbf{q}^2 + \mathbf{q}'^2) \right] - \mathbf{d} (\tau_j^\gamma \epsilon^{\alpha\beta\gamma} \vec{\sigma}_j \cdot \mathbf{q} \times \mathbf{q}'),$$

- Low momentum expansion of πN Scattering amplitudes
- Cut-off of $F_{\pi NN}$: fit to B.E. of ${}^3\text{H}$

Urbana-IX 3NF

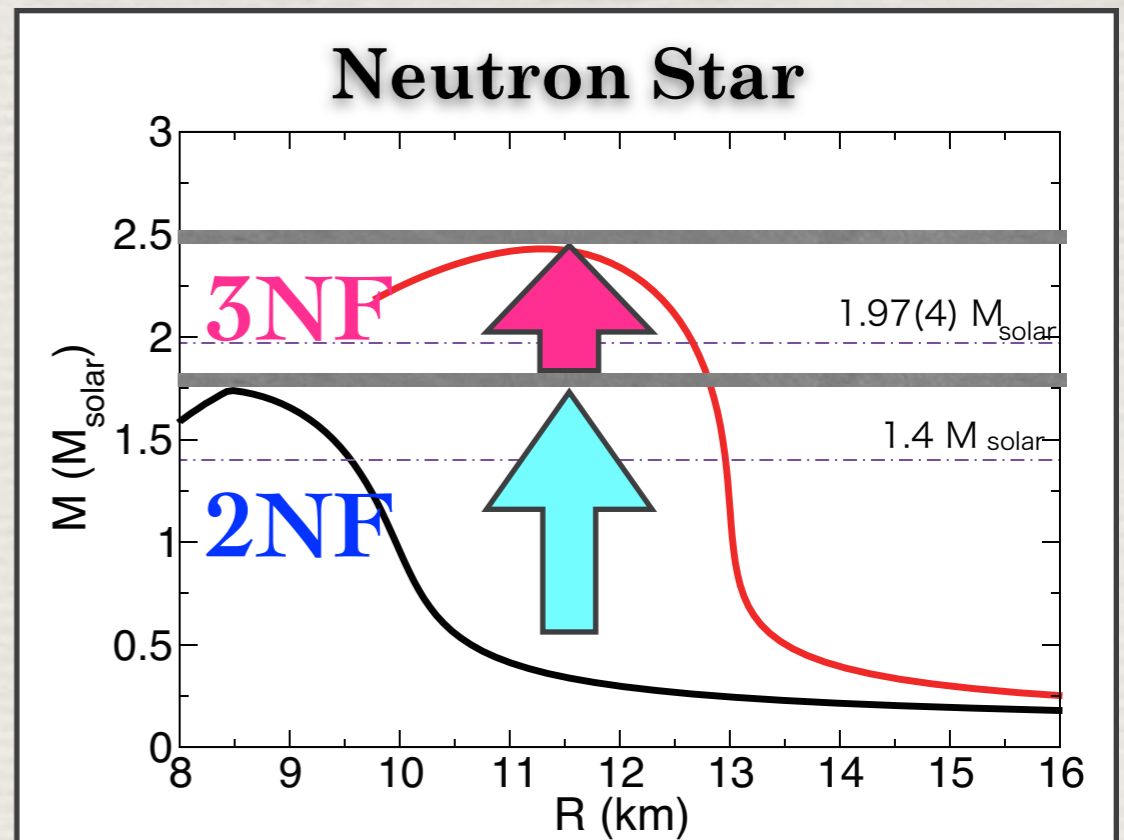
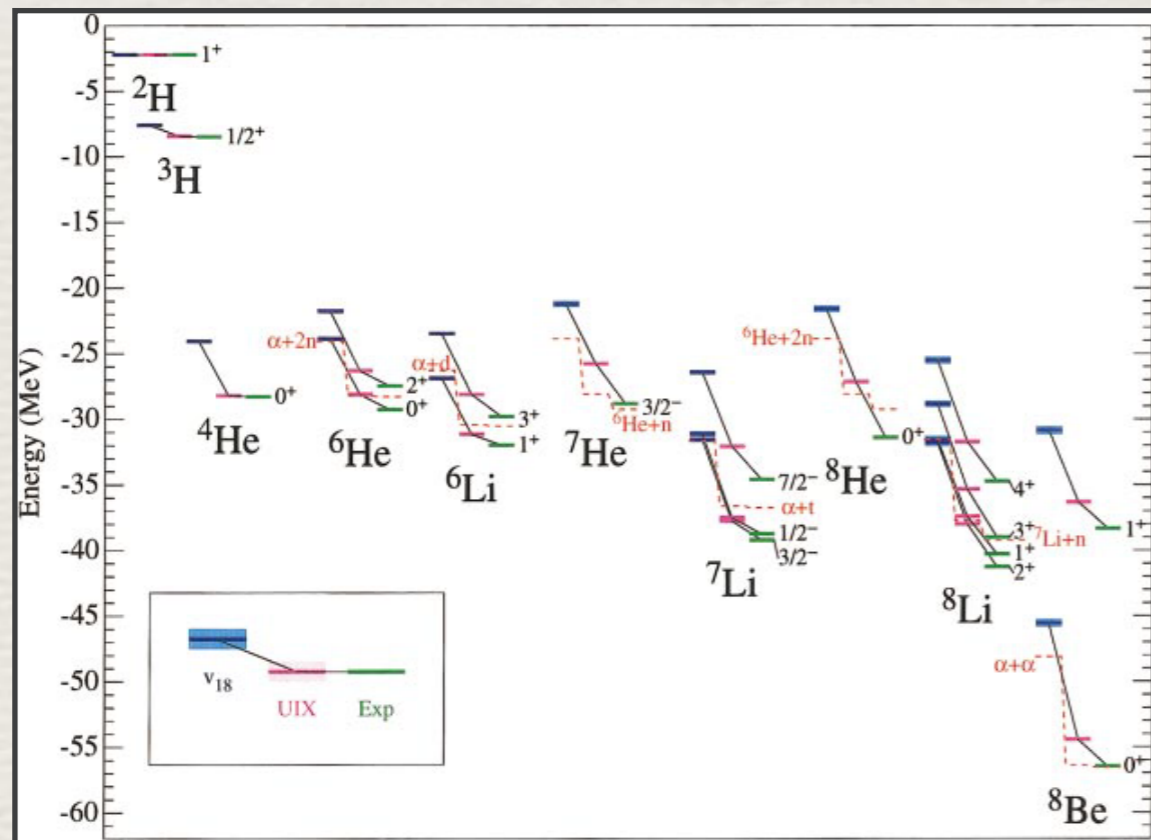
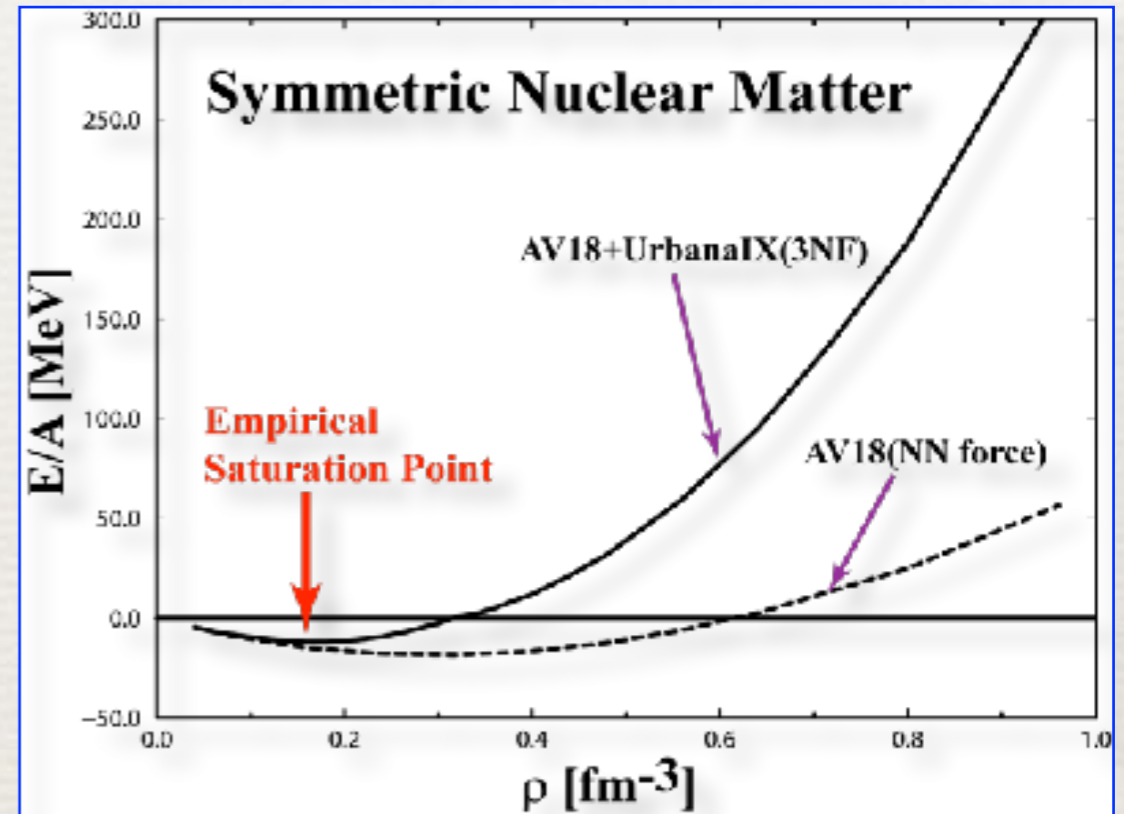
B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper, and R. B. Wiringa, *Phys. Rev. C* **56**, 1720 (1997).

- 2π -exchange (Fujita-Miyazawa 3NF)
- + phenomenological short-range 3NF $V_{2\pi} + V_R$

- 2 parameters

$A_{2\pi}$: Binding Energy of ^3H

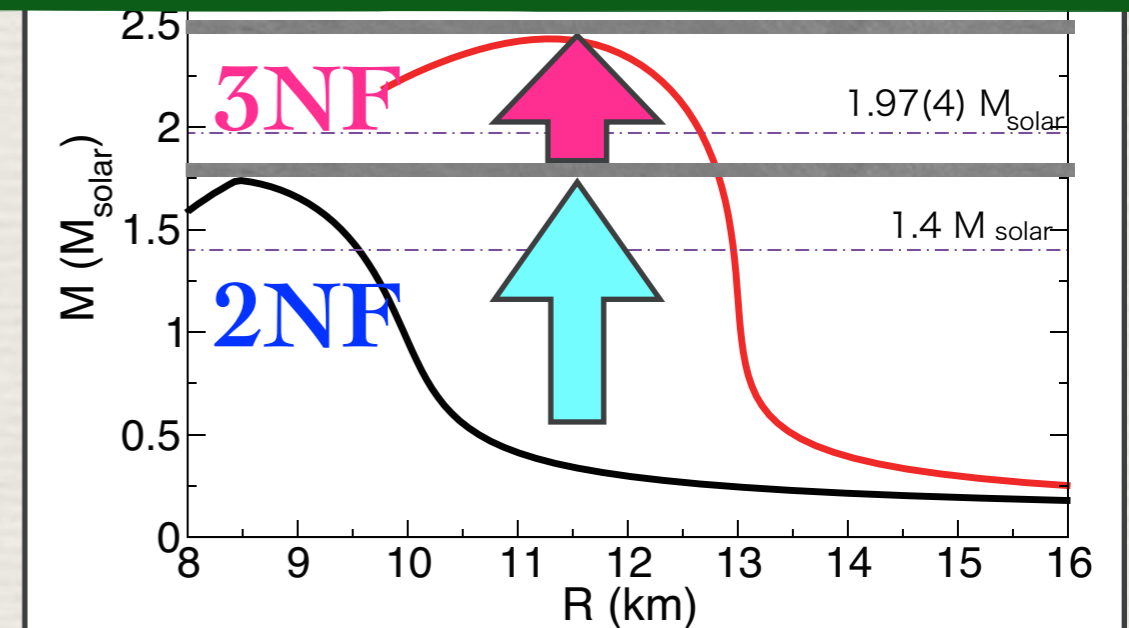
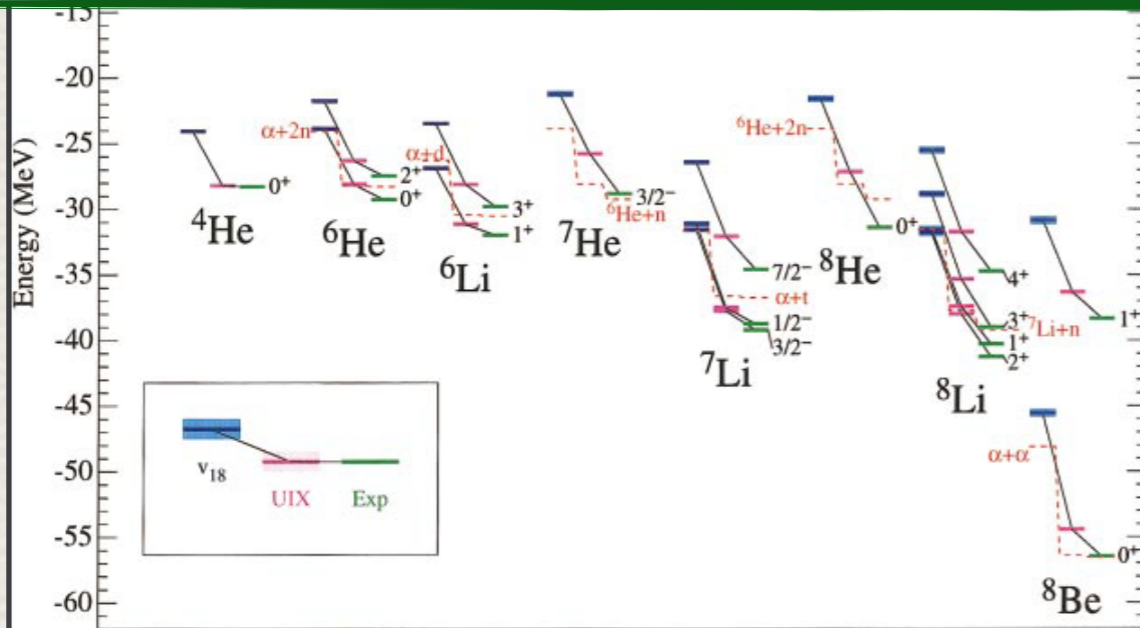
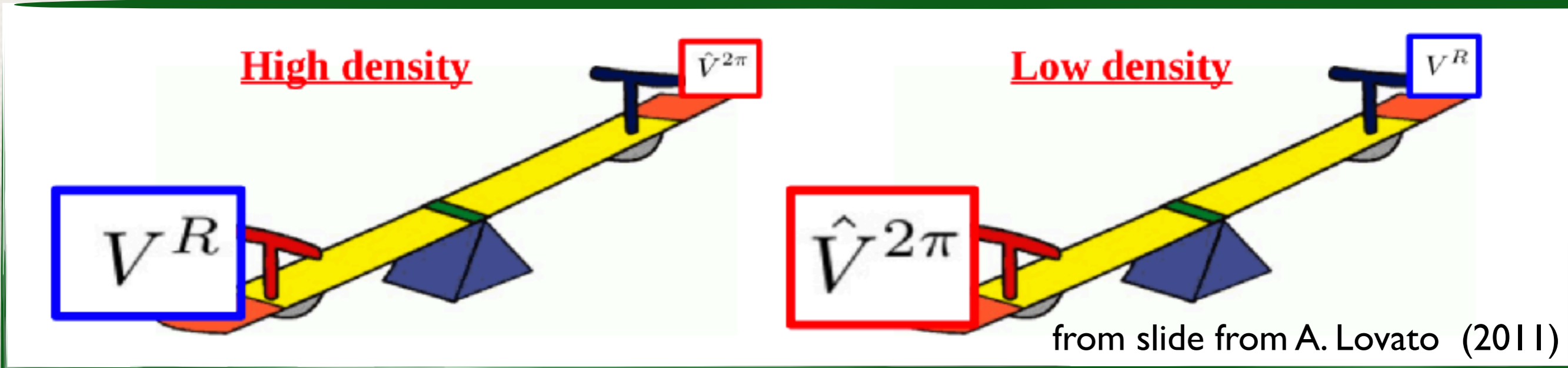
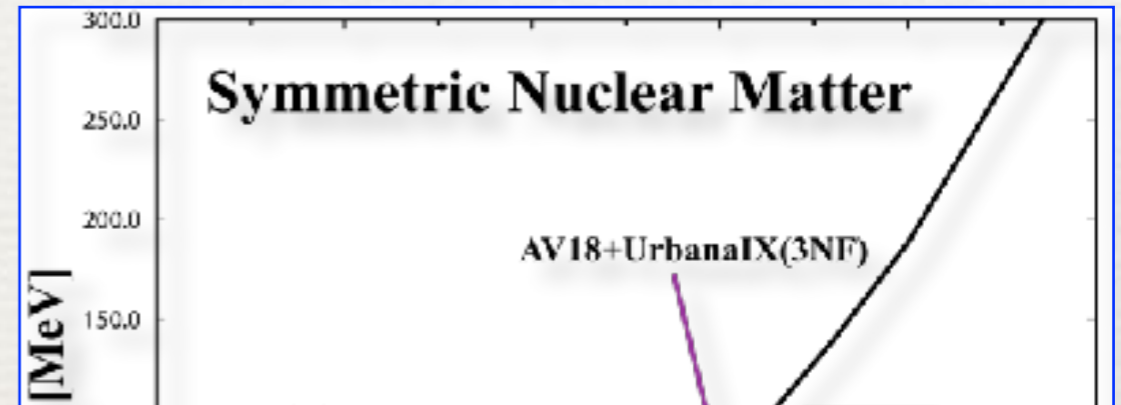
U_0 : Saturation Point of Symmetric Nuclear matter



Urbana-IX 3NF

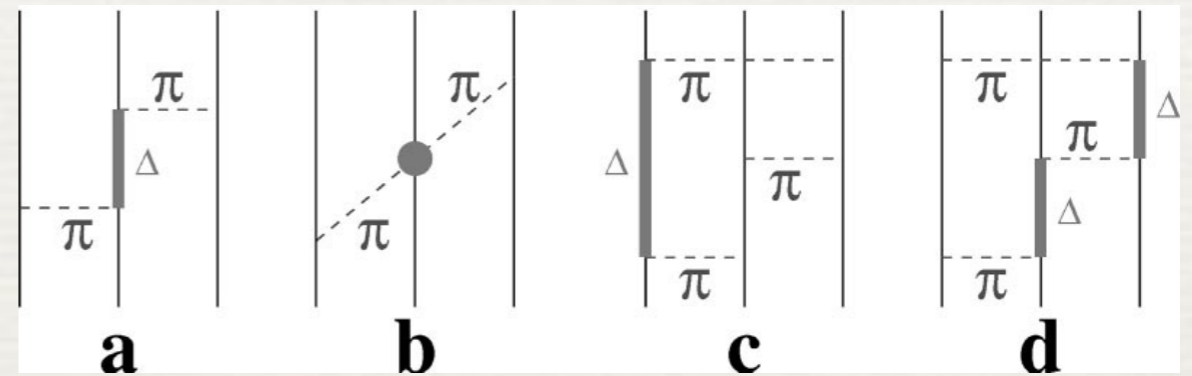
B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper, and R. B. Wiringa, *Phys. Rev. C* **56**, 1720 (1997).

- 2π -exchange (Fujita-Miyazawa 3NF)
- + phenomenological short-range $V_{2\pi} + V_R$
- 2 parameters



Illinois-3NF

- Extension of Urbana-IX 3NF
- 2π -exchange (πN scattering S-wave : b)
 (πN scattering P-wave : a)
 + 3π -ring with one Δ (c, d)

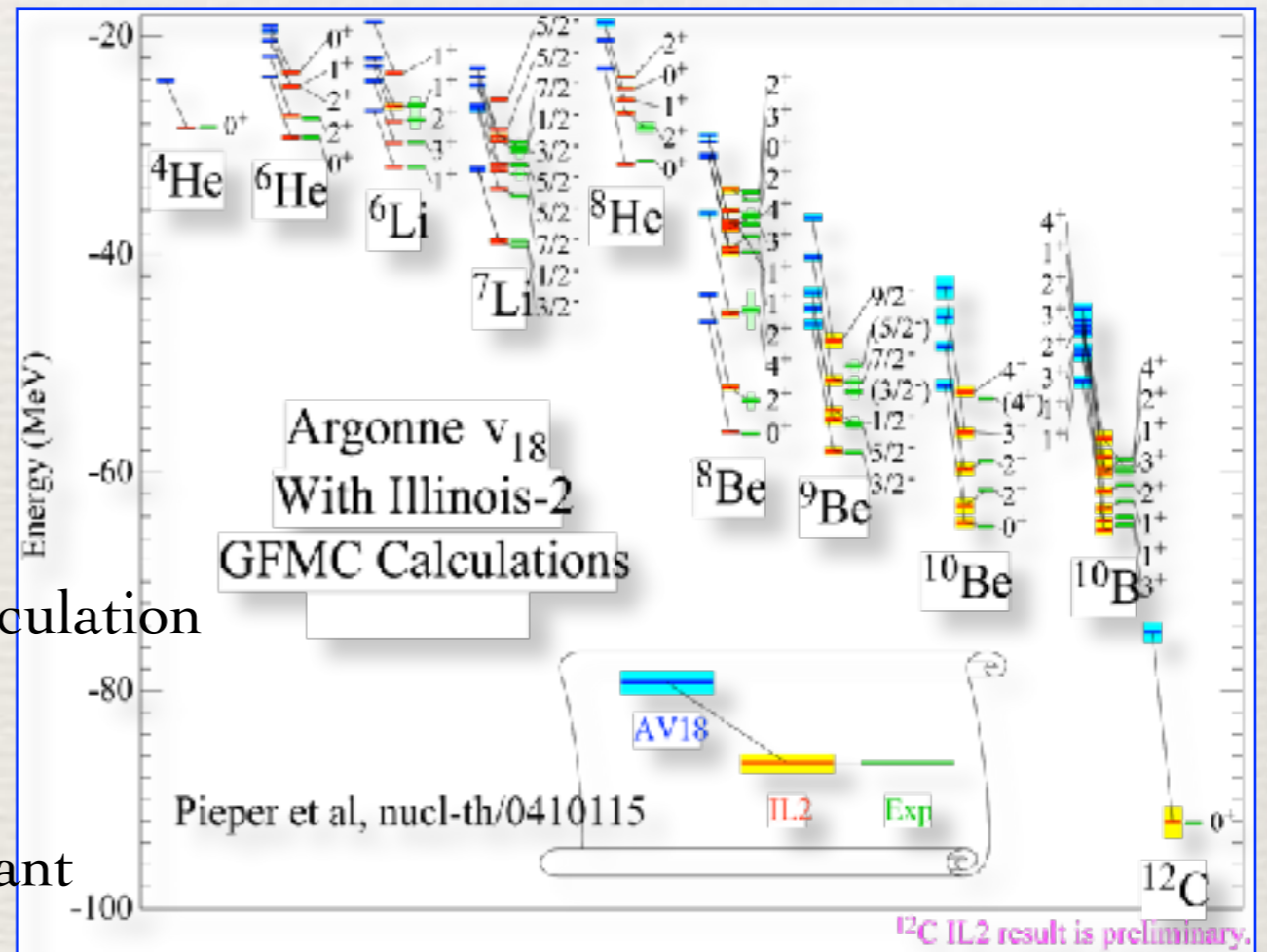


+ phenomenological short range

- 5 parameters (including short-range 3NF)
 Binding Energies of 17 light nuclei

- Applied to light nuclei $A \leq 12$
 with combination of AV18 NN potential
 → Green Function Monte Carlo (GFMC) Calculation

- p-shell Nuclei
 - Isospin channels of $T=3/2$ 3NF are important in neutron-rich nuclei ?

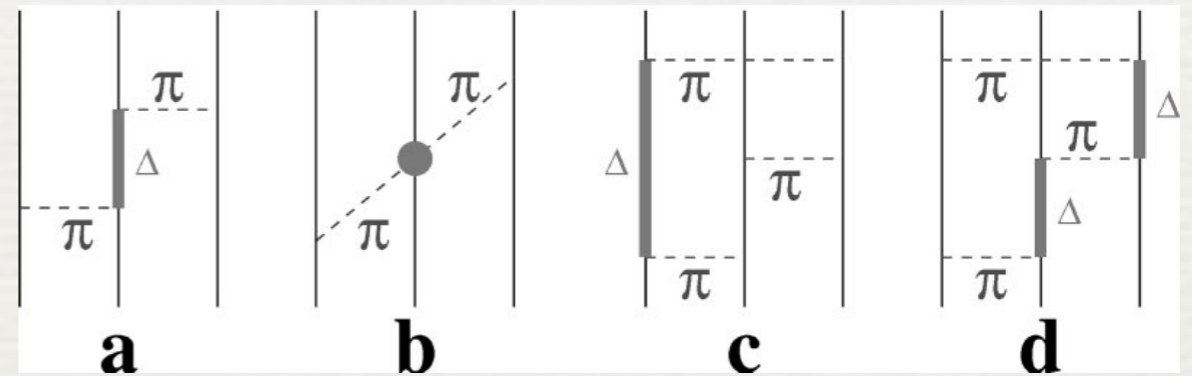


- 3π -ring 3NF : remarkable $T=3/2$ 3NFs

S.C.Pieper, K. Varga, and R. B. Wiringa,
 Phys. Rev. C 66, 44310 (2002)

Illinois-3NF

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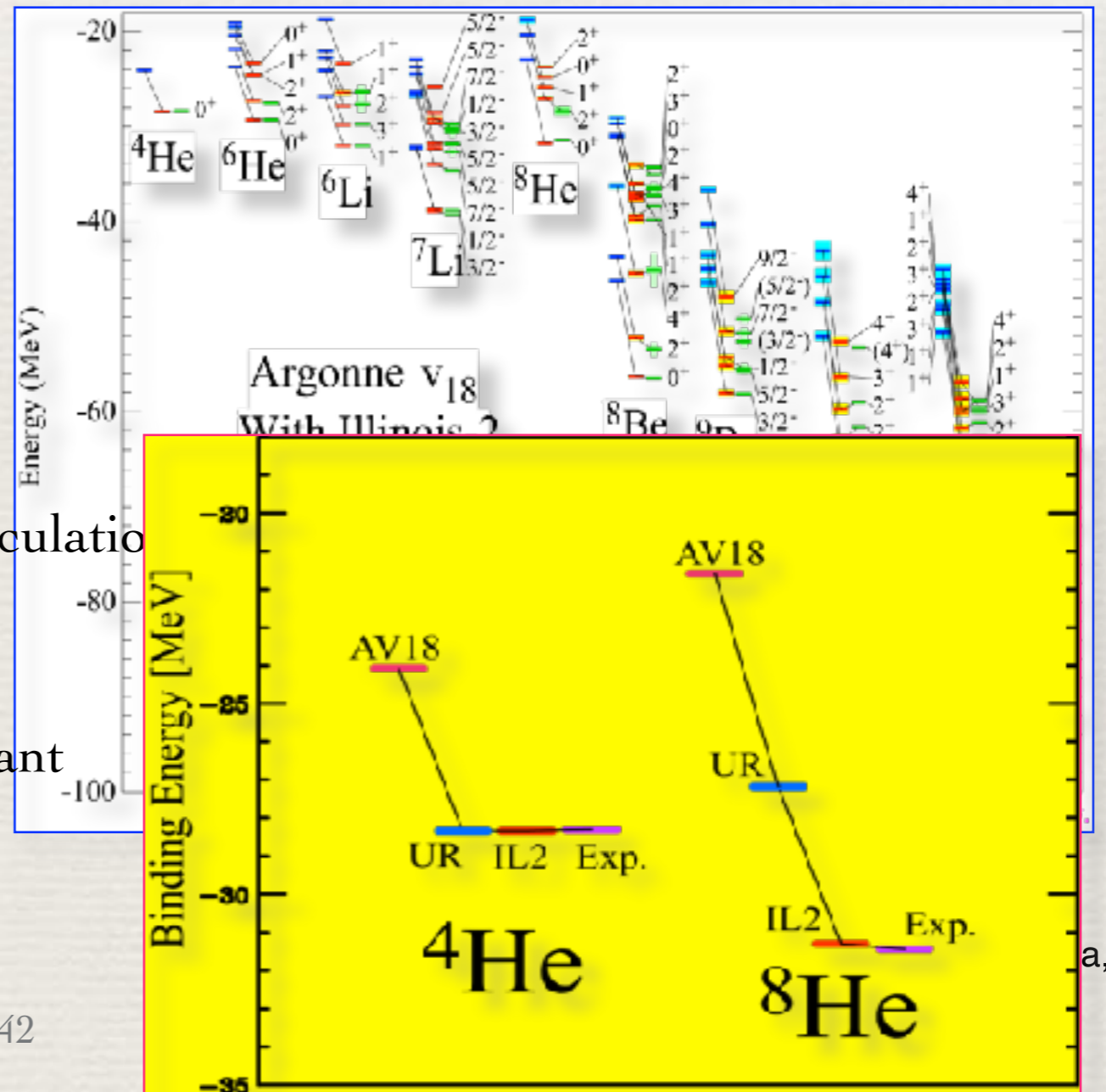


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 - 3π -ring 3NF : remarkable $T=3/2$ 3NFs



2-meson exchange 3NF in ${}^3\text{H}$ B.E.

SHORT-RANGE THREE-NUCLEON FORCES AND LOW...

PHYSICAL REVIEW C **69**, 034008 (2004)

TABLE IV. Triton binding energies and their differences (in MeV) calculated for various model Hamiltonians with different NN potentials and contributions to the $3N$ force added consecutively. All πNN vertices in the $3N$ forces of this table are calculated in PV coupling. The columns labeled E_t show the triton binding energies, while the ones labeled ΔE_t indicate the differences between the binding energies of consecutive rows, indicating the effect of the corresponding $3N$ force component.

3NF	Reid		Paris		Nijmegen 93		Bonn B	
	E_t	ΔE_t	E_t	ΔE_t	E_t	ΔE_t	E_t	ΔE_t
No 3NF	-7.230		-7.383		-7.756		-8.100	
$+\pi\pi(a')$	-7.279	-0.049	-7.439	-0.056	-7.811	-0.055	-8.159	-0.059
$+\pi\pi(b)$	-8.739	-1.460	-8.939	-1.500	-9.471	-1.660	-9.624	-1.465
$+\pi\pi(d)$	-9.100	-0.361	-9.220	-0.281	-9.782	-0.311	-9.847	-0.223
$+\pi\rho(\text{KR})$	-9.017	0.083	-9.118	0.102	-9.635	0.147	-9.672	0.175
$+\pi\rho(\Delta^+)$	-8.849	0.168	-8.961	0.157	-9.464	0.171	-9.506	0.166
$+\pi\rho(\Delta^-)$	-8.747	0.102	-8.821	0.140	-9.285	0.179	-9.325	0.181
$+\pi\rho(T)$	-8.772	-0.025	-8.850	-0.029	-9.316	-0.031	-9.352	-0.027
$+\pi\sigma(Z)$	-8.273	0.499	-8.213	0.637	-8.663	0.653	-8.658	0.694
$+\pi\sigma(N^*)$	-8.711	-0.438	-8.610	-0.397	-9.145	-0.482	-9.055	-0.397
$+\pi\omega(Z)$	-9.213	-0.502	-9.380	-0.770	-9.977	-0.832	-9.956	-0.901
$+\pi\omega(N^*)$	-8.735	0.478	-8.898	0.482	-9.370	0.607	-9.524	0.432

Nuclear Forces linked to QCD

Nuclear Forces based on Chiral Effective Field Theory

- Link to QCD

Lagrangian :

includes all the terms consistent with the assumed symmetries :

Lorentz and iso-spin Invariance

& **Spontaneously Broken Chiral Symmetry**

Interactions :

π + Nucleon + contact terms

- **Nuclear forces** (2NF, 3NF, ...)

and currents are **derived**










in a **consistent way**.

- **Hierarchy of Nuclear Forces** :

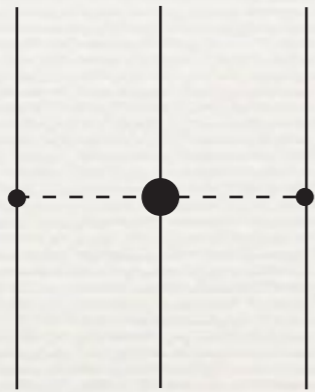
$2NF > 3NF > 4NF$

The first 3NF appears in NNLO.

Layout of χ EFT Nuclear Forces

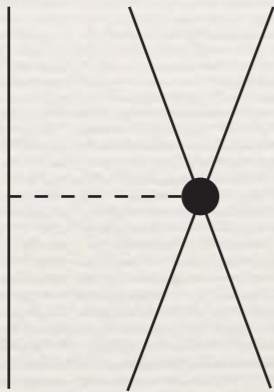
	2N Force	3N Force	4N Force
LO $(Q/\Lambda_\chi)^0$		—	—
NLO $(Q/\Lambda_\chi)^2$		—	—
N2LO $(Q/\Lambda_\chi)^3$			—
N3LO $(Q/\Lambda_\chi)^4$			
N4LO $(Q/\Lambda_\chi)^5$			

3NFs in χ EFT (N2LO)

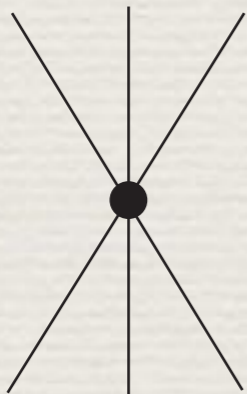


$$V_{2\text{PE}}^{3\text{NF}} = \left(\frac{g_A}{2f_\pi} \right)^2 \sum_{i \neq j \neq k} \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(q_i^2 + m_\pi^2)(q_j^2 + m_\pi^2)} F_{ijk}^{ab} \tau_i^a \tau_j^b$$

$$F_{ijk}^{ab} \tau_i^a \tau_j^b = \delta^{ab} \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2} \vec{q}_i \cdot \vec{q}_j \right] + \frac{c_4}{f_\pi^2} \sum_c \epsilon^{qbc} \tau_k^c \vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j]$$



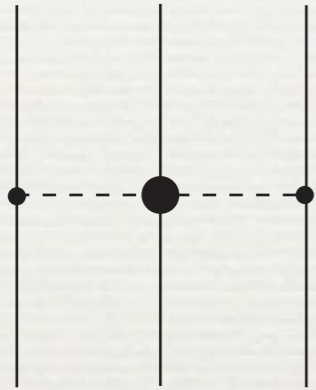
$$V_{1\text{PE}}^{3\text{NF}} = -\textcircled{D} \frac{g_A}{8f_\pi^2} \sum_{i \neq j \neq k} \frac{\vec{\sigma}_i \cdot \vec{q}_j}{q_j^2 + m_\pi^2} (\vec{\tau}_i \cdot \vec{\tau}_j) (\vec{\sigma}_i \cdot \vec{q}_j)$$



$$V_{\text{ct}}^{3\text{NF}} = \textcircled{E} \frac{1}{2} \sum_{j \neq k} \tau_j \cdot \tau_k$$

Parameters D, E to be determined by Experimental data

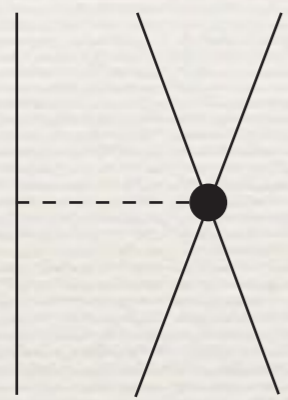
3NFs in χ EFT (N2LO)



$$V_{2PE}^{3NF} = \left(\frac{g_A}{2c} \right)^2 \sum_{i,j,k} \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(\dots)} F_{ijk}^{ab} \tau_i^a \tau_j^b$$

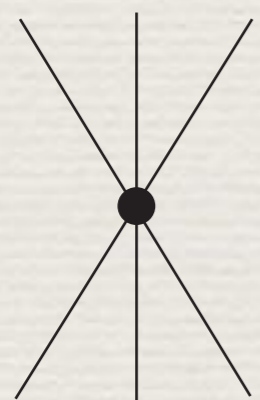
central + tensor + spin orbit + rank 3

$$F_{ijk}^{ab} \tau_i^a \tau_j^b = \delta^{ab} \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2} \vec{q}_i \cdot \vec{q}_j \right] + \frac{c_4}{f_\pi^2} \sum_c \epsilon^{qbc} \tau_k^c \vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j]$$



V_{1H}^3 central + tensor

$$\frac{c_5}{f_\pi} \sum_{i \neq j \neq k} \tau_i^a \tau_j^b \tau_k^c \dots$$



V_c^3 central

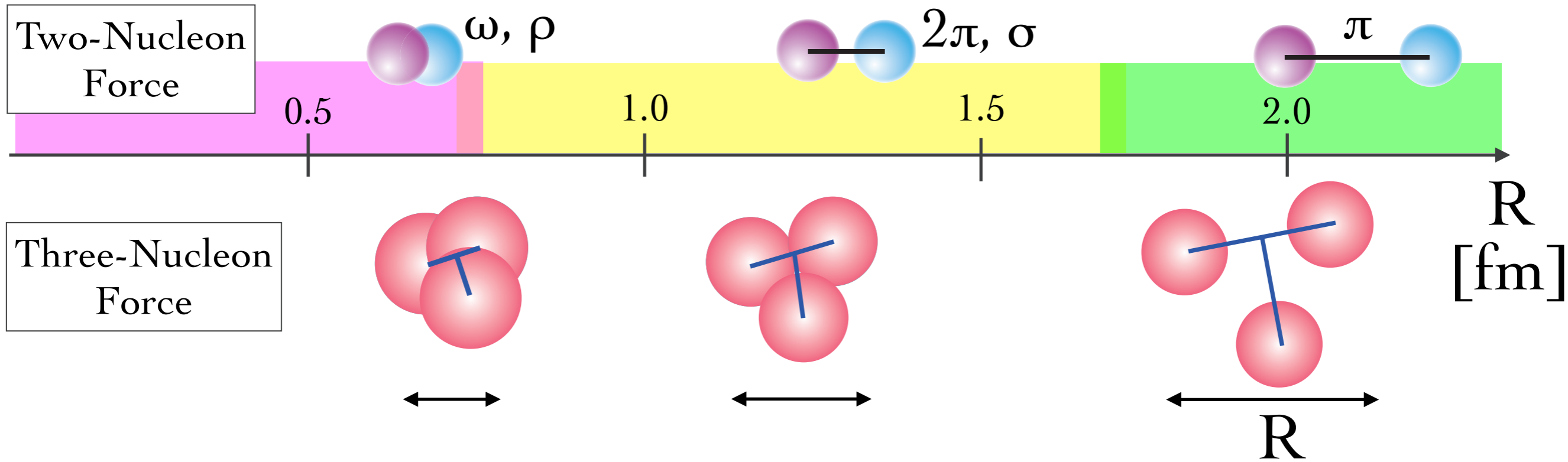
T. Fukui, by courtesy

Two & Three-Nucleon Force

①. Repulsive
-Short Range-

②. Attractive (strong)
-Intermediate Range-

③. Attractive (weak)
- Long Range -



3NFs are momentum, spin, and iso-spin dependent.

