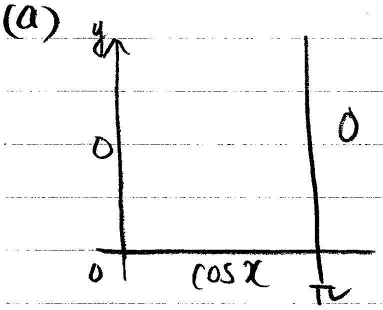
R1.

(a)



$$\nabla^2 T = \frac{1}{\alpha^2}\frac{\partial T}{\partial t}$$

As $t \to \infty$, T approaches steady state, Ts.
At steady state, $\frac{\partial T_s}{\partial t} = 0$.

$\therefore \nabla^2 T_s = 0$ (Laplace equation) $\Rightarrow T_s(x,y) = X(x)Y(y)$

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} = 0$$

$$\frac{1}{X}\frac{d^2X}{dx^2} = -k^2 \Rightarrow X(x) = A\cos kx + B\sin kx$$

From the boundary conditions

$X(0) = A = 0$

$X(\pi) = B\sin k\pi = 0$

$\therefore k = 0, \pm 1, \pm 2, \ldots$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = k^2 \Rightarrow Y(y) = Ce^{ky} + De^{-ky}$$

Let $k > 0$.

Then $Y(y) < \infty$ as $y \to \infty$.

$\therefore C = 0$ and $Y(y) = De^{-ky}$

$T_s^m(x,y) = \sin mx \, e^{-my}$ where $m = 1, 2, 3, \ldots$

$$T_s(x,y) = \sum_{m=1}^{\infty} C_m \sin mx \, e^{-my}$$

Another boundary condition is $T(x,0) = \cos x$

$$T_s(x,0) = \sum_{m=1}^{\infty} C_m \sin mx = \cos x$$

$$C_m = \frac{2}{\pi}\int_0^{\pi} \cos x \sin(mx)\, dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\frac{1}{2}[\sin(x+mx)-\sin(x-mx)]dx$$

$$= \frac{2}{\pi}\frac{1}{2}\int_0^{\pi}[\sin(m+1)x+\sin(m-1)x]dx$$

$$= \frac{1}{\pi}\ \frac{-\cos(m+1)x}{m+1} - \frac{\cos(m-1)x}{m-1}\Bigg|_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{1-\cos(m+1)\pi}{m+1}+\frac{1-\cos(m-1)\pi}{m-1}\right]$$

If $m=\text{odd}$, $= \frac{1}{\pi}\left[\frac{1-1}{m-1}+\frac{1-1}{m-1}\right]=0$

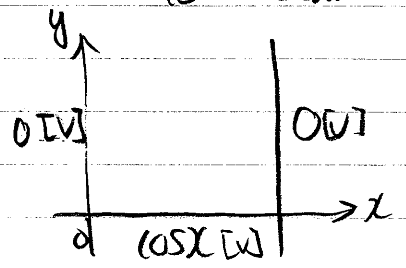
If $m=\text{even}$, $= \frac{1}{\pi}\left[\frac{1+1}{m+1}+\frac{1+1}{m-1}\right]=\frac{4}{\pi}\frac{m}{m^2-1}$

Finally, $T_s(x,y)=\sum_{m=1}^{\infty}\frac{1}{\pi}\frac{m}{m^2-1}\sin(mx)e^{-my}$

(b) $V_s(x,y)=\sum_{m=1}^{\infty}\frac{1}{\pi}\frac{m}{m^2-1}\sin(mx)e^{-my}$ for electrical potential.