Nuclear Matter
Neutron Star

Nuclear Structure

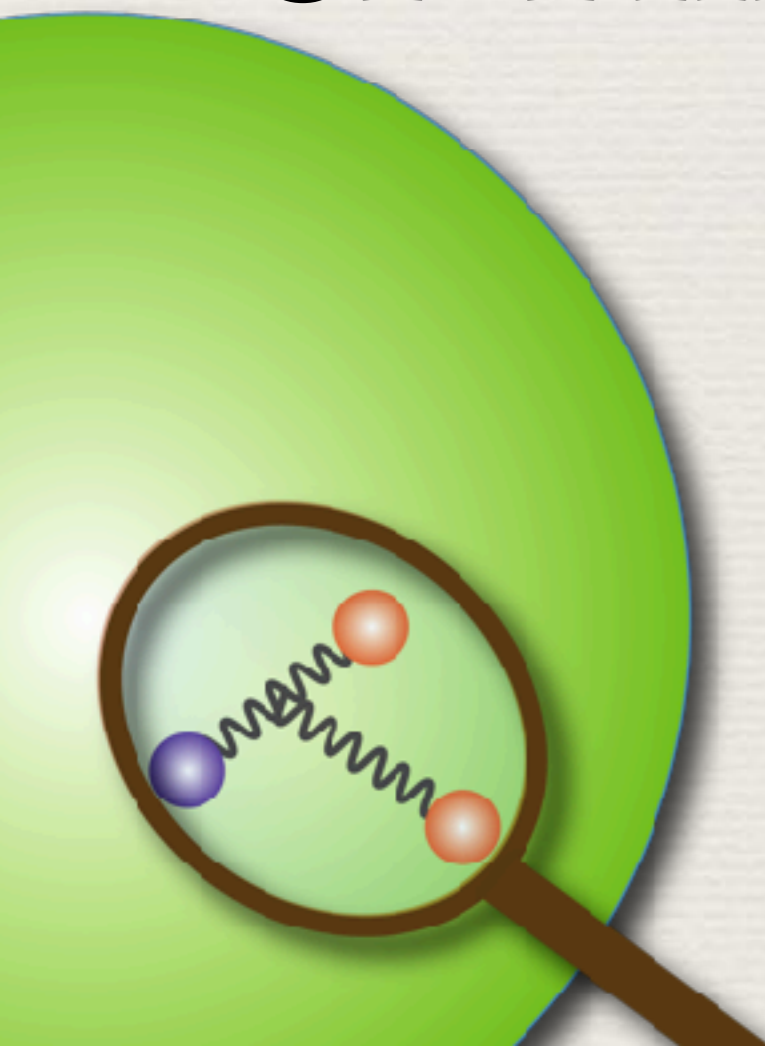
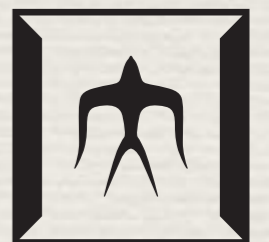
End of Part 1

Experimental Study of Three-Nucleon Forces

Part II

Department of Physics
Tokyo Institute of Technology

Kimiko Sekiguchi



Frontier of Nuclear Force Study

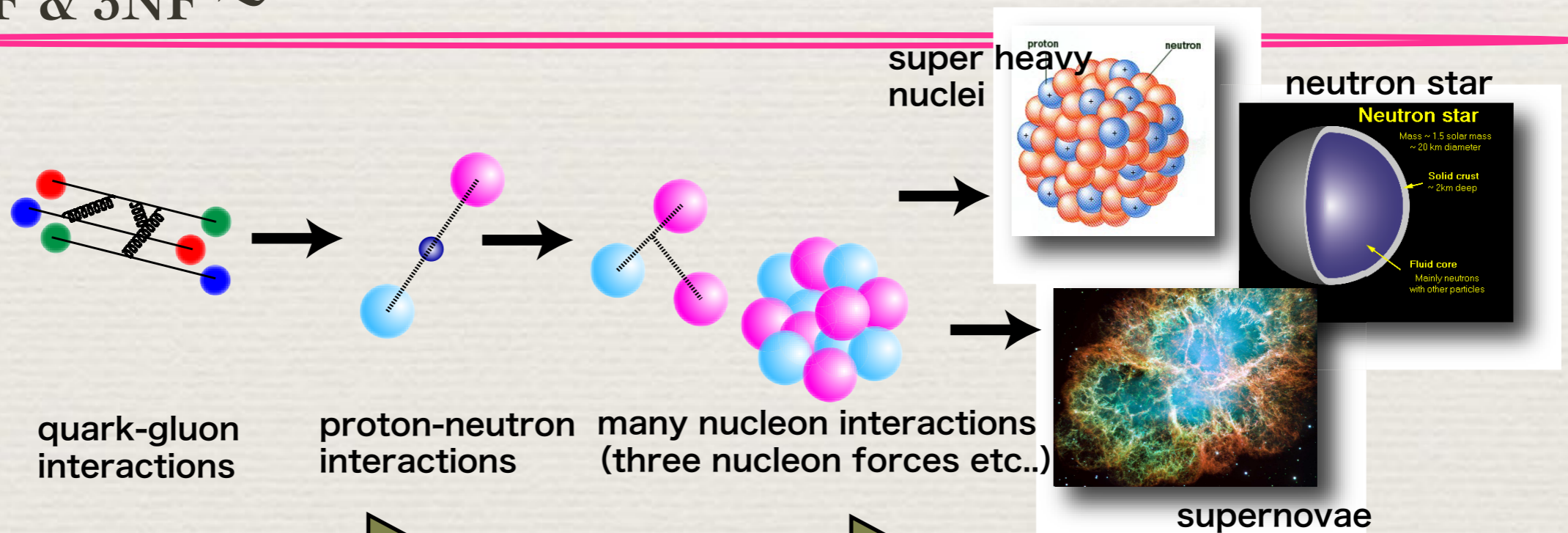
1990's Realistic Modern Nucleon-Nucleon Forces (2NFs)

→ We have "reliable" two nucleon forces.

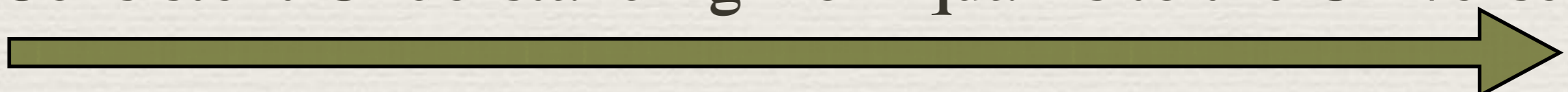
• To understand Nuclear Forces from Quarks (elementary particles)

• To understand Nuclei and Nuclear Matter from bare Nuclear Forces

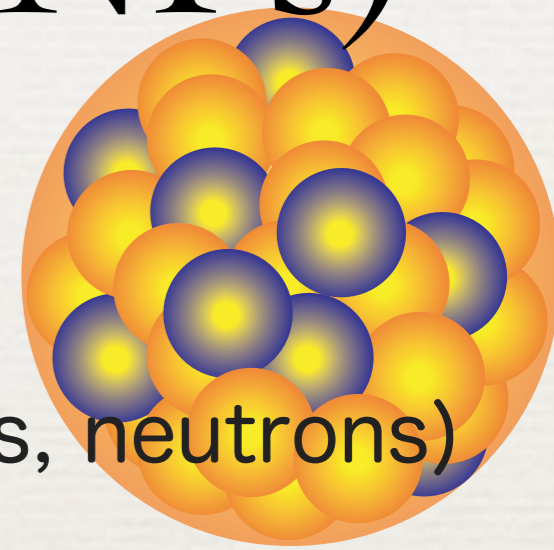
~ 2NF & 3NF ~



Consistent Understanding from quarks to the Universe



Three-Nucleon Forces (3NFs) in Nuclei



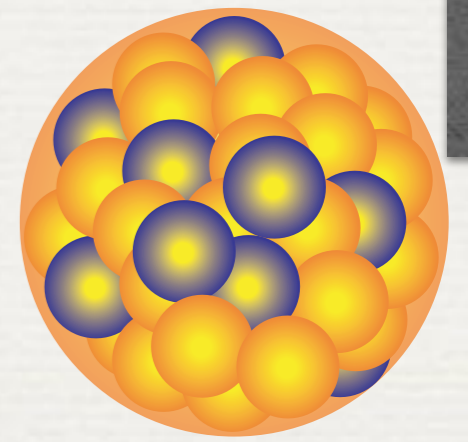
- Nucleus : a compact system of nucleons (protons, neutrons)
- Nuclear Force : Strong Interaction ... Short and Strong
- Effects of Three Body Forces in Nuclei

— Where and How ? —

	Solar System	Atom	Nucleus
Length	10^8 m	10^{-10} m	10^{-15} m
Interaction	Gravity	Electro-Magnetic	Strong
Coupling Constant	10^{-38}	10^{-2}	1
$\frac{V(3BF)}{V(2BF)}$	0.001%	a few %	?

Where ?

3NFs in $A > 3$ - ① -



3NFs in Finite Nuclei

Ab Initio Calculations for Light Nuclei ($A \lesssim 12$): ${}^4\text{He}$ to ${}^{12}\text{C}$

- Green's Function Monte Carlo
- No-Core Shell Model etc..
- 2NF provide less binding energies
- 3NF : well reproduce the data

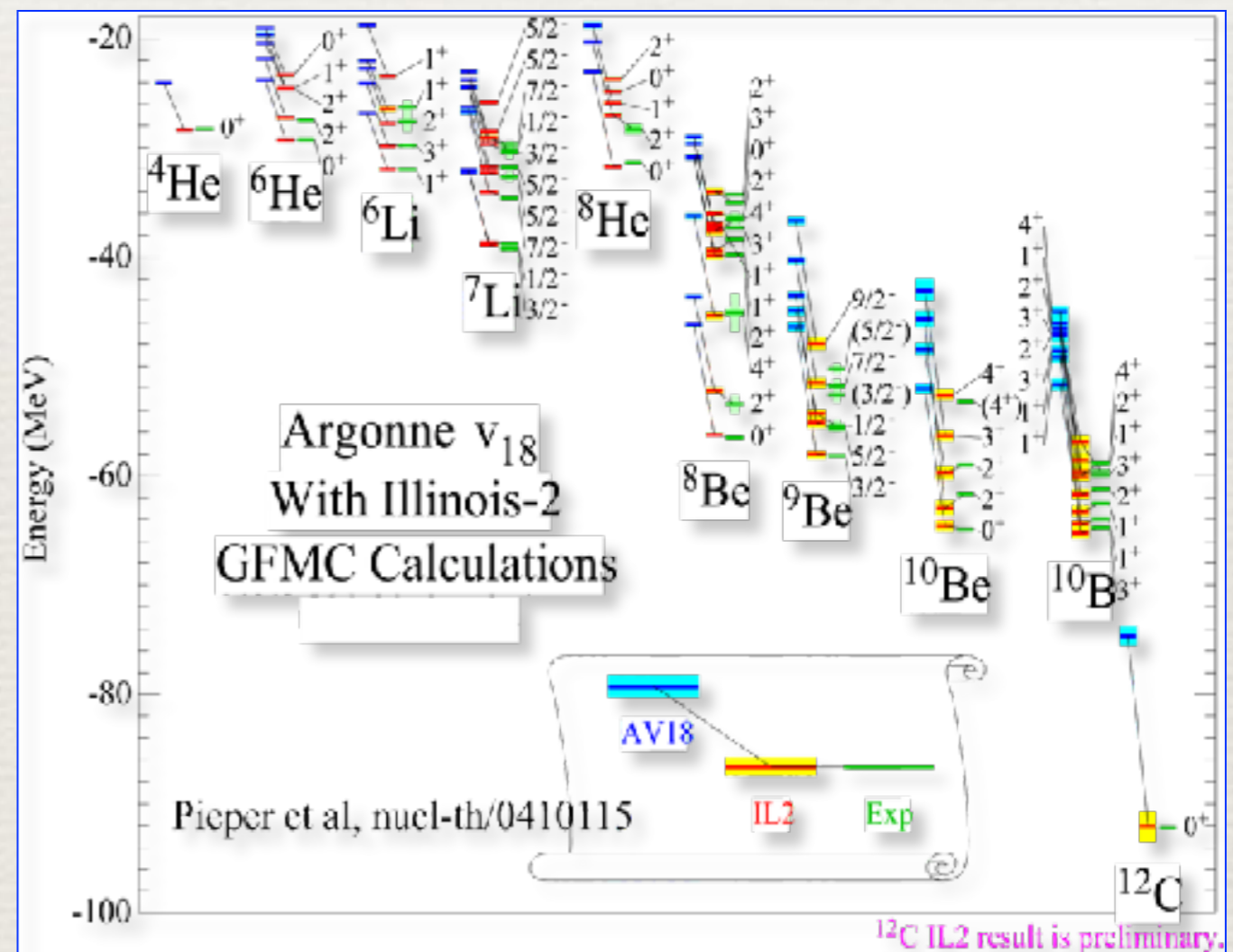
IL2 3NF (Illinois-II 3NF) :
 2π -exchange 3NF
+ 3π -ring with Δ -isobar

3NF effects in B.E.

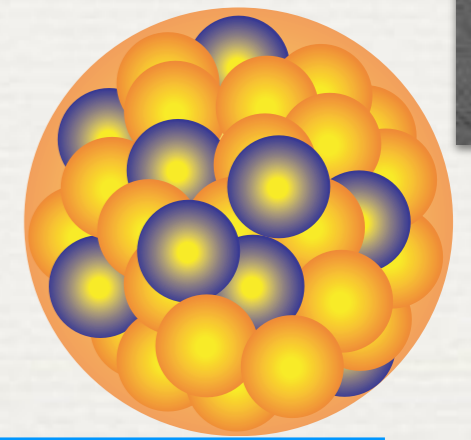
- 10-25%
- Attractive

Note :

T=3/2 3NFs (three-neutron force)
play important roles to explain B.E.
in neutron rich nuclei.



3NFs in $A > 3$ - ① -



3NFs in Finite Nuclei

Ab Initio Calculations for Light Nuclei ($A \lesssim 12$): ${}^4\text{He}$ to ${}^{12}\text{C}$

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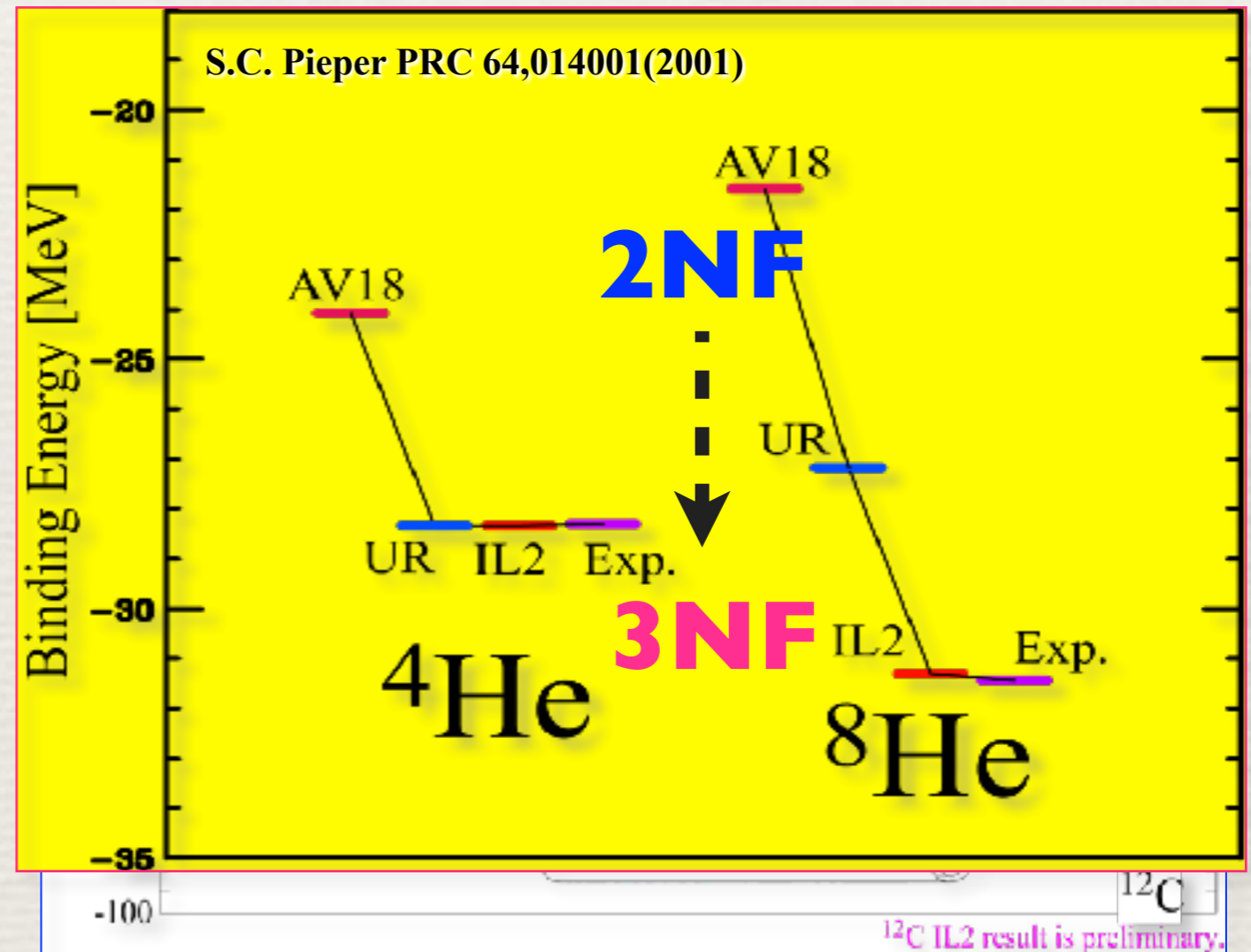
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3NF effects in B.E.

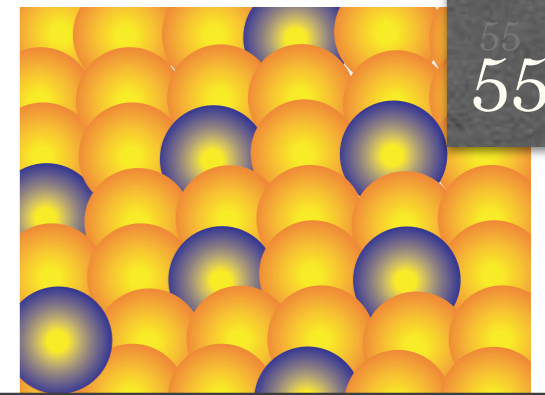
- 10-25%
- Attractive

Note :

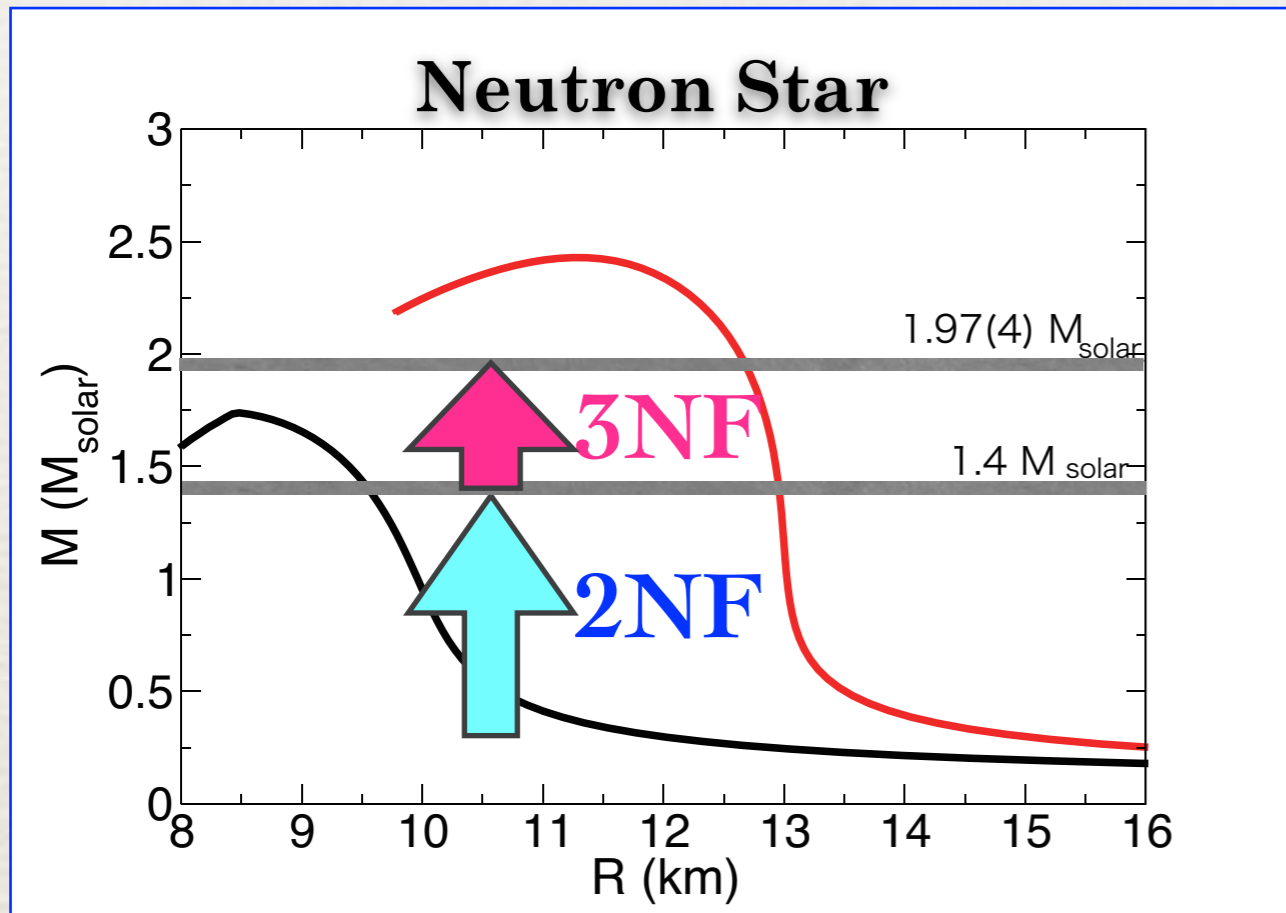
T=3/2 3NFs (three-neutron force) play important roles to explain B.E. in neutron rich nuclei.



3NFs in $A > 3$ - ② -



3NFs in Infinite Nuclei - Neutron Star -



A. Akmal et al., PRC 58, 1804('98)

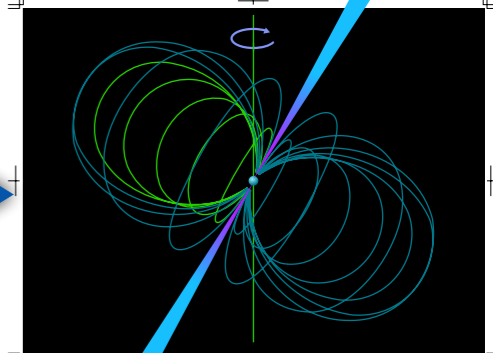
- 3NF in Nuclei is required...
 - Short & Repulsive
- Large effects at high density.

“Endpoint of stellar evolution”

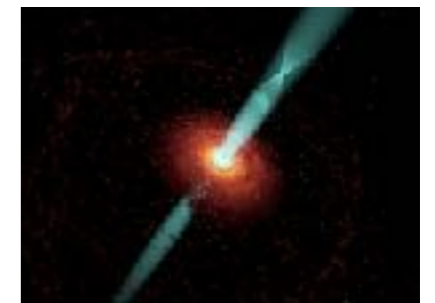
Supernovae
Explosion



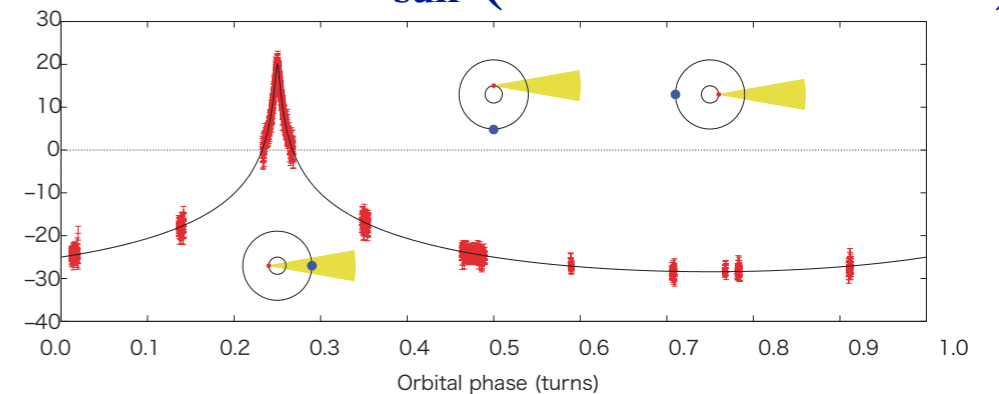
Neutron Star



Black Hole



Discovery of Heaviest Neutron Star with 2 solar-mass M_{sun} (PSR J1614-2230)



Nature 467 1081 (2010)

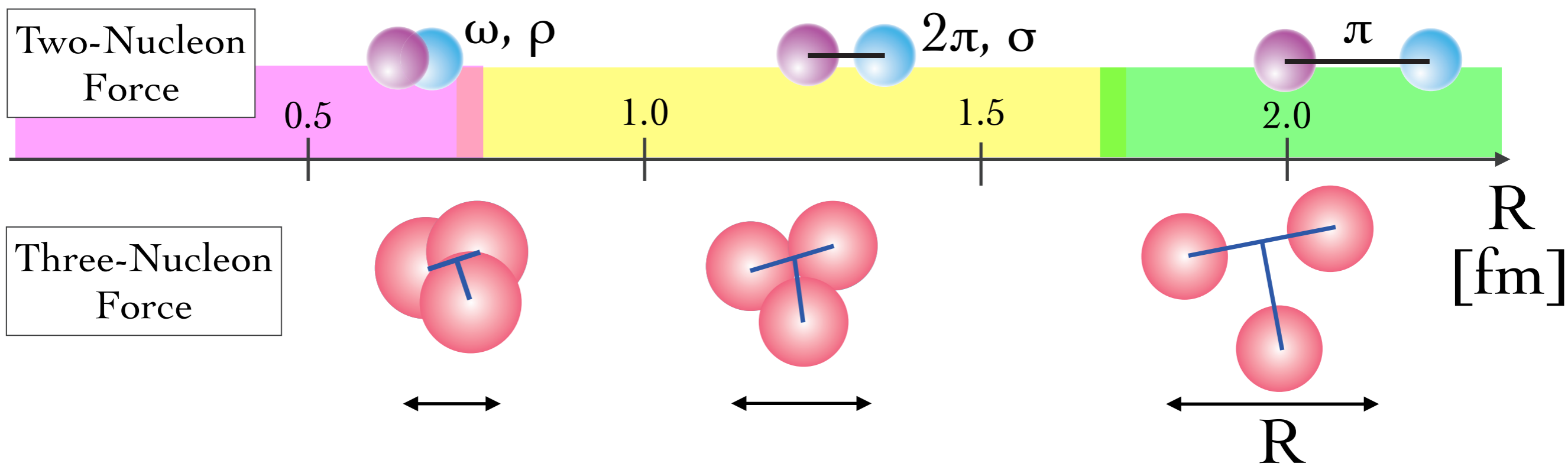
How ?

Two & Three-Nucleon Force

①. Repulsive
-Short Range-

②. Attractive (strong)
-Intermediate Range-

③. Attractive (weak)
- Long Range -



3NFs are momentum, spin, and iso-spin dependent.

Nuclear Matter
Neutron Star

Nuclear Structure

Few-Nucleon Systems

How to approach Three-Nucleon Forces ?

Direct Comparison between Theory and Experiment

1. Exact Solution of Three-Nucleon System

Faddeev Theory : Exact solution of three-body systems

2. Establishment of Two-Nucleon Forces

Realistic 2NFs :

reproduce 3500 NN scattering exp. data
with high precision, $\chi^2 \sim 1$.

3. High Precision Experiment

e.g. Our experiment

Extract information of Three-Nucleon Forces

Three Body Problem

in Quantum Mechanics

CAN BE SOLVED EXACTLY!

■ Uncertainty Principle by Heisenberg

$$\Delta p \Delta x \sim \hbar = h/2\pi \quad \rightarrow \text{reduce 'Degrees of Freedom'}$$

for Equations of Motion

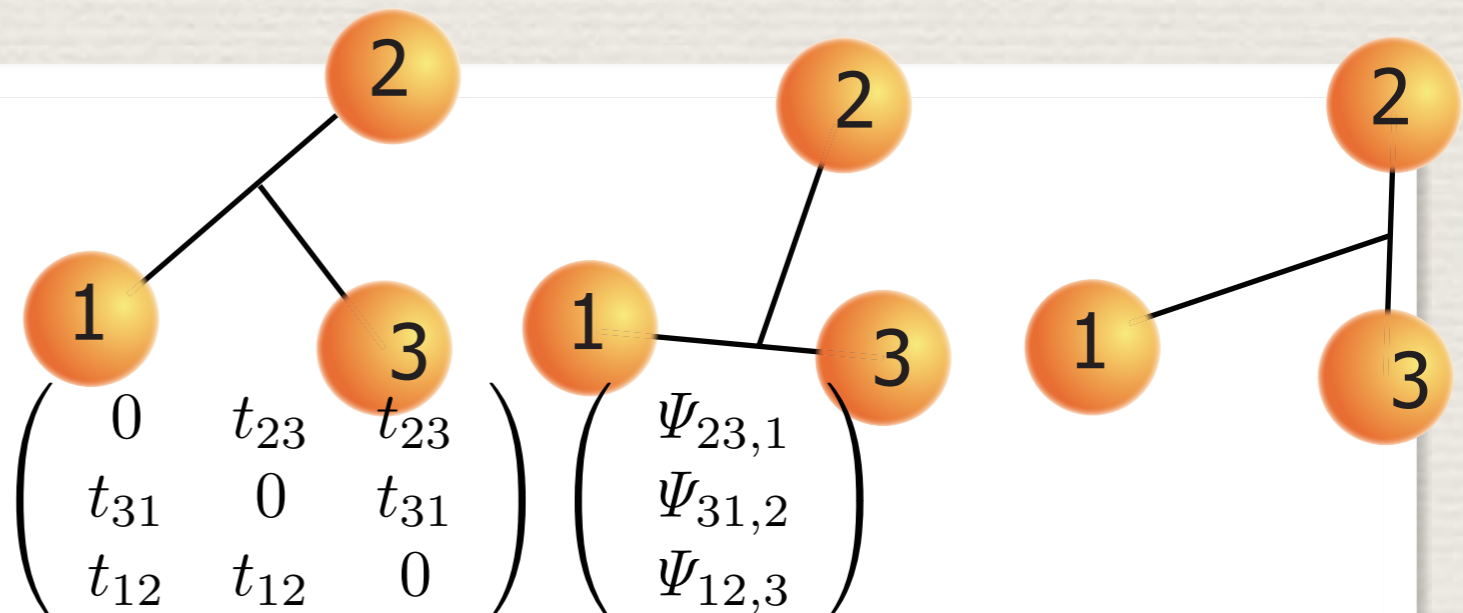
■ Faddeev Theory (L.D. Faddeev, 1961)

Exact solution of three body system in Q.M.

$$H = H_0 + V_{12}^{NN} + V_{23}^{NN} + V_{31}^{NN}$$

$$\Psi = \Psi_{23,1} + \Psi_{31,2} + \Psi_{12,3}$$

$$\begin{pmatrix} \Psi_{23,1} \\ \Psi_{31,2} \\ \Psi_{12,3} \end{pmatrix} = \begin{pmatrix} \phi_{23,1} \\ \phi_{31,2} \\ \phi_{12,3} \end{pmatrix} + G_0$$



Triton (${}^3\text{H}$) Binding Energy

Triton (${}^3\text{H}$)

- $A=3$ ($Z=1$, $N=2$)
- 2NF provides less binding energies by $0.5 \sim 1 \text{ MeV}$.
- 3NF fill the gap between the data and the calculations based on 2NFs.
- The cut-off Λ is determined to reproduce ${}^3\text{H}$ binding energy.

$$F_{\pi NN}(q^2) = \frac{\Lambda^2 - m_\pi^2}{\Lambda + q^2}$$

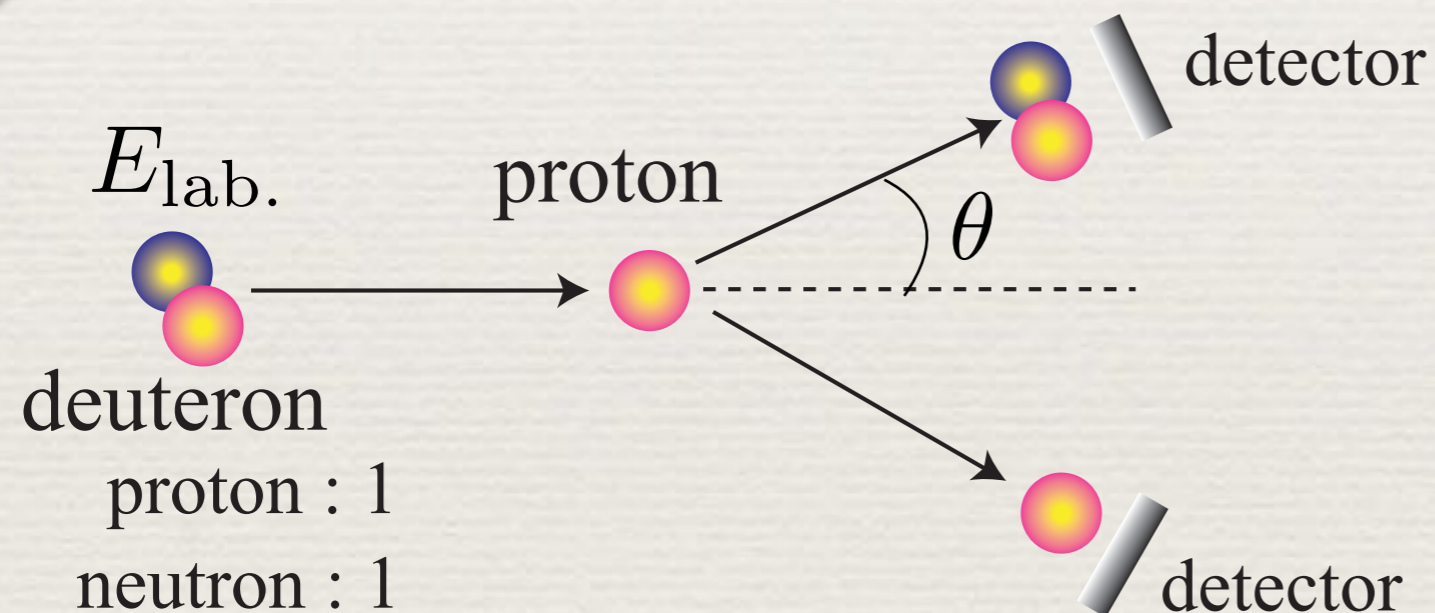
Potential	E_B [MeV] (w/o 3NF)	E_B [MeV] (with 3NF)	Λ/m_π
CDBonn	7.953	8.483	4.856
AV18	7.576	8.479	5.215
Nijm I	7.731	8.480	5.147
Nijm II	7.709	8.477	4.990
Nijm 93	7.664	8.480	5.207
Exp.	8.481821(4) [MeV]		

A. Nogga *et al.*, Phys. Rev. C **65**, 054003 (2002).

Three-Nucleon Scattering

a good probe to study
the dynamical aspects of 3NFs.