To obtain electric field $E=-\nabla V_s$

$$= -\frac{\partial V_s}{\partial x}\hat{i}+\frac{\partial V_s}{\partial y}\hat{j}$$



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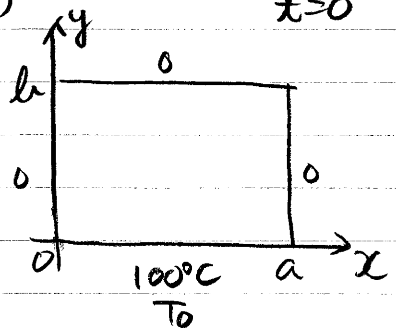
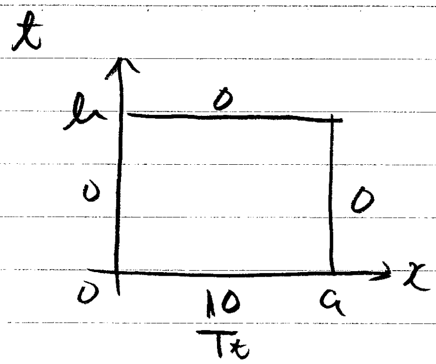




$$\nabla^2 T = \frac{1}{\alpha^2}\frac{\partial T}{\partial t}$$

First find the steady state solution.

As $t \to \infty$, $T \to T_s$ and $\frac{\partial T}{\partial t} = 0$

$$\nabla^2 T_s = 0$$

As in (a), $X(x) = A\cos kx + B\sin kx$

$$X(0) = A = 0$$

$$X(a) = B\sin ka = 0$$

$$ka = m\pi \quad \text{where } m = \pm 1, \pm 2, \pm 3, \dots$$

$$k = \frac{m\pi}{a}$$

$$X(x) = \sin\left(\frac{m\pi}{a}x\right)$$

For $Y(y)$, $\frac{1}{Y}\frac{d^2Y}{dy^2} = k^2 \Rightarrow Y(y) = Ce^{ky} + De^{-ky}$

From the boundary condition $Y(b) = 0$,

$$Y(b) = Ce^{kb} + De^{-kb} = 0 \quad -①$$

$$T_s^m(x,y) = \sum_{m=1}^{\infty}\left(C_m e^{\frac{m\pi y}{a}} + D_m e^{-\frac{m\pi y}{a}}\right)\sin\left(\frac{m\pi}{a}x\right)$$

At $x=0$, $T_s(x,0) = T_t = \sum_{m=1}^{\infty}(C_m + D_m)\sin\left(\frac{m\pi}{a}x\right) \quad -②$

$$C_m + D_m = \frac{2}{a}\int_0^a T_t \sin\left(\frac{m\pi x}{a}\right)dx$$

$$= \frac{2T_t}{m\pi}\left[1-(-1)^m\right] \quad -②'$$

Writing ① and ②' together, we get

$$\begin{cases} C_m e^{\frac{m\pi b}{a}} + D_m e^{-\frac{m\pi b}{a}} = 0 & -① \\ C_m + D_m = \frac{2T_t}{m\pi}[1-(-1)^m] & -②' \end{cases}$$

$① - ②' e^{\frac{m\pi b}{a}}$

$$D_m e^{-\frac{m\pi}{a}b} - D_m e^{\frac{m\pi b}{a}} = -\frac{2T_t}{m\pi}[1-(-1)^m]e^{\frac{m\pi b}{a}}$$

$$-2D_m \sinh\left(\frac{m\pi b}{a}\right) = -2\frac{T_t}{m\pi}[1-(-1)^m]e^{\frac{m\pi b}{a}}$$

$$\begin{cases} D_m = \frac{T_t}{m\pi}\frac{1}{\sinh\left(\frac{m\pi b}{a}\right)}[1-(-1)^m]e^{\frac{m\pi b}{a}} \\ C_m = -D_m e^{-2\frac{m\pi b}{a}} = -\frac{T_t}{m\pi}\frac{1}{\sinh\left(\frac{m\pi b}{a}\right)}[1-(-1)^m]e^{-\frac{m\pi b}{a}} \end{cases}$$

Finally, $T_s(x,y) = \sum_{m=1}^{\infty} \frac{T_t}{m\pi}\frac{[1-(-1)^m]}{\sinh\left(\frac{m\pi b}{a}\right)}\left\{-e^{\frac{m\pi}{a}(y-b)} + e^{-\frac{m\pi}{a}(y-b)}\right\}$

$$= \sum_{m=1}^{\infty} \frac{T_t}{m\pi}\frac{[1-(-1)^m]}{\sinh\left(\frac{m\pi b}{a}\right)}\left(-2\sinh\frac{m\pi}{a}(y-b)\right)$$

$$\boxed{T_s(x,y) = \sum_{m=1}^{\infty} \frac{2T_t}{m\pi}\frac{[1-(-1)^m]}{\sinh\left(\frac{m\pi b}{a}\right)}\sinh\frac{m\pi}{a}(b-y)}$$

Now we will proceed to find the full solution with the initial condition

$$T(x,y,t) = T_p(x,y,t) + T_s(x,y)$$

where $T_p(x,y,t)$ is a transient (time dependent) part of $T(x,y,t)$.

$$T_p(x,y,t) = T(x,y,t) - T_s(x,y)$$

Now look at the boundary conditions for T_p.

$$T_p(0,y,t) = 0 - 0 = 0$$

$T_p(a, y, t) = 0 - 0 = 0$
$T_p(x, 0, t) = T_t - T_t = 0$
$T_p(x, b, t) = 0 - 0 = 0$

Thus, at the boundary $T_p(x, y, t) = 0$

Now the problem to solve is

$$\boxed{\begin{array}{l} \nabla^2 T_p = \frac{1}{\alpha^2}\frac{\partial T_p}{\partial t} \\ T_p(x,y,t) = 0 \quad \text{at the boundary.} \\ T_p(x,y,0) = T_0 - T_s(x,y) \quad \text{as the initial condition} \end{array}}$$

Let $T_p(x,y,t) = X_p(x) Y_p(y) f(t)$, then

$$\frac{1}{X_p}\frac{d^2X_p}{dx^2} + \frac{1}{Y_p}\frac{d^2Y_p}{dy^2} = \frac{1}{\alpha^2}\frac{1}{f}\frac{df}{dt} = -k^2$$

$$f(t) = A e^{-\alpha^2 k^2 t} \Rightarrow f(0) = A = T_0 = 100°C$$

$$\frac{1}{X_p}\frac{d^2X_p}{dx^2} = -k_x^2 \Rightarrow X_p(x) = B\cos k_x x + C\sin k_x x$$

$$X_p(0) = B = 0$$
$$X_p(a) = C\sin k_x a = 0 \quad \therefore\ k_x = \frac{n\pi}{a}$$

$$\therefore\ X_p(x) = \sin\frac{n\pi}{a}x$$

Similarly, $Y_p(y) = \sin\left(\frac{m\pi}{b}y\right)$ where $\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 = k^2$

Then we get
$$T_p^{m,n}(x,y,t) = \sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{b}y\right)e^{-\alpha^2\left\{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right\}t}$$

$$T_p(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{m,n}\, T_p^{m,n}(x,y,t) \quad -②$$

From the initial condition

$$T_p(x,y,0) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{m,n}\, T_p^{m,n}(x,y,0) = T_0 - T_s(x,y)$$

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{m,n} \sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi y}{b}\right) = T_0 - T_s(x,y)$$

$$T_0 - T_s(x,y) = \sum_{n=1}^{\infty}\left[\sum_{m=1}^{\infty} C_{m,n}\sin\left(\frac{m\pi y}{b}\right)\right]\sin\left(\frac{n\pi}{a}x\right)$$

$$\sum_{m=1}^{\infty} C_{m,n}\sin\left(\frac{m\pi y}{b}\right) = \underbrace{\frac{2}{a}\int_0^a [T - T_s(x,y)]\sin\left(\frac{n\pi}{a}x\right)dx}_{d_n(y)}$$