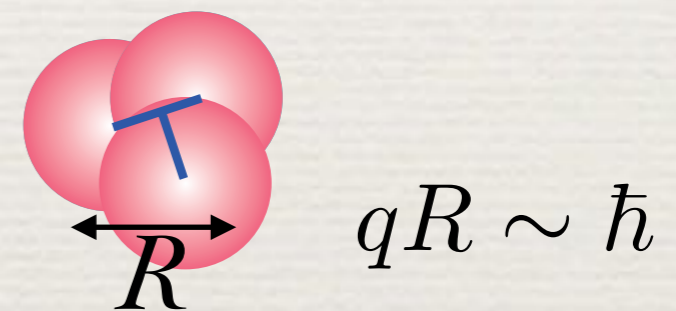
- ✓ Momentum dependence (R-dependence)
- ✓ Spin-dependence



$$\frac{d\sigma}{d\Omega}(\theta) \propto \left| \int \exp(-\mathbf{q} \cdot \mathbf{r}') V(r) d\mathbf{r}' \right|^2$$

Nuclear Potential

Angular Dependent Cross Section :
Fourier Transform of Nuclear Potential



$$R \sim 1 \times 10 \text{ fm}$$

$$q \sim 200 \text{ MeV}/c$$

$$E_{lab.} \sim 100 \text{ MeV}/A$$

One can change R
by adjusting $E_{lab.}$

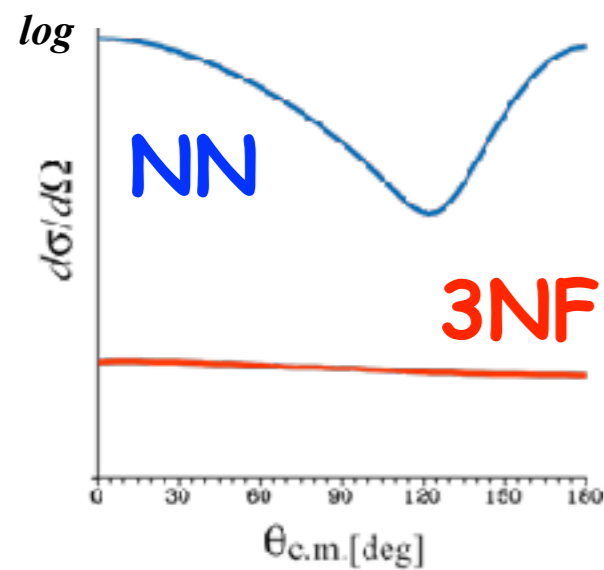
Where is the hot spot for 3NFs ?

Nucleon-Deuteron Scattering - 3N Scattering -

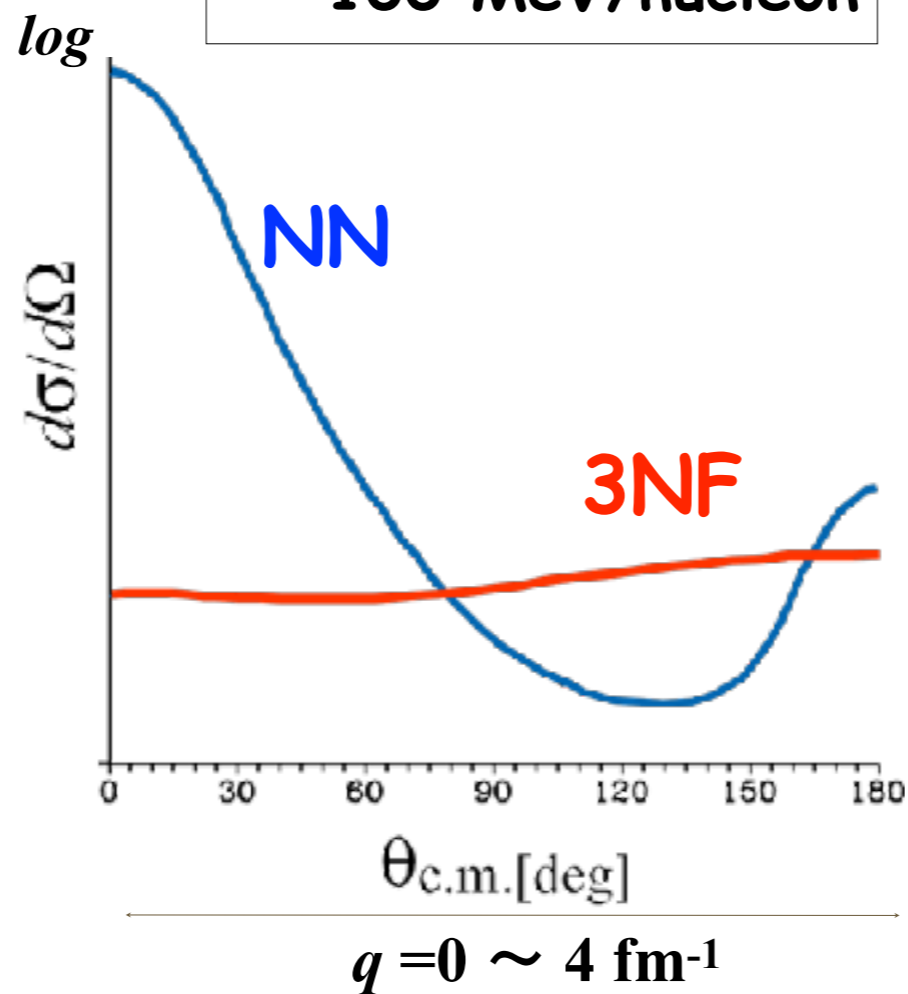
Predictions by H. Witala et al. (1998)

Cross Section minimum for Nd Scattering at ~ 100 MeV/nucleon

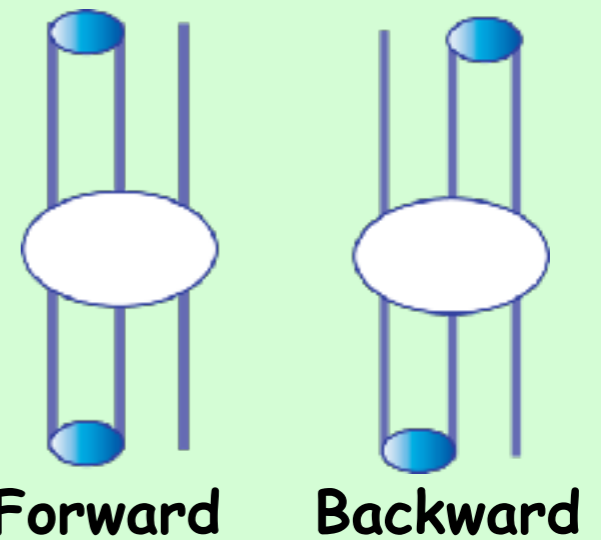
~ 10 MeV/nucleon



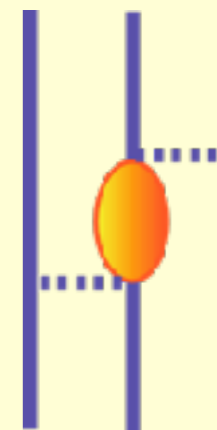
~ 100 MeV/nucleon



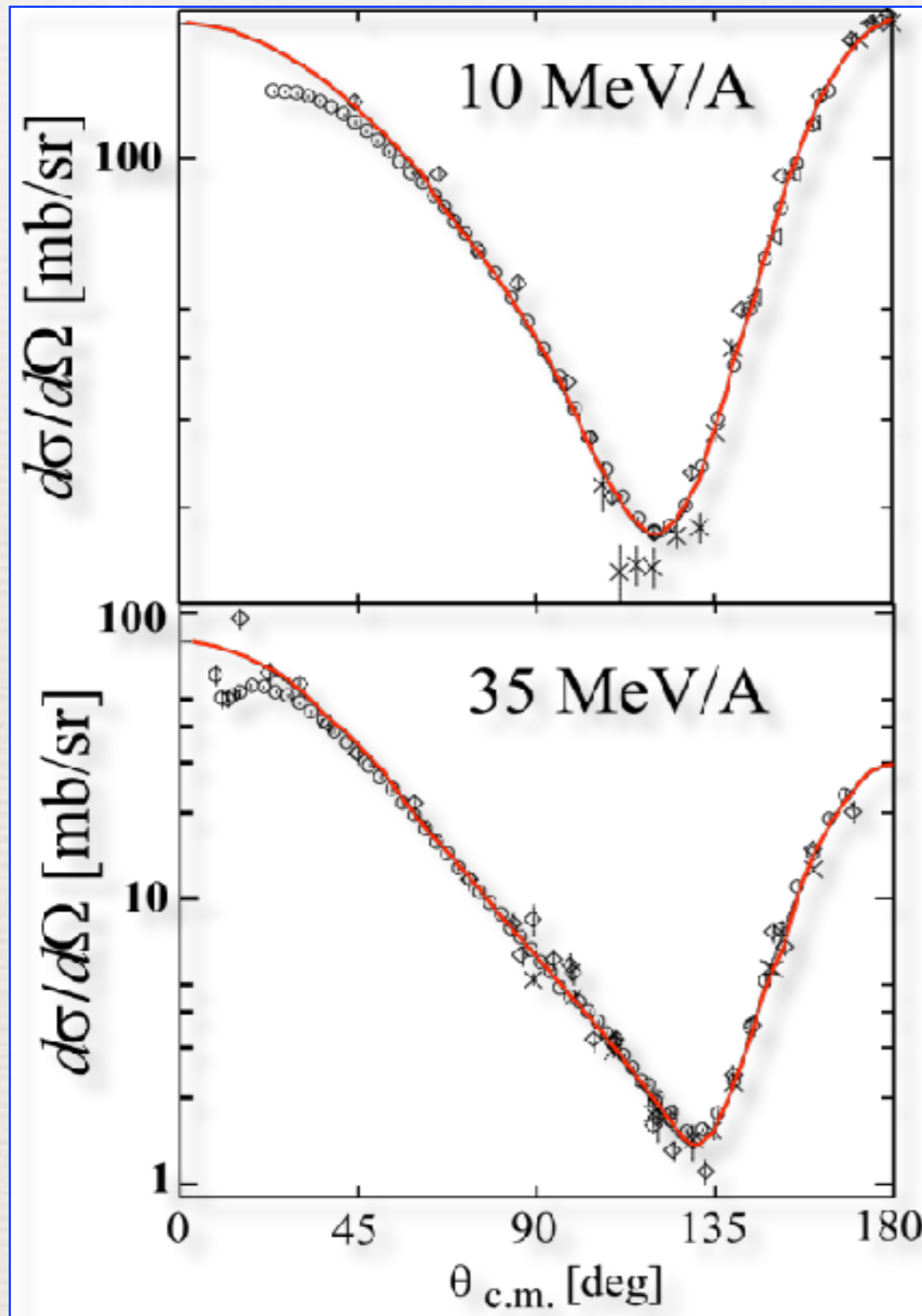
Nd scattering



3NF



Nd Scattering at Low Energies ($E \leq 30$ MeV/A)



- High precision data are explained by Faddeev calculations based on 2NF. (Exception : A_y, iT_{11})

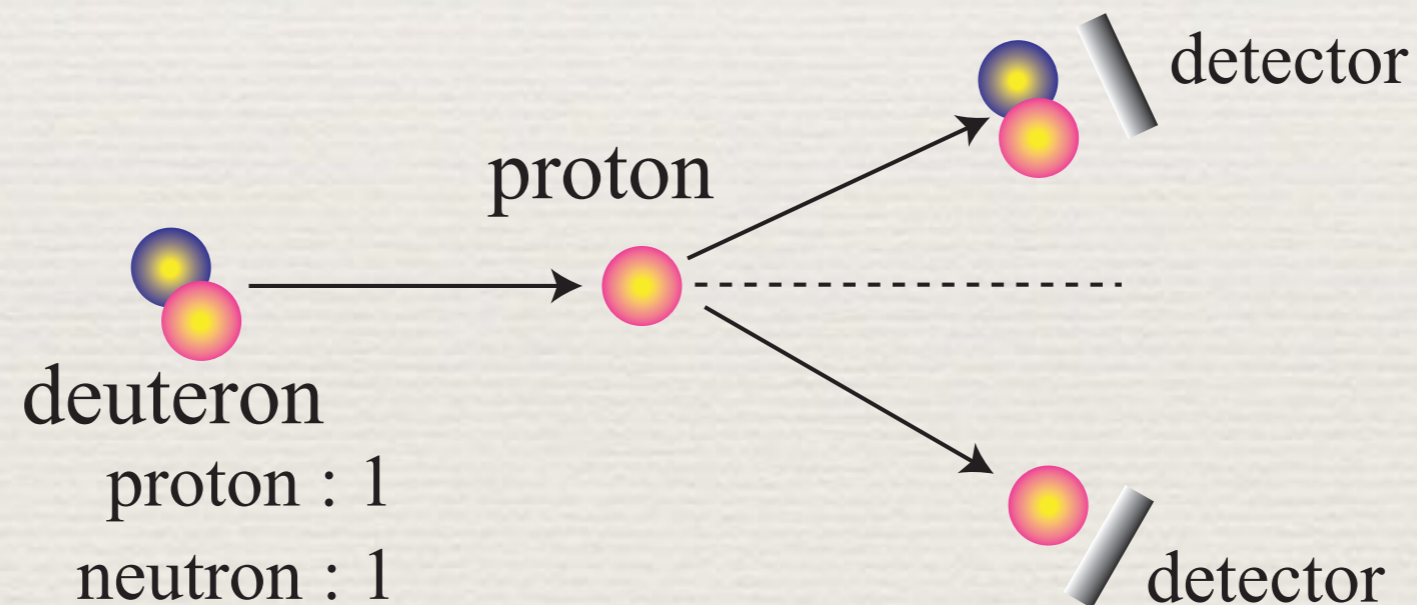
No signatures of 3NF

Exp. Data from
Kyushu, TUNL, Cologne etc..

W. Glöckle et al., Phys. Rep. 274, 107 (1996).

deuteron-nucleon scattering

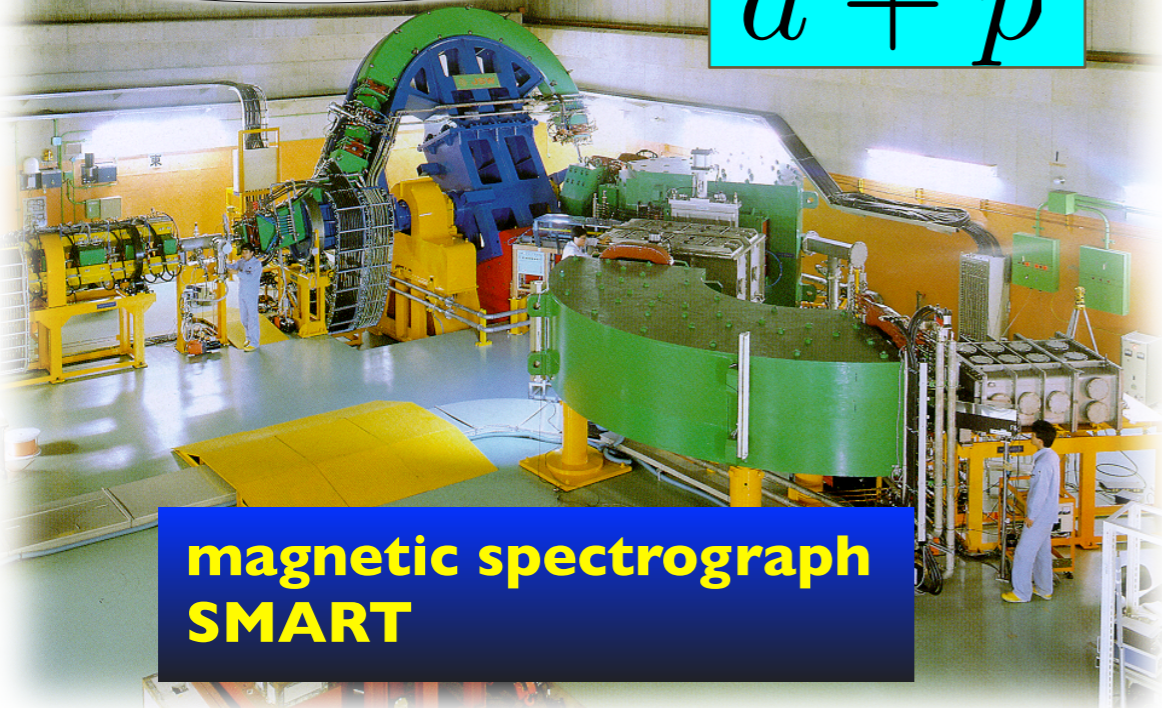
at ~ 100 MeV/nucleon



Facilities

RIKEN

$$\vec{d} + p$$

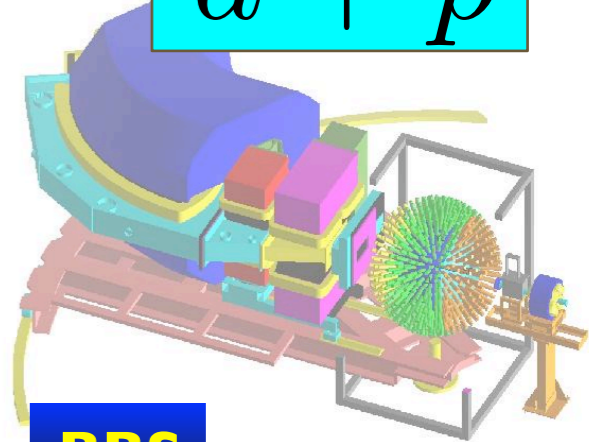


magnetic spectrograph
SMART

**BINA
& SALAD**



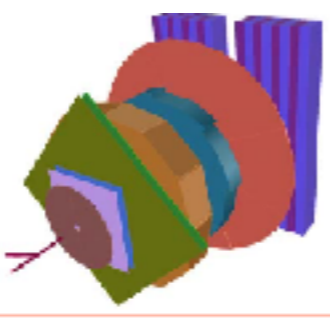
$$\vec{p} + d$$
$$\vec{d} + p$$



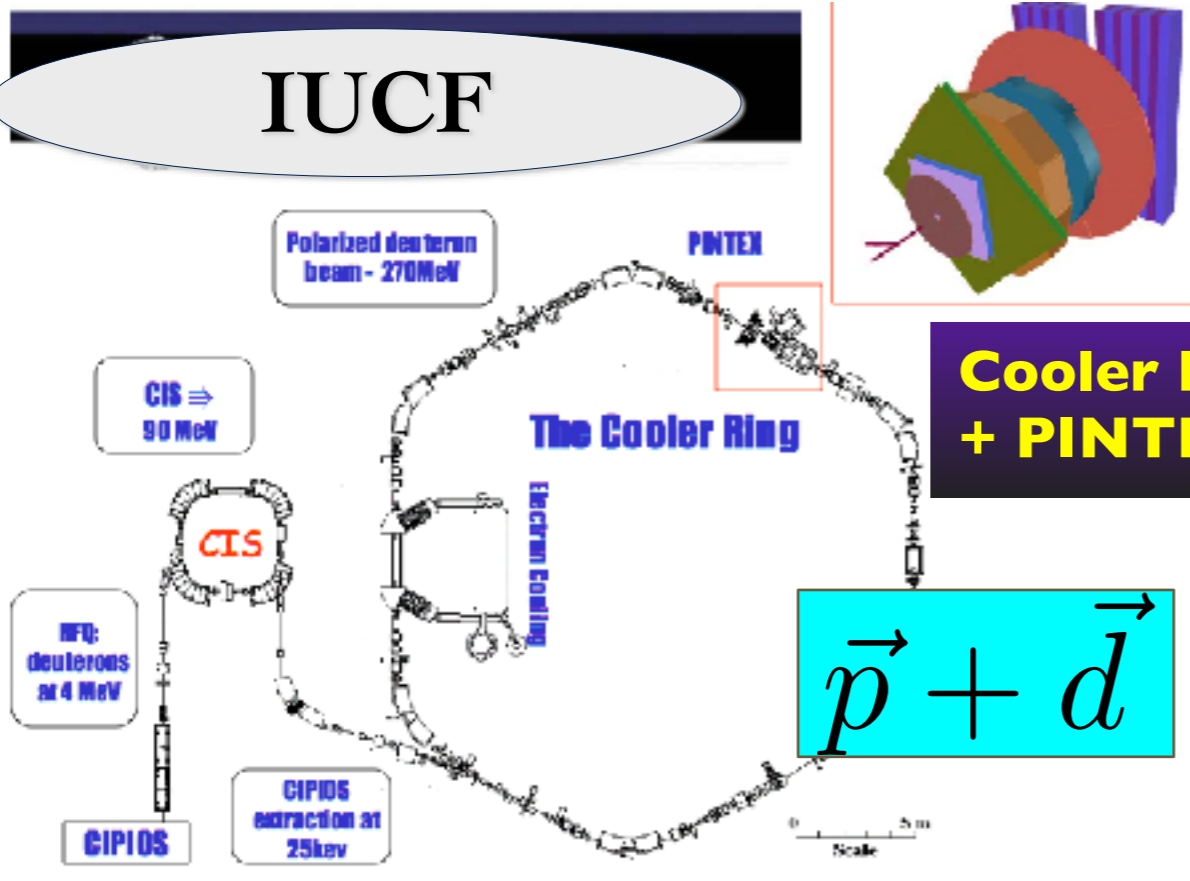
BBS

KVI

IUCF



**Cooler Ring
+ PINTEX**



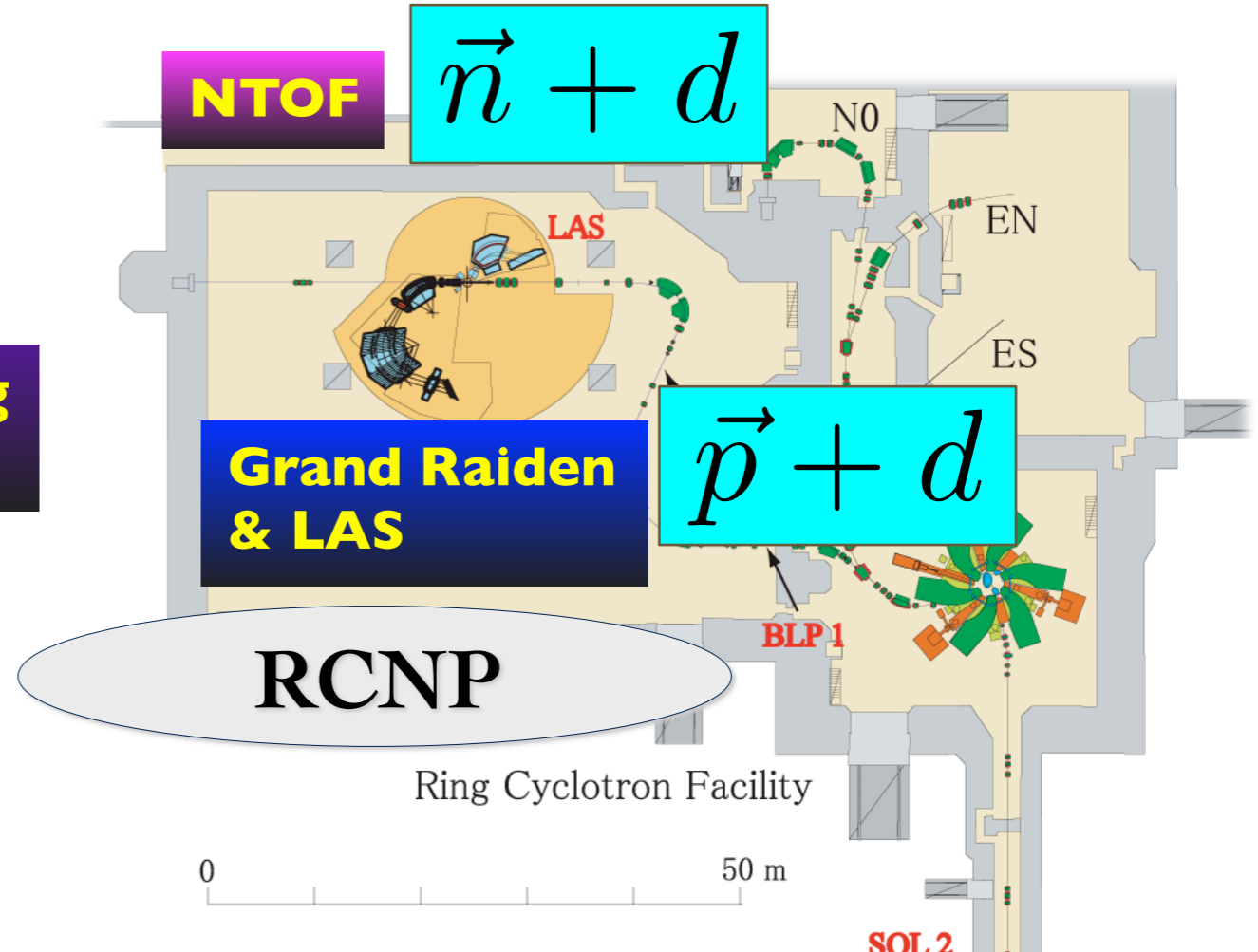
$$\vec{p} + \vec{d}$$

NTOF

$$\vec{n} + d$$

**Grand Raiden
& LAS**

$$\vec{p} + d$$



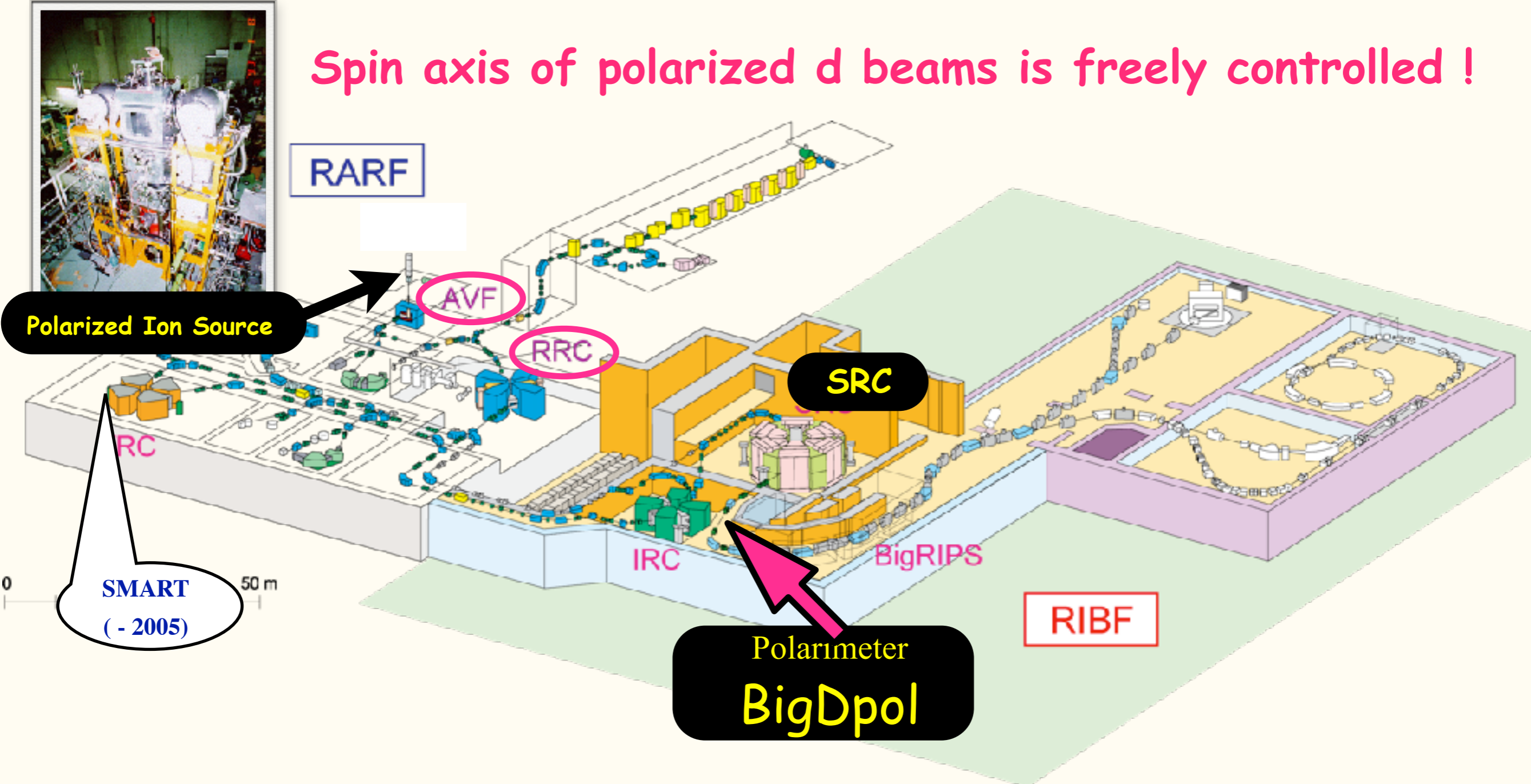
RCNP

Ring Cyclotron Facility

RIKEN RI Beam Factory (RIBF)

- Polarized *d* beam
 - acceleration by AVF+RRC : 65-135 MeV/nucleon
 - acceleration by AVF+RRC+SRC : 190-300 MeV/nucleon
 - polarization : 60-80% of theoretical maximum values
- Beam Intensity : < 100 nA

Spin axis of polarized d beams is freely controlled !

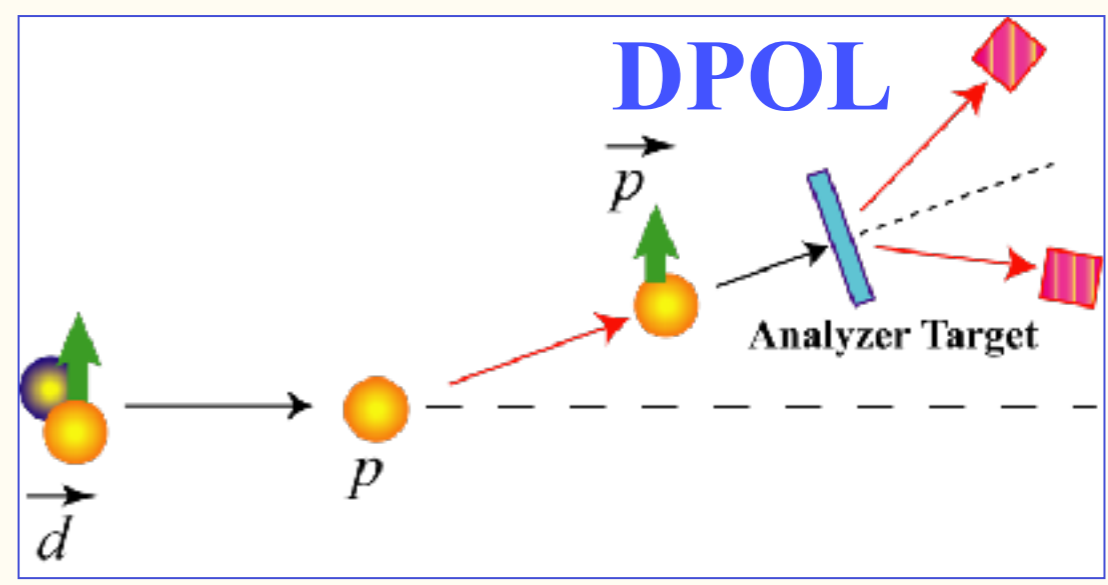
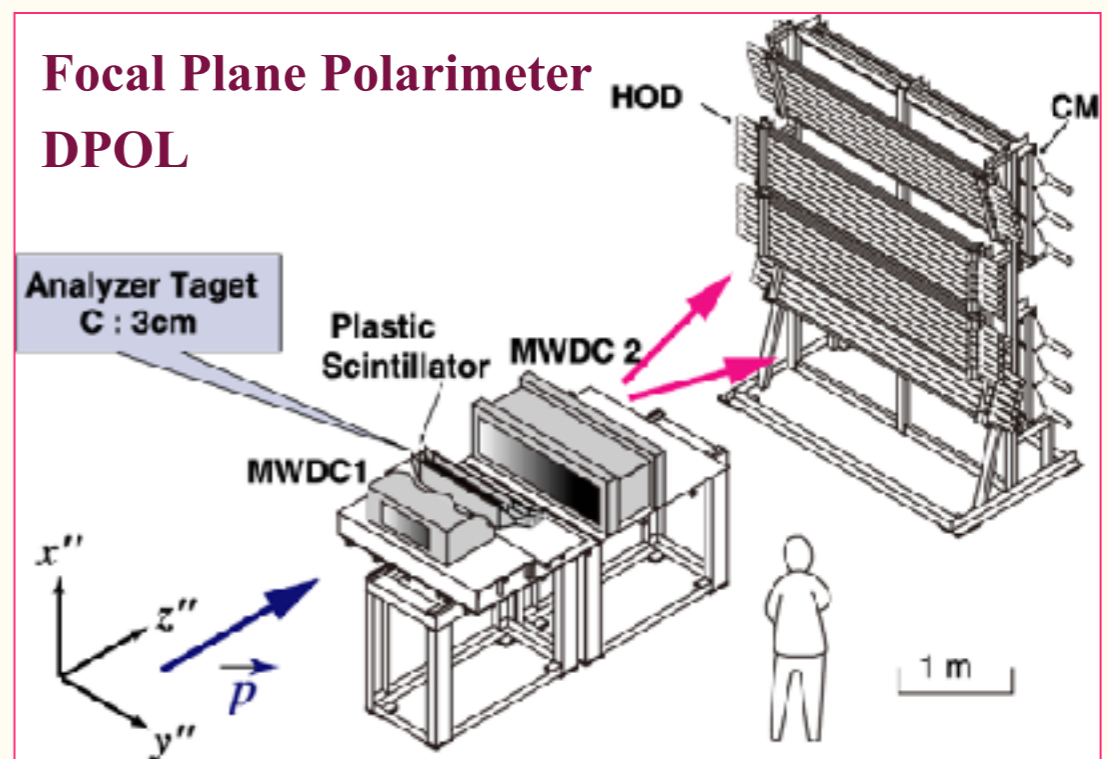
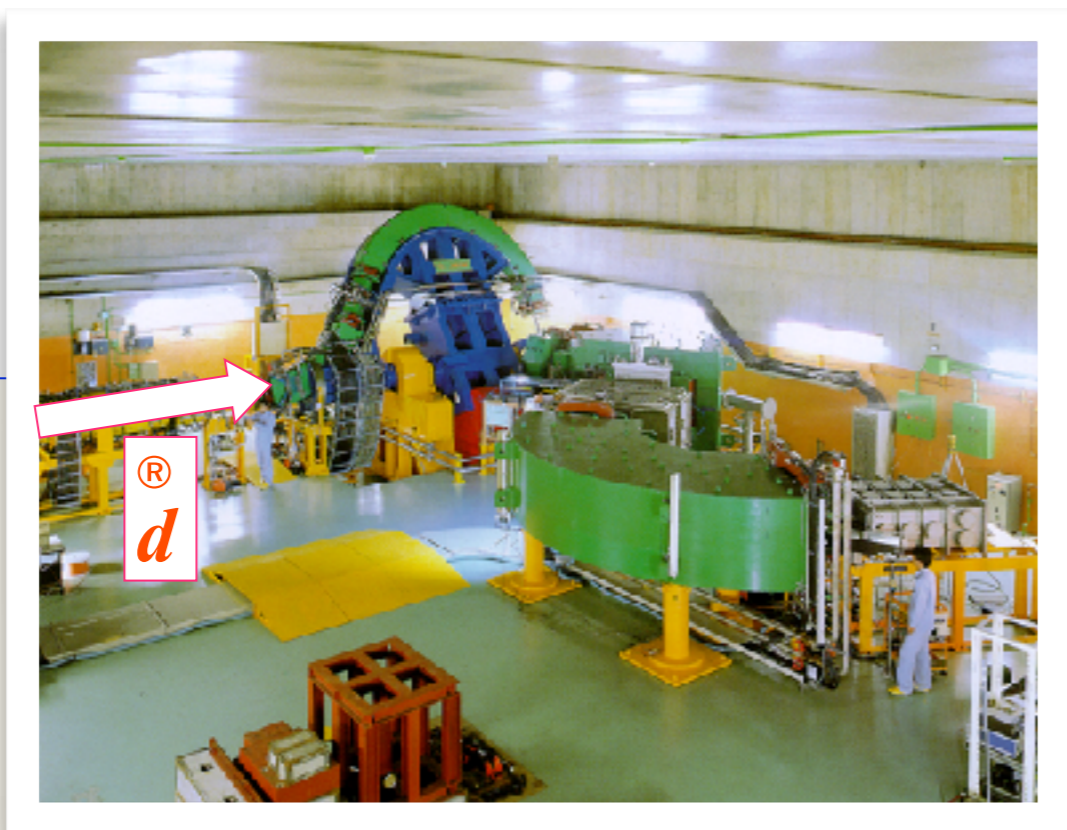
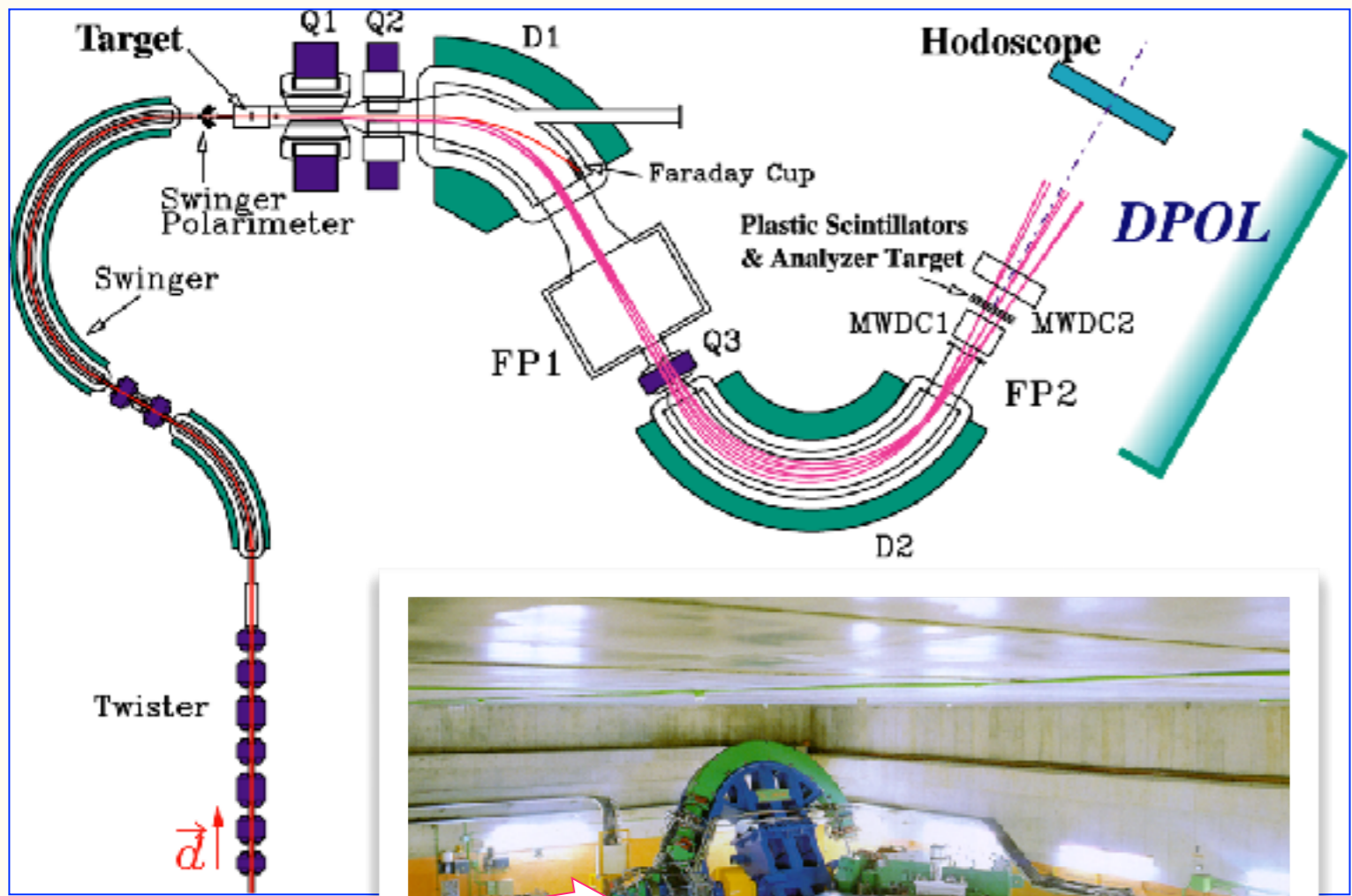


N. Nakamoto et al., Phys. Lett. B 367, 60 (1996),
 H. Sakai et al., Phys. Rev. Lett. 84, 5288 (2000),
 K. S. et al., Phys. Rev. C 65, 034003 (2002),
 K. S. et al., Phys. Rev. C 70, 014001 (2004),
 K. S. et al., Phys. Rev. Lett. 95, 162301 (2005),
 K. S. et al., Phys. Rev. C 79, 054008 (2009)

SMART at RIKEN (- 2005)

Swinger and Magnetic Analyzer with Rotator and Twister

- ❖ Differential Cross Section at 70, 135 MeV/nucleon
- ❖ All Deuteron Analyzing Powers at 70, 100, 135 MeV/nucleon
- ❖ Deuteron to Proton Polarization Transfer Coefficients at 135 MeV/nucleon



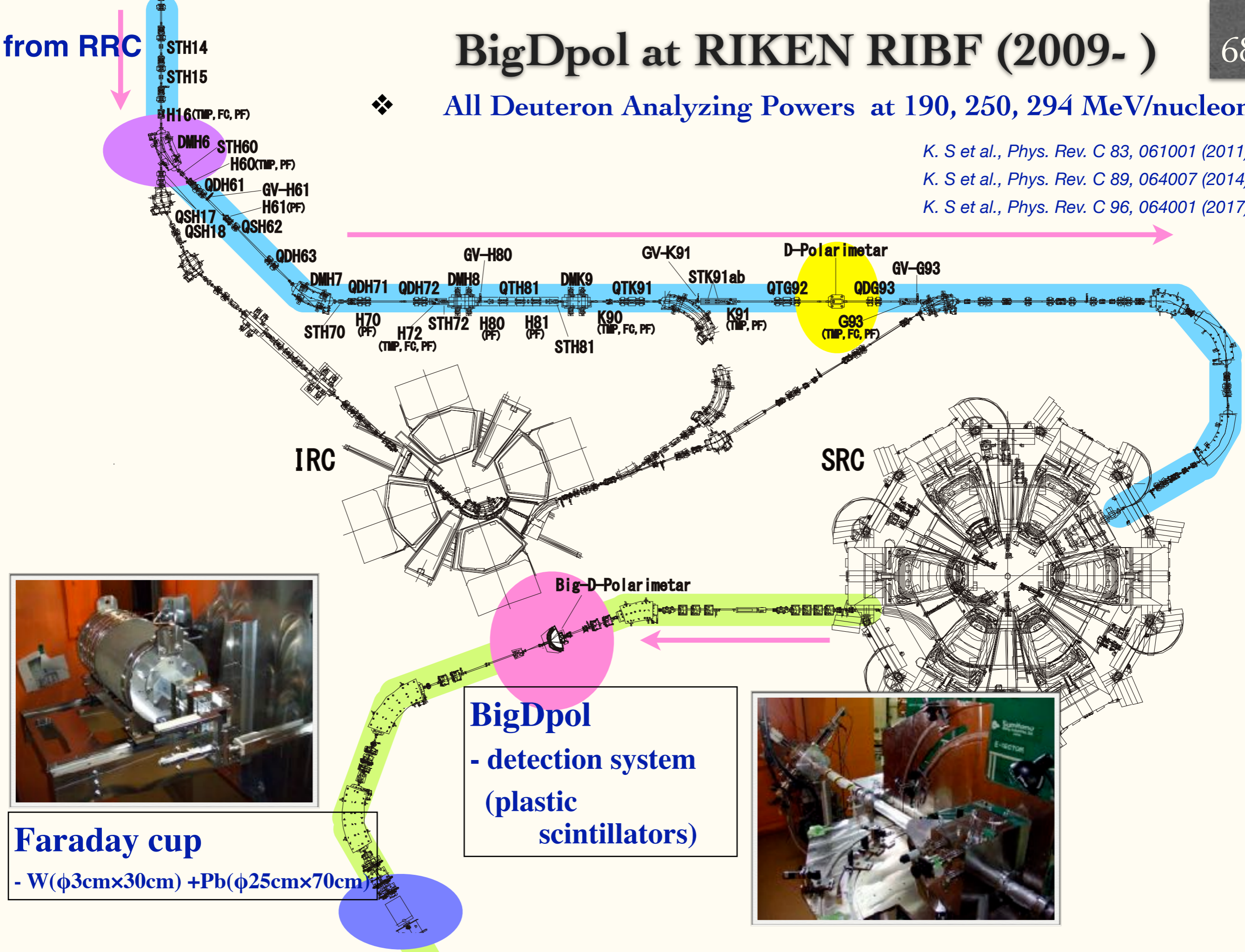
BigDpol at RIKEN RIBF (2009-)

❖ All Deuteron Analyzing Powers at 190, 250, 294 MeV/nucleon

K. S et al., Phys. Rev. C 83, 061001 (2011)

K. S et al., Phys. Rev. C 89, 064007 (2014)

K. S et al., Phys. Rev. C 96, 064001 (2017)



from RRC

STH14
STH15
H16 (TMP, FC, PF)

DMH6 STH60
H60 (TMP, PF)
QDH61 GV-H61
H61 (PF)
QSH17 QSH18 QSH62

QDH63
DMH7 QDH71 QDH72
STH70 H70 (PF)
H72 (TMP, FC, PF)
STH72 H80 (PF)
H81 (PF)
STH81

GV-H80
DMH8 QTH81
DMK9

GV-K91
QTK91
STK91ab
K90 (TMP, FC, PF)
K91 (TMP, PF)

D-Polarimeter
QTG92
QDG93
GV-G93
G93 (TMP, FC, PF)

IRC

SRC

Big-D-Polarimeter

BigDpol
- detection system
(plastic scintillators)

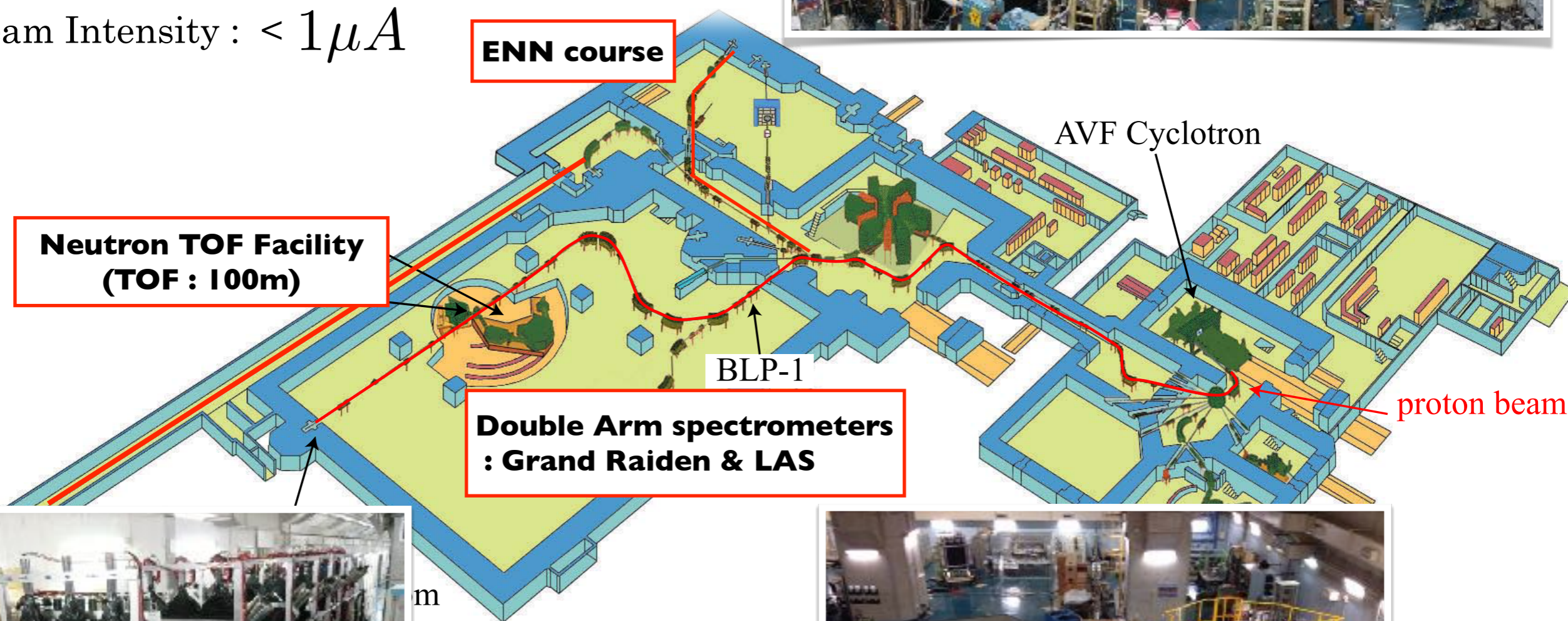
Faraday cup

- W($\phi 3\text{cm} \times 30\text{cm}$) + Pb($\phi 25\text{cm} \times 70\text{cm}$)



RCNP, Osaka University

- Polarized p beam : 10 - 420 MeV/nucleon
- Polarized d beam : 5 - 100 MeV/nucleon
 - Polarizations : < 70 %
- (pol.) Neutron beams by ${}^7\text{Li}(p,n)$
- Beam Intensity : < $1\mu\text{A}$



Summary of Precise Measurement of *Nd* Elastic Scattering at RIKEN/RCNP

$d + p$

RIKEN

1. **Differential Cross Section** at 70, 135 MeV/nucleon
2. **All Deuteron Analyzing Powers** (iT_{11} , T_{20} , T_{21} , T_{22})
at 70, 100, 135, 190, 250, 300 MeV/nucleon
3. **Deuteron to Proton Polarization Transfer Coefficients** at 135 MeV/nucleon

N. Sakamoto et al., Phys. Lett. B 367, 60 (1996), H. Sakai et al., Phys. Rev. Lett. 84, 5288 (2000), K. S. et al., Phys. Rev. C 65, 034003 (2002), K. S. et al., Phys. Rev. C 70, 014001 (2004), K. S. et al., Phys. Rev. C 83, 061001 (2011), K. S. et al., Phys. Rev. C 89, 064007 (2014), K.S. et al., Phys. Rev. C 96, 064001 (2017).

$p + d$

RCNP

1. **Differential Cross Section** at 135, 250 MeV
2. **Proton Analyzing Powers** at 250 MeV
3. **Proton to Proton Polarization Transfer Coefficients** at 250 MeV

*K. Hatanaka et al., Phys. Rev. C. 66, 044002 (2002)
K. S. et al., Phys. Rev. Lett. 95, 162301 (2005)*

$n + d$

RCNP

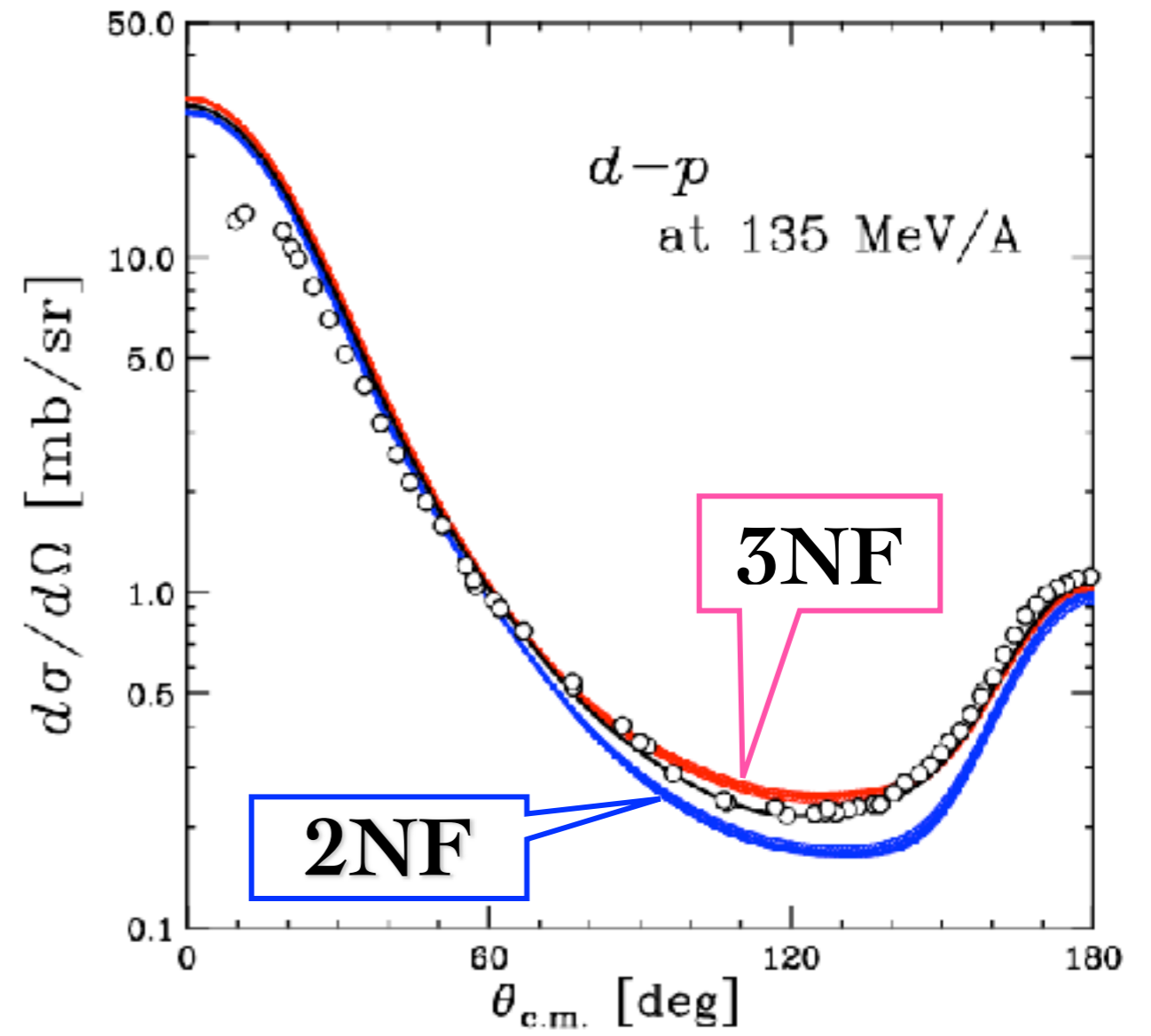
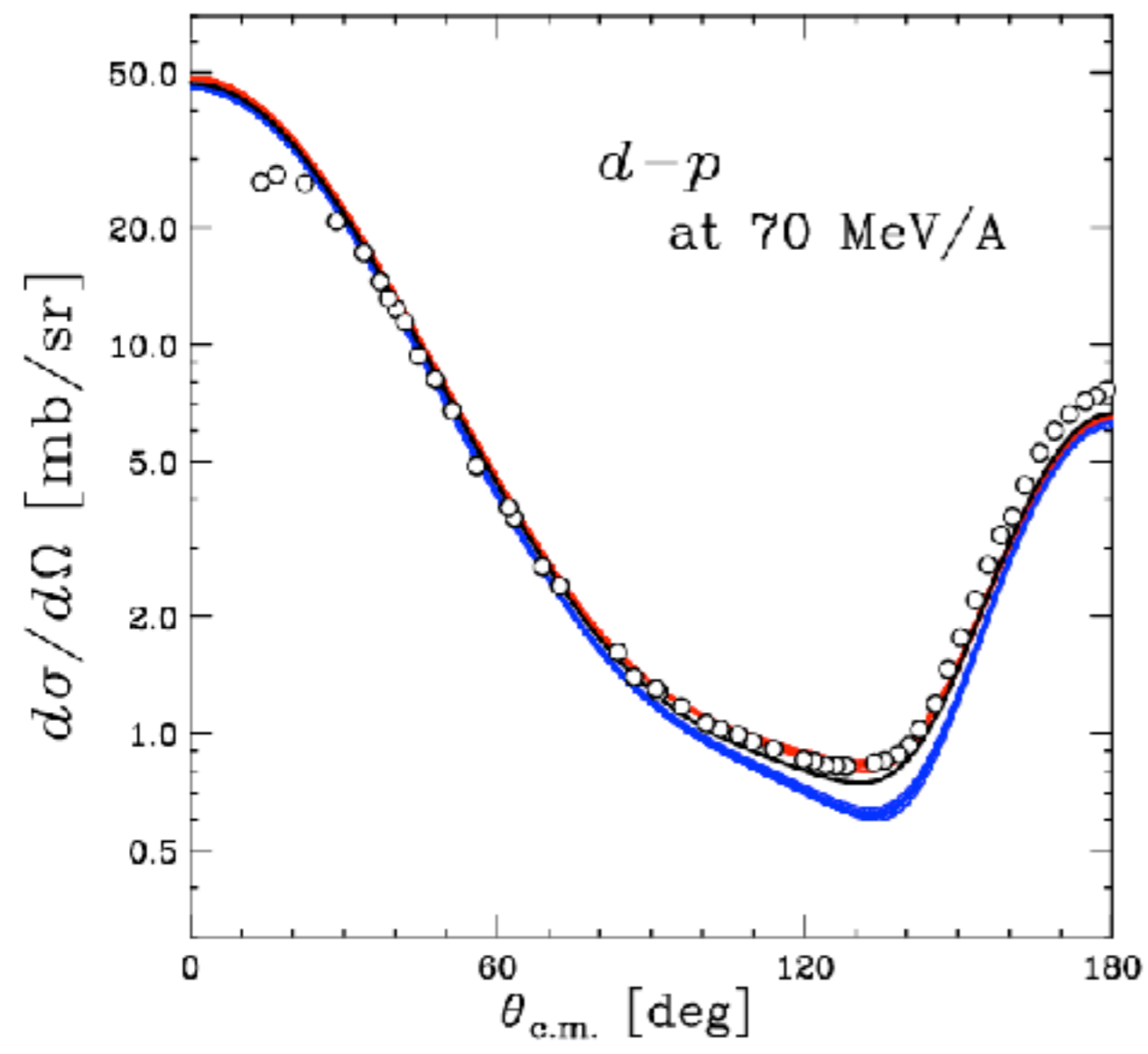
1. **Differential Cross Section** at 250 MeV
2. **Neutron Analyzing Powers** at 250 MeV

Y. Maeda et al., Phys. Rev. C 76, 014004 (2007)

d-p elastic scattering
Differential Cross section

Calculations by Bochum-Cracow Gr.

- NN (CDBonn, AV18, Nijm I,II)
- TM'(99) 3NF + NN(CD Bonn, AV18, Nijm I,II)
- Urbana IX 3NF+AV18

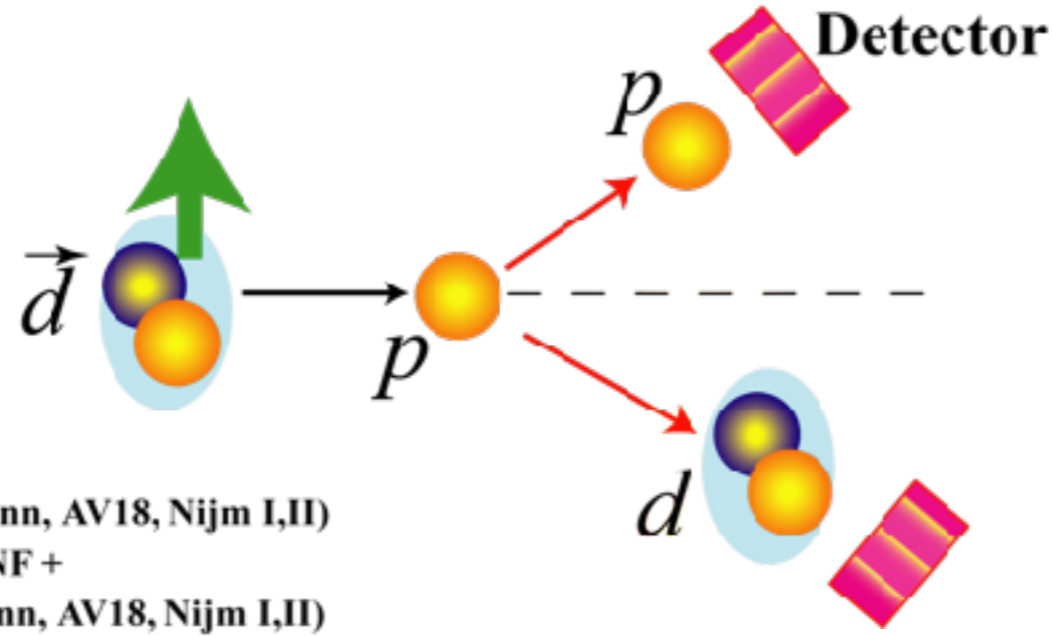


2NF (CDBonn, AV18, Nijmegen I,II)
: Large discrepancy in Cross Section Minimum (~ 30%)

2π-exchange 3NFs (Tucson-Melbourne, Urbana IX) : Good Agreement
: First Clear Signatures of 3NF effects in 3-Nucleon Scattering

d-p elastic scattering
Spin Observables

Analyzing Powers



- █ NN (CDBonn, AV18, Nijm I,II)
- █ TM'(99) 3NF + NN(CD Bonn, AV18, Nijm I,II)
- █ with Urbana IX 3NF+AV18
- █ with Δ -isobar + CDBonn

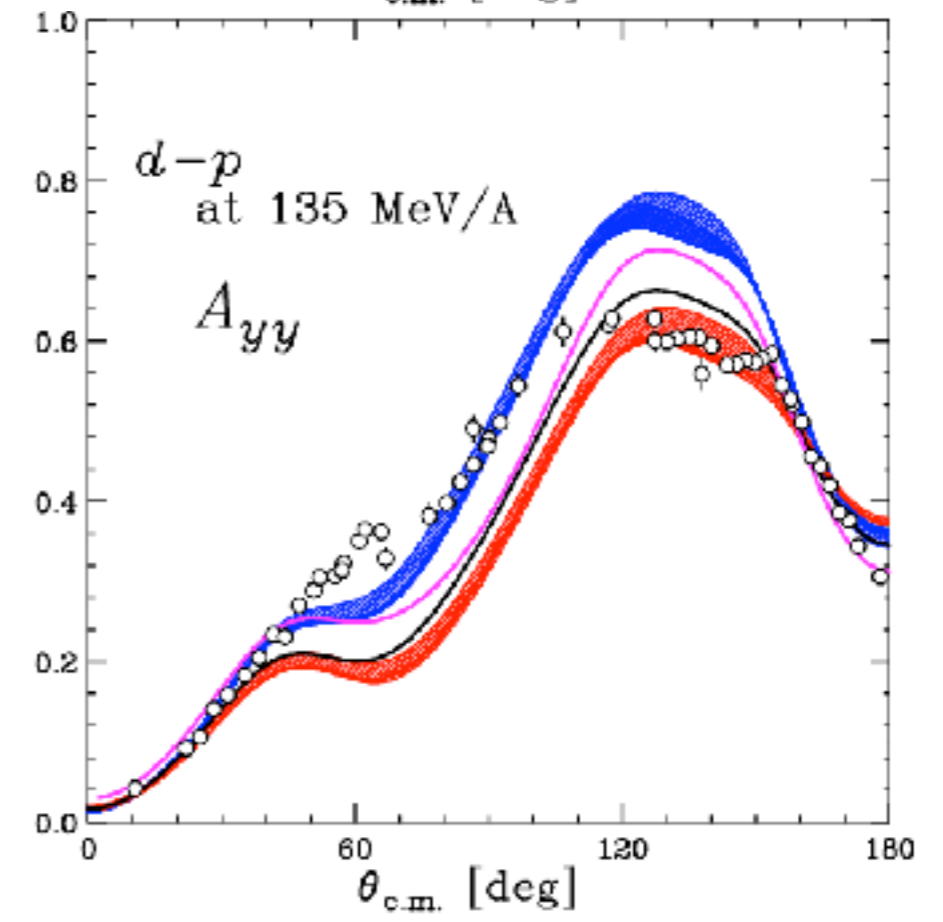
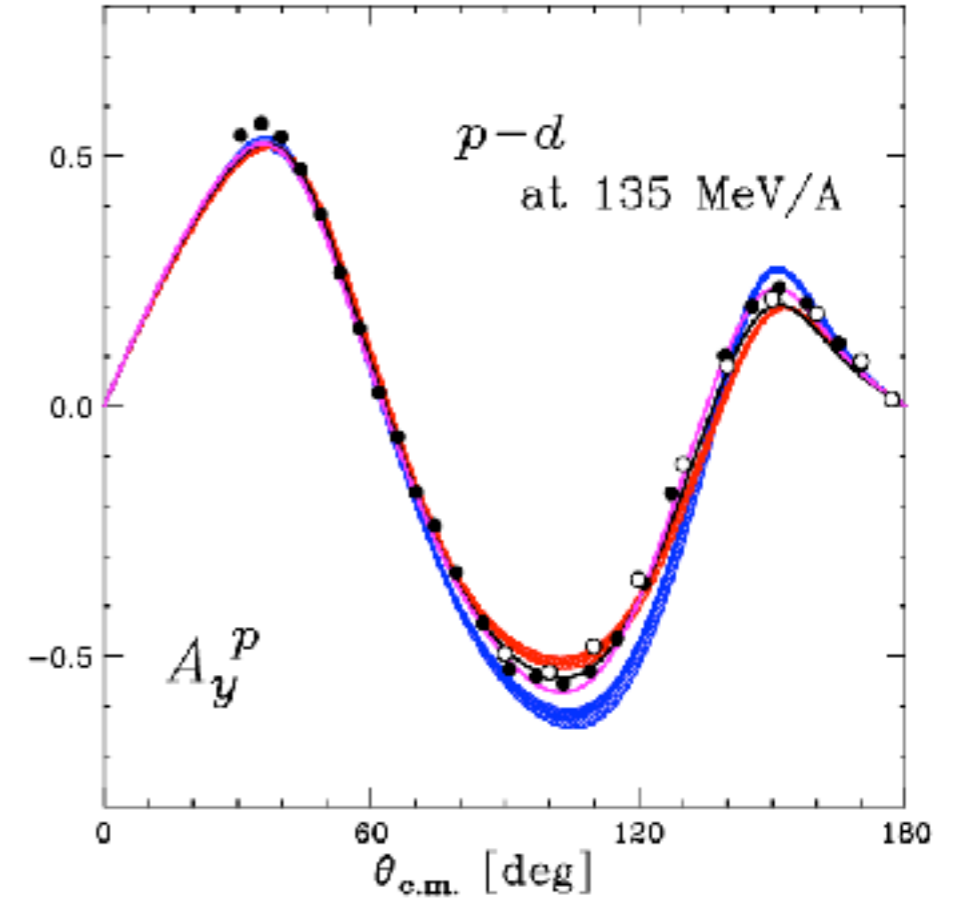
2NF (CDBonn, AV18, Nijmegen I,II) :
Large discrepancy
in Cross Section Minimum

3NF (Tucson-Melbourne, Urbana IX, Δ -isobar) :

Vector Analyzing Power A_y^p
: Good Agreement

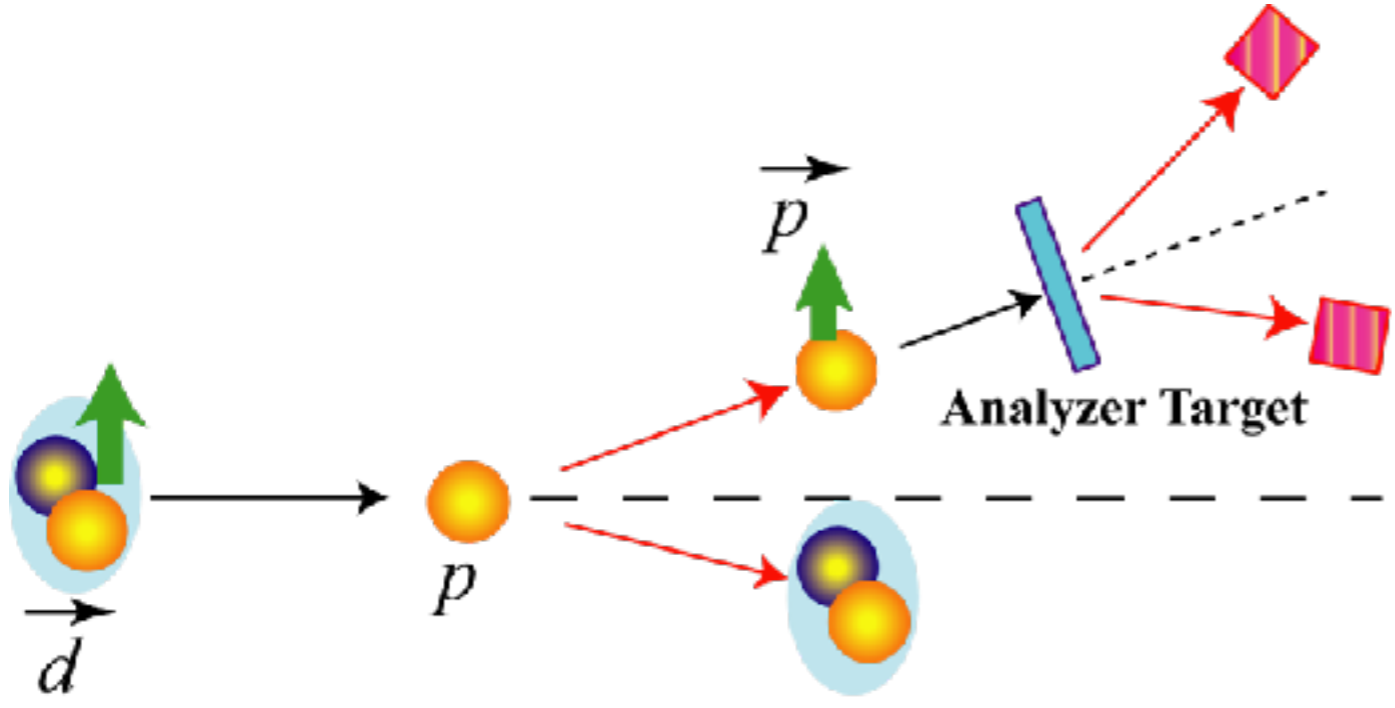
Tensor Analyzing Power A_{yy}
: No superiority

K. Sekiguchi et al. PRC 65, 034003 (2002)

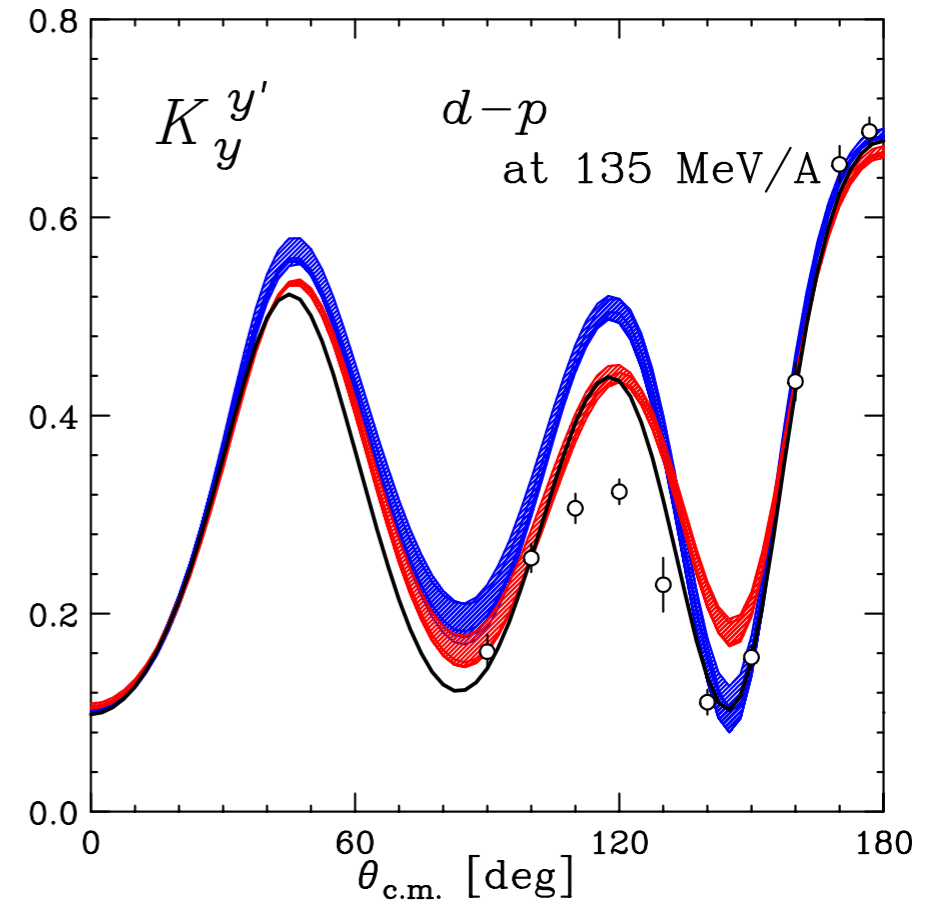
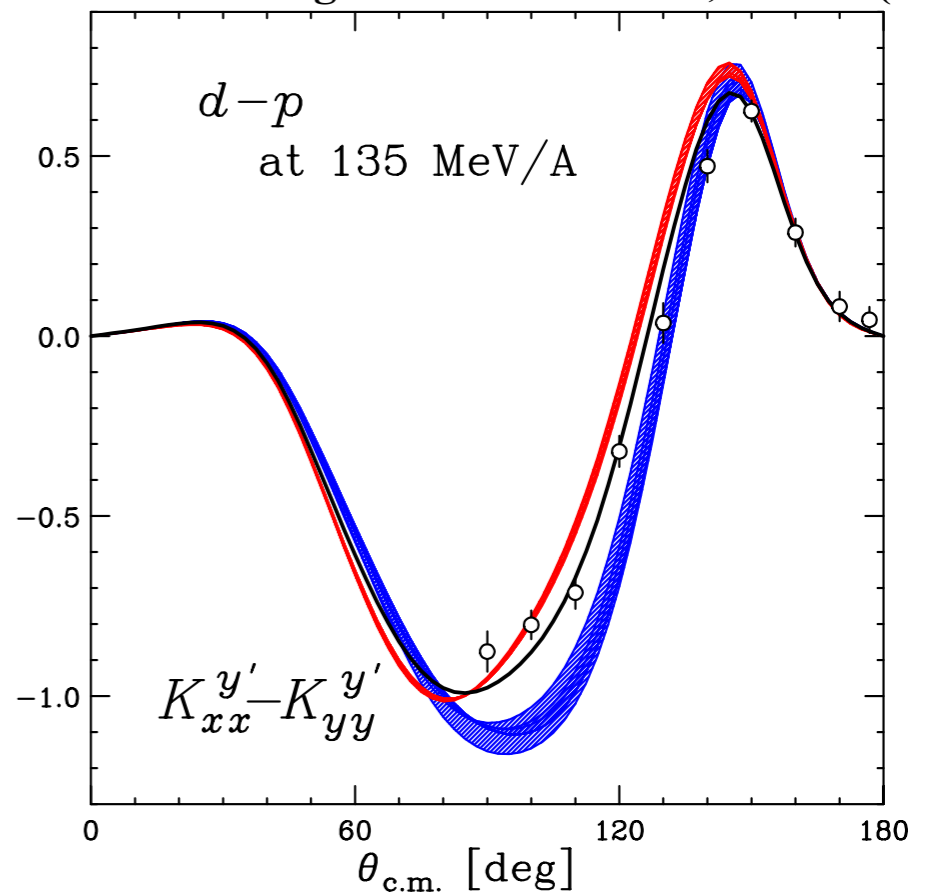