$$d(x) = \sum_{m=1}^{\infty} C_{m,n}\sin\left(\frac{m\pi y}{b}\right)$$

$$C_{m,n} = \frac{2}{b}\int_0^b d_n(y)\sin\left(\frac{m\pi y}{b}\right)dy$$

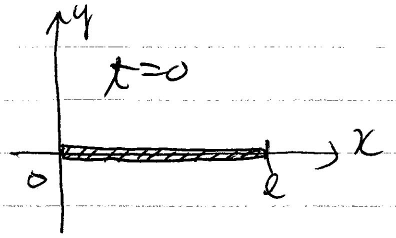
$$= \frac{2}{b}\int_0^b \frac{2}{a}\int_0^a [T - T_s(x,y)]\sin\left(\frac{n\pi}{a}x\right)dx\,\sin\left(\frac{m\pi y}{b}\right)dy$$

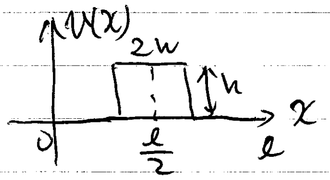
$$= \frac{4}{ab}\int_0^b\int_0^a (T - T_s(x,y))\sin\left(\frac{n\pi}{a}x\right)dx\,\sin\left(\frac{m\pi}{b}y\right)dy$$

Finally

$$\boxed{\begin{aligned} &T(x,y,t) = T_p(x,y,t) - T_s(x,y) \\ &T_p = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{m,n}\sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi y}{b}\right)e^{-\alpha^2\left\{\left(\frac{n\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right\}t} \\ &C_{m,n} = \frac{4}{ab}\int_0^b\int_0^a (T - T_s(x,y))\sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{b}y\right)dx\,dy \\ &T_s(x,y) = \sum_{m=1}^{\infty} \frac{2T_t}{m\pi}\frac{[1-(-1)^m]}{\sinh\left(\frac{m\pi b}{a}\right)}\sinh\frac{m\pi}{a}(b-y) \end{aligned}}$$

R2.





Wave equation

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2}$$

Use separation of variable, $\varphi(x,t) = X(x) f(t)$

$$\frac{d^2 X(x)}{dx^2} f(t) = \frac{1}{c^2}\frac{d^2 f(t)}{dt^2} X(x)$$

$$\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \frac{1}{c^2}\frac{1}{f(t)}\frac{d^2 f(t)}{dt^2} = -k^2$$

$$\frac{d^2X(x)}{dx^2} = -k^2 X(x) \Rightarrow X(x) = A\cos kx + B\sin kx$$

$X(0) = A = 0$

$X(l) = B\sin kl = 0 \Rightarrow kl = n\pi \Rightarrow k = \frac{n\pi}{l}$

$$X_n(x) = \sin\left(\frac{n\pi}{l}x\right)$$

$$\frac{1}{c^2}\frac{1}{f(t)}\frac{d^2 f(t)}{dt^2} = -k^2 \Rightarrow f(t) = C\cos kct + D\sin kct$$

$$k_n c = \frac{n\pi}{l}c = \omega_n$$

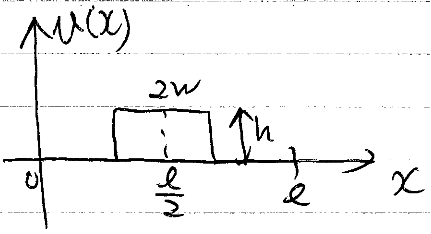
The initial condition is equivalent to hitting the string at $t=0$ with $\left.\frac{\partial \varphi}{\partial t}\right|_{t=0} = v(x)$

Thus, $\varphi(x,0) = 0 \Rightarrow f(0) = C = 0$.