Polarization Transfer Coefficients

K. Sekiguchi et al. PRC 70, 014001(2004)



3NF :
 $K_{xx}^{y'} - K_{yy}^{y'}$: Good Agreement
 $K_y^{y'}$: Direction : O.K.
 Magnitude : not enough

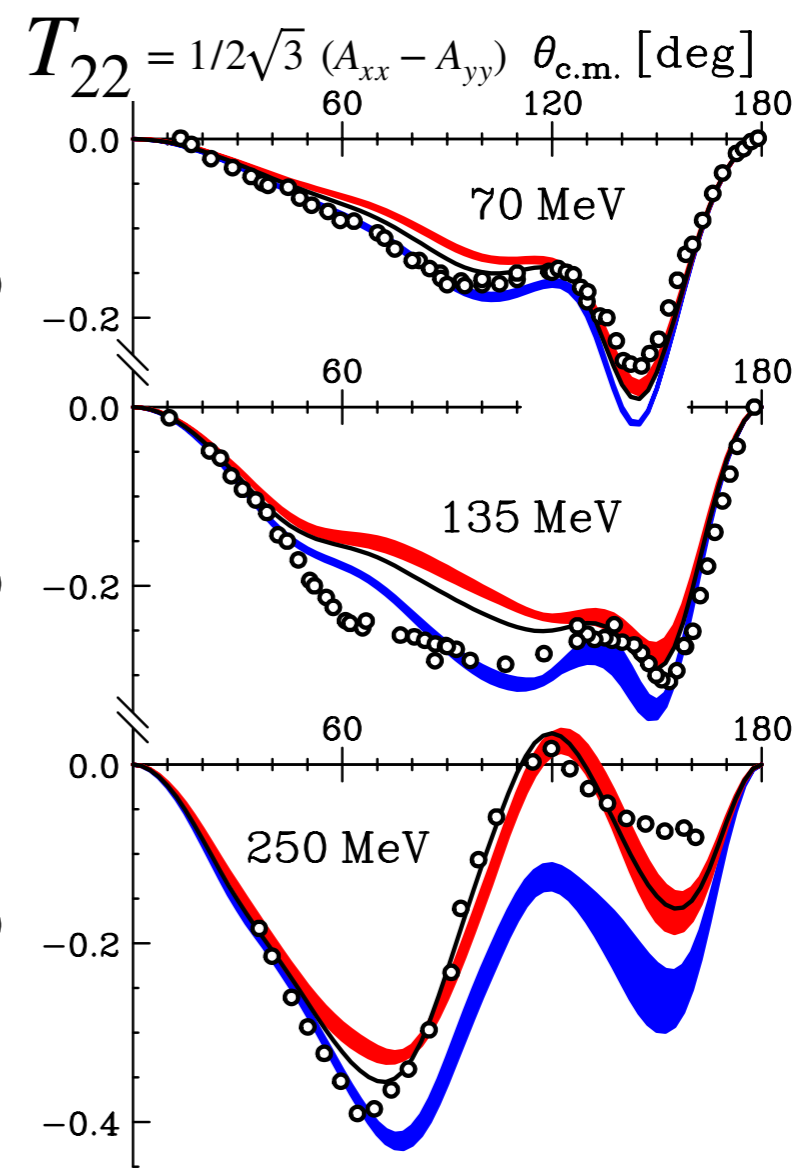
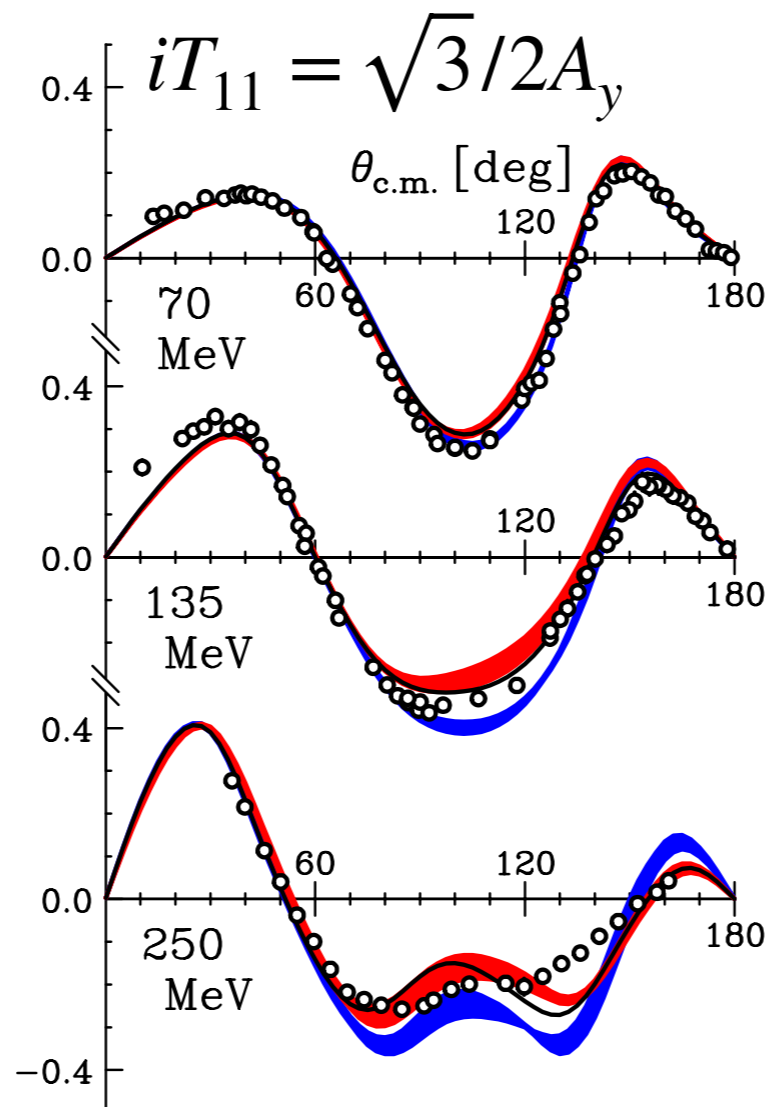
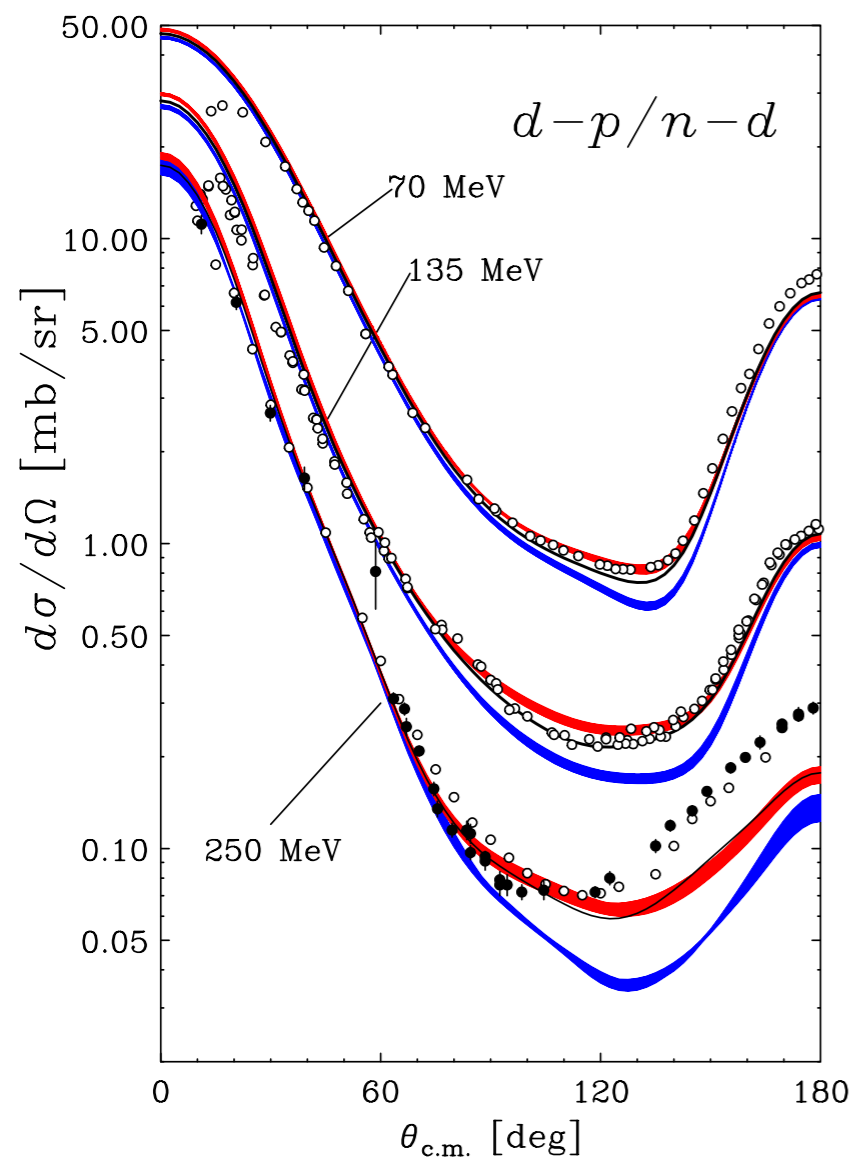


d-p elastic scattering
Energy Dependence

Energy Dependent Study for dp Scattering

- Cross Section & Analyzing Powers -

K. Hatanaka et al., Phys. Rev. C. 66, 044002 (2002)
 Y. Maeda et al., Phys. Rev. C 76, 014004 (2007)
 K. S. et al., Phys. Rev. C 83, 061001 (2011)
 K. S. et al., Phys. Rev. C 89, 064007 (2014)



Serious discrepancies exist at very backward angles.

- NN (CDBonn, AV18, Nijm I,II)
- TM'(99) 3NF+
NN(CD Bonn, AV18, Nijm I,II)
- Urbana IX 3NF+AV18

Summary of Results of Comparison for dp elastic scattering

- Cross section at ~ 100 MeV/nucleon
 - First clear signature of 3NF effects in 3N scattering
 - Magnitudes of 3NFs is O.K. .

- Spin observables
 - Not always described by 2π -3NFs
 - Defects of spin-dependent parts of 3NFs

- At higher energies ...
 - Serious discrepancy at backward angles
 - Short Range 3NFs are required.

Nd Elastic Scattering Data at Intermediate Energies

pd and *nd* Elastic Scattering at 70–400 MeV/A

~1998

Observable	100	200	300	400
$\frac{d\sigma}{d\Omega}$				
\vec{p} A_y^p \vec{n} A_y^n				
\vec{d} A_y^d A_{yy} A_{xx} A_{xz}				
$\vec{p} \rightarrow \vec{p}$ $K_y^{y'}$ $K_x^{x'}$ $K_x^{z'}$ $K_z^{x'}$ $K_z^{z'}$				
$\vec{d} \rightarrow \vec{p}$ $K_y^{y'}$ $K_{xx}^{y'}$ $K_{yy}^{y'}$ $K_{xz}^{y'}$				
$\vec{p} \rightarrow \vec{d}$ $K_y^{y'}$				
$\vec{p}\vec{d}$ C_{yy} C_{ij}				

π threshold

Nd Elastic Scattering Data at Intermediate Energies

pd and nd Elastic Scattering at 70–400 MeV/nucleon

Observable	100	200	300	400
$\frac{d\sigma}{d\Omega}$	•	•••••	•	•
\vec{p} A_y^p \vec{n} A_y^n		•••••	•	•
\vec{d} iT_{11} T_{29} T_{22} T_{21}	•••••	•••••	•••••	•
$\vec{p} \rightarrow \vec{p}$ $K_y^{y'}$ $K_x^{x'}$ $K_x^{z'}$ $K_z^{x'}$ $K_z^{z'}$				•••••
$\vec{d} \rightarrow \vec{p}$ $K_y^{y'}$ $K_{xx}^{y'}$ $K_{yy}^{y'}$ $K_{xz}^{y'}$	•	•••••		
$\vec{p} \rightarrow \vec{d}$ $K_y^{y'}$				•
$\vec{p}\vec{d}$ C_{ij} $C_{ij,k}$		••	••	

~2022

- High precision data set of $d\sigma/d\Omega$ & Analyzing Powers from RIKEN, RCNP, KVI, IUCF

After about **90** Years of Yukawa's Meson Theory (1935) &

After **65** Years of Fujita-Miyazawa 3NF (1957)

Quantitative discussions on 3NFs start via Theor. & Exp. .

Recent Study and Future Aspects

in Progress : Theory

Quantitative discussions on three-nucleon forces start via Theor. & Exp. .

Three-Nucleon Force is one key element to understand fundamental properties of nuclei.

- Nuclear Forces linked to QCD
- Three Nucleon Forces in $A > 3$

Nuclear Forces linked to QCD

Nuclear Forces based on Chiral Effective Field Theory

- Link to QCD

Lagrangian :

includes all the terms consistent with the assumed symmetries :
Lorentz and iso-spin Invariance

& **Spontaneously Broken Chiral Symmetry**

Interactions :

π + Nucleon + contact terms

- **Nuclear forces** (2NF, 3NF, ...)










and currents are **derived**
in a consistent way.

- **Hierarchy of Nuclear Forces :**

$2NF > 3NF > 4NF$

The first 3NF appears in NNLO.

Layout of χ EFT Nuclear Forces

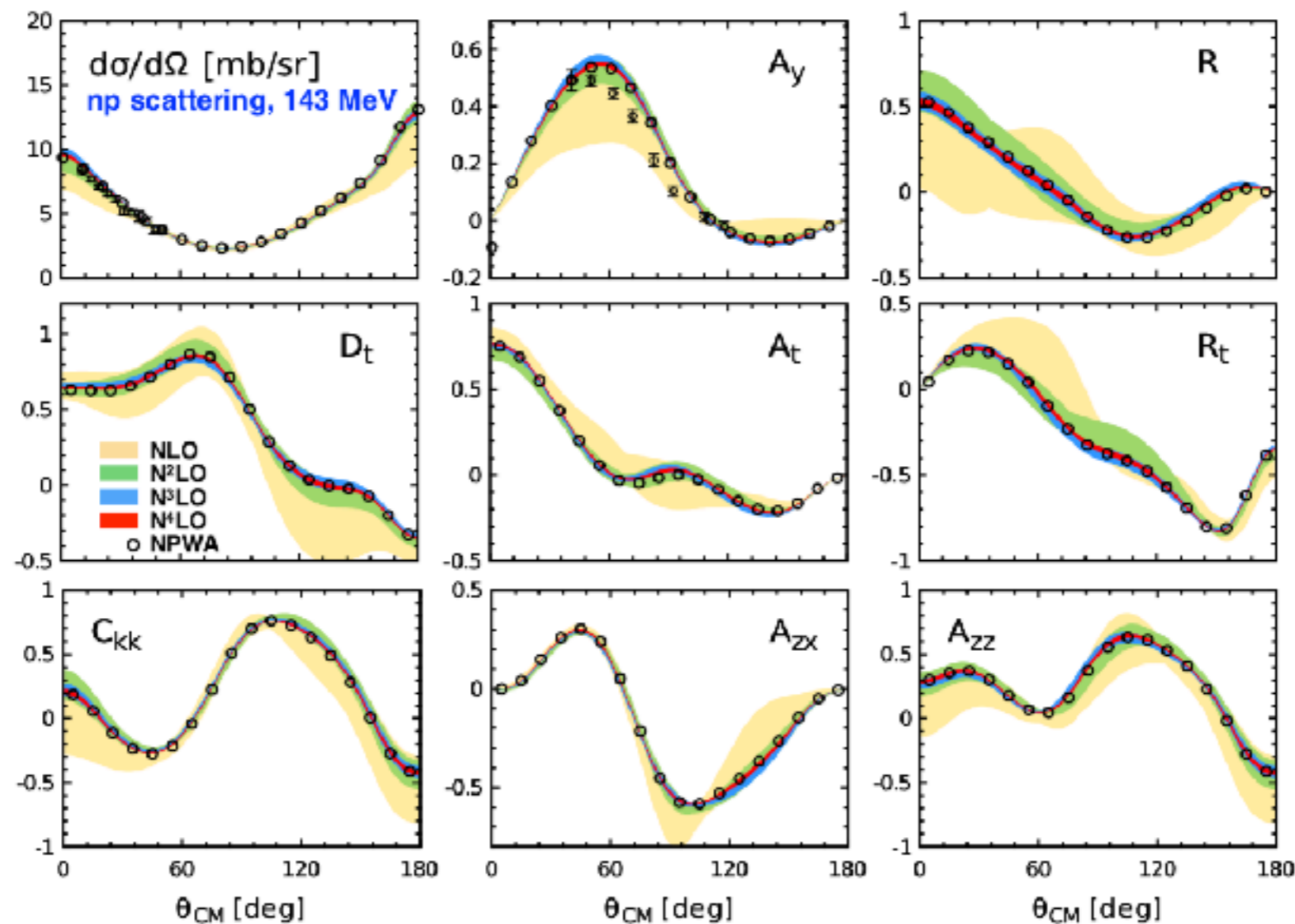
	2N Force	3N Force	4N Force
LO (Q/Λ_χ) ⁰		—	—
NLO (Q/Λ_χ) ²		—	—
N2LO (Q/Λ_χ) ³			—
N3LO (Q/Λ_χ) ⁴			
N4LO (Q/Λ_χ) ⁵			

The 2N system

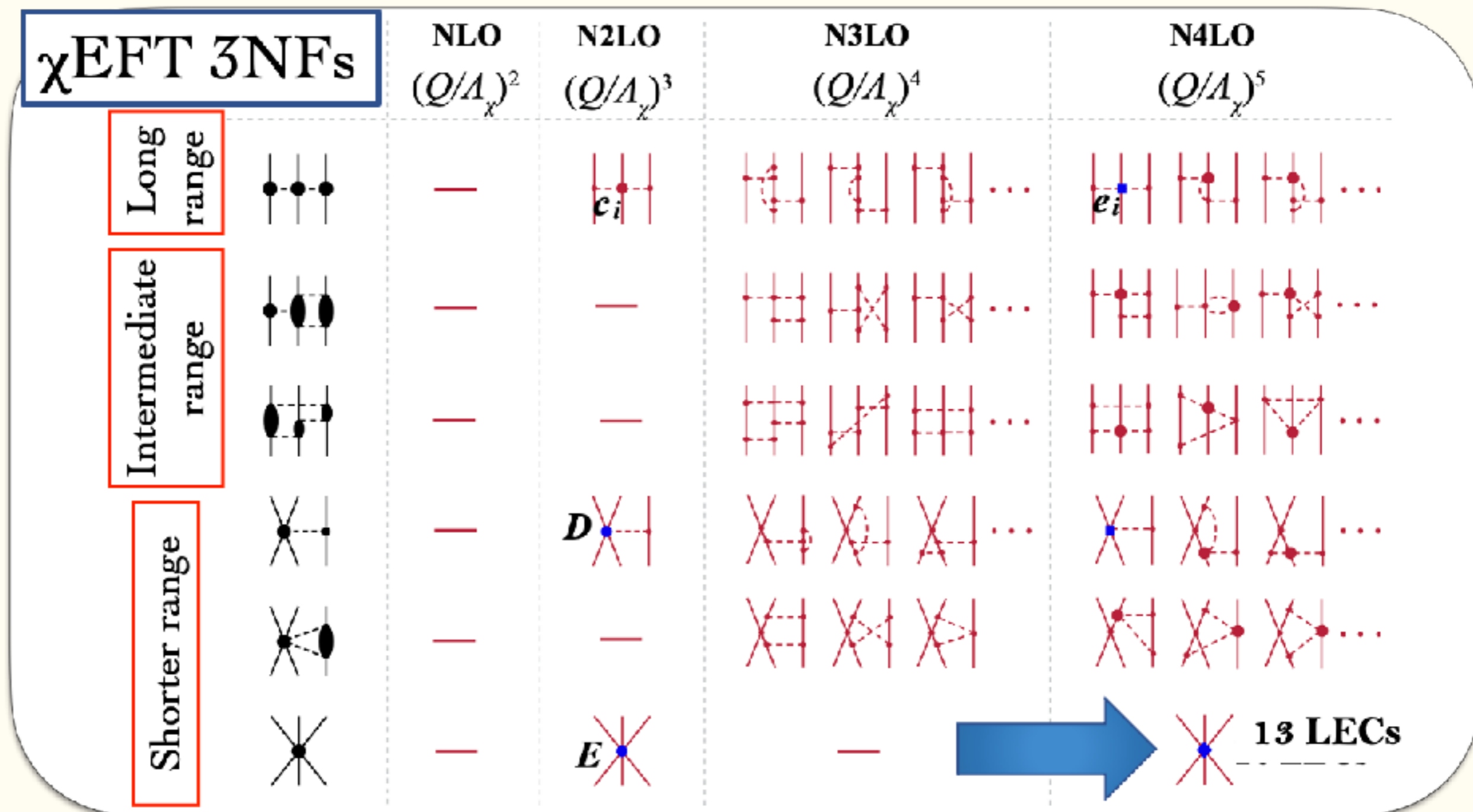
Experience in the 2N sector: **how far should one go to obtain a precise description of data?**

	LO	NLO	N ² LO	N ³ LO	N ⁴ LO ⁺
χ^2/datum (<i>np</i> , 0 – 300 MeV)	75	14	4.1	2.01	1.06
χ^2/datum (<i>pp</i> , 0 – 300 MeV)	1380	91	41	3.43	1.00

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88



χ^{EFT} N⁴LO 2NF has achieved to **high precision**.
Number of parameters to be fitted to NN data : 28



L. Girlanda, et al., Phys. Rev. C 84, 014001 (2011)

L. Girlanda, et al., Phys. Rev. C 102, 019903 (2020).

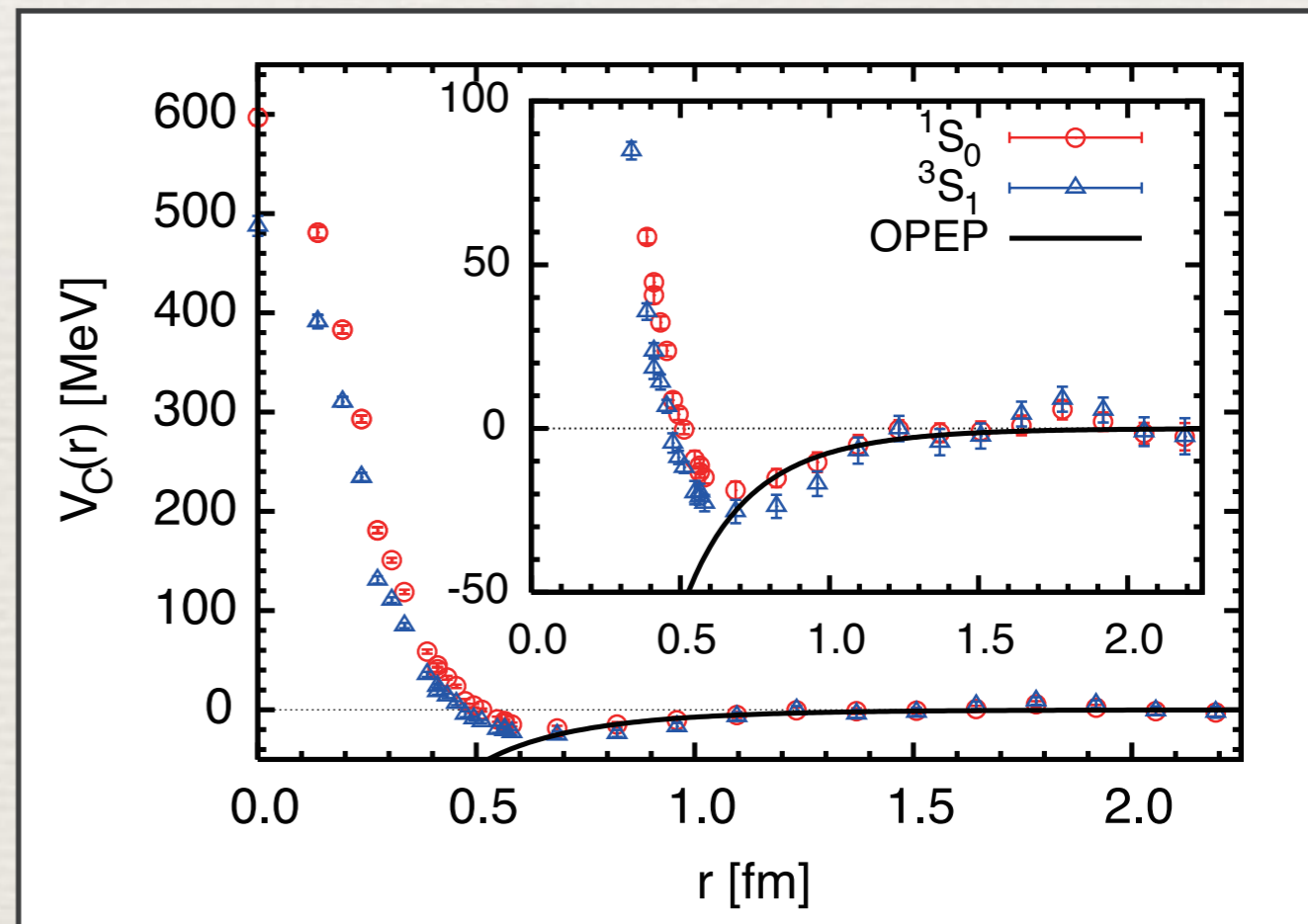
Nuclear Forces linked to QCD

● Nuclear Forces from Lattice QCD

Ishii, Aoki, Hatsuda, Phys. Rev. Lett. 99, 022001 (2007)

🔊 Lattice QCD simulations succeeded in providing bulk properties of nucleon-nucleon forces.

🔊 Study of 3NFs from Lattice QCD is in progress.
Doi et al (HAL QCD Coll.)
Prog. Theor. Phys. 127, 723 (2012)



Three Nucleon Force in $A > 3$

3NFs in $A > 3$ - ① -

3NFs in Finite Nuclei

Ab Initio Calculations for Light Nuclei

- Green's Function Monte Carlo
 - No-Core Shell Model, etc..
- 2NF provide less binding energies

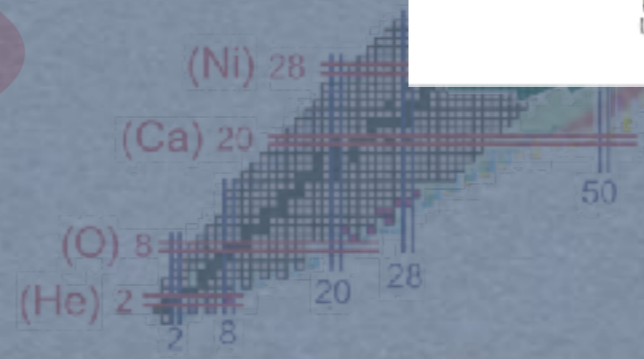
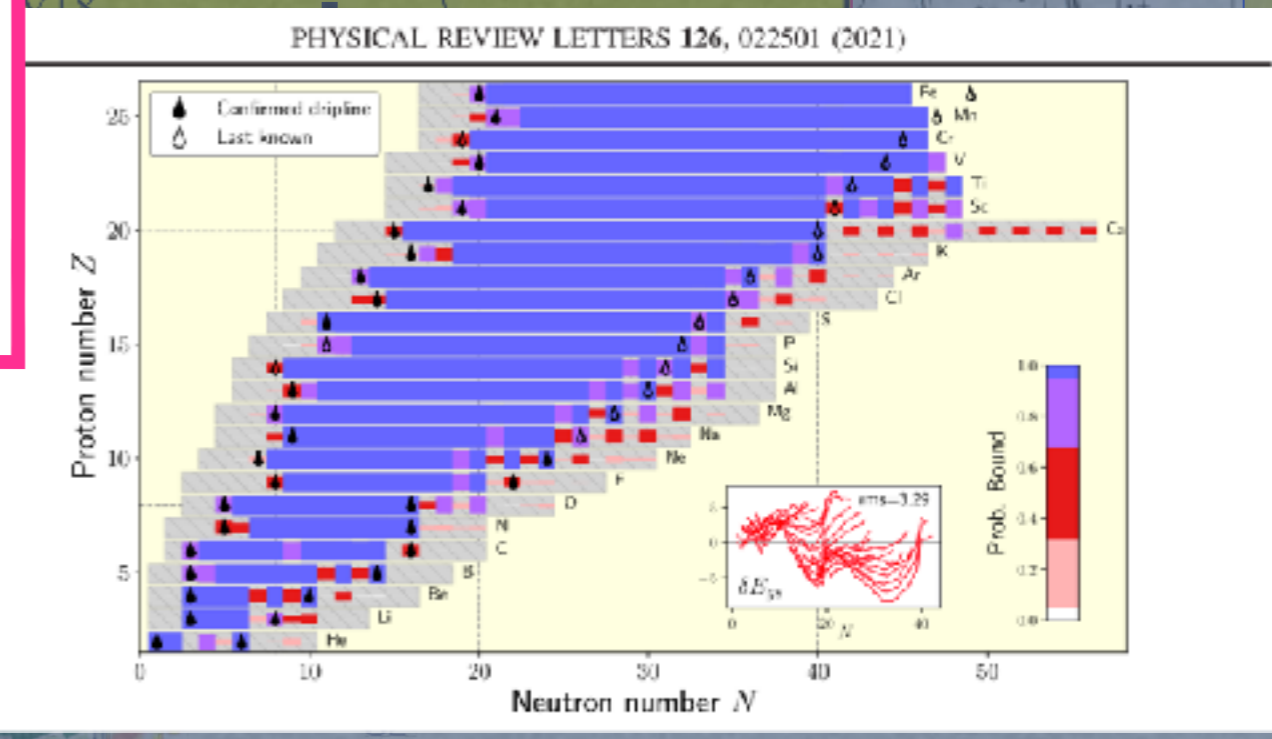
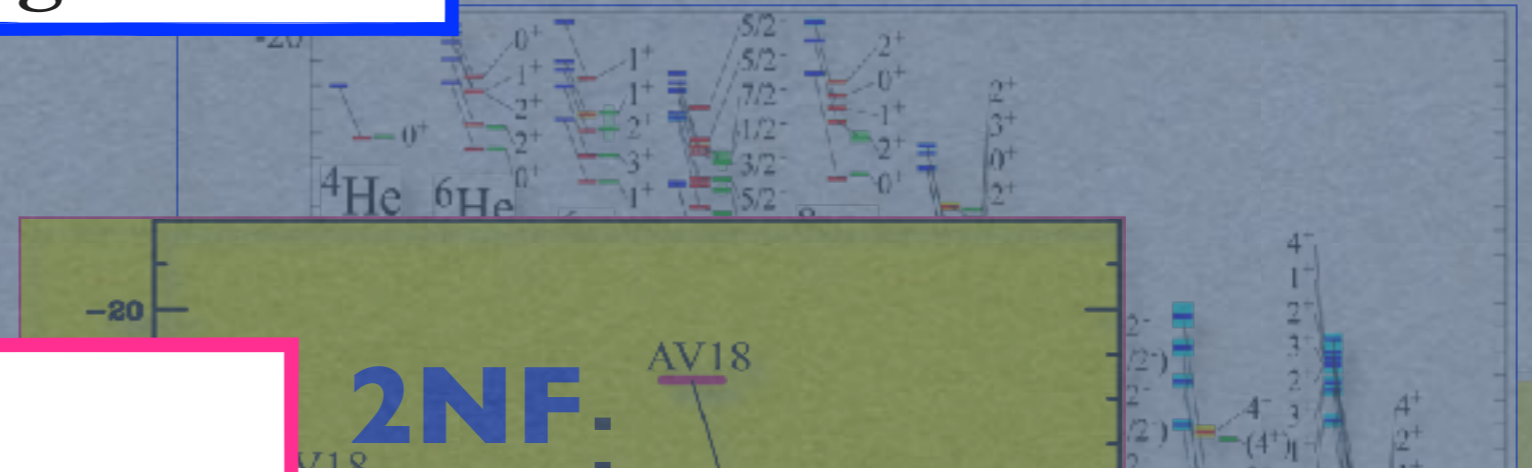


Medium Mass Nuclei
 3NFs provide key mechanisms,
 e.g. shell-evolution,
 boundaries of nuclear stability.

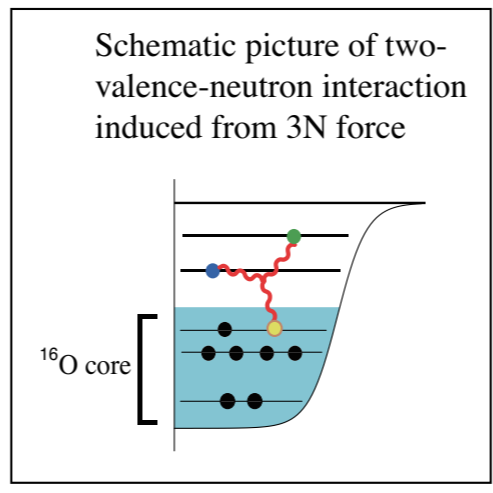
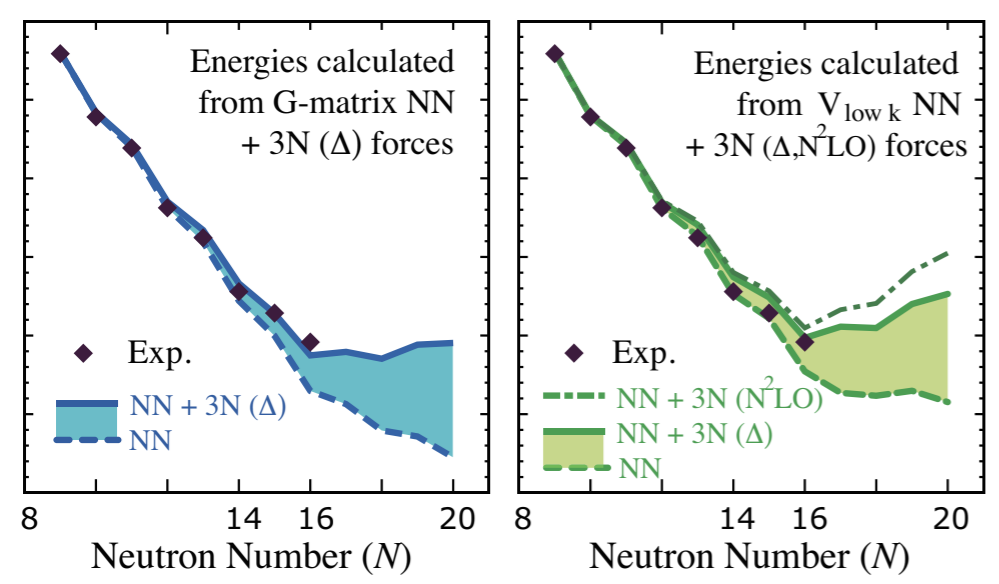
roles to explain B.E.
 in neutron rich nuclei.

3NF effects in B.E.

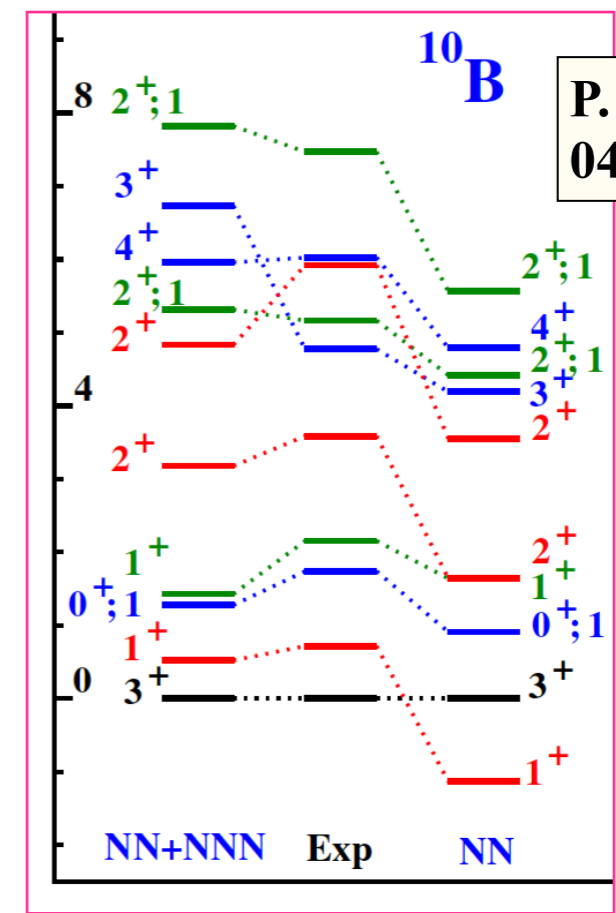
- 10-25%
- Attractive



Nuclear Structure and χ EFT Nuclear Potential



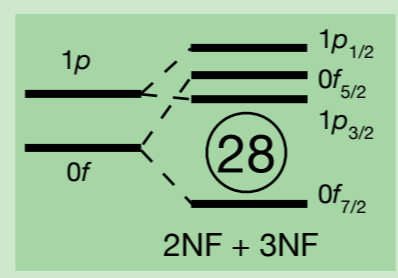
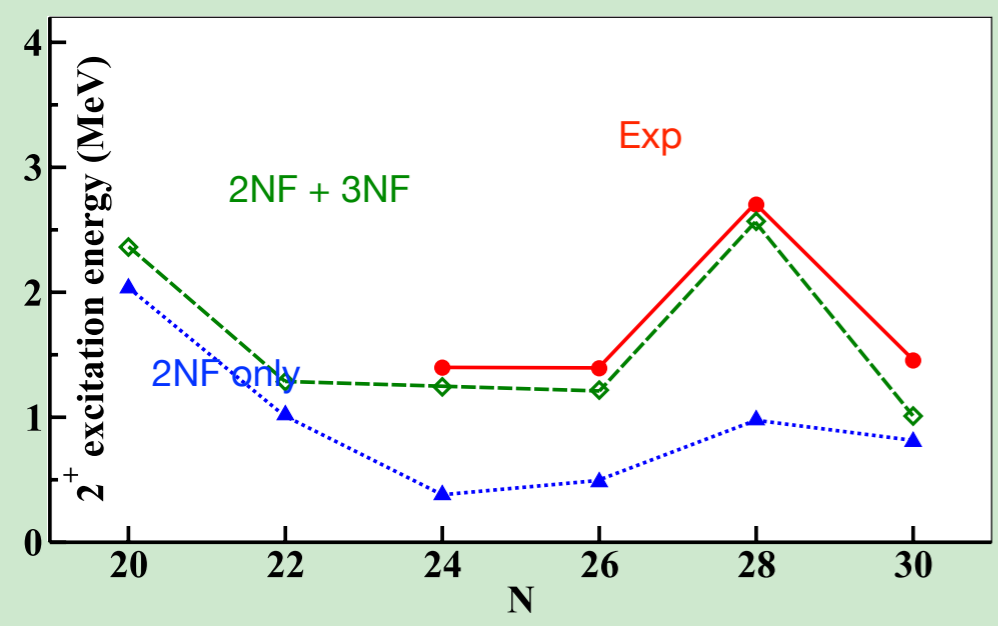
Otsuka et al., Phys. Rev. Lett. 105, 032501 (2010)



P. Navratil, PRL 99, 042501 (2007)

pf-shell nuclei (Ni isotopes)

Ma, Fukui et al., PRC 100, 034324 (2019)



Contribution of Each term of the N2LO 3NF in ^6Li
T. Fuku et al. EPJ web of conference 223, 01018 (2019)

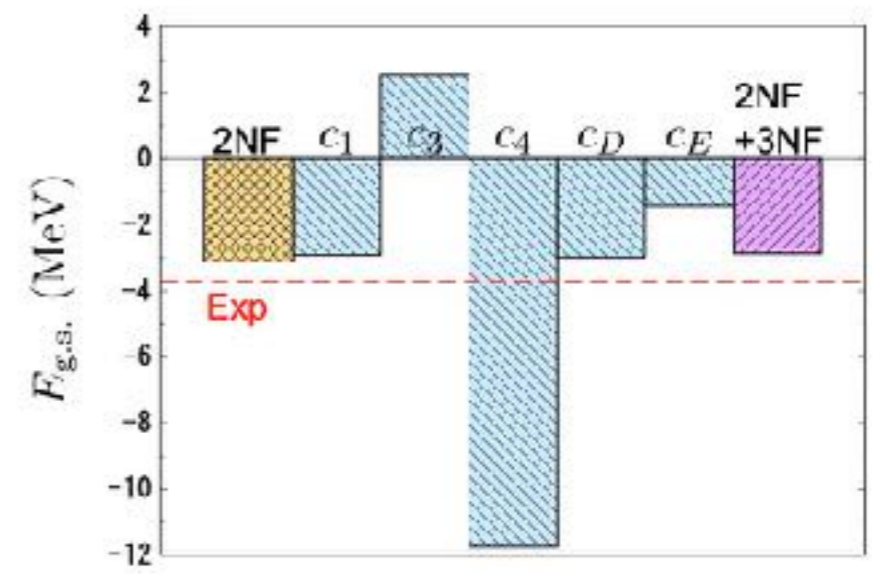
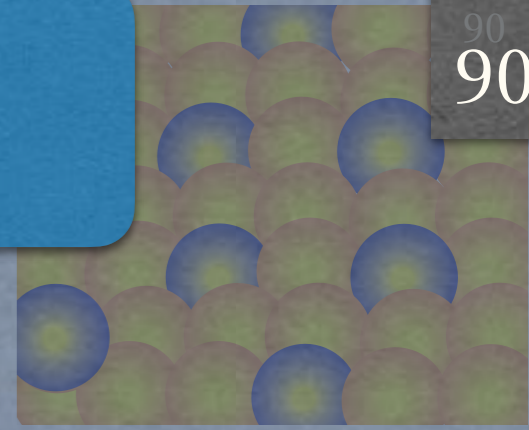


Figure 3. The individual contribution of each term of the 3NF to $E_{g.s.}$ of ^6Li . See text for details.

$3NFs$ in $A > 3$ - ② -



3NFs in Infinite Nuclei - Neutron Star -

核物質の状態方程式

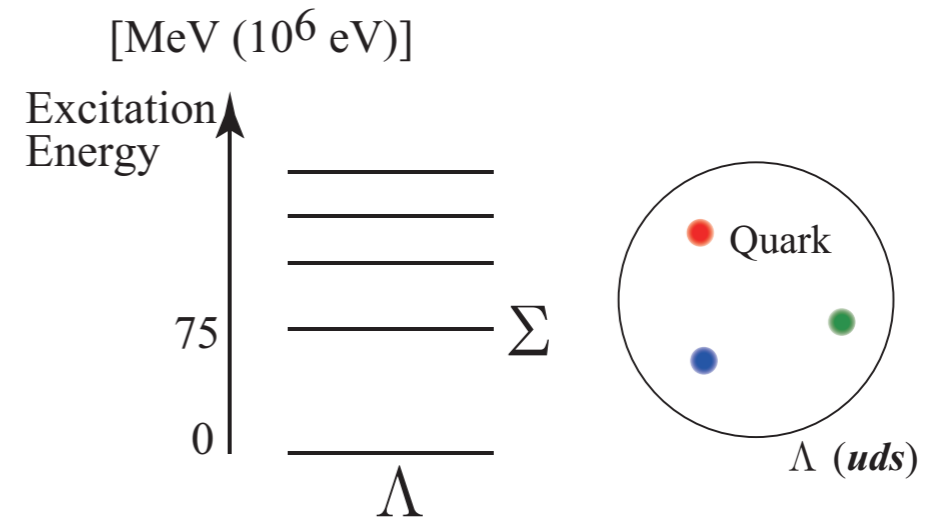
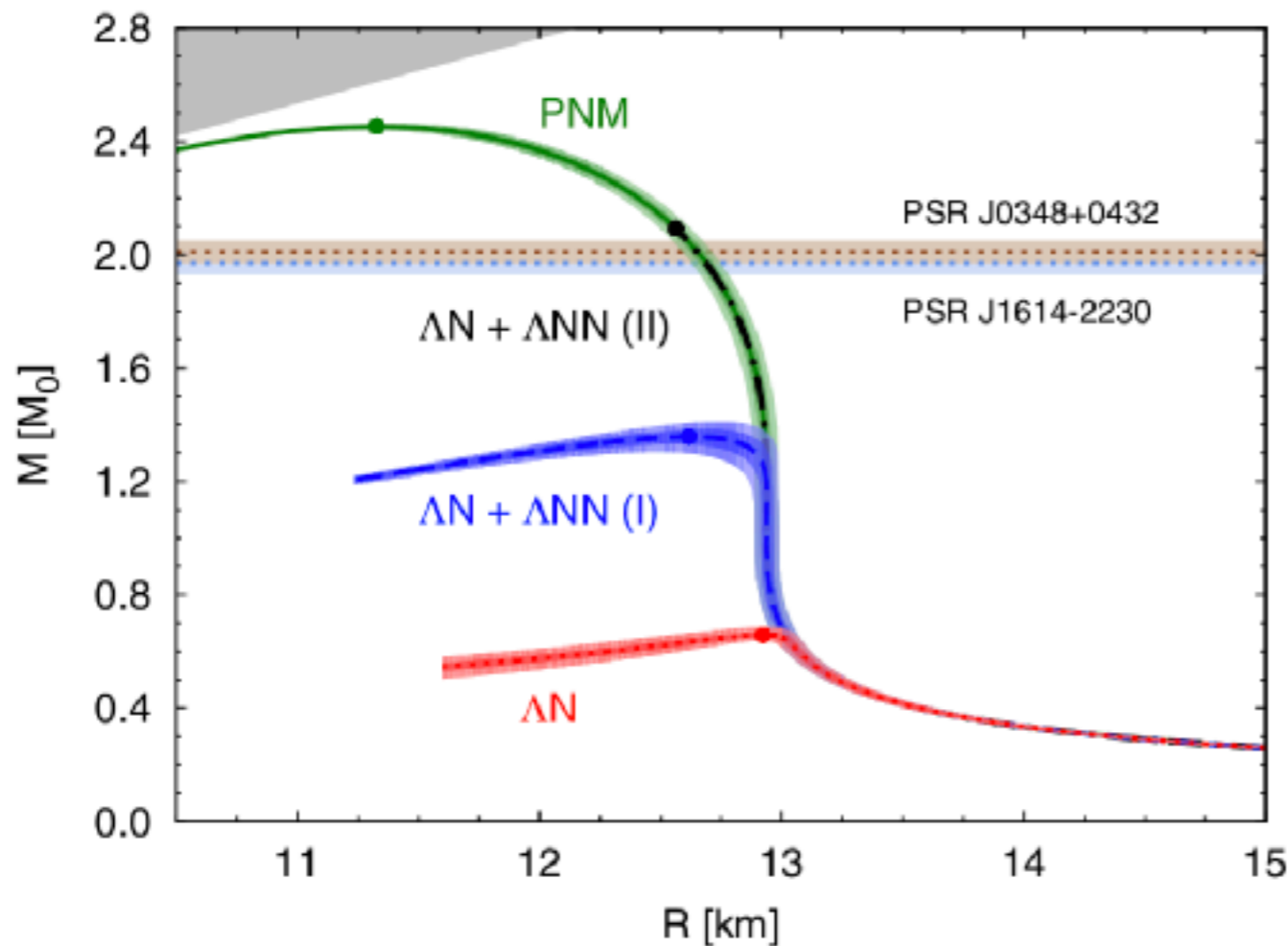


A. Akmal et al., PRC 58, 1804('98)

Symmetric Nuclear Matter

Neutron Star

NNN(nnn) + $NNA\Lambda$ in Infinite Nuclei - Neutron Star -



D. Lonaldoni et al., Phys. Rev. Lett. 114, 092301 (2015)

in Progress : Experiment

So far ...

Nucleon-Deuteron Scattering at ~ 100 MeV/nucleon

- First Evidence of 3NF effects
- Defects of existing 3NF models

from here ...

 *d-p* Scattering at ~ 100 MeV/nucleon : *Golden window of 3NFs*

- Determine 3NFs based on χ EFT Nuclear Potential
- High-precision measurement of Spin Correlation Coefficients

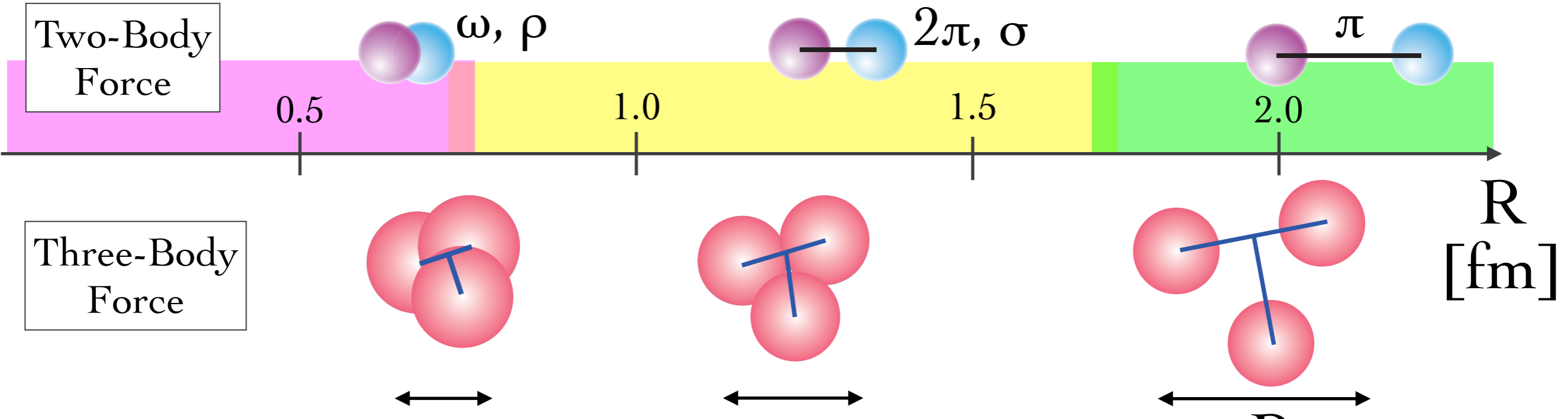
 Proton- ^3He Scattering at ~ 100 MeV/nucleon : *New Probe of 3NF Study*

- First Step from Few to Many
- 3NFs of isospin channel of $T=3/2$

①. Repulsive
-Short Range-

②. Attractive (strong)
-Intermediate Range-

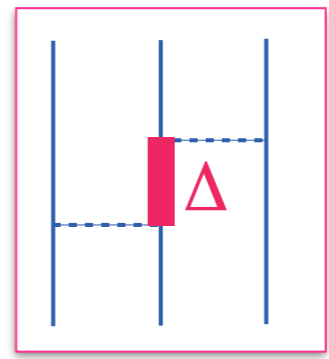
③. Attractive (weak)
- Long Range -



Two-Body Force

Three-Body Force

R [fm]



Intermediate State
 Δ
-Fujita-Miyazawa-

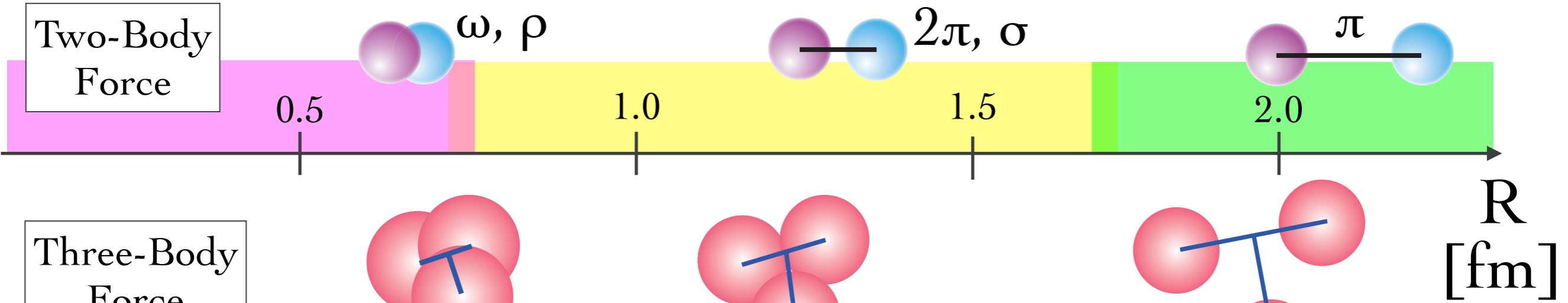
Nuclear Matter
Neutron Star

Nuclear Structure

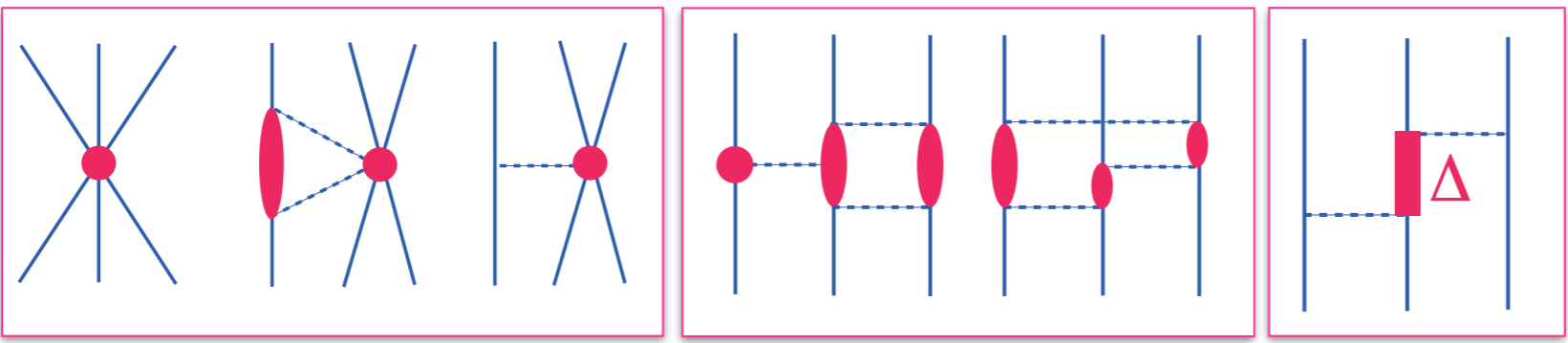
①. Repulsive
-Short Range-

②. Attractive (strong)
-Intermediate Range-

③. Attractive (weak)
- Long Range -



← quark & gluon picture



Intermediate State
 $N^*, \Delta\Delta$ etc...

$T=3/2$ $3NF$

Intermediate State
 Δ
-Fujita-Miyazawa-

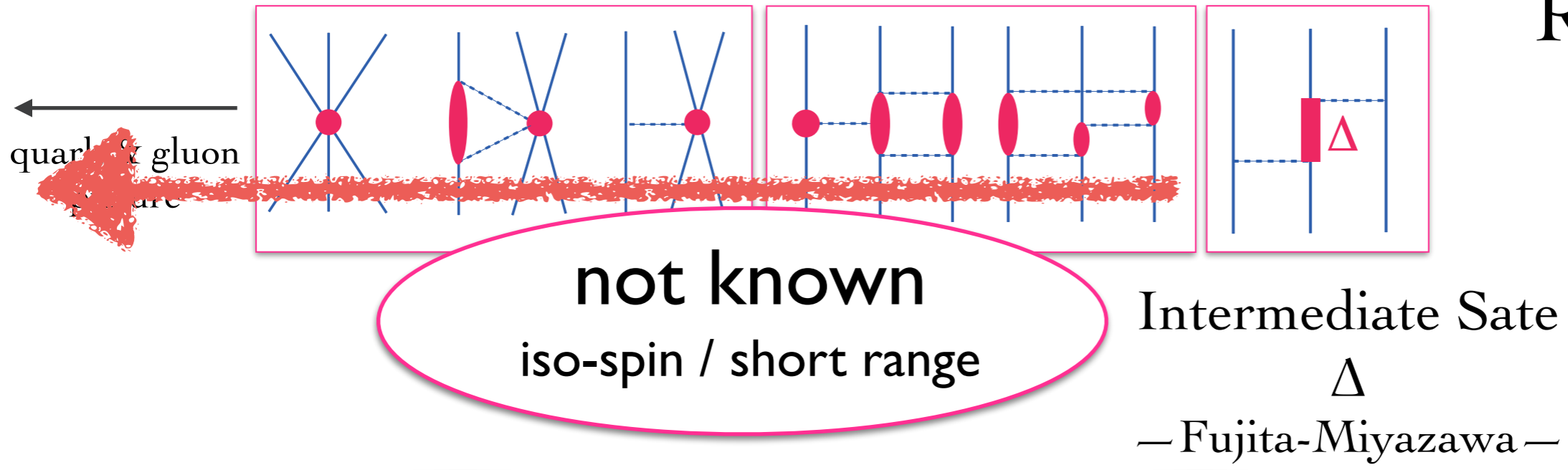
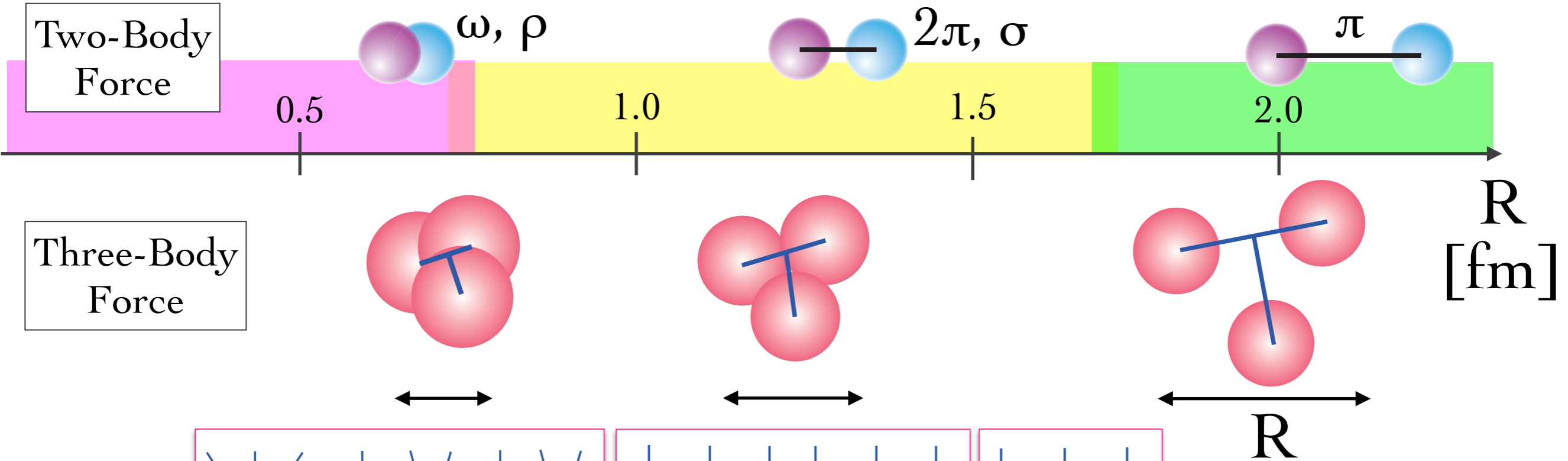
Nuclear Matter
Neutron Star

Nuclear Structure

①. Repulsive
-Short Range-

②. Attractive (strong)
-Intermediate Range-

③. Attractive (weak)
- Long Range -



Nuclear Matter
Neutron Star

Nuclear Structure

χ EFT & dp elastic scattering

χ EFT 2NFs have achieved to high-precision.

5th order of NN potentials (N4LO+) reproduce pp(np) data with $\chi^2/\text{datum}=1.00$

P. Reinert, H. Krebs, E. Epelbaum EPJA 54, 86 (2018)

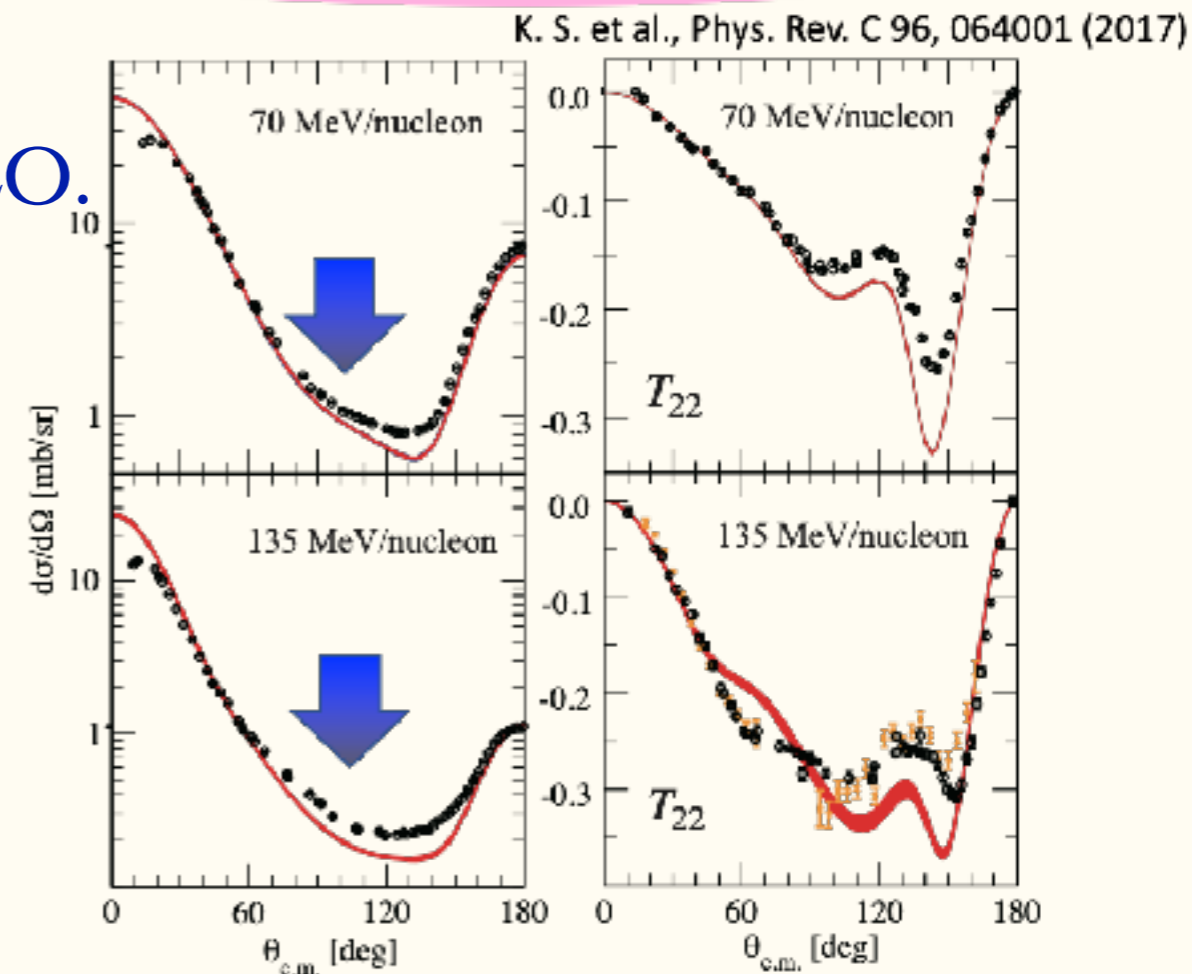
dp elastic scattering data show necessities of 3NFs up to the N4LO.



Cross Section minimum region for dp elastic scattering at $\sim 100\text{MeV}/\text{nucleon}$ are “Golden windows for N4LO 3NFs”.

LENPIC collaboration,
Phys. Rev. C 98, 014002 (2018)

dp scattering & N4LO χ EFT 2NFs



NN Interactions with $R = 0.9$ fm
E. Epelbaum, H. Krebs, and U.-G. Meißner,
Phys. Rev. Lett. 115, 122301 (2015)

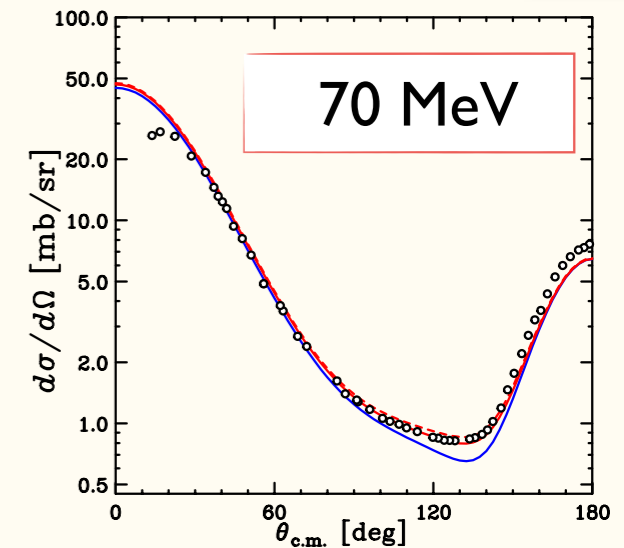
χ EFT & dp elastic scattering

Calculations of N4LO⁺ NN with N2LO 3NF
(Preliminary)

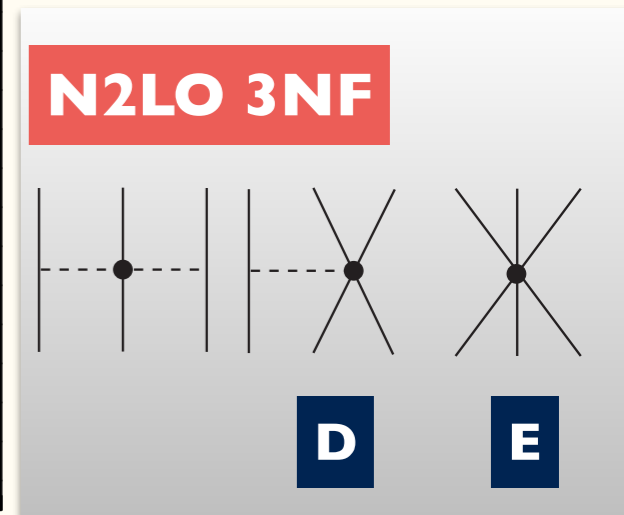
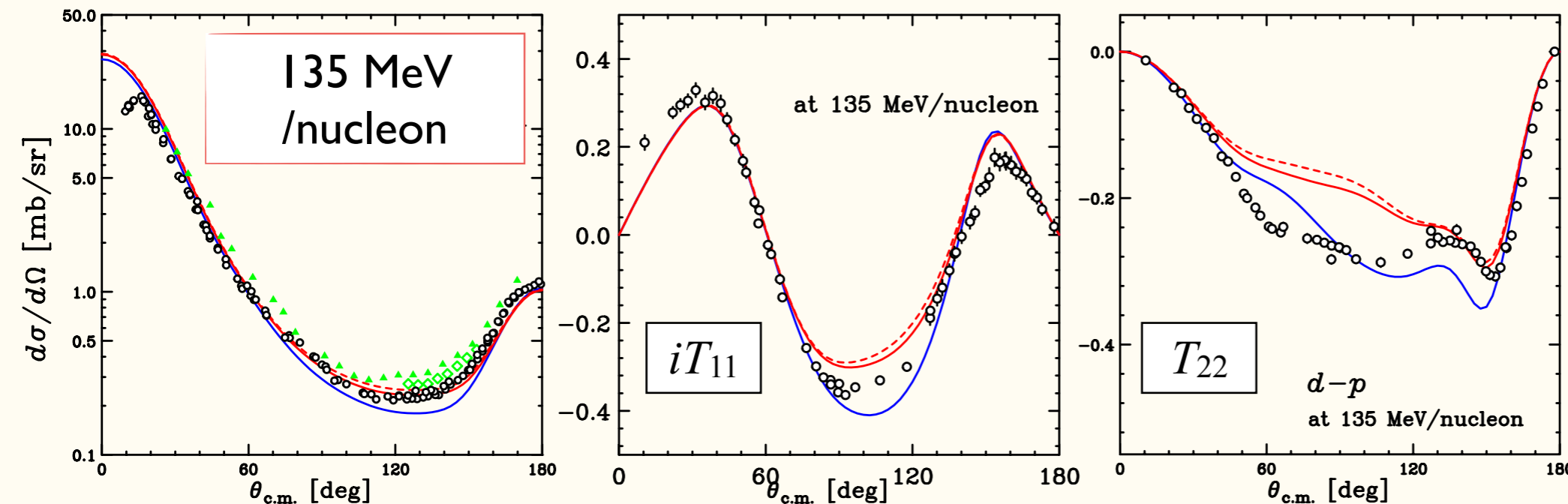
2NF : Semi-local Momentum-Space regularized Chiral NN potentials

P. Reinert, H. Krebs, E. Epelbaum EPJA 54, 86 (2018)

3NF : LECs of N2LO 3NF (D & E terms) are determined
by ³H B.E. & cross section minimum for *Nd* @ 70MeV.



— N4LO+, $\Lambda=450$ MeV
— $C_D=2.0, C_E=0.286$
 $C_D=4.0, C_E=0.499$



calculations : H. Witala private communications.

- Spin observables & C.S. at higher energies : N3LO&N4LO 3NFs are needed.
- Cross section minimum region : Golden Window for the higher-order 3NFs.