$$\varphi(x,t) = \sum_{n=1}^{\infty} D_n \sin k_n c t \sin\left(\frac{n\pi}{l}x\right)$$

$$= \sum_{n=1}^{\infty} D_n \sin\left(\frac{n\pi}{l}x\right)\sin\omega_n t$$

$$v(x) = \left.\frac{\partial \varphi}{\partial t}\right|_{t=0} = \sum_{n=1}^{\infty} \omega_n D_n \sin\left(\frac{n\pi}{l}x\right)$$



$$v(x) = \begin{cases} 0 & 0 \le x < \ell/2 \\ h & \frac{\ell}{2} - w \le x \le \frac{\ell}{2} + w \\ 0 & \frac{\ell}{2} + w < x \le \ell \end{cases}$$

$$\omega_n D_n = \frac{2}{\ell}\int_0^{\ell} v(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$$

$$= \frac{2}{\ell}\left[\int_0^{\ell/2-w}(0)\sin\left(\frac{n\pi}{\ell}x\right)dx + \int_{\ell/2-w}^{\ell/2+w} h\sin\left(\frac{n\pi}{\ell}x\right)dx + \int_{\ell/2+w}^{\ell}(0)\sin\left(\frac{n\pi}{\ell}x\right)dx\right]$$

$$= \frac{2h}{\ell}\int_{\ell/2-w}^{\ell/2+w}\sin\left(\frac{n\pi x}{\ell}\right)dx$$

$$= \frac{2h}{\ell}\left(-\frac{\ell}{n\pi}\right)\left[\cos\left(\frac{n\pi}{\ell}x\right)\right]_{\ell/2-w}^{\ell/2+w}$$

$$\omega_n D_n = -\frac{2h}{n\pi}\left(\cos\left[\frac{n\pi}{\ell}\left(\frac{\ell}{2}+w\right)\right] - \cos\left[\frac{n\pi}{\ell}\left(\frac{\ell}{2}-w\right)\right]\right)$$

$$= \frac{-2h}{n\pi}(-2)\sin\left(\frac{1}{2}\frac{2n\pi}{\ell}\frac{\ell}{2}\right)\sin\left(\frac{1}{2}\frac{n\pi}{\ell}2w\right)$$

$$D_n = \frac{4h}{n\pi}\frac{1}{\omega_n}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi w}{\ell}\right)$$

$D_n = 0$ when $n = 2m$

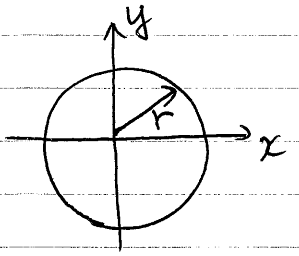
$$D_{2m+1} = \frac{4h}{n\pi}\frac{1}{\omega_n}\sin\left(m\pi + \frac{\pi}{2}\right)\sin\left((2m+1)\frac{\pi w}{\ell}\right)$$

$$= \frac{4h}{n\pi}\frac{\ell}{n\pi c}\sin\left((2m+1)\frac{\pi w}{\ell}\right)(-1)^m$$

where $m = 0, 1, 2, 3 \ldots$

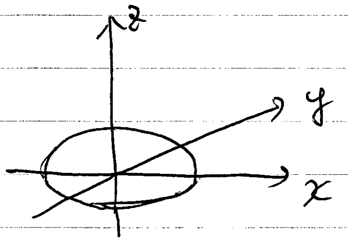
Finally
$$\boxed{\begin{gathered}\varphi(x,t) = \sum_{m=1}^{\infty}(-1)^m\frac{4h\ell}{n^2\pi^2}\frac{1}{c}\sin\left((2m+1)\frac{\pi w}{\ell}\right)\sin(\omega_{2m+1}t) \\ \text{where } \omega_{2m+1} = (2m+1)\frac{\pi}{\ell}c\end{gathered}}$$

C2.



Let the displacement of the membrane be $z(r,\theta)$ in polar coordinate.