Determination of χ EFT N⁴LO 3NFs from dp elastic scattering

➔ “High precision 2N+3N forces”

Project of Theory

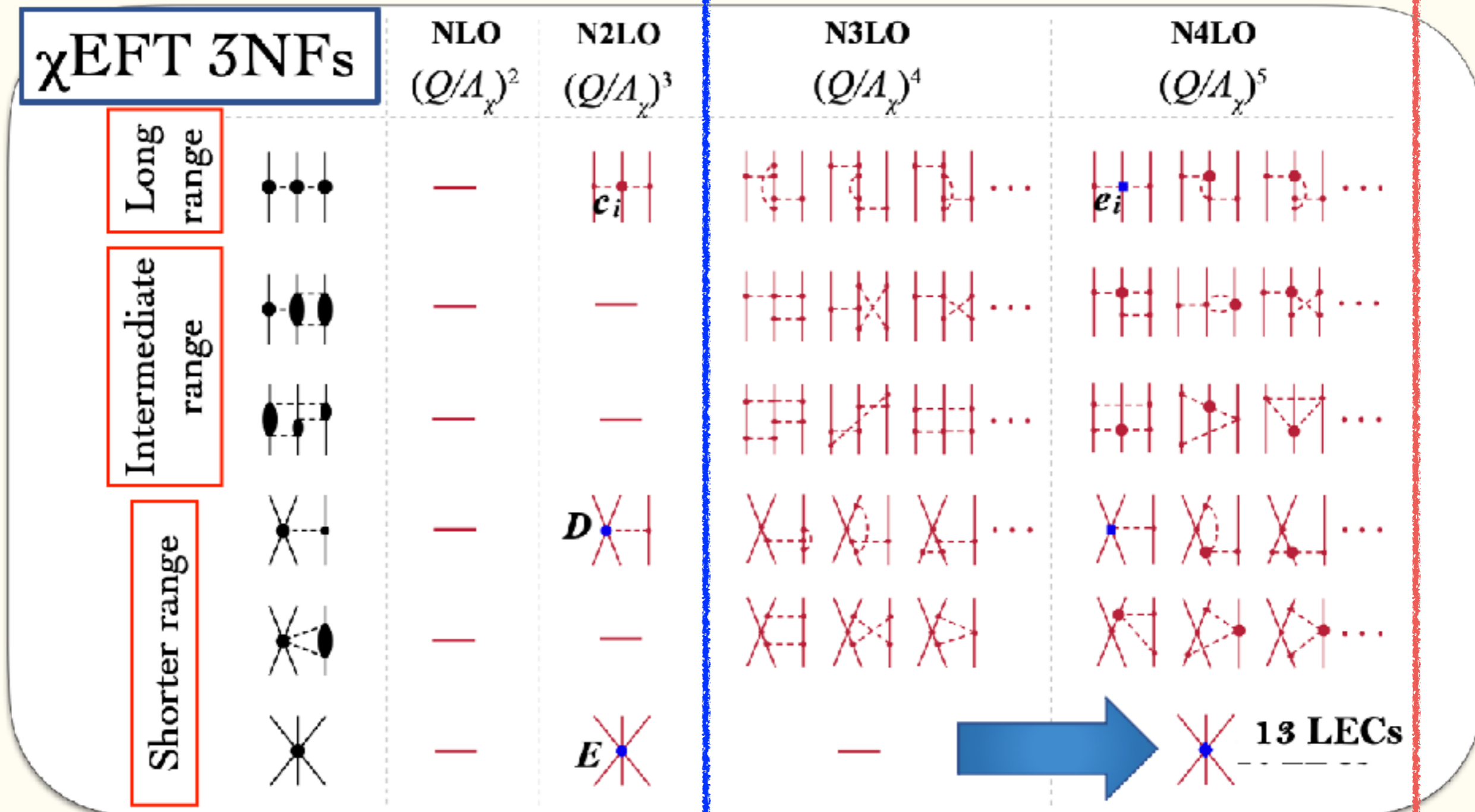
Partial Wave Analysis of Nd scattering in the framework of χ EFT

PI : E. Epelbaum (Bochum / LENPIC Collaboration)

Project of Experiment

Complete set of spin correlation coefficient for dp elastic scattering at ~ 100 MeV/nucleon at RIBF

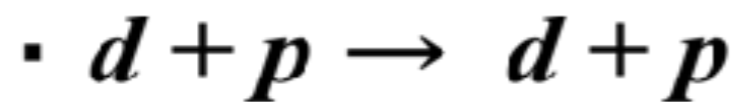
- ✓ To determine Low Energy Constants
- ✓ To test “2N+3N forces”



L. Girlanda, et al., Phys. Rev. C 84, 014001 (2011)

L. Girlanda, et al., Phys. Rev. C 102, 019903 (2020).

dp Scattering

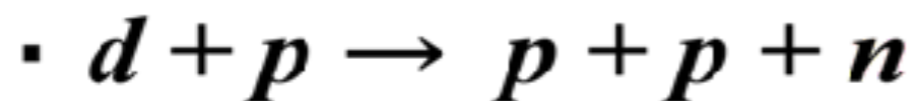
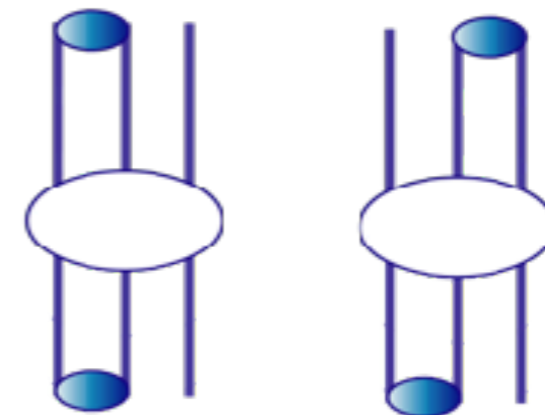
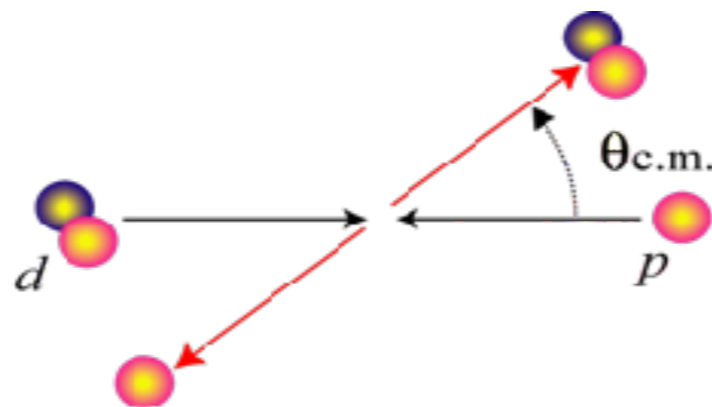


$$\theta_{\text{c.m.}} = 0^\circ \sim 180^\circ$$

Momentum transfer

$$q = 0 - 3.4 \text{ fm}^{-1}$$

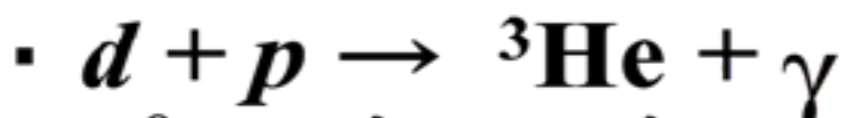
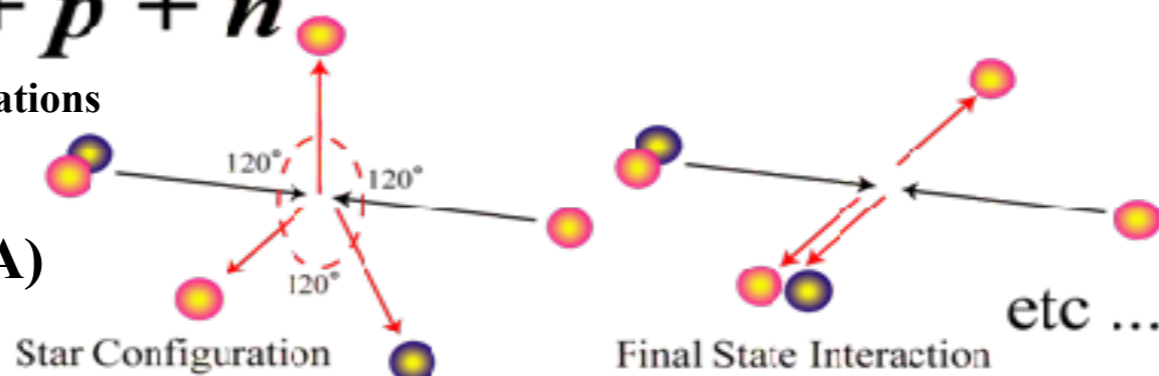
(at $E = 135 \text{ MeV/A}$)



Many kinematical configurations

$$q = 0 - 3 \text{ fm}^{-1}$$

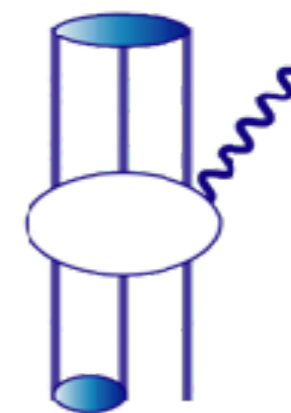
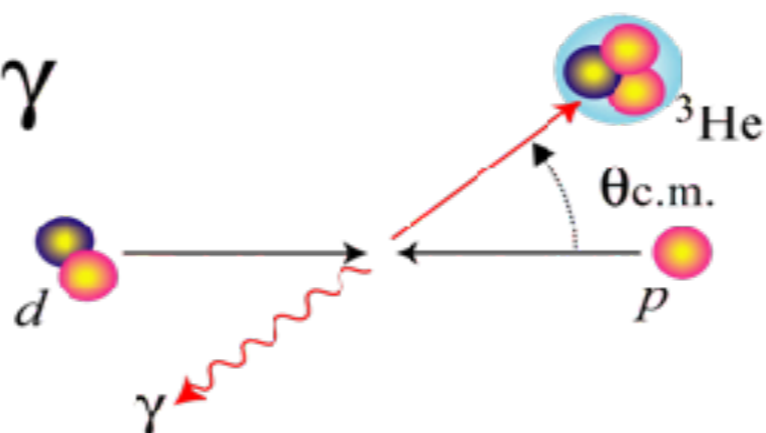
(at $E = 135 \text{ MeV/A}$)



$$\theta_{\text{c.m.}} = 0^\circ \sim 180^\circ$$

$$q = 1.5 - 2.5 \text{ fm}^{-1}$$

(at $E = 135 \text{ MeV/A}$)



p - ^3He scattering

4-nucleon scattering

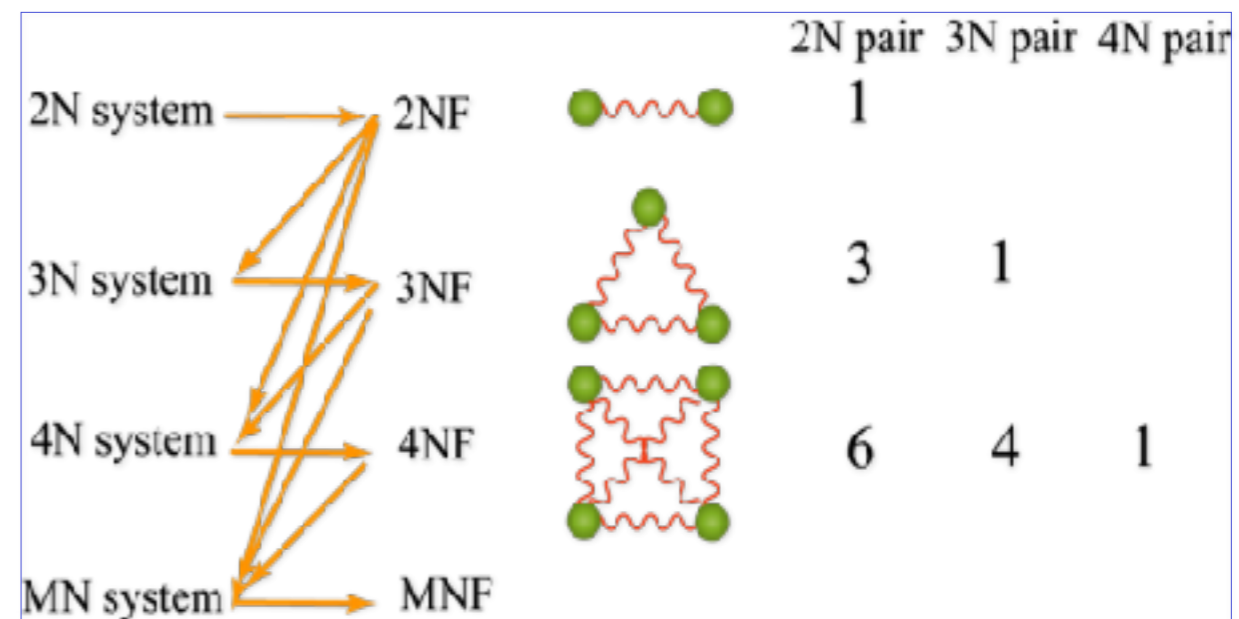
First Step from Few to Many

Larger effects of 3NFs ?

Approach to iso-spin dependence of 3NFs

$T=3/2$ 3NFs

4NF effects



p - ^3He scattering

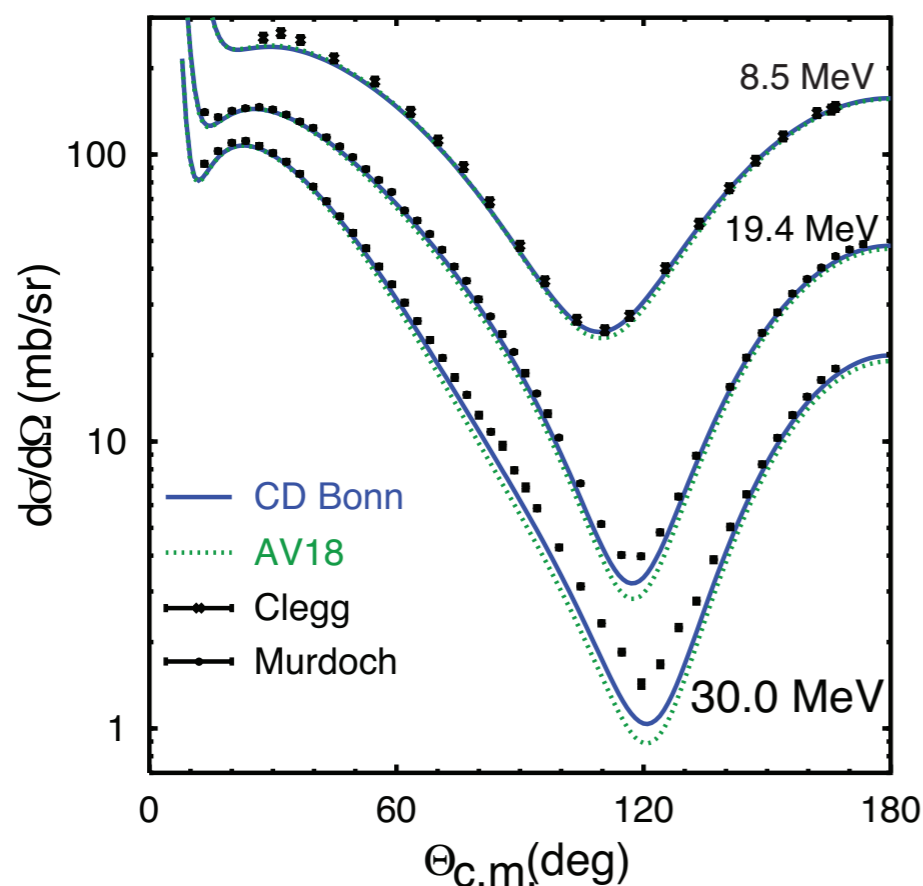
Theory in Progress

Calculations above 4-nucleon breakup threshold energy

open new possibilities of 3NF study in 4N-scattering.

up to 35 MeV

A. Deltuva and A.C. Fonseca
Phys. Rev. C 87, 054002 (2013)

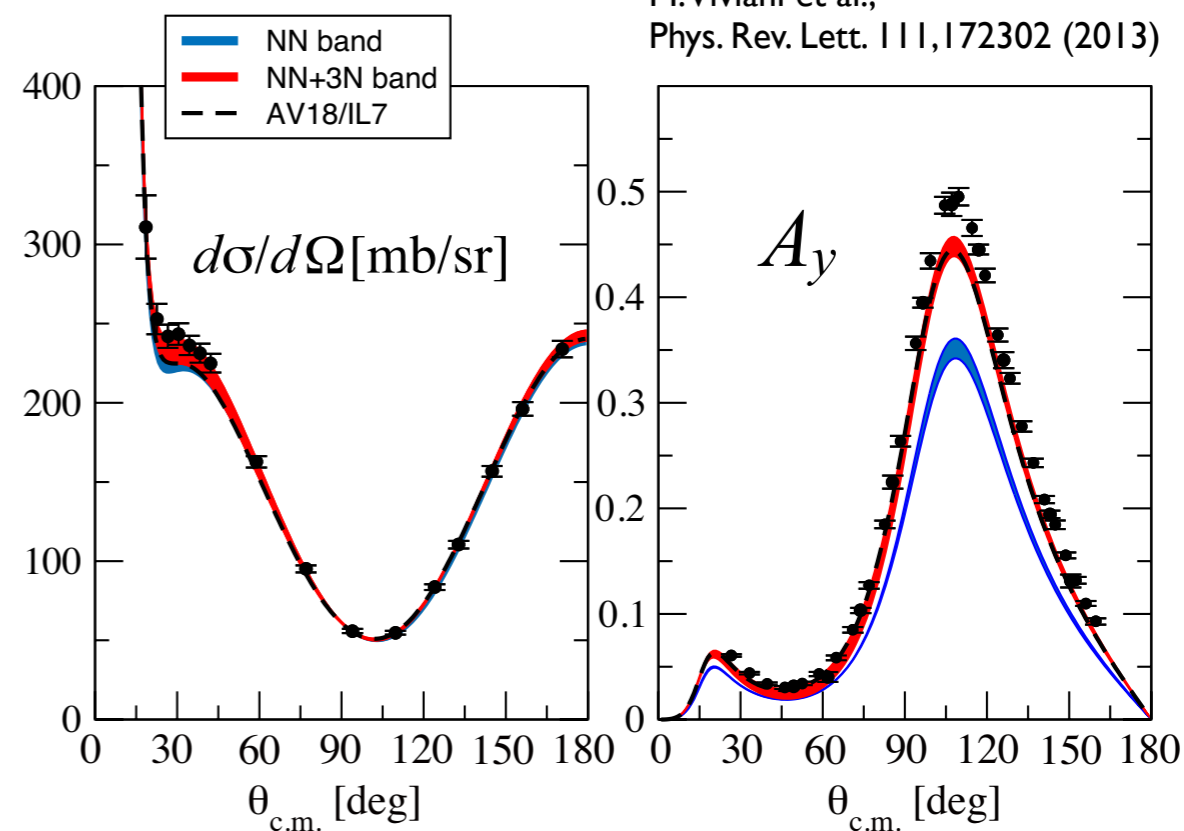


Discrepancies in cross section minimum
at higher energies

New rooms for 3NF study

at 5.54 MeV

M. Viviani et al.,
Phys. Rev. Lett. 111, 172302 (2013)

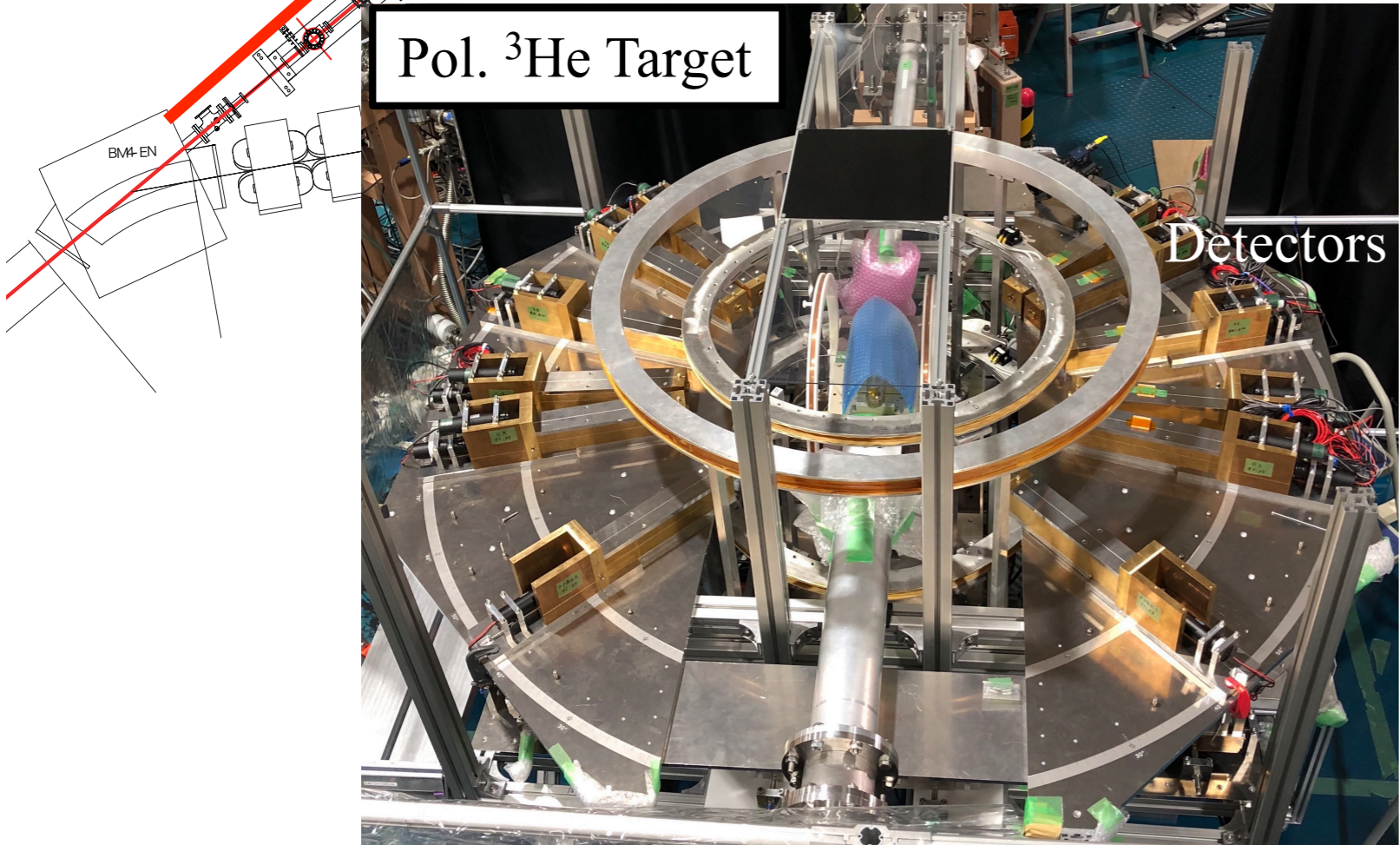
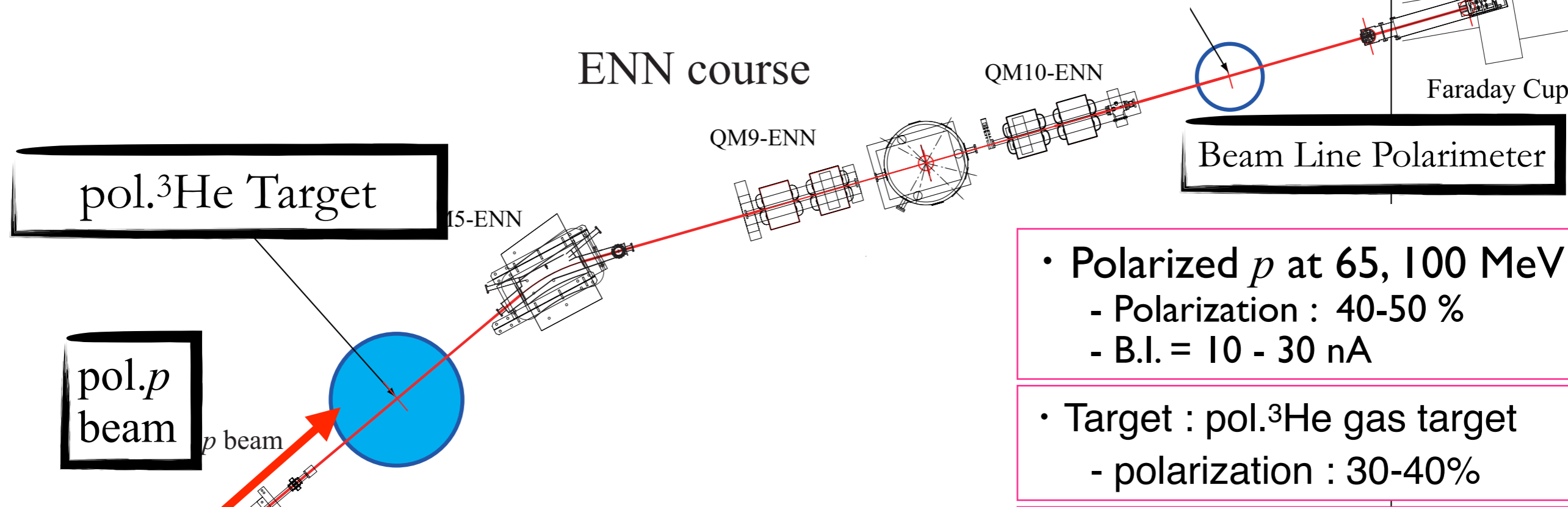


- No signature of 3NFs in cross section

- $A_y(p)$ puzzle : 3NFs sensitive to p -shell nuclei
improve the agreement to the data.

How about spin observables at higher energy?

pol.p+pol.³He experiment at RCNP



- Polarized p at 65, 100 MeV
 - Polarization : 40-50 %
 - B.I. = 10 - 30 nA

- Target : pol.³He gas target
 - polarization : 30-40%

- Observables :

$$A_y(p), A_y(^3\text{He}), C_{y,y}$$

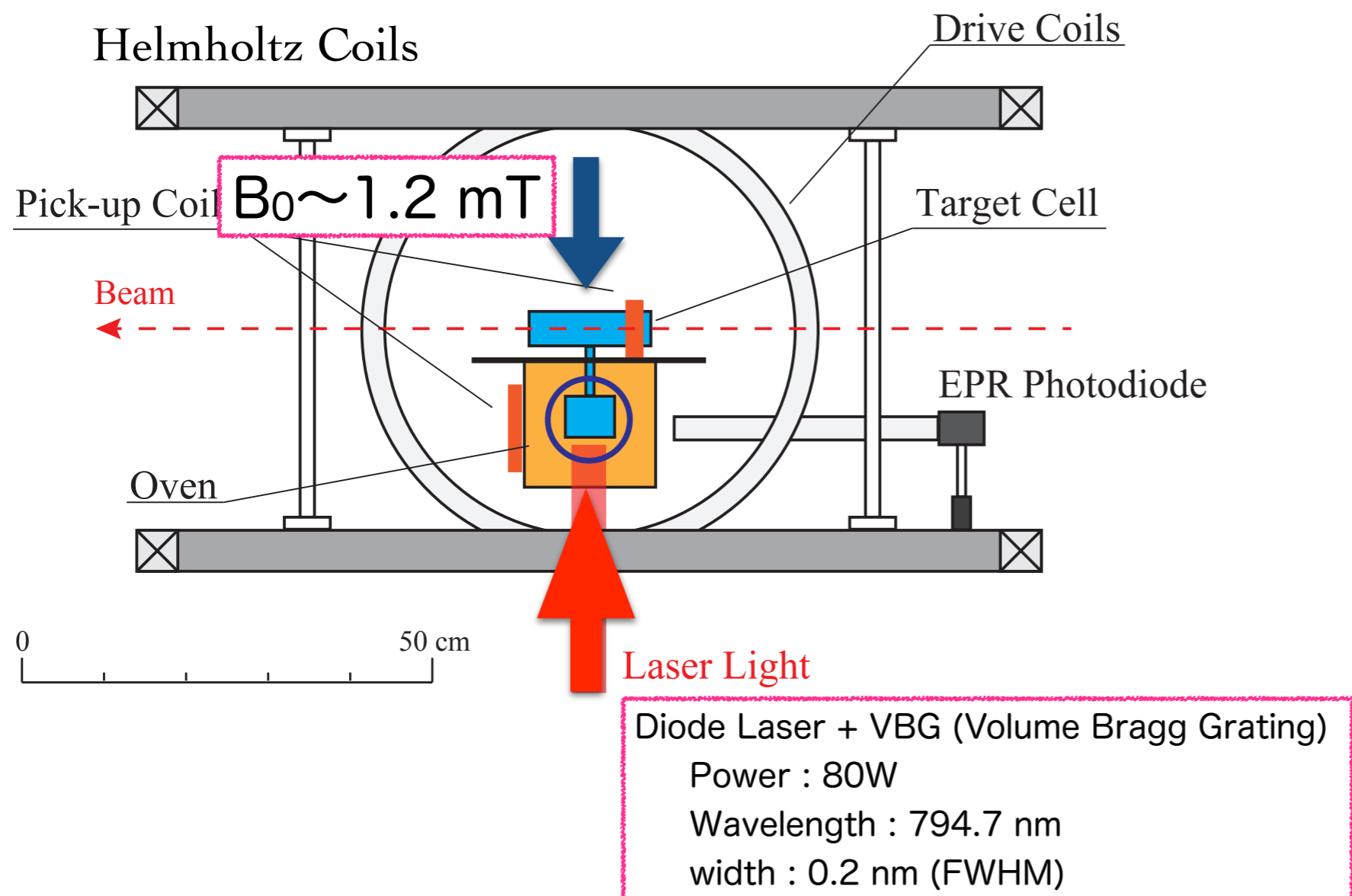
- Measured Angles

$$\theta_{\text{c.m.}} = 47^\circ - 156^\circ$$

- Detectors
 - dE : Plastic Scintillator (0.5 mm^t)
 - E : NaI(Tl) (55 mm^t)
 - 2 sets × 6 angles = 12 sets

Polarized ^3He Target System

- Polarization Method :
 - (Alkali-Hybrid) Spin Exchange Optical Pumping
- Polarization (current) : 60%, Relaxation time : about 40 hrs
- Calibration of absolute values : EPR & neutron-transmission



Summary of Measurements for $p+^3\text{He}$

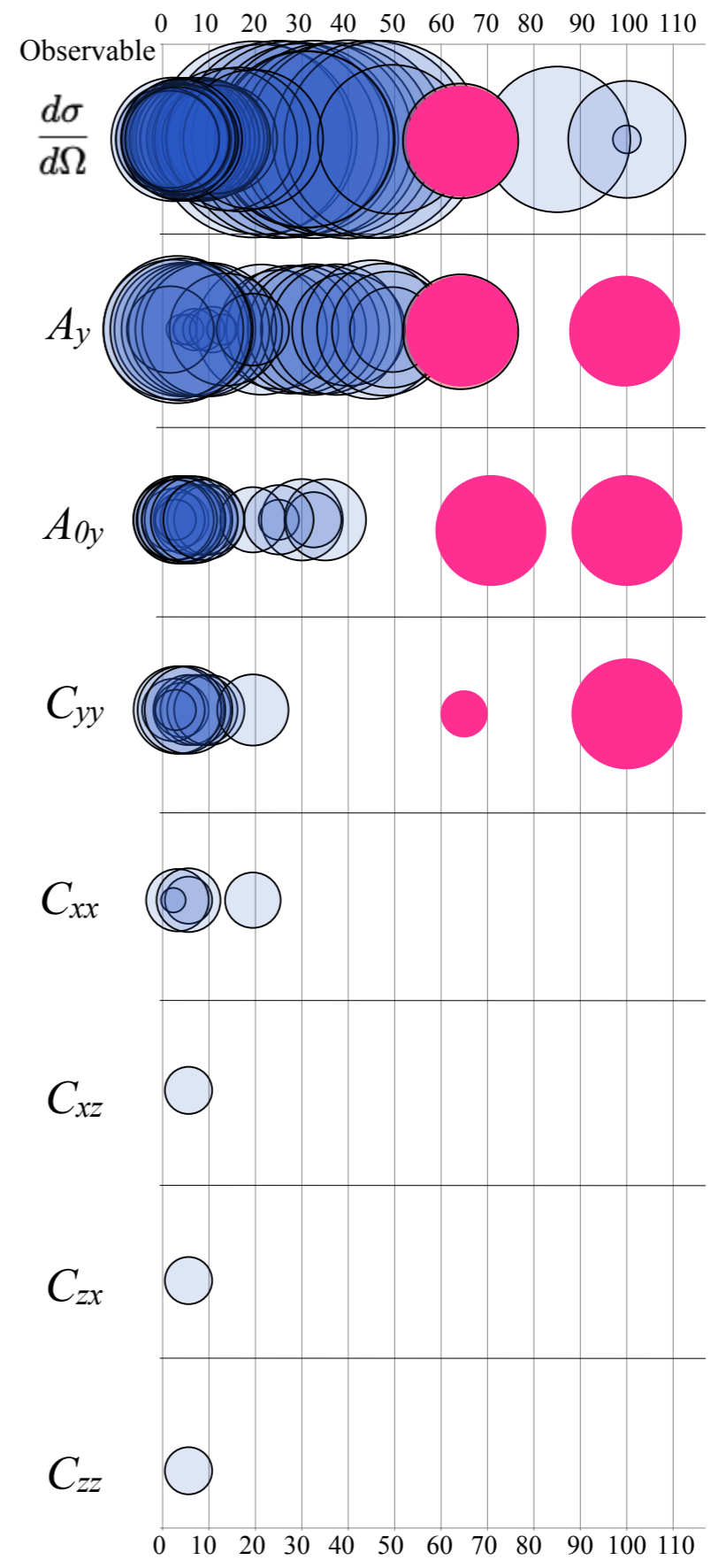
Incident Energy	70 MeV	50 MeV	65 MeV	65 MeV	100 MeV
Beam	p	p	pol. p	pol. p	pol. p
Observables	A_{0y}	A_{0y}	$d\sigma/d\Omega, A_y$	$A_y, A_{0y}, C_{y,y}$	$A_y, A_{0y}, C_{y,y}$
Measured Angles ($\theta_{\text{c.m.}}$)	$46^\circ - 141^\circ$	$47^\circ - 120^\circ$	$27^\circ - 170^\circ$	$47^\circ - 133^\circ$	$47^\circ - 149^\circ$
Facility	CYRIC, Tohoku Univ.	CYRIC, Tohoku Univ.	RCNP, Osaka Univ.	RCNP, Osaka Univ.	RCNP, Osaka Univ.
Exp. Course	41 course	41 course	WS course	ENN course	ENN course

Summary of Measurements for $p+{}^3\text{He}$

Incident Energy	70 MeV	50 MeV
Beam	p	p
Observables	A_{0y}	A_{0y}
Measured Angles ($\theta_{\text{c.m.}}$)	$46^\circ - 141^\circ$	$47^\circ - 120^\circ$
Facility	CYRIC, Tohoku Univ.	CYRIC, Tohoku Univ.
Exp. Course	41 course	41 course

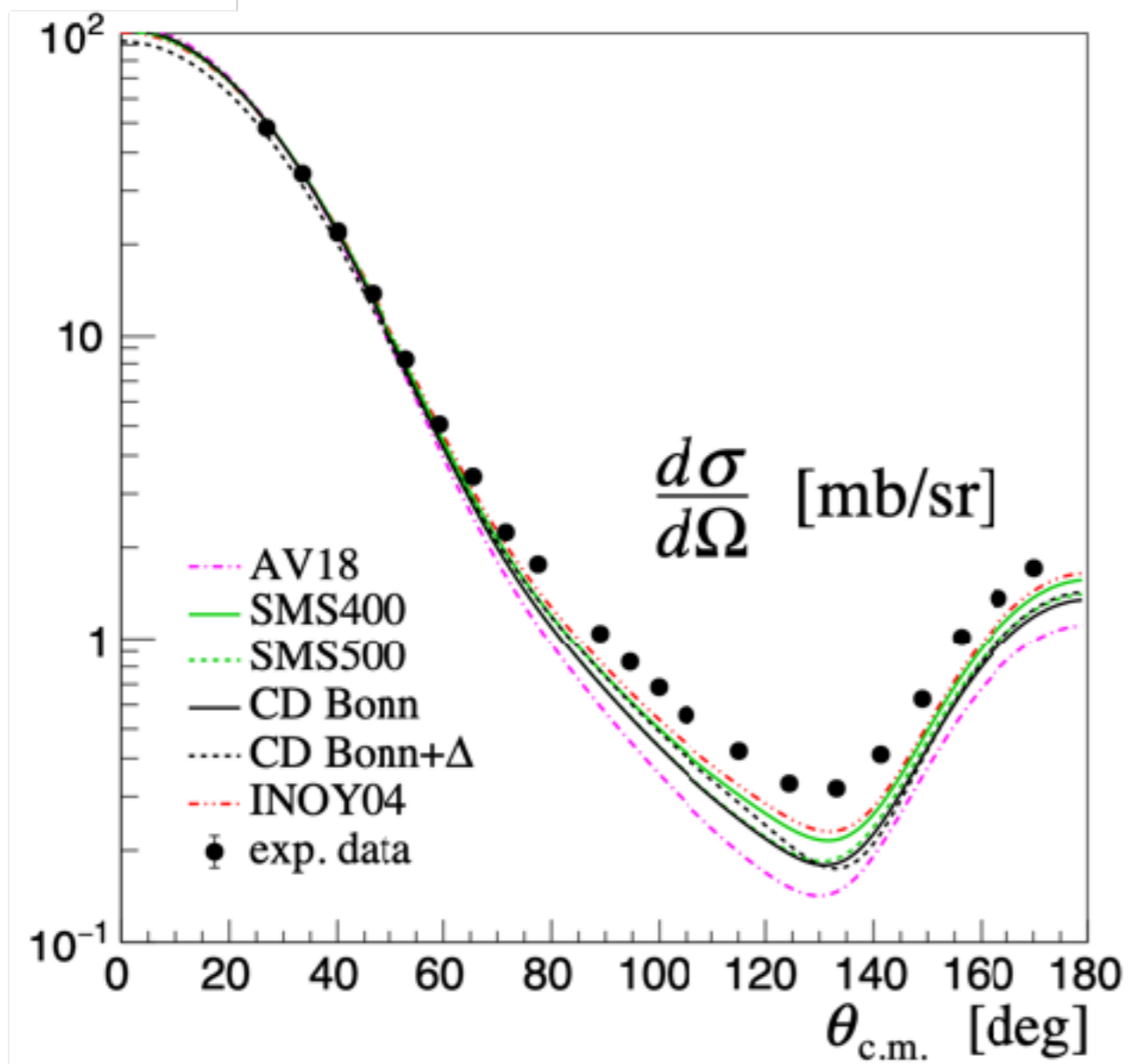
● data from RCNP/CYRIC

Data Table E_p [MeV]

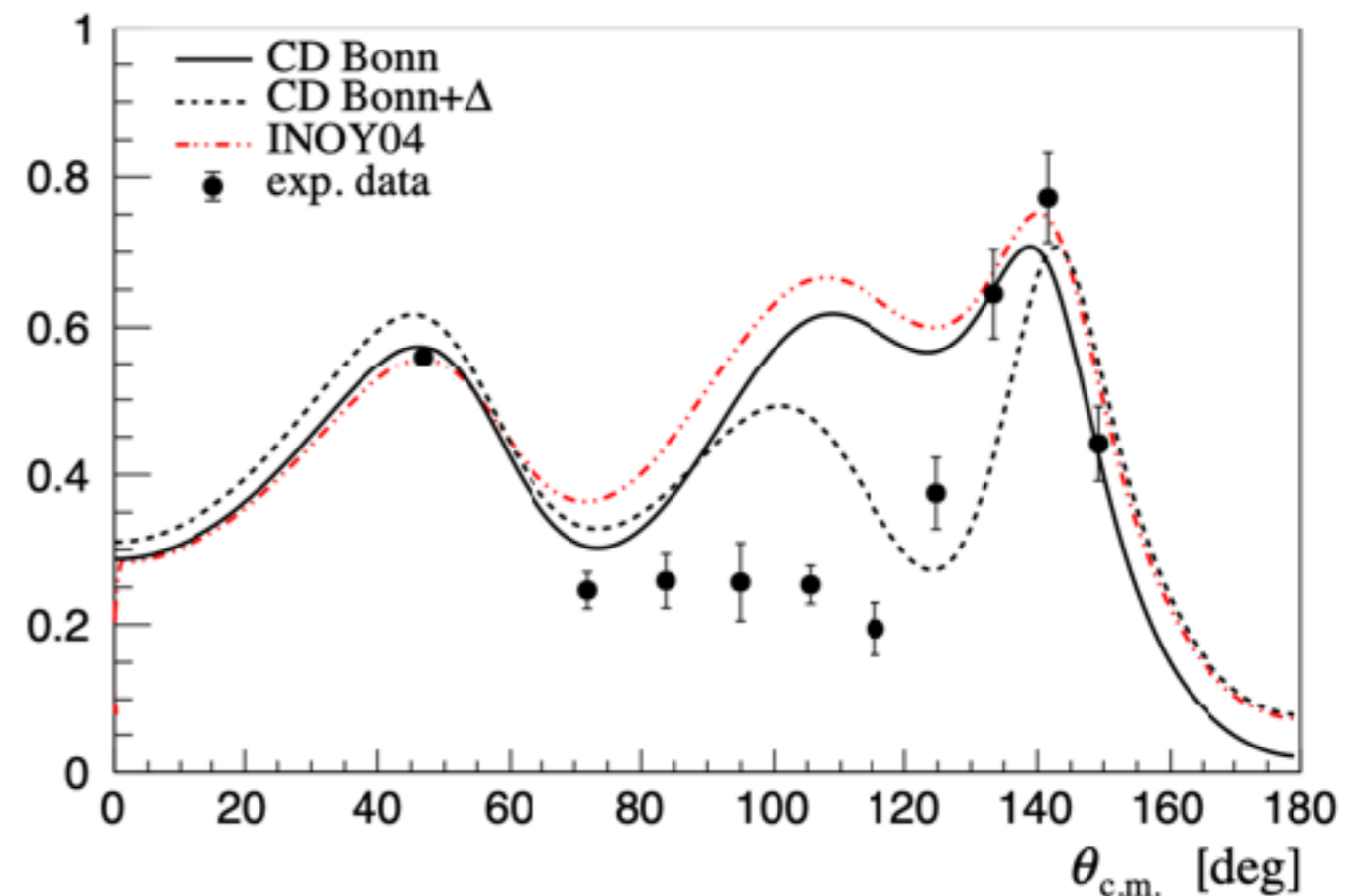


New Data of $p+{}^3\text{He}$ at Intermediate Energies

Cross Section at 65 MeV

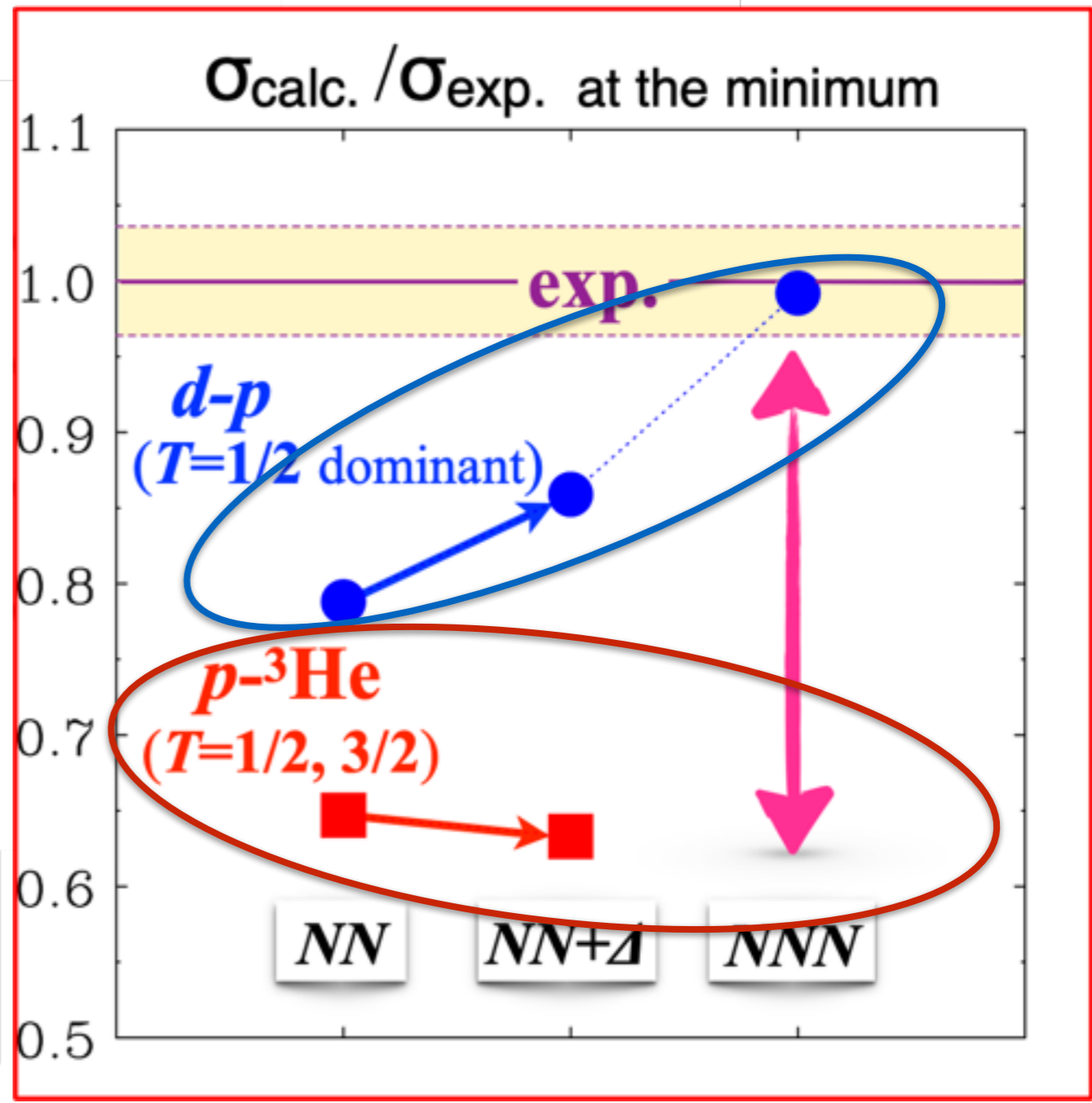
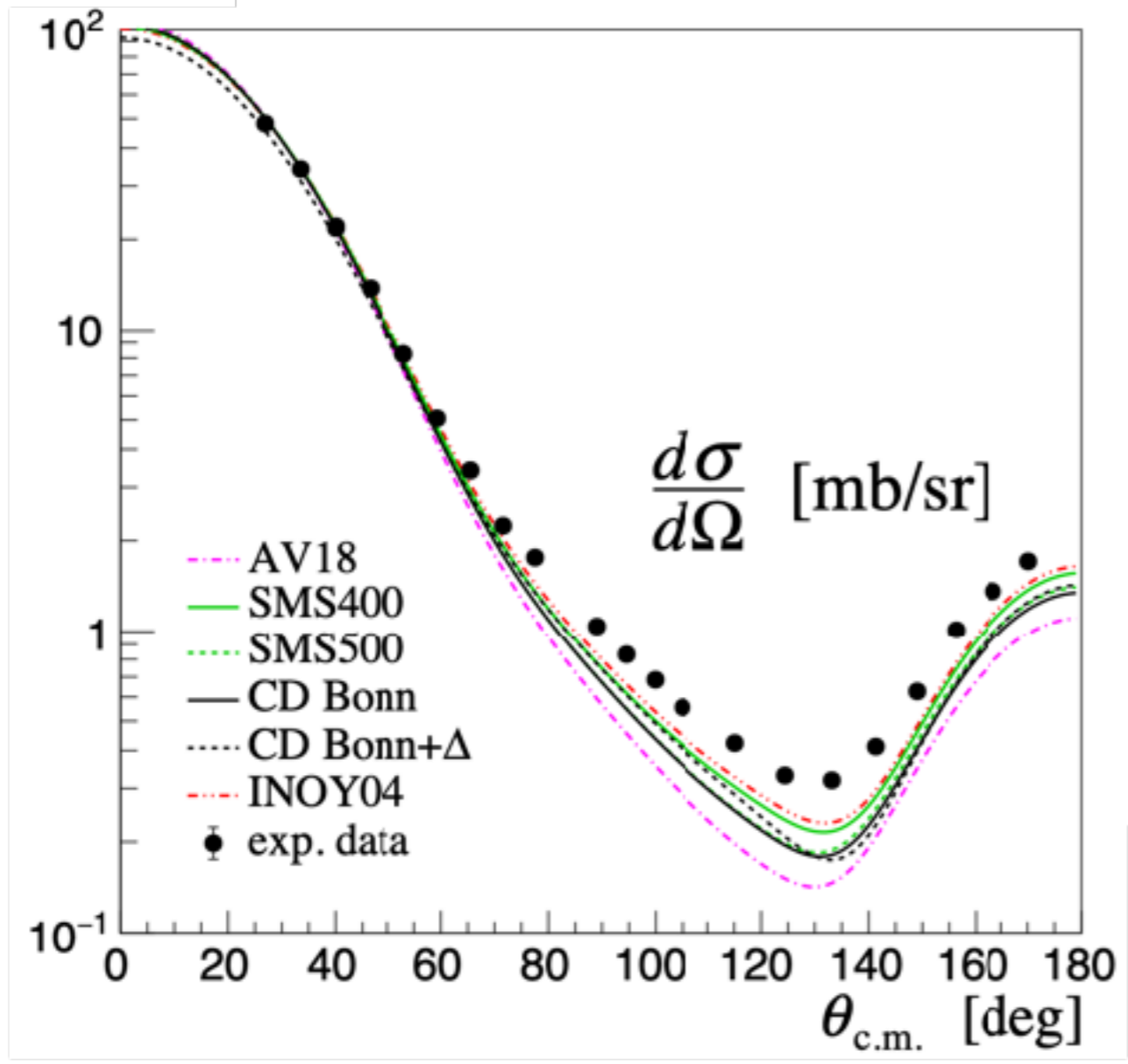


$C_{y,y}$ at 100 MeV



New Data of $p+^3\text{He}$ at Intermediate Energies

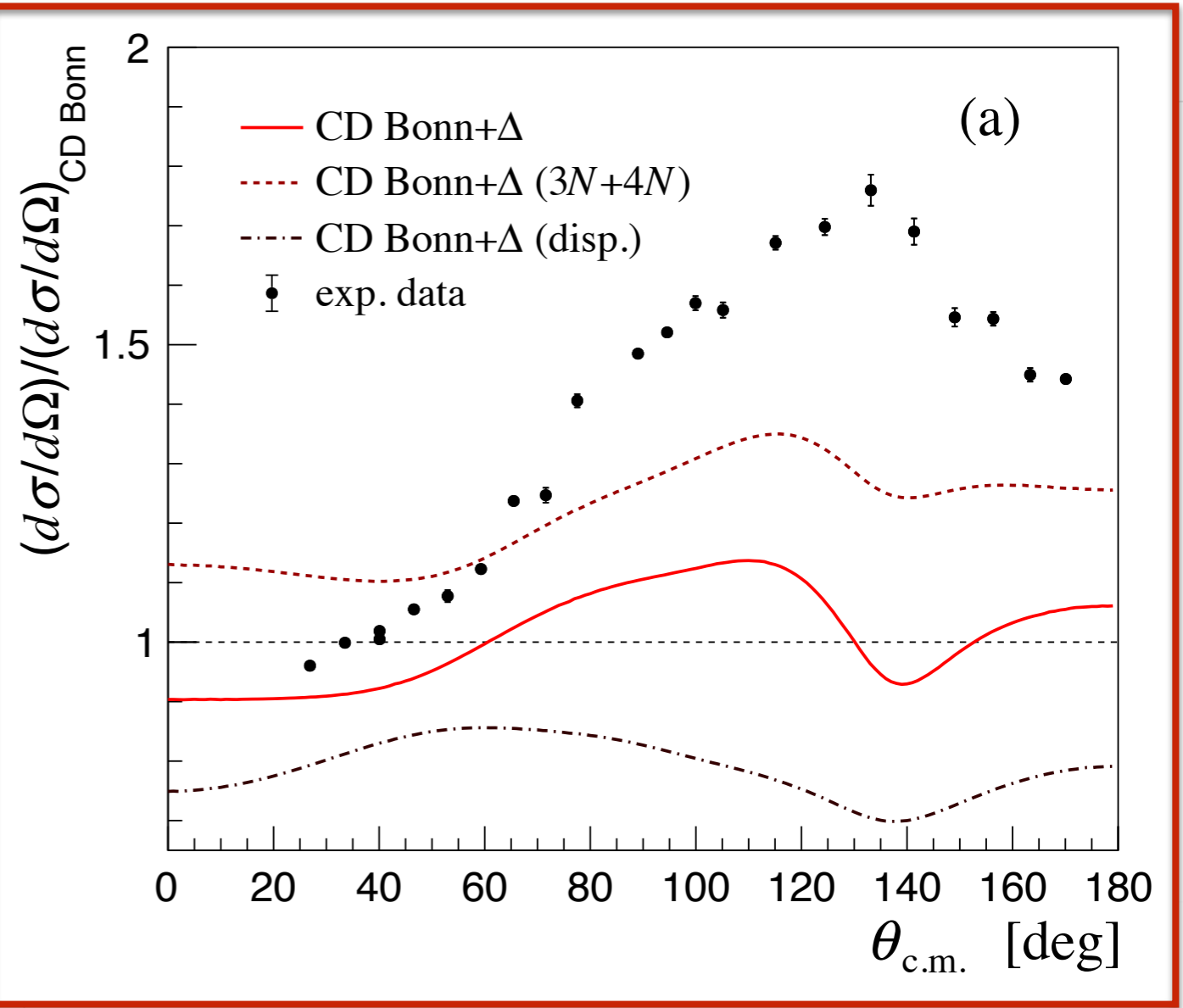
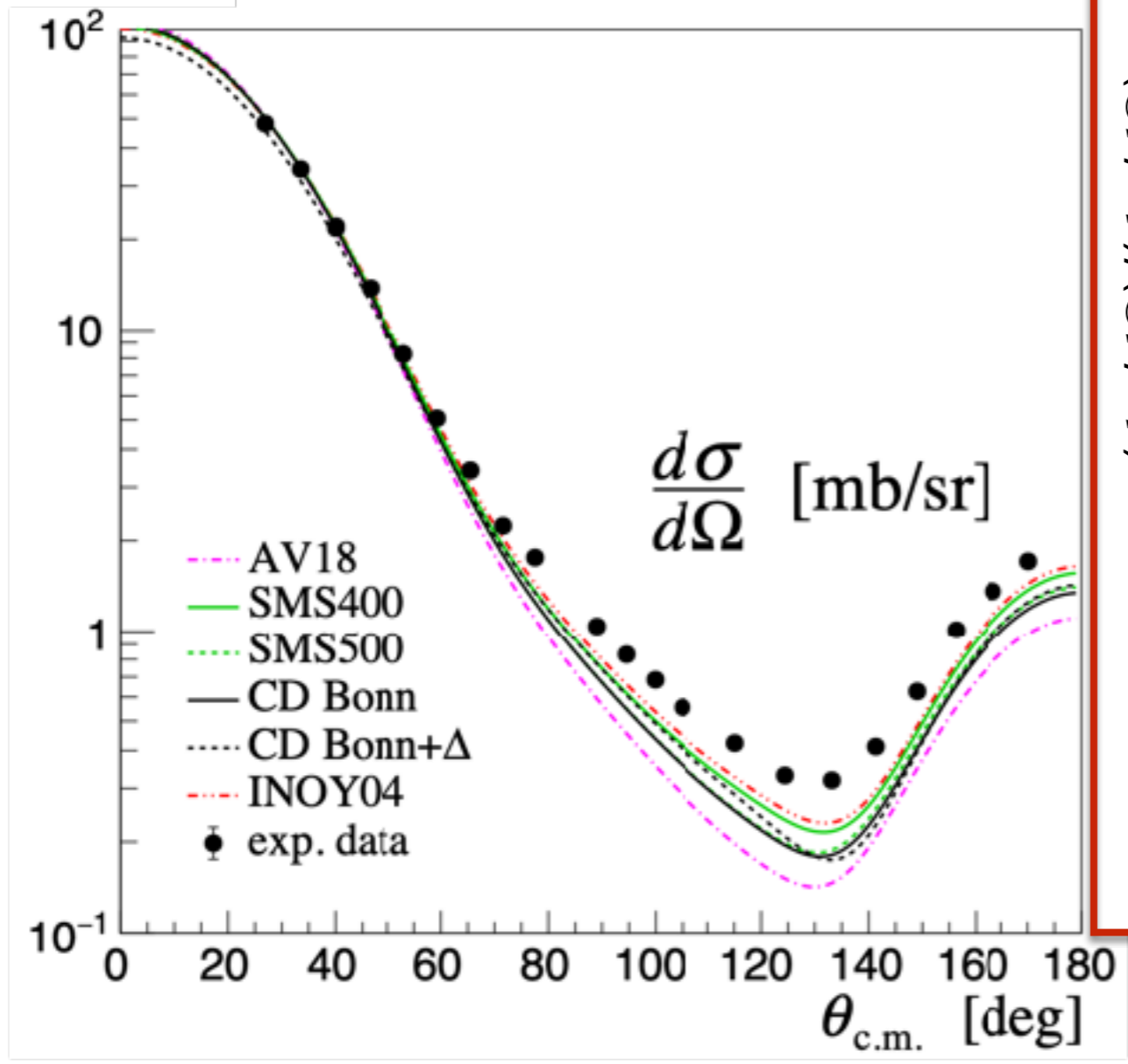
Cross Section at 65 MeV



A. Watanabe, S. Nakai, et al. , Phys. Rev. C 103, 044001 (2021)
Selected as Editors' Suggestion

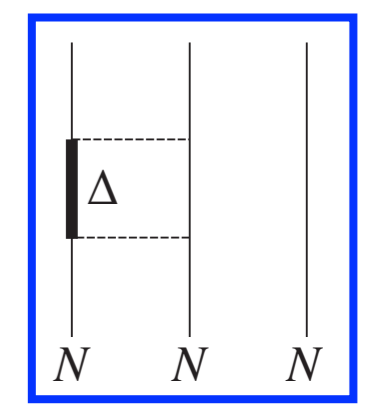
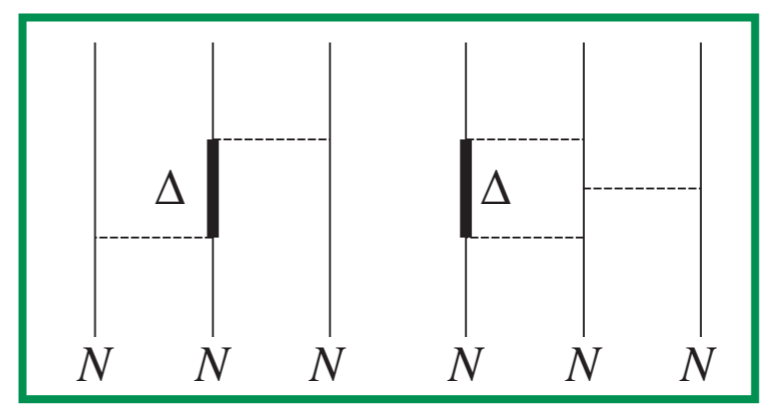
New Data of $p+^3\text{He}$ at Intermediate Energies

Cross Section at 65 MeV



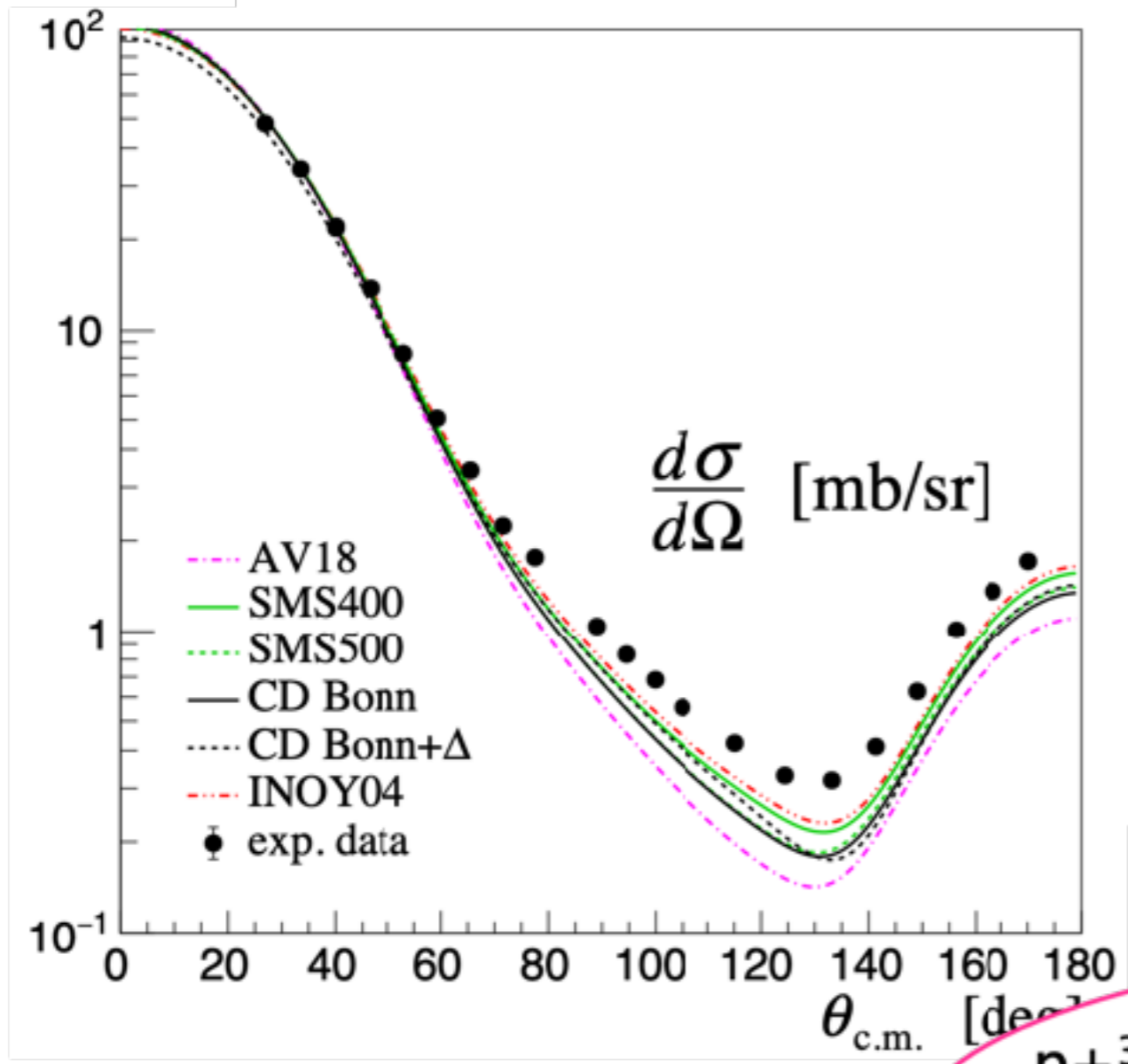
Effective 3, 4NFs

2N dispersion

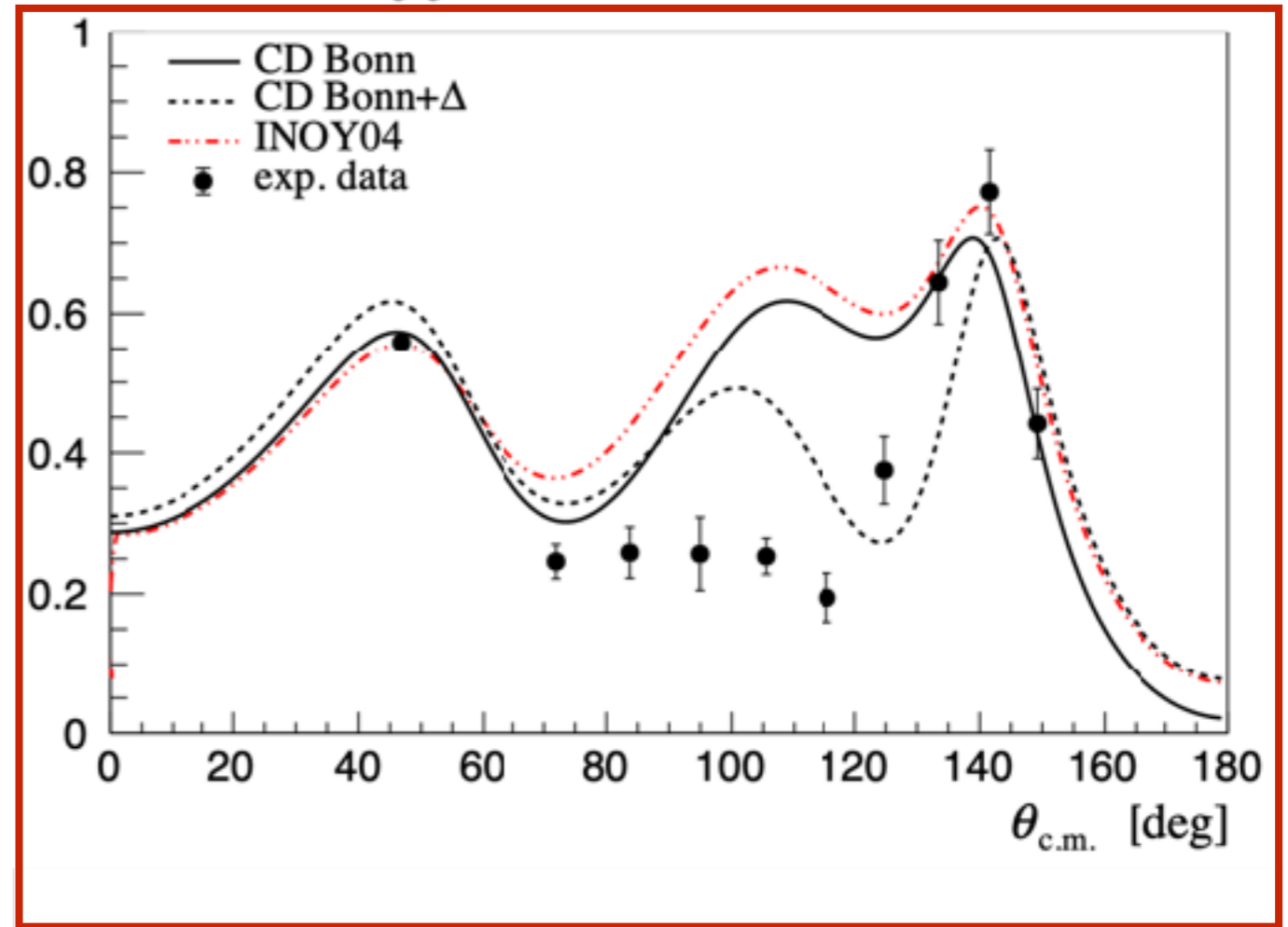


New Data of $p+^3\text{He}$ at Intermediate Energies

Cross Section at 65 MeV



$C_{y,y}$ at 100 MeV



$p+^3\text{He}$ scattering at intermediate energies is an excellent tool to explore nuclear interactions not accessible by Nd scattering.

Summary

To understand nuclear forces is a hot topic of nuclear physics.

Frontiers of nuclear force study
to understand nuclear forces from quarks
to understand nuclei/matter from 2 & 3-NFs

3NFs are key elements to fully understand nuclear properties;
a few-, many-, and infinite-nucleon systems.

dp scattering at ~ 100 MeV/nucleon inspires
quantitative discussions on 3NFs.

Determination of 3NFs based on χ EFT
from few-nucleon scattering data is about to start.

- dp scattering : LECs of 3NFs
- p - ^3He scattering : test χ EFT 3NFs, including iso-spin dependence

towards **High Precision 2N and 3NFs**

Perspective of 3NF Study

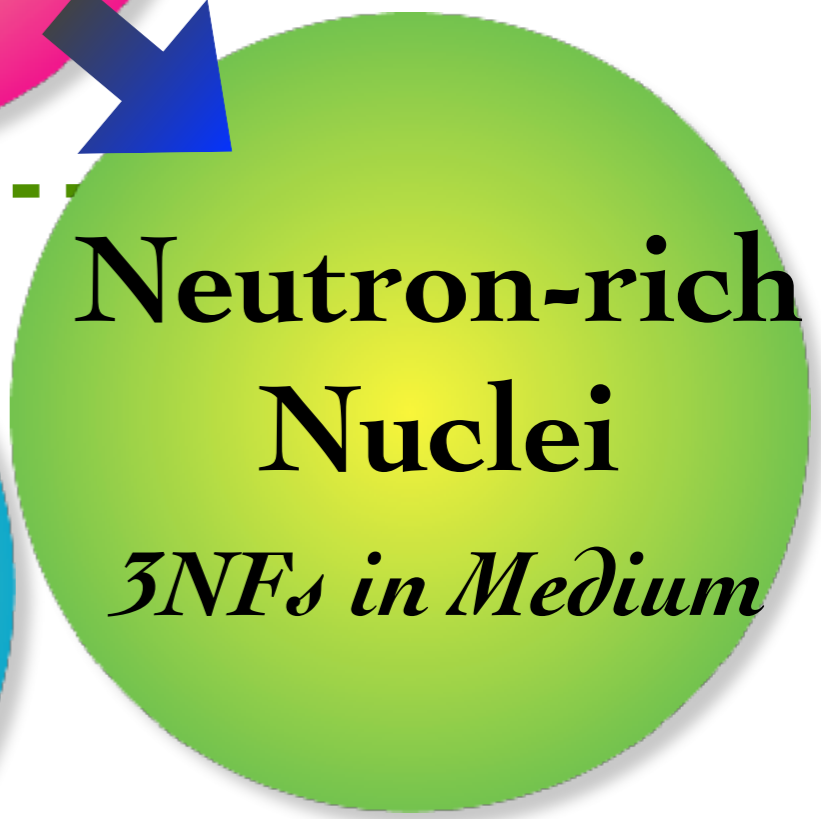
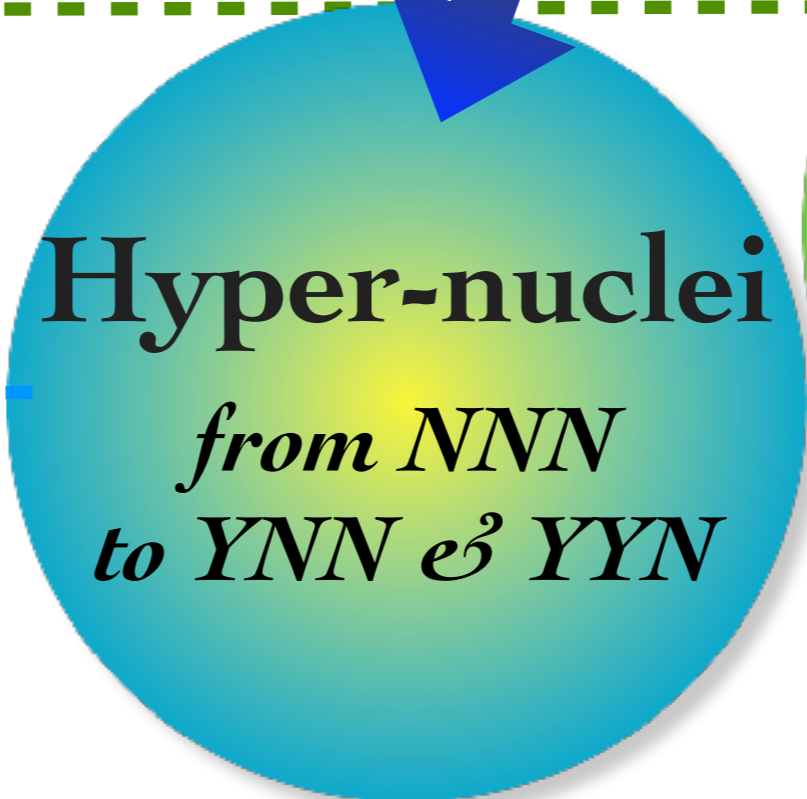
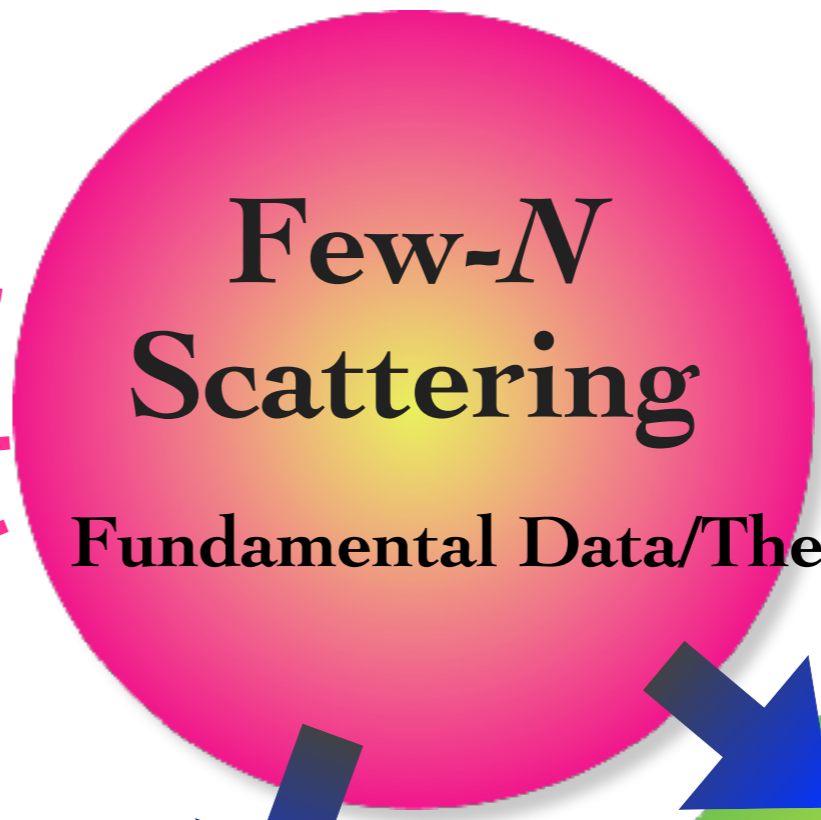
~Consistent Understanding From Quarks to the Universe~

Momentum

Spin

Isospin

Strangeness



RIKEN RIBF-*d*. Collaboration

Department of Physics, Tohoku University

K. Sekiguchi, K. Miki, Y. Wada, A. Watanabe, D. Eto, T. Akieda, H. Kon,
J. Miyazaki, T. Taguchi, U. Gebauer, K. Takahashi, T. Mashiko

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N. Sakamoto, H. Sakai, T. Uesaka, M. Sasano, Y. Shimizu

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RCNP, Osaka University

H. Okamura

Kyungpook National University

S. Chebotaryov, E. Milman

Experiment at RIKEN RIBF (2015)



p - ^3He Collaboration

Department of Physics, Tohoku University

K. Sekiguchi, **Y. Wada**, Y. Shiokawa, **A. Watanabe**, **S. Nakai**, K. Miki,
T. Mukai, S. Shibuya, M. Watanabe, K. Kawahara, D. Sakai,
T. Taguchi, D. Eto, T. Akieda, H. Kon, M. Inoue, Y. Utsuki

Experiment at CYRIC, Tohoku Univ. (2016)

CYRIC, Tohoku University

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K. Hatanaka, A. Tamii, H.J. Ong, H. Kanda,
N. Kobayashi, A. Inoue, S. Nakamura, D. T. Tran

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H. Kasahara, S. Mitsumoto, H. Oshiro

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RIKEN Nishina Center

H. Sakai

RIKEN RANS

Y. Otake, A. Taketani, Y. Wakabayashi

NIRS

T. Wakui



Experiment at RCNP, Osaka Univ. (2018)

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H. Witała, J. Góolak, R. Skibinski

Kyushu Institute of Technology

H. Kamada

Forschungszentrum of Jülich

A. Nogga

Vilnius University

A. Deltuva

Hannover University

P. U. Sauer, S. Nemoto

Lisbon University

A. Sa. Fonseca

Hosei University

S. Ishikawa



Bad Honnef (2006)