The wave equation for this 2D problem is $\nabla^2 z = \frac{1}{C^2}\frac{\partial^2 z}{\partial t^2}$ — ①



Let $z = F(r,\theta) f(t)$ and put it into ①.

$$(\nabla^2 F(r,\theta)) f(t) = \frac{1}{C^2}\frac{d^2 f(t)}{dt^2} F(r,\theta)$$

$$\frac{\nabla^2 F}{F} = \frac{1}{C^2}\frac{d^2 f}{dt^2}\frac{1}{f} = -k^2$$

$$\frac{d^2 f(t)}{dt^2} = -k^2 C^2 f(t) \Rightarrow f(t) = A\cos kCt + B\sin kCt$$

∇^2 in polar cooridinate is given as

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

Let $F(r,\theta) = R(r)\Theta(\theta)$

Then $\nabla^2 F + k^2 F = 0$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)\Theta(\theta) + \frac{1}{r^2}\frac{d^2\Theta}{d\theta^2}R(r) = -k^2 R(r)\Theta(\theta)$$

$$\frac{1}{R(r)}\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{1}{r^2}\frac{d^2\Theta}{d\theta^2}\frac{1}{\Theta(\theta)} = -k^2$$

$$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + k^2 r^2 = -\frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2} = +m^2 \quad (m = 0, 1, 2, \cdots$$

$$\therefore \quad -\frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2} = m^2 \Rightarrow \Theta(\theta) = A\cos m\theta + B\sin m\theta$$

For r, $r\frac{d}{dr}\left(r\frac{dR}{dr}\right) + (k^2 r^2 - m^2)R = 0$

This equation is called Bessel's equation and its solution is a Bessel function $J_m(kr)$.

Therefore $\quad z(r,\theta,t) = J_m(kr)(A\cos m\theta + B\sin m\theta)(\cos kct + \sin kct)$,

The boundary condition is $\quad z(r=1,\theta,t)=0$.

$$\therefore \quad J_m(k) = 0 \quad — ②$$

The k values must satisfy ② (k is an eigenvalue)

These k will be denoted as k_{mn}

$$J_m(k_{m0}) = J_m(k_{m1}) = \cdots = J(k_{mn}) = \cdots = 0$$

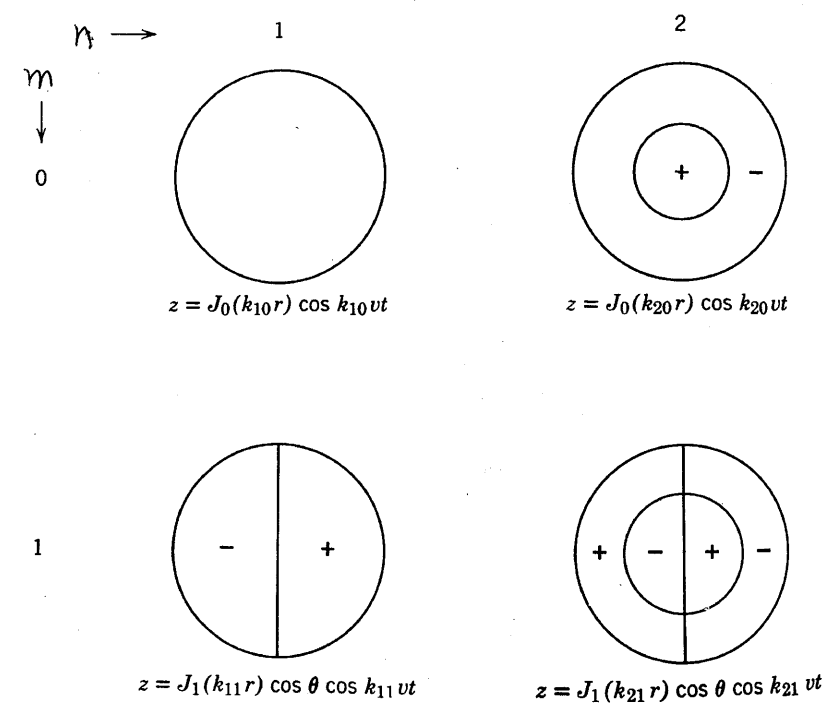
Frequency $\quad \nu = \dfrac{\omega}{2\pi} = \dfrac{kc}{2\pi} \Rightarrow \nu_{nm} = \dfrac{k_{mn}c}{2\pi}$

ν_{nm} : characteristic vibration frequency of 2D vibration (notice you need two indicies now)

For the case of $J_m(kr)\cos m\theta \cos kct$, four eigenmodes are sketched.

The black circles are the nodal lines where $z=0$.

$+$ and $-$ indicate opposite displacement of z.



$z = J_0(k_{10}r)\cos k_{10}vt$ $\qquad$ $z = J_0(k_{20}r)\cos k_{20}vt$

$z = J_1(k_{11}r)\cos\theta\cos k_{11}vt$ $\qquad$ $z = J_1(k_{21}r)\cos\theta\cos k_{21}vt$

S1. Schrodinger Equation (time-independent)

$$\nabla^2 \psi(r,\theta,\phi) + V(r,\theta,\phi)\psi = E\psi \quad — \bigstar$$

Seperation of variable

$$\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi) \quad — ①$$

$\nabla^2\psi$ in splerical coordinate is given

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} \quad — ②$$

From ① and ② (★) becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right)\Theta(\theta)\Phi(\phi) + \frac{1}{r^2\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)R(r)\Phi(\phi) + \frac{1}{r^2\sin^2\theta}\frac{d^2\Phi}{d\phi^2}R(r)\Theta(\theta)$$
$$= (E - V(r))r^2$$

By mulitplying both sides with $\frac{r^2\sin^2\theta}{R(r)\Theta(\theta)\Phi(\phi)}$

$$\sin^2\theta\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right)\frac{1}{R(r)} + \frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)\frac{\sin\theta}{\Theta(\theta)} + \frac{d^2\Phi}{d\phi^2}\frac{1}{\Phi(\phi)} = (E - V(r))r^2\sin^2\theta$$

$$\sin^2\theta\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right)\frac{1}{R(r)} + (V(r)-E)r^2\sin^2\theta + \frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)\frac{\sin\theta}{\Theta(\theta)} = -\frac{d^2\Phi}{d\phi^2}\frac{1}{\Phi(\phi)} = m^2 \quad — ③$$

$$\therefore \frac{d^2\Phi}{d\phi^2} = -m^2\Phi(\phi) \Rightarrow \Phi(\phi) \propto e^{\pm im\phi} \text{ with } m = 0, 1, 2, \cdots \quad — ④$$

From ③ ÷ $\sin^2\theta$

$$\frac{1}{R(r)}\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right) + (V(r) - E) = \frac{m^2}{\sin^2\theta} - \frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)\frac{1}{\Theta(\theta)}\frac{1}{\sin\theta} = \ell(\ell+1)$$

$$\therefore \frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right) + (V(r) - \ell(\ell+1) - E)R = 0 \quad — ⑤$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) + \left[\ell(\ell+1) - \frac{m^2}{\sin^2\theta}\right]\Theta(\theta) = 0 \quad — ⑥$$

The solution to ⑥ is

$$\Theta = P_\ell^m(\cos\theta) \qquad P_\ell^m(\cos\theta) : \text{associated Legendre function}$$

So $$\varphi(r,\theta,\phi) = R(r)\underbrace{P_\ell^m(\cos\theta)e^{im\phi}}_{\text{spherical harmonics } Y_\ell^m(\theta,\phi)}$$

For a hydrogen atom

$$V(r) = -\frac{e^2}{r}$$

So $-\frac{\hbar^2}{2m}\nabla^2\varphi - \frac{e^2}{r}\varphi = E\varphi$ is the Schrodinger equation for a hydrogen atom. If you solve this, then you get $\varphi(r,\theta,\phi)$, a wave function for an electron orbiting a proton, which is a hydrogen atom